Optimal Global Carbon Management with Ocean Sequestration☆

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Abstract

We investigate the socially optimal anthropogenic intervention into the global carbon cycle. The limiting factor for this intervention is the accumulation of carbon in the atmosphere, which causes global warming. We apply a two-box model to incorporate aspects of the global carbon cycle in a more appropriate way than the usual proportional decay assumption does. Anthropogenic intervention into the global carbon cycle enters the model as the amount of CO_2 emitted into the atmosphere and the amount of CO_2 injected into the deep ocean for purposes of sequestration. We derive a critical level for ocean sequestration costs above which ocean sequestration is just a temporary option or below which it is the long-run option allowing extended use of fossil fuels. The latter option involves higher atmospheric stabilization levels, whereby the efficiency of ocean sequestration depends on the time preference and the inertia of the carbon cycle.

Keywords: climate change, global carbon cycle, ocean sequestration
JEL: Q30, Q54

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1. Introduction

Today, society has recognized the far-reaching consequences of the increase in atmospheric carbon to above its preindustrial level because this contributes to a large extent to global warming. Nevertheless, the carbon stock in the atmosphere is growing continuously and its growth rate increased even further in the period 2000-2006 (Canadell et al., 2007). This growth depends not only on global economic activity and the carbon intensity of the economy but also on the effectiveness of the natural carbon sinks, namely the terrestrial biosphere and the ocean. The change in atmospheric carbon in a given year relative to that year's total carbon emissions constitutes the airborne fraction (AF) of carbon. This ratio is currently substantially lower than 1, which indicates that the natural sinks are removing anthropogenic carbon from the atmosphere. Canadell et al. (2007) estimate the AF to be 0.45 for the period 2000 to 2006, whereby the ocean sink removed around 0.25 and natural sinks on land removed around 0.3. But the uptake by the land sinks is subject to strong fluctuations that are not well understood yet. Looking at the change in the various carbon reservoirs from 1800 to 1994, Sabine et al. (2004) show that the terrestrial biosphere has been a total net source of 39 (±28) Gt C.[1] They conclude that "the ocean has constituted the only true net sink for anthropogenic CO_2 over the past 200 years "(p.370). The absolute value of the long-run atmospheric carbon stabilization level and the time pattern of its achievement will depend crucially on the marine carbon cycle, which is therefore perceived to be the most import cycle with regard to the climate (Najjar, 1992). Therefore, any approach to manage the global carbon cycle or to mitigate global warming that ignores the ocean ignores optimization potential.

In this paper, we address the question how the inclusion of the largest carbon reservoir of the carbon cycle, the ocean, changes the optimal path of carbon emissions and whether its inclusion allows additional optimization potential. Since natural forces transport carbon into the deep ocean, where it cannot affect society as adversely as when in the atmosphere, the logical question is: At which cost level would it be beneficial to accelerate the process of downward carbon transfer by injecting carbon into the deep ocean? CO_2 could be transported via pipelines or ships to an ocean storage site, where it would be injected into the water column of the ocean or at the seafloor. This way, it would also become part of the global carbon cycle, but would enter the cycle in a more favorable way (Marchetti, 1977; Ozaki et al., 2001; IPCC, 2005; Keeling, 2009).

The growing knowledge about the importance of the marine carbon cycle for the mitigation of global warming has not just lead to the inclusion of the oceanic sink in the pioneering integrated assessment model (see e.g. modifications from DICE 94, Nordhaus (1994), to DICE 99, Nordhaus and Boyer (2000)), but to the development of highly sophisticated computer models that include complex coupled atmosphere-ocean general circulation models to represent the global carbon cycle and the climate system of the world (for an overview of integrated assessment models, see Tol (2006) or Kelly and Kolstad (1999)). However, many general properties about the stock externality problem were derived in microeconomic optimal control models, like Plourde (1976) or Forster (1980). In the application of these optimal control models to climate change problems, the global carbon cycle is only roughly approximated. The majority of the models apply a

[1] Based on the net increase of 165 Gt carbon in the atmosphere from 1800 to 1994, Sabine et al. (2004) subtract their ocean inventory estimate of 118 (± 19) Gt carbon from the total of fossil-fuel-emitted carbon, which amounted to 244(±20) Gt during this period. As a consequence, the terrestrial biosphere has to be considered a net source of carbon if the carbon budget is to be balanced.

constant rate of decay, which yields a proportional decay of carbon in the atmosphere. As a result, in these models, global warming presents itself merely as a problem of temporary duration and the atmospheric carbon stock is represented as a completely renewable resource (see, e.g., Tahvonen (1997)). The other extremum is to model the atmospheric carbon stock without decay, whereby it becomes a completely non-renewable resource (see, e.g., Section 4 in Hoel (1978) and Farzin (1996)). The atmospheric carbon stock is neither appropriately represented by a completely renewable description nor by a completely non-renewable description. Whereas the completely renewable description clearly overestimates the storing capacity of the global carbon cycle, the completely non-renewable description underestimates the storing capacity of the global carbon cycle. The first description implies a complete oceanic carbon sink, the second description neglects the oceanic carbon sink. Taking into account this oceanic uptake, the atmospheric carbon stock could be considered as a partial non-renewable resource. This approach is followed by Farzin and Tahvonen (1996). Based on a paper by Maier-Raimer and Hasselmann (1987), they divide the atmospheric carbon stock within their dynamic system artificially into two different stocks, one with a constant rate of decay and the other without. Given that the proportion of emissions to the nondecaying stock is equal to the long-run equilibrium, Farzin and Tahvonen's model captures important aspects of the carbon cycle. Nevertheless, the only management option in their dynamic model is to control the amount of emissions released into the atmosphere, which is proportional to the amount of extracted fossil fuels. Herzog et al. (2003) consider the injection of CO_2 into the deep ocean. They calculate the effectiveness of this activity, measured as the ratio between the net benefit gained from temporary storage and the benefit gained from permanent storage. They find "that the value of relatively deep ocean sequestration is nearly equivalent to permanent sequestration if marginal damage (i.e., carbon prices) remains constant or if there is a backstop technology that caps the abatement cost in the not too distant future" (p. 306). However, their calculation of ocean sequestration is not embedded within an optimal control framework, and therefore is not the result of a combined extraction and sequestration decision.

We analyze the optimal amount of extraction whereby the related emissions can be released into the atmosphere and injected into the deep ocean for purposes of ocean sequestration in the light of global warming in a microeconomic partial analysis framework. In Section 2 we explain how we include the oceanic carbon stock in the optimization problem by applying a two-box model representation for the global carbon cycle. Thereby, we replace the constant or nonconstant decay assumption and capture the essential nonrenewable aspects of the global carbon cycle without artificially dividing the atmospheric carbon stock. In Section 3 we presents our results. In Subsection 3.1 we derive the general optimality conditions for the solution, before we start in Subsection 3.2 by analyzing the scenario, where only one control variable is available, extraction and consumption of fossil fuel with related emissions released to the atmosphere. We show, that the two-box model provides the same results regarding atmospheric stabilization levels as the partially non-renewable model of Farzin and Tahvonen (1996), but clarify that the inclusion of the non-renewable aspects of the global carbon cycle allows only two possible emission and ocean sequestration tax paths, a monotonically increasing and U-shaped path, if extraction costs are constant. In Subsection 3.3 we analyze the scenario, where both control options are available, extraction and consumption of fossil fuels with related emissions released into the atmosphere or injected into the deep ocean for purposes of ocean sequestration. The relation of the start-up costs for ocean sequestration to its critical cost level indicates whether ocean sequestration leads to extended use of fossil fuels and therefore to higher long-run atmospheric and oceanic carbon stabilization levels. The critical cost level indicates the effectiveness of ocean sequestration due to the injection depth and the time preference. Including the control option ocean sequestration, it is not possible to obtain an interior solution within our model setup and therefore to derive clear statement about the tax paths before the final control regime into the steady state has been approached. In Subsection 3.4 we analyze for quadratic-linear functional forms the policy relevant case, where the start-up costs of ocean sequestration are below the critical level and the initial levels of atmospheric carbon concentration are still below atmospheric stabilization targets. Finally, Section 4 concludes.

2. Anthropogenic intervention into the global carbon cycle

We investigate the optimal anthropogenic intervention into the carbon cycle in the light of global warming as a social planner's problem in which the planer needs to determine the global optimal amount of fossil fuel extraction and consumption with the related emissions released to the atmosphere, $q(t)$, and the global optimal amount of fossil fuel extraction and consumption with the related emissions injected into the deep ocean for purpose of sequestration, $a(t)$. Consequently, total amount of fossil fuel extraction and consumption is, $x(t) = q(t) + a(t)$. The social welfare function can be formalized as follows:

$$\max_{q(t),a(t)} \int_0^\infty (U(q(t) + a(t)) - A(a(t)) - D(S(t)))e^{-\rho t} dt, \tag{1}$$

$$\text{with} \quad a(t), q(t) \geq 0, \tag{2}$$

which has to be maximized subject to the constraints:

$$\dot{S} = q(t) - \gamma(S(t) - \omega W(t)) \quad \text{with} \quad S(t_0) = S_0, \tag{3}$$

$$\dot{W} = a(t) + \gamma(S(t) - \omega W(t)) \quad \text{with} \quad W(t_0) = W_0, \tag{4}$$

$$\dot{R} = -q(t) - a(t) \quad \text{with} \quad R(t_0) = R_0. \tag{5}$$

The total amount of fossil fuel extraction and consumption, $q(t) + a(t)$, generates gross utility in the social welfare function at any instant in time. The gross utility of total fossil fuel consumption is described by $U(q(t) + a(t))$, which has the properties $U' > 0$, $U'' < 0$, and $U'(0) = b < \infty$. The last property implies that there is a choke price or a backstop price. We assume that the costs of fossil fuel extraction are constant and are included in $U(x(t))$. The proportional amount of carbon emissions related to total fossil fuel consumption (the proportionality factor is one) can be released directly to the atmosphere, $q(t)$ (emissions) or injected into the deep ocean, $a(t)$ (ocean sequestration). Ocean sequestration generates additional costs in the social welfare function at any instant in time. The costs of ocean sequestration are described by $A(a(t))$, which has the properties $A' > 0$ and $A'' > 0$ and is measured in the same units as utility. Ocean sequestration summarizes the activities of capturing CO_2 generated from the use of fossil fuels, of transporting the captured CO_2 via pipelines or ships to an ocean storage site, and of injecting it into the deep ocean. The IPCC (2005) special report on carbon dioxide capture and storage provides cost ranges for CO_2 capture. The ranges indicate that sequestration costs vary by differences in the design of CO_2 capture systems and by differences in the operating and financing of the reference plant to which the capture technology is applied (p.27). Additionally, their estimates show that the costs are increasing in the transportation distance on land and on sea (p.31 Figure TS.6 and p.39 Table TS.8). We assume that carbon capture would be first applied to the most cost efficient plants and to plants located nearest to the shore. However, in order to increase the amount of ocean sequestration, the capture technology has to applied to less efficient plants and to plants located far away from the shore. Consequently, by including ocean sequestration into such a microeconomic partial analysis framework such as ours, the most appropriate representation of the costs becomes a convex function.

Both control variables, $q(t)$ and $a(t)$, increase the amount of carbon in the global carbon cycle whereas only the atmospheric carbon stock influences the objective function. The increase in the atmospheric carbon stock to above preindustrial levels leads to global warming and thereby causes social costs for society at any instant in time. The social costs of global warming are denoted as damage and are described by the strictly convex function $D(S(t))$, with the properties $D' > 0, \quad D'' > 0$, and $D'(0) = 0$.

Equations (3) and (4) constitute the two-box model representation of the atmosphere and ocean (see Figure 1), whereby the boxes entail the carbons stocks in the atmosphere and the ocean, respectively. Equation (5) incorporates the endowment of fossil reserves, $R(t)$. The equations (3) to (5) describe the dynamics of the global carbon cycle as a consequence of the anthropogenic intervention. The upper box

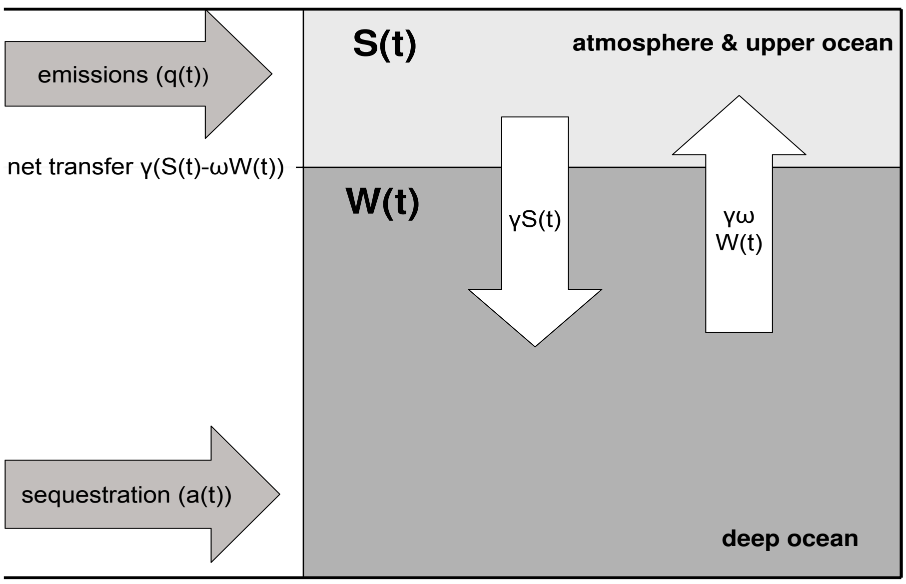

Figure 1: Two-Box Model

aggregates the carbon stocks in the atmosphere and in the upper mixed layer of the ocean. There is a net transfer of carbon between the atmosphere and the upper mixed layer of the ocean if there is a difference in the partial pressure of carbon dioxide (pCO_2) between these two reservoirs. The equilibration time for the upper layer of the ocean with the atmosphere takes around one year.[2] However, only a small fraction of the ocean is involved in direct exchange with the atmosphere and the uptake bottleneck is the transport

[2]Most of the CO_2 dissolves in water, forming carbon acid first and then bicarbonate (HCO_3^-) and carbonate ions (CO_3^{2-}).

of anthropogenic carbon to the deeper parts of the ocean. Consequently, we assume that the atmosphere and the upper mixed layer are always in equilibrium and that the stock of carbon in the atmosphere is a constant fraction of the carbon stock in the upper box, $S(t)$.

In order to model the transport of anthropogenic carbon to the deeper parts of the ocean, we include the carbon stock in the deep ocean, $W(t)$, in the lower box. The transport of anthropogenic carbon to the deeper parts of the oceans is effected by the biological pump and especially by the solubility pump. The biological pump is driven by two cycles: the organic matter cycle and the calcium carbonate cycle. Due to their influence on the amount of dissolved inorganic carbon and alkalinity and hence pCO2, both cycles are important factors for the chemical equilibration of carbon between the atmosphere and the upper layer of the ocean. The term biological pump designates the small fraction of organic matter and skeletons within these two cycles that survives remineralization in the euphotic zone and sinks to deeper layers. The main contribution to the transport of anthropogenic carbon is provided by the solubility pump. The solubility pump is driven by two phenomena: thermohaline circulation and the solubility of CO_2. Surface water in equilibrium with atmospheric CO_2 takes up additional CO_2 on its way to the poles, as the decreasing temperature increases the solubility of CO_2. The formation of deep seawater is driven by thermohaline circulation, which transports cold and high-solubility high-latitude surface waters into the deep ocean. Consequently, these two phenomena act together to pump carbon from the atmosphere into the ocean's deeper layers until the deep ocean is saturated with respect to upper layer. At this point in time, the up-welling water in the mid-latitudes transports anthropogenic carbon back to the upper layer.

The downward flux of carbon from the upper box to the lower box is represented by the fraction $\gamma S(t)$ and the upwards flux of carbon from the lower box to the upper box by the fraction $\gamma\omega W(t)$. These two fluxes are represented in Figure 1 by the two white vertical arrows between the boxes. Both arrows have the same size, indicating that the upward flux is balanced by the downward flux. Putting these two fluxes together, we obtain the net transfer between the boxes, $\gamma(S(t) - \omega W(t))$. There will be a net flux between these two boxes if there is a difference between the relative stock sizes. An increase in the stock size in the upper box causes a downward transfer of excess carbon into the deep ocean, whereas up-welling water is still free of excess carbon, so that we observe a net transfer from the upper box into the lower box. The upper box is relatively small in comparison to the lower box. Consequently, ω is the proportionality factor to scale the stock of carbon in the lower box with respect to the upper box and γ is the turnover factor to describe the speed of the adjustment process.[3] The anthropogenic intervention into the carbon cycle, the amount of emissions and the amount of ocean sequestration are depicted by the grey horizontal arrows in Figure 1. The amount of emissions enters the the upper box and the amount of ocean sequestration enters the lower box.[4] Even though, the carbon stock in upper box entails atmospheric and oceanic carbon, we refer to it as the atmospheric carbon stock.

The sum of these three elements describes the total amount of carbon in the ocean, called dissolved inorganic carbon (DIC). The amount of DIC in the ocean consists to 89.1% of bicarbonate ions, to 10.4% of carbonate ions and only to 0.5% of CO_2 (Najjar, 1992). Regarding the last figure, the atmosphere "sees" only a tiny fraction of the carbon present in ocean surface water within the chemical process of pCO_2 equilibration between the atmosphere and the ocean.

[3]Currently, this turnover speed is mainly limiting the uptake process by the ocean so that the total ocean is estimated to be undersaturated for a long time (order of 10^3 years) (Körtzinger and Wallace (2002)).

[4]Note, it would also be possible to apply the control variables, $x(t)$ and $a(t)$ instead of $q(t)$ and $a(t)$. As a result, only the net emissions, $x(t) - a(t)$ would be released to the upper box and the control constraints, (2), would change to $x(t) - a(t) \geq 0$ and $a(t) \geq 0$. Releasing the first constraint by allowing $x(t) - a(t) \leq 0$ would imply the option of air capture. This possibility if further investigated in Lontzek and Rickels (2008).

3. Results

3.1. Optimal solution conditions

The corresponding current value Hamiltonian from (1) and (3) to (5) is

$$H_c = U(q+a) - A(a) - D(S)) - \psi \dot{S} - \pi \dot{W} + \mu \dot{R}, \tag{6}$$

$$\text{where} \quad \lim_{t\to\infty} S(t) \geq 0, \quad \lim_{t\to\infty} W(t) \geq 0, \quad \lim_{t\to\infty} R(t) \geq R_0. \tag{7}$$

Note that from now on we drop the time variable whenever it is convenient. We have changed the signs of the costate variables, ψ and π, in order to facilitate their economic interpretation as taxes. Together with the two Lagrange multipliers for the control contraints (2), θ_1 and θ_2, we obtain the current value Lagrangian:

$$L_c = H_c - \theta_1(-q) - \theta_2(-a). \tag{8}$$

According to Proposition 6.2 and Propostion 7.5 in Feichtinger and Hartl (1986) the admissible solution candidate has to fulfill the necessary first-order conditions (FOC),

$$\frac{\partial L_c}{\partial q} = 0 \quad \Rightarrow \quad U' - \psi - \mu + \theta_1 = 0, \tag{9}$$

$$\frac{\partial L_c}{\partial a} = 0 \quad \Rightarrow \quad U' - A' - \pi - \mu + \theta_2 = 0, \tag{10}$$

$$-\frac{\partial L_c}{\partial S} = -\dot{\psi} + \rho\psi \quad \Rightarrow \quad -D'(S) + \gamma\psi - \gamma\pi = \dot{\psi} - \rho\psi, \tag{11}$$

$$-\frac{\partial L_c}{\partial W} = -\dot{\pi} + \rho\pi \quad \Rightarrow \quad -\gamma\omega\psi + \gamma\omega\pi = \dot{\pi} - \rho\pi, \tag{12}$$

$$-\frac{\partial L_c}{\partial R} = \dot{\mu} - \rho\mu \quad \Rightarrow \quad 0 = \dot{\mu} - \rho\mu, \tag{13}$$

$$\frac{\partial L_c}{\partial \theta_1} \geq 0 \quad \theta_1 \geq 0 \quad \theta_1(-q) = 0, \tag{14}$$

$$\frac{\partial L_c}{\partial \theta_2} \geq 0 \quad \theta_2 \geq 0 \quad \theta_2(-a) = 0, \tag{15}$$

as well as the transversality conditions,

$$\lim_{t\to\infty} e^{-\rho t}\psi = 0, \quad \lim_{t\to\infty} e^{-\rho t}\pi = 0, \quad \lim_{t\to\infty} e^{-\rho t}\mu R = 0, \tag{16}$$

and the constraint qualification for the control constraints (see Appendix A). As any admissible path for the state and costate variables is non-negative and as any admissible path for the state variables is bounded due to the description of the carbon cycle as close system, the fulfillment of the transversality conditions, (16) is sufficient for the fulfillment of the general transversality conditions in a infinity horizon problem (Feichtinger and Hartl, 1986, Remark 2.9 and Remark 7.5). The fulfillment of the necessary conditions (9) to (16) provides the optimal solution, because our carbon cycle is described by linear equations, (3) to (5), the control constraints (2) are quasi-concave, and in Appendix A we show, that the maximized Hamiltonian is concave in the state variables and the Hamiltonian is strict concave in the control variables (Feichtinger and Hartl, 1986, Proposition 7.5). The strict concavity of Hamiltonian in the control variables implies that the Hamiltonian is regular and that therefore the controls are continuous, in particular at switching point (Feichtinger and Hartl, 1986, corollary 6.2). Note, the equations (3) to (5) constitute a closed system, which means that no carbon vanishes the cycle, $\dot{S} + \dot{W} + \dot{R} = 0$ and one state variable, e.g. $W(t) = R_0 + S_0 + W_0 - R(t) - S(t)$, and the corresponding costate variable can be dropped. Consequently, the modified Hamiltonian dynamic system with a full rank is a 4x4 dynamic system. The remaining two costate variables measure then as well the influence of the omitted state variable on the objective function. To facilitate the interpretation, the analysis is based on the system with all state variables (Full system), whereas some technical arguments and the calculations in the Appendix are based on the system with only two state variables (Redux system).

The optimal amounts of the control variables, q and a, are determined by the costate variables, μ, ψ, and π, where μ measures the shadow resource scarcity rent, ψ measures the shadow environmental scarcity rent of the atmospheric carbon stock and π measures the shadow environmental scarcity rent of the oceanic carbon stock (Farzin, 1996). The two costate variables ψ and π can be interpreted as the optimal tax values throughout time for an implementation of the social optimal solution in a decentralized economy. The costate variable that corresponds to the carbon stock in the upper box, ψ, denotes an emission tax, and the costate variable that corresponds to the carbon stock in the lower box, π, denotes an ocean sequestration tax. FOCs (11) and (12) indicate that both taxes are always positive and that the emission tax is always larger than the ocean sequestration tax, otherwise the transversality conditions (16) would be violated. This can be seen by solving the equation of motion for the tax difference, $\lambda = \psi - \pi$, which coincides with the emission tax in the redux system, ψ_R:

$$\dot{\psi}_R = \dot{\lambda}\psi(\rho + \gamma + \gamma\omega) - D'(S) \quad \Rightarrow \quad \psi_R(t) = \lambda(t) = \int_t^{\infty} D'_S e^{-(\rho+\gamma+\omega)(\tau-t)d\tau}. \tag{17}$$

Consequently, if anthropogenic intervention takes place, the tax difference is positive. For an unconstraint solution, $\theta_1 = \theta_2 = 0$, this can directly be seen by simplifying FOC (10) to $A' = \psi - \pi$. The marginal damage, D', enters FOC (11) with a negative sign, which implies that a higher marginal damage corresponds to a higher value of ψ. The costate variable ψ enters the FOC (12) with a negative sign, and therefore it also transfers the impact of an higher marginal damage into the equation for the determination of π. However, the transfer is muted because ψ is multiplied with $\gamma\omega$. The economic interpretation is, that the amount of emissions directly increases the harmful carbon stock in the upper box, while the amount of ocean sequestration does only indirectly increases the carbon stock in the upper box via the natural transfer. Nevertheless, as the ocean sequestration tax is positive, we see that ocean sequestration does cause social costs due to its temporary storage characteristics and therefore does not completely offset emissions into the atmosphere.

3.2. Scenario 1: Optimal extraction without the option of ocean sequestration

We start investigating the implication of the representation of the carbon cycle as a two-box model by considering a scenario, where only one control variable is available, q, the extraction of fossil fuels with related emissions released to the atmosphere (Scenario 1). The anthropogenic intervention into the global carbon cycle ends, when no further carbon is added due to the fact that either the marginal damage caused by carbon in the atmosphere has increased to such an extent that the choke price has been hit by the emission tax (Situation A) or the stock of fossil reserves is completely exploited (Situation B). In Situation

A, the solution approaches a steady state as $t \to \infty$ and the steady state values for the costate variables, which fulfill the transversality conditions (16), are

$$\psi_\infty^A = \frac{D'(S_\infty)}{\gamma+\rho+\gamma\omega} + \frac{\gamma\omega D'(S_\infty)}{\rho(\gamma+\rho+\gamma\omega)}, \quad \pi_\infty^A = \frac{\gamma\omega D'(S_\infty)}{\rho(\gamma+\rho+\gamma\omega)}, \quad \mu_\infty^A = 0. \tag{18}$$

Using the steady state values for the costate variables, we can derive from the FOC (9) in which the Kuhn-Tucker multiplier, θ_1, is zero, the level of the steady state atmospheric carbon stock:

$$S_\infty^A = D'^{-1}(b\frac{\rho(\gamma+\rho+\gamma\omega)}{\rho+\gamma\omega}). \tag{19}$$

Equations (3) and (4) indicate that the two-box model is a non-renewable resource model, because no carbon vanishes or decays. Therefore, the release of carbon will increase the stock in both boxes forever and the two-box model approaches a new equilibrium with an atmospheric carbon stabilization level above the preindustrial one. The comparable atmospheric stabilization levels obtained with a renewable description of the global carbon cycle (R) like in Tahvonen (1997), obtained with a non-renewable description of the global carbon cycle (NR) like in Hoel (1978), and obtained with a partially non-renewable description of the global carbon cycle (PNR) like in Farzin and Tahvonen (1996) are[5]

$$S_\infty^R = S_0, \quad S_\infty^{NR} = D'^{-1}(b\rho), \quad S_\infty^{PNR} = D'^{-1}(b\frac{\rho(\alpha+\rho}{a-\alpha a+\rho}). \tag{20}$$

Intuitively, the renewable description implies that the atmospheric carbon stock returns to its preindustrial level when the release of carbon emissions has ended due to physically ($R_\infty = 0$) or economically ($R_\infty \geq 0$) exhaustion of the fossil reserve. The atmospheric stabilization levels of the non-renewable and partially non-renewable description are only compared if as well situation A applies, that it is not optimal to completely exploit the fossil reserve. The non-renewable description provides lower atmospheric carbon stabilization levels than the two-box model, because all carbon emissions remain in the atmosphere. The partially non-renewable description provides the same atmospheric stabilization levels as the two-box model, if a in S_∞^{PNR} is chosen to be $\frac{\gamma+\omega}{\rho+\gamma+\omega}$ and α in S_∞^{PNR} is chosen to be γ, whereby the parameter a describes the fraction of emissions which add to the decaying carbon stock and the parameter α describes the decay fraction of the decaying carbon stock. The renewable description implies a complete oceanic carbon sink, the non-renewable description neglects the oceanic carbon sink. The reality is somewhere in between: 15% (Körtzinger and Wallace, 2002) to 20% (IPCC, 2005) of all anthropogenic CO_2 will remain in the atmosphere within a new carbon cycle equilibrium. Consequently, the most appropriate description seems to be the partially non-renewable one. The comparison with the partially non-renewable description in Farzin and Tahvonen's model shows, that the two-box model has no advantage in itself in representing the global carbon cycle.

[5] In section 4 of Hoel's model, he assumes that the amount of the substitute has no effect on the harmful stock and denotes therefore a backstop technology. The cost of the backstop technology is c and $U'(0) = c$ describes the situation where it is optimal to switch to the backstop. The steady state is then described by a situation where the backstop is used at a constant rate. There is no intermediate time of simultaneous positive amounts of extraction and substitute due to the fact that the harmful stock does not decay. The price of the backstop, c, can be compared to our choke price, b. In Farzin and Tahvonen's model, they divide the atmospheric carbon stock artificially into two different stocks, one with a constant rate of decay and the other without. Their objective function includes stock dependent extraction costs as well, which we have set to zero in order to compare the steady states.

However, it becomes indispensable if further options to release carbon into the carbon cycle are considered (see Subsection 3.3).

The new equilibrium of the two-box model implies that there is no net transfer between the boxes and that $S_\infty = \omega W_\infty$ is fulfilled. Consequently, we can derive the critical initial level of the fossil reserve for the steady state in situation A to be feasible:

$$R^{A1}_{crit} = (\frac{\omega+1}{\omega}) D'^{-1}(b\frac{\rho(\gamma+\rho+\gamma\omega)}{\rho+\gamma\omega}) - S_0 - W_0. \tag{21}$$

If $R_0 < R^{A1}_{crit}$, extraction stops in finite time, because $U'(0) = b < \infty$ (Farzin and Tahvonen, 1996). The corresponding atmospheric and oceanic carbon stabilization levels are then

$$S^B_\infty = \frac{\omega}{1+\omega} R_0 + S0, \quad W^B_\infty = \frac{1}{1+\omega} R_0 + W0, \tag{22}$$

which are lower than the carbon stabilization levels in Situation A. Note, the stabilization levels in situation B are not approached at the point in time when extraction stops, but with $t \to \infty$. According to Meinshausen et al. (2009) the emission of carbon from all proven fossil fuel reserves would exceed the atmospheric stabilization levels corresponding to a 2°C temperature increase above preindustrial levels, which has been accepted by by most countries as maximum tolerable limit for global warming. We focus therefore in our analysis on Situation A, where the limiting factor for optimal extraction is not the endowment of the fossil reserve, but rather harmful levels of atmospheric carbon concentration and impose therefore

Assumption 1. $R_0 \geq R^{A1}_{crit}$.

We show in Appendix B, that the steady state of the 4x4 Hamiltonian dynamic system is saddle point with four real eigenvalues, two being positive and two being negative. If Assumption 1 is fulfilled, the steady state is feasible and the optimal solution is the unique saddle path converging to the steady state. Additionally, it can be seen from the steady state levels of the costate variable in Situation A (18) and the FOC (13) that $\mu(t) = 0$ for $t\epsilon[0, \infty)$. The fulfillment of Assumption 1 implies that the fossil resource is not scarce for the optimal solution in Situation A. The optimal path of extraction is therefore only determined by the emission tax, ψ.

Proposition 1. *In Scenario 1 (related emissions can only be released to the atmosphere) the optimal path for the emission tax, ψ, is either monotonically increasing or U-shaped, if extraction costs are constant and if Assumption 1 and $U'(0) = b < \infty$ are fulfilled.*

Proof. Because of the saddle path property with 4 real eigenvalues, the optimal path of the emission tax towards the steady state is determined by two exponential terms and can therefore only entail one extremum. As a result, the set of optimal paths is limited to a monotonically increasing, a monotonically decreasing, an U-shaped, and an inversely U-shaped path. The fulfillment of FOC (9) in the steady state with $q_\infty = 0$ requires $\psi_\infty = U'(0) = b \leq \infty$. Therefore, the control constraint $q(t) \geq 0$ for $t\epsilon[0, \infty)$ allows only tax paths which increase into the steady state. □

Proposition 2. *In Scenario 1 (related emissions can only be released to the atmosphere) the optimal path for the ocean sequestration tax, ψ, and the tax difference, λ, is either monotonically increasing or U-shaped, if extraction costs are constant and if Assumption 1 and $U'(0) = b < \infty$ are fulfilled.*

Proof. Again, the set of optimal paths for the ocean sequestration tax and the tax difference is limited to a monotonically increasing, a monotonically decreasing, an U-shaped, and an inversely U-shaped path due to the saddle path property with real eigenvalues. It is not possible for the path of π to approach its steady state value from above. Such a path would imply that π is decreasing, whereas ψ is increasing. As a consequence, π could not achieve a steady state, as it would continue to decrease. The steady state assumption and the transversality condition (16) would be violated. For the tax difference, the monotonically decreasing and inversely U-shaped path imply a monotonically increasing and U-shaped path for $\pi - \psi$, respectively, which indicates that the transversality condition for π in (16) would be violated. □

The two excluded paths from the set of optimal paths, the monotonically decreasing and inversely U-shaped path, require an additional term, which increases within the movement to the steady state so that FOC (9) allows a declining amount of extraction even if the emission tax is decreasing as well. The requirement of such a term can be fulfilled, by modeling extraction costs not constant as in our model, but stock dependent like in Farzin and Tahvonen (1996), $qC(R)$ with $C' < 0$.[6] The inclusion of stock dependent extraction costs does allow to consider besides physical exhaustibility as well economically exhaustibility, which implies $U'(0) = C(R_\infty)$ with $R_\infty \geq 0$. However, our formulation with constant extraction costs allows to clarify the implications of the description of the global carbon cycle. The renewable description of the global carbon cycle allows two tax paths, an monotonically decreasing and inversely U-shaped path (e.g., Tahvonen, 1997). The non-renewable description of the global carbon cycle allows only one tax path, a monotonically increasing path (e.g., Farzin, 1996). The partially non-renewable description allows compared to the non-renewable description only one additional possible path, an U-shaped path. Note, Farzin and Tahvonen (1996) observe as well the two paths from the renewable description within their partially non-renewable description, but not as consequence of the partially non-renewable description but due to the inclusion of stock dependent extraction costs. The result is confirmed by the fact, that the description with the two-box model allows as well to observe the two additional paths from the renewable description if extraction costs are modeled to be stock dependent (Lontzek and Rickels, 2008).

Additionally, Farzin and Tahvonen (1996) show that for the partially non-renewable description and for a specific initial level of the atmospheric carbon stock the possibility of a stationary emission tax with a stationary atmospheric carbon level exists, given that there is no steady state, $U'(0) \to \infty$. The stationary atmospheric carbon level requires that the decaying atmospheric carbon stock in the sum of the total atmospheric carbon stock declines at the rate at which the non-decaying atmospheric carbon stock increases. The decreasing decaying atmospheric carbon stock implies lower decay so that extraction is decreasing at a constant rate. Such a stationary atmospheric carbon stock whereas the amount of emission is constantly decreasing is as well possible with the two-box model. It requires that the amount of carbon emissions decreases at the same rate as the net transfer between the two boxes decreases due to the carbon accumulation in the deep ocean. The amount of emission would decline according to

$$q(t) = \gamma\omega(\frac{\bar{S}}{\omega} - W_0)e^{-\gamma\omega t} \tag{23}$$

[6] An alternative formulation is $C(X)$ with $C' > 0$, where X measures the cumulative amount of extracted fossil fuels (Farzin, 1992).

and the oceanic carbon stock would increase according to

$$W(t) = \frac{\bar{S}}{\omega} + (W_0 - \frac{\bar{S}}{\omega})e^{-\gamma\omega t}, \tag{24}$$

where $\bar{S}$ denotes the stationary atmospheric carbon level. Additionally, by changing our model formulation so that $U'(0) \to \infty$ is valid and stock dependent extraction costs are included, the result of a constant emission tax with fossil fuel endowment as a specific feature of the description of the carbon cycle as partially non-renewable in Farzin and Tahvonen (1996) could be confirmed. However, the constant path requires first particular functional forms for $U(q)$ and $C(R)$ so that a constant percentage of the resource stock is extracted which implies that extraction declines at this constant percentage rate (p.523) and second that the constant percentage coincides with the parameters of the carbon cycle description.[7] Consequently, the partially non-renewable description of carbon cycle is the precondition for such a path to happen, but the occurrence of such path is rather the consequence of a specific extraction path than the consequence of the carbon cycle description. We focus in our analysis on Situation A, where the global carbon cycle approaches a steady state with non-constant taxes.

3.3. Scenario 2: Optimal extraction with the option of ocean sequestration

We return to the scenario, where both control variables are available, q and a, the extraction and consumption of fossil fuels with related emissions released to the atmosphere and the extraction and consumption of fossil fuels with related emissions injected into the deep ocean (Scenario 2). As in the previous section, the anthropogenic intervention into the global carbon cycle ends, when no further carbon is added due to the fact that either the marginal damage caused by carbon in the atmosphere has increased to such an extent that the choke price has been hit or even exceed by the emission tax (Situation A) or the stock of fossil reserves is completely exploited (Situation B). Again, in Situation A the solution approaches a steady state as $t \to \infty$ and the steady state values for the costate variables are given by (18) from Section 3.2. However, in comparison to the previous section, not just FOC (9) has to be fulfilled, but as well FOC (10), which can be simplified to

$$A' + \theta_1 - \theta_2 = \psi - \pi. \tag{25}$$

The condition (25) shows that the amount of ocean sequestration is determined by the difference between the two taxes, because by injecting the carbon emission into the ocean, one saves the emission tax, but has instead to bear the ocean sequestration tax. We already discussed in the previous section, that Situation A requires an increasing emission tax for FOC (9), $U'(0) = \psi_\infty$, to be fulfilled in the steady state. However, the amount of ocean sequestration is increasing in ψ due to the convexity of the ocean sequestration cost function, therefore we cannot further assume the solution to be unconstraint. Using the steady state levels of the taxes (18), we can derive two conditions for the steady state atmospheric carbon stock to fulfill FOC

[7] In Farzin and Tahvonen (1996) the constant percentage is given by αa, the decay parameter and the fraction adding to non-decaying stock, whereas in our model the constant percentage is given by the turnover speed between the boxes, γ, and the proportionality parameter, ω.

(9) (a) and FOC (10) (b):

$$S_\infty^{A2a} = D'^{-1}((b+\theta_1)\frac{\rho(\gamma+\rho+\gamma\omega)}{\rho+\gamma\omega}), \quad S_\infty^{A2b} = D'^{-1}((b-A'(0)+\theta_2)\frac{\rho(\gamma+\rho+\gamma\omega)}{\gamma\omega}). \tag{26}$$

By equating S_∞^{A2a} and S_∞^{A2b}, we obtain

$$A'(0) = b\frac{\rho}{\rho+\gamma\omega} - \frac{\gamma\omega}{\rho+\gamma\omega}\theta_1 + \theta_2, \tag{27}$$

by which we distinguish 3 cases for the steady state:[8]

$$\text{Case 1:} \quad A'(0) > b\tfrac{\rho}{\rho+\gamma\omega} \Rightarrow \theta_1 = 0 \quad \theta_2 > 0, \tag{28}$$

$$\text{Case 2:} \quad A'(0) = b\tfrac{\rho}{\rho+\gamma\omega} \Rightarrow \theta_1 = 0 \quad \theta_2 = 0, \tag{29}$$

$$\text{Case 3:} \quad A'(0) < b\tfrac{\rho}{\rho+\gamma\omega} \Rightarrow \theta_1 > 0 \quad \theta_2 = 0. \tag{30}$$

The value of $A'(0)$, is somehow the counterpart of the choke price b. Whereas b denotes the maximum value for marginal utility, $A'(0)$ is the minimum level of the marginal sequestration costs. Therefore, $A'(0)$ can be interpreted as the start-up cost for ocean sequestration. Due to the importance of $A'(0)$ within Case 1 to 3, we define

$$A_{crit} = b\frac{\rho}{\rho+\gamma\omega}. \tag{31}$$

Proposition 3. *In Scenario 2 (related emissions can be released to the atmosphere and injected into the deep ocean), the atmospheric stabilization level increases compared to Scenario 1, if $A'(0) < A_{crit}$ is fulfilled.*

Proof. From FOC (10) we can derive the atmospheric stabilization level as

$$S_\infty^{A2} = D'^{-1}(b - A'(0) + \theta_2)\frac{\rho(\gamma+\rho+\gamma\omega)}{\gamma\omega}. \tag{32}$$

If $A'(0) = A_{crit}$, θ_2 is zero and (32) can be simplified to

$$S_\infty^{A1} = S_\infty^{A2} = D'^{-1}(b\frac{\rho(\gamma+\rho+\gamma\omega)}{\rho+\gamma\omega}). \tag{33}$$

If $A'(0) < A_{crit}$, θ_2 remains zero, but the function argument in (32) increases and consequently the atmospheric stabilization level increases so that $S_\infty^{A2} > S_\infty^{A1}$. □

If the atmospheric carbon stabilization level increases compared to Scenario 1, so does the oceanic carbon stabilization level in order to satisfy the carbon cycle equilibrium condition $S_\infty = \omega W_\infty$. Consequently, we can again derive the critical initial level of the fossil reserve for the steady state in Situation A to be feasible:

$$R_{crit}^{A2}(A'(0)) = (\frac{\omega+1}{\omega})D'^{-1}((b-A'(0))\frac{\rho(\gamma+\rho+\gamma\omega)}{\gamma\omega}) - S_0 - W_0, \tag{34}$$

[8]We only consider carbon cycle equilibriums which are approached via anthropogenic intervention into the global carbon cycle and do not consider a potential Case 4 with both Kuhn-Tucker multipliers being positive, because in such a case the carbon cycle equilibrium is only determined by the initial levels, S_0 and W_0.

whereas $R^2_{crit}(A'(0)) > R^1_{crit}$ requires $A'(0) < A_{crit}$ to be fulfilled. If $R_0 < R^{A2}_{crit}(A'(0))$, extraction stops again in finite time as in Section 3.2, because $U'(0) = b < \infty$ (Situation B). The corresponding atmospheric and oceanic carbon stabilization levels are only determined by the initial level of the fossil reserve, R_0 (see (22)). It is possible to observe $R^{A2}_{crit}(A'(0)) > R_0 > R^{A1}_{crit}$ if $A'(0) < A_{crit}$ so that the opportunity of injecting fossil fuel consumption related carbon emissions directly into the ocean would lead to the complete exploitation of the fossil reserve (Situation B), whereas without this opportunity the fossil reserve would not be completely exploited (Situation A). We concentrate for Scenario 2 as well on Situation A and extend therefore Assumption 1

Assumption 2. $R_0 \geq R^{A2}_{crit}(A'(0)$.

Again, if Assumption 2 is fulfilled, it can be seen from the steady state levels of the costate variable in Situation A (18) and the FOC (13) that $\mu(t) = 0$ for $t\epsilon[0, \infty)$. The fulfillment of Assumption 2 implies that the fossil resource is not scarce for the optimal solution in Situation A. The optimal path of extraction is therefore only determined by the emission tax, ψ, and the ocean sequestration tax, π. The steady states in Case 1, (28), and Case 3, (30), have positive Kuhn-Tucker multipliers so that within the movement into the steady state a point in time emerges, t_s, from which on the dynamic system is either described by $q(t) \geq 0$ and $a(t) = 0$ for $t\epsilon[t_s, \infty)$ (Case 1) or by $q(t) = 0$ and $a(t) \geq 0$ for $t\epsilon[t_s, \infty)$ (Case 3). Note, from the point in time t_s, the dynamic system leading toward the steady state coincides with a dynamic system which allows only extraction with related emissions released into the atmosphere (Case 1) or with a dynamic system which allows only extraction with related emissions injected into the deep ocean (Case 3). To formalize this idea, we define 3 control regimes:[9]

Regime 1: *No sequestration:* $a = 0$ and $q \geq 0$ ($\theta_1 = 0$ and $\theta_2 > 0$),

Regime 2: *Sequestration:* $a \geq 0$ and $q \geq 0$ ($\theta_1 = 0$ and $\theta_2 = 0$),

Regime 3: *Only sequestration:* $a \geq 0$ and $q = 0$ ($\theta_1 > 0$ and $\theta_2 = 0$).

Proposition 4. *In Scenario 2 (related emissions can be released to the atmosphere and injected into the deep ocean), the movement into the steady state is not an interior solution, but is from some point in time, t_s, described either by Regime 1 ($A'(0) \geq A_{crit}$, $R_0 > R^{A2}_{crit}(A'(0)) = R^{A1}_{crit}$), or by Regime 3 ($A'(0) < A_{crit}$, $R_0 > R^{A2}_{crit}(A'(0)) > R^{A1}_{crit}$), if extraction costs are constant, if Assumption 2 is fulfilled and if $U'(0) = b < \infty$.*

Proof. The movement into the steady cannot be determined by Regime 2, because $q(t) \to 0$ requires an increasing emission tax, whereas $a(t) \to 0$ requires a decreasing emission tax. Consequently, the movement into the steady state requires at least one control constraint to be active (either θ_1 or θ_2). □

We already referred in Section 3.2 to Appendix B, where we show that Regime 1 (which is equal to Scenario 1), obeys saddle path properties with real eigenvalues. In Appendix B we show as well, that Regime 3 obeys saddle path properties. For the eigenvalues to be real in Regime 3, the condition

$$\frac{1}{4}(\gamma(1+\omega)(\rho+\gamma+\gamma\omega) - D''(a'_\mu) + a'_\pi)^2 > -\gamma\omega(\gamma+\rho+\gamma\omega)D''a'_u \tag{35}$$

[9] We only consider control regimes with anthropogenic intervention into the global carbon cycle and do not consider a potential Regime 4 with $q = a = 0$ ($\theta_1 > 0$ and $\theta_2 > 0$).

has to be fulfilled in the steady state.

Proposition 5. *In Scenario 2 (related emissions can be released to the atmosphere and injected into the deep ocean), the optimal path for the emission and ocean sequestration tax is after t_s (final regime) either monotonically increasing or U-shaped ($A'(0) \geq A_{crit}$) or monotonically increasing ($A'(0) < A_{crit}$), if extraction costs are constant and if Assumption 2 and $U'(0) = b < \infty$ are fulfilled.*

Proof. After point t_s the optimal path is either determined by Regime 1 or Regime 3. We showed already within within Proposition 1 and 2 that the optimal path is either monotonically increasing or U-shaped for Regime 1. For Regime 3, we see from FOC (10), $U'(a) - A'(a) = \pi$, that only ocean sequestration tax paths with increasing parts towards the end fulfill the condition due to concavity of the utility function and the convexity of the ocean sequestration cost function. We can therefore exclude paths, which decrease into the steady state like an inversely U-shaped path. Additionally, at t_s, $U'(a(t_s)) = \psi(t_s)$ and $U'(a(t_s)) - A'(a(t_s))) = \pi(t_s)$ has to be fulfilled with $q(t_s) = 0$. We know from (12) that an U-shaped ocean sequestration tax requires as well an U-shaped emission tax for the transversality conditions to be fulfilled, (16). The decreasing emission tax $\psi(t)$ on the U-shaped paths contradicts $q(t) = 0$ for $t\epsilon[t_s, t^*]$, because $a(t)$ would be decreasing $t\epsilon[t_s, t^*]$ due to the decreasing tax difference (see Proposition 2) and therefore the LHS in $U'(a(t)) = \psi(t)$ would be increasing whereas the RHS would be deceasing. Consequently, if $A'(0) < A_{crit}$ only monotonically increasing tax paths are possible for $t\epsilon[t_s, \infty]$. □

Proposition 5 is only valid for $t\epsilon[t_s, \infty)$. Before the point t_s is approached, various successions of regimes are possible, so that the possible set of optimal emission and ocean sequestration tax paths becomes more complex. Additionally, the dynamics in the regimes before the final regime are not any longer determined just by the negative eigenvectors, but by the full set of eigenvectors. The reason is, that the negative eigenvectors describe the optimal path towards the steady state corresponding to the regime (saddle path). However, in regimes prior to the final regime, the corresponding steady is not feasible, as a result the path towards such a non-feasible steady state cannot describe the optimal path towards the regime switching point.

3.4. Utilizing ocean sequestration within a global carbon management strategy

Consider the situation, where the initial values for atmospheric and oceanic carbon stocks, S_0 and W_0, are low, the initial value for the fossil reserve, R_0, fulfills Assumption 1, and the start-up costs for ocean sequestration are at least equal to the critical level, $A'(0) \geq A_{crit}$. In this situation, the optimal solution is completely described by Regime 1 for $t\epsilon[0, \infty)$. The tax difference between the emission and ocean sequestration tax is never sufficient to bear the additional costs of ocean sequestration. The tax difference, determining the amount of ocean sequestration, $A'(a) = \psi - \pi$, approaches not before the steady state the critical level, A_{crit}, so that with $A'(0) \geq A_{crit}$ ocean sequestration is not beneficial. Obviously, if the initial levels for the carbon stocks are low, there is no difference between ocean sequestration too costly ($A'(0) > A_{crit}$ in Scenario 2) and between ocean sequestration not available or prohibited (Scenario 1). We refer to this situation as Policy 1.

Consider the situation, where the initial values for atmospheric and oceanic carbon stocks, S_0 and W_0, are low, the initial value for the fossil reserve, R_0, fulfills Assumption 2, and the start-up costs for ocean sequestration are below the critical level, $A'(0) < A_{crit}$. In this situation, the optimal solution might be

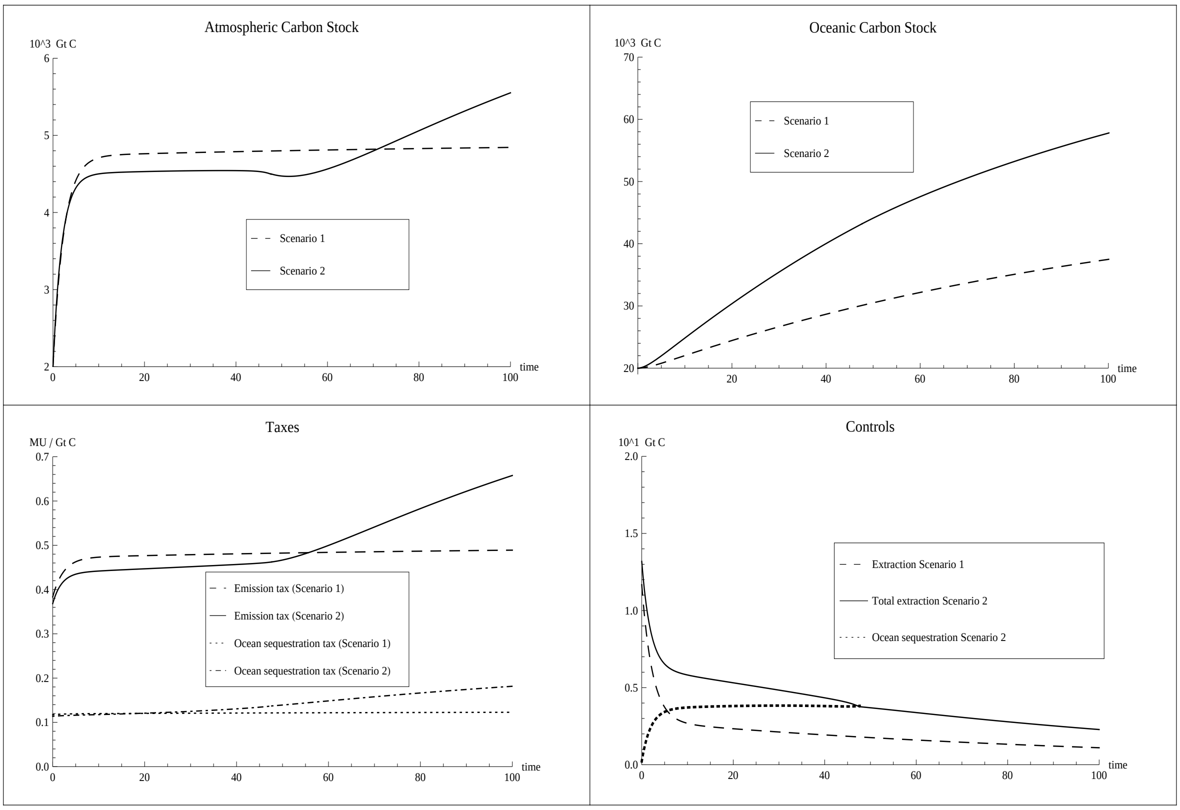

Figure 2: Dynamics for low initial atmospheric carbon stocks with constant extraction costs

described by succession of various regimes, but each succession involves Regime 3 for $t\epsilon[t_s, \infty)$. Even though, a succession of various regimes is possible before t_s, the non-renewable description of the carbon cycle by the two-box model and the low initial level imply tax paths which are increasing in direction towards t_s. Consequently, the tax difference will be at some point in time, $t < t_s$, above the start-up costs for ocean sequestration. Then it becomes beneficial to pay the ocean sequestration costs for some fraction of the emissions but to save the emission tax (Regime 1 to Regime 2). With the tax difference increasing into the steady state, the overall amount of fossil fuel consumption decreases, but the fraction of sequestering the related emissions into the ocean increases. Consequently, at t_s the complete amount of fossil fuel consumption is injected into the deep ocean (Regime 2 to Regime 3). We see that from t_s onwards ocean sequestration is declining until $b - \pi_\infty = A'(0)$ is fulfilled and that the emission tax increases above the choke price, $b = \psi_{ss} - \theta_1$ with $\theta_1 > 0$. The increasing tax paths prevent backward regime switches, e.g. Regime 2 to Regime 1. Consequently, if ocean sequestration is not too costly, more than one regime can be involved within the optimal solution. We refer to this situation as Policy 2.

In Figure 2 we show the dynamics of the atmospheric and oceanic carbon stock, the emission and ocean sequestration tax and the controls for Policy 1 and Policy 2 for low initial levels by using simple quadratic linear functional forms.[10] Policy 2 shows succession from Regime 2 to Regime 3. We see in the upper left window, that the atmospheric carbon stock increases slower in the beginning with Policy 2 than with Policy

[10]The utility function is $U(q) = bq - u_2q^2$, the ocean sequestration cost function is $A(a) = a_1a + a_2a^2$, and the damage

1. With Policy 2 carbon emissions are not only released into the atmosphere but as well injected into the deep ocean, consequently, the oceanic carbon stock increases faster with Policy 2 than with Policy 1, where it only increases by the natural transfer (upper right window). Due to slower increasing atmospheric carbon concentration, fossil fuel consumption declines slower with Policy 2 than with Policy 1 (lower right window). The fraction of ocean sequestration for the related emissions increases until the point where all emissions are captured and injected into the deep ocean. This occurs, before upper and lower box have equilibrated, which can seen by a slight temporary decrease in the atmospheric carbon stock. However, in the long run, the atmospheric carbon concentration increases stronger with Policy 2 due to the extended use of the fossil reserve and by a positive net transfer from the ocean to the atmosphere. Both taxes are increasing with Policy 2 in the long run above the levels which are obtained with Policy 1.

Even though, we observe in Figure 2 with Policy 2 a switch from Regime 2 to Regime 3, the dynamics are not characterized by a significant increase in volatility. One reason is that the long-run dynamics are mainly influenced by the natural transfer parameters, γ and ω, which are rather low in order to represent the inertia of the carbon cycle. Consequently, the eigenvalues have different magnitude and the Hamiltonian dynamic system is rather stiff. To demonstrate this effect, we show again Policy 1 and Policy 2 with a switch from Regime 2 to Regime 3 in Figure 3, but this time with stock dependent extraction costs included.[11] However, instead of the emission tax and the ocean sequestration tax in Figure 2 we show in Figure 3 the difference between the two taxes. Due to the presence of stock dependent extraction costs, the use of fossil fuels cannot be extended in such a way through ocean sequestration than without stock dependent extraction costs. Consequently, the atmospheric and oceanic stabilization levels are not affected by that magnitude as in Figure 2. Total fossil fuel consumption is in Scenario 2 rather similar to fossil fuel consumption in Scenario 1 and at some point in time even slightly lower (lower right window). Note, even with rather similar paths for total fossil fuel consumption, atmospheric peak concentration is significantly lower due to the presence of ocean sequestration. Additionally, we see that both atmospheric carbon stock and tax difference show a inverted S-shape with Policy 2 and confirm that the inclusion of ocean sequestration extends the set of possible tax paths so far discussed in the literature. The influence of ocean sequestration on atmospheric peak concentration and as well the possibility of an interior solution due to inversely U-shaped tax paths (lower left window) are further investigated in Lontzek and Rickels (2008).

It remains to briefly consider the situation, where the initial values for the atmospheric carbon, S_0, stock is high, whereas the initial level of the oceanic carbon stock, W_0, is low and the initial value for the fossil reserve, R_0, fulfills Assumption 2. In this situation, the optimal solution might be described by succession of various regimes, even with $A'(0) \lesseqqgtr A_{crit}$. In contrast to the non-renewable atmospheric carbon stock models (e.g. Hoel, 1978; Farzin, 1996), the non-renewable two-box model allows periods of time where the atmospheric carbon stock is decreasing. A decreasing atmospheric carbon stock implies that the natural downward transfer into the deep ocean exceeds the amount of emissions released into the atmosphere. If the harmful carbon stock in the atmosphere is initially high, the emission tax starts at high initial level so that only small amounts of emissions are released into the atmosphere and the atmospheric carbon stock can

function is $D(S) = v_1(sS - A_{preind})^2$. As a result of the linear-quadratic functional forms, the start-up costs, $A'(0)$, simplify to the parameter value a_1. The parameter values are $b = 5/10$, $\gamma = 1/10$, $\omega = 1/10$, $\rho = 3/100$, $a_1 = 1/4$ $a_2 = 1/10$, $u_2 = 1/20$, $v_1 = 0.1$, $s = 3/10$, and $A_{preind} = 6/10$, $S_0 = 2$, $W_0 = 20$. These parameter values yield $a_{crit} = 3/8$ and $R^{A2}_{crit}(a_1) = 64.1667$.

[11] The stock dependent extraction cost function is $c_1 - c_2 * R(t)$, with the parameter values c_1=5/10, $c_2 = 1/200$, and $R_0 = 100$. Note, the parameter value for a_1 has to be lower to 1/8 in order to still observe a final Regime 3, because with the stock dependent extraction costs the critical level for the start-up costs changes to $A_{crit} = (b - C(R_\infty))\frac{\rho}{\rho+\gamma\omega}$.

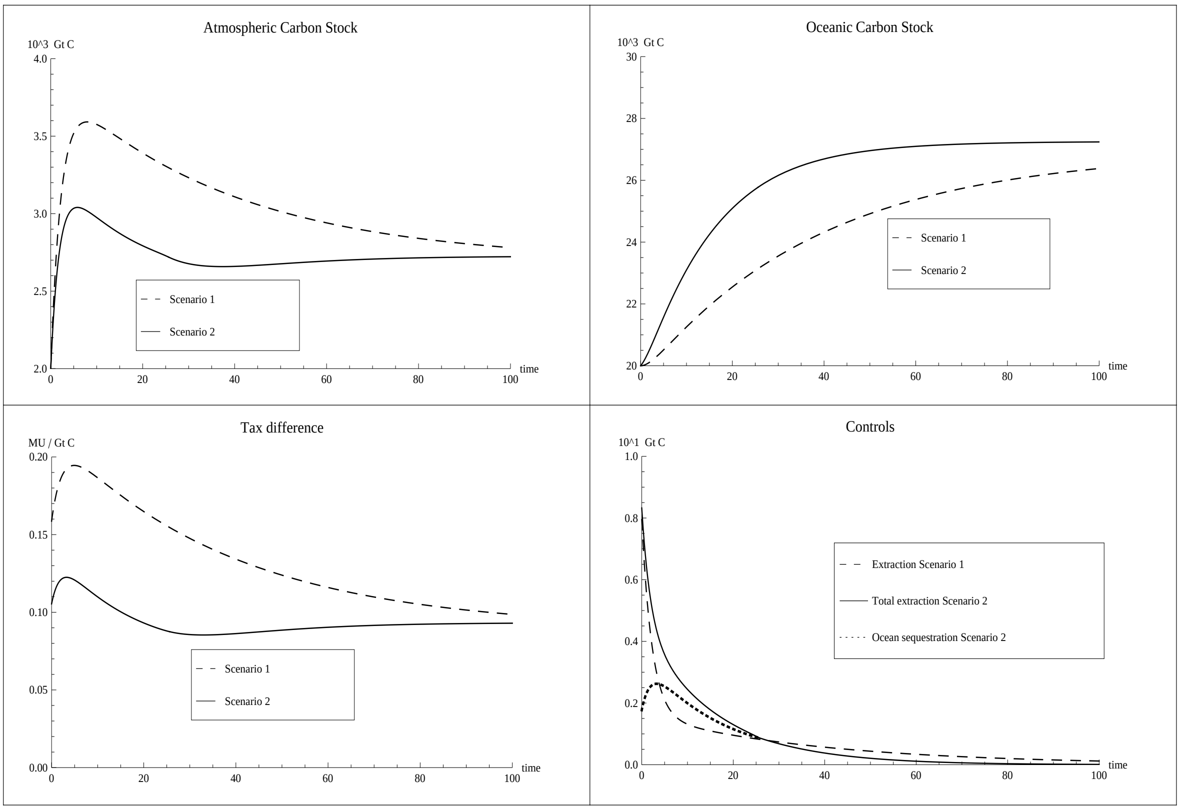

Figure 3: Dynamics for low initial atmospheric carbon stocks with stock dependent extraction costs

equilibrate with the oceanic carbon stock by the natural transfer while the emission tax is declining. However, whereas high emission tax levels imply low extraction with related emissions released to the atmosphere, they favor the utilization of ocean sequestration for the related emissions. Consequently, given that the tax difference starts decreasing at initial levels higher than its steady state level, $\lambda(t) > \lambda_\infty$ for $t\epsilon[0, t_a)$, ocean sequestration is utilized, if $\lambda(t) \geq A'(0) \geq A_{crit} = \lambda_\infty$ for $t\epsilon[0, t_a)$ with t_a defined by $\lambda(t_a) = \lambda_\infty$. With the start-up costs above the critical level, ocean sequestration can only be utilized until t_a, thereafter the dynamics are again complete described by Regime 1. However, if the initial levels for the atmospheric carbon stock are high, there is a difference between ocean sequestration too costly ($A'(0) > A_{crit}$ in Scenario 2) and between ocean sequestration not available or prohibited (Scenario 1). Even though ocean sequestration is not an option for the long-run management of the global carbon cycle, it might be utilized within the consumption of fossil fuels for some period time, if the atmospheric carbon stock is rather high, but the oceanic carbon stock is still rather low. If the start-up costs are below the critical level, ocean sequestration will be utilized beyond t_a, but not necessarily as the only control option. If the emission tax decreases sufficiently on the U-shaped path, it might be beneficial to switch back to Regime 2 and release some of the emissions again to the atmosphere or even back to Regime 1 and release all of the emissions again to the atmosphere. However, as the atmospheric carbon stock will start increasing again at some point in time so will the emission tax and therefore the dynamic system will return at t_s to Regime 3 if $A'(0) < A_{crit}$.

Independent of the realized tax paths, the effectiveness of ocean sequestration in this two-box model depends crucially on generating utility by using fossil fuels while delaying the resulting damage due to increased levels of carbon in the atmosphere into the future. As result, the effectiveness of sequestration depends on the time preferences and the adjustment times of the two boxes. The critical level, A_{crit}, is determined by the discount factor and the adjustment parameters of the two-box model. When the discount factor decreases, the critical level decreases as well. As a result, the effectiveness of sequestration decreases because, with a lower discount factor, delaying damages pays off less. When γ and ω decrease, the critical level increases. As a result, the effectiveness of sequestration increases because, with lower adjustment factors, the adjustment time of the two boxes decreases. A smaller value for γ implies a slower mixing of the two boxes. A smaller value for ω implies a greater lower box, which in turn implies that the lower box can contain greater amounts of carbon. In the context of our two-box model, variations in the amount of carbon active in the lower box can be used to approximate various injection depths for ocean sequestration. A deeper injection depth goes along with a greater lower box and a slower adjustment process. Therefore, the effectiveness of ocean sequestration depends on the injection depth.

4. Conclusions

In this paper, we investigate optimal anthropogenic intervention into the global carbon cycle, which enters the carbon cycle as the amount of carbon emitted into the atmosphere and the amount of carbon injected into the deep ocean for purposes of ocean sequestration. Fossil fuel consumption is halted when dangerous levels of atmospheric carbon are achieved. To capture the complete accumulation of carbon in the global carbon cycle, we include, besides the atmospheric carbon stock, the oceanic carbon stock in a two-box model representation. Using a two-box model to model the global carbon cycle does not in itself provide different optimal atmospheric stabilization levels than models with a partially non-renewable description of the atmospheric carbon stock. However, factoring in ocean sequestration may do so. Thus it is important to account for the "decayed" amount of carbon by including the oceanic sink in the lower box. By doing so, we can show that ocean sequestration does not serve as a complete offset for a carbon emission tax, but has a price itself, the ocean sequestration tax. Without ocean sequestration, the optimal tax paths towards the steady state are limited to a monotonically increasing and an U-shaped path, if extraction costs are constant and the utility function entails a choke price. With ocean sequestration, we cannot obtain an interior solution if extraction costs are constant and the utility function entails a choke price. Therefore, the constraint solution can involve succession of various control regimes and does prevent a clear statement regarding the optimal tax paths beside that they have to be increasing into the steady state.

Nevertheless, by the deriving the critical level for the start-up costs for ocean sequestration we could determine the role of ocean sequestration in a global carbon management strategy. In the situation where the start-up costs of ocean sequestration are lower than the critical level, ocean sequestration is the long-term option for the extended use of fossil fuels going along with higher atmospheric and oceanic stabilization levels. In the situation where the start-up costs of ocean sequestration are equal or larger than the critical level, ocean sequestration is never an option for the management of the global carbon cycle, if the atmospheric carbon level is below it long-run stabilization levels. However, if initial levels imply that the atmospheric carbon level has already peaked above its long-run stabilization level, ocean sequestration might be utilized for some period until the atmospheric carbon stock has decreased sufficiently by the natural adjustment with the ocean. Given a climate policy, which formulates an atmospheric carbon stabilization goal, ocean sequestration cannot increase the total amount of fossil fuel consumption. Carbon injected into the deep ocean in excess of the atmosphere-ocean equilibrium amount corresponding to the atmospheric stabilization goal, is expected to leak back to the atmosphere, because the ocean becomes supersaturated in relation to the atmosphere. However, the option of ocean sequestration does extend the period that fossil fuels can be extracted in reasonable amounts, whereas without ocean sequestration the amounts of extraction would have to decline much earlier due to the inertia of the carbon cycle. Consequently, ocean sequestration constitutes a serious option to buy time to deal with the atmospheric carbon accumulation problem. The effectiveness of this option depends on the injection depth of the sequestered carbon and the time preference of society.

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A. Necessary and sufficient optimality conditions

For the two constraints, $g_1(q,d) = -q \leq 0$ and $g_2(g,d) = -d \leq 0$, the constraint qualification is fulfilled, if the matrix

$$\begin{pmatrix} \frac{\partial g_1}{\partial q} & \frac{\partial g_1}{\partial d} & g_1 & 0 \\ \frac{\partial g_1}{\partial q} & \frac{\partial g_1}{\partial d} & 0 & g_2 \end{pmatrix} \tag{A.1}$$

has the full row rank (Feichtinger and Hartl, 1986, 6.2), which can be seen to be fulfilled from

$$\begin{pmatrix} -1 & 0 & -q & 0 \\ 0 & -1 & 0 & -d \end{pmatrix} \tag{A.2}$$

The concavity of the maximized Hamiltonian follows from the negative semi-definiteness of the Hessian matrix of the Hamiltonian (Feichtinger and Hartl, 1986, Remark 2.4). For the calculation of the Hessian matrix we eliminate the state variable $W(t)$ so that the carbon cycle equations (3) to (5) simplify to

$$\dot{S} = q - \gamma(S - \omega(S_0 + R_0 + W_0 - S - R) \tag{A.3}$$
$$\dot{R} = q - a. \tag{A.4}$$

Consequently, the Current Value Hamiltonian is $H^c = U(q+a) - A(a) - D(S) - \mu\dot{R} - \psi\dot{S}$ and we can calculate the Hessian matrix:

$$\begin{pmatrix} H_{SS} & H_{SR} & H_{Sq} & H_{Sa} \\ H_{RS} & H_{RR} & H_{Rq} & H_{Ra} \\ H_{qS} & H_{qR} & H_{qq} & H_{qa} \\ H_{aS} & H_{aR} & H_{aq} & H_{aa} \end{pmatrix} = \begin{pmatrix} -D'' & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & U'' & U'' \\ 0 & 0 & U'' & U'' - A'' \end{pmatrix}, \tag{A.5}$$

which has the eigenvalues

$$\sigma_{1,2,34} = \Big(-D'', \ \ \tfrac{1}{2}(2U'' - A'' - \sqrt{4(U'')^2 + (A'')^2}), \ \ \tfrac{1}{2}(-U'' - A'' + \sqrt{4(U'')^2 + (A'')^2}), \ \ 0 \Big). \tag{A.6}$$

The Hessian matrix being negative semi-definite requires $\sigma_{1,2,3,4} \leq 0$. Taking into account our function properties, $A'' > 0$, $U'' < 0$, and $D'' > 0$, the negativeness of the third eigenvalue can be see from:

$$A'' - 2U'' > \sqrt{(A'')^2 + 4(U'')^2}, \quad \text{and} \quad -4A''U'' > 0, \tag{A.7}$$

whereas the first and second eigenvalue are obviously negative and the fourth eigenvalue is zero.

The regularity of the Hamiltonian follows from the strict concavity of the Hamiltonian in the control variables. The strict concavity can be seen from the lower right bloc matrix in the Hesse matrix (A), because the first leading principal minor is negative ($U'' < 0$) and the determinant of the lower right bloc matrix is positive, $-A''U'' > 0$.

B. Saddle path properties for Regime 1 and Regime 3

Following Dockner (1985, Theorem 3) the fulfillment of first $K < 0$ and second $0 < Det(HDS) < (K/2)^2$ is necessary and sufficient for the eigenvalues to be real, two being negative and two being positive. HDS abbreviates Hamiltonian dynamic system and K is defined as

$$K = Det\begin{pmatrix} \frac{\partial \dot{x}}{\partial x} & \frac{\partial \dot{x}}{\partial \lambda} \\ \frac{\partial \dot{\lambda}}{\partial x} & \frac{\partial \dot{\lambda}}{\partial \lambda} \end{pmatrix} + Det\begin{pmatrix} \frac{\partial \dot{y}}{\partial y} & \frac{\partial \dot{y}}{\partial \mu} \\ \frac{\partial \dot{\mu}}{\partial y} & \frac{\partial \dot{\mu}}{\partial \mu} \end{pmatrix} + 2Det\begin{pmatrix} \frac{\partial \dot{x}}{\partial y} & \frac{\partial \dot{x}}{\partial \mu} \\ \frac{\partial \dot{\lambda}}{\partial y} & \frac{\partial \dot{\lambda}}{\partial \mu} \end{pmatrix}. \tag{B.1}$$

B.1. Regime 1

The HDS for Scenario 1 as well as for Regime 1 is, again based on the system with full rank and eliminated state variable $W(t)$:

$$\begin{aligned} \dot{R} &= -q(\mu,\psi), \\ \dot{S} &= +q(\mu,\psi) - \gamma(S - \omega(S_0 + R_0 + W_0 - S - R), \\ \dot{\mu} &= \rho\mu - \gamma\omega\psi, \\ \dot{\psi} &= (\rho + \gamma + \gamma\omega)\psi - D', \end{aligned} \tag{B.2}$$

and the corresponding Jacobian is

$$J_{R1} = \begin{pmatrix} 0 & 0 & -q'_\mu & -q'_\psi \\ -\gamma\omega & -\gamma - \gamma\omega & q'_\mu & q'_\psi \\ 0 & 0 & \rho & -\gamma\omega \\ 0 & -D'' & 0 & \rho + \gamma + \gamma\omega \end{pmatrix}. \tag{B.3}$$

We see that the determinate of $det(J_{R1}) = -\gamma^2\omega^2 D'' q'_\mu - \gamma\rho\omega D'' q_\psi$ and $K_{R1} = -\gamma(1+\omega)(\rho+\gamma+\gamma\omega) + D'' q'_\psi$ fulfill the conditions $K_{R1} < 0$ and $det(J_{R1}) > 0$, because $q'_\psi < 0$ and $q'_\mu < 0$. Additionally, $det(J_{R1}) < (K/2)^2$ is fulfilled, because

$$(K/2)^2 - det(J_{R1}) = \gamma\omega D''(\gamma\omega q'_\mu + \rho q'_\psi + \frac{1}{4}(\gamma(1+\omega)(\rho+\gamma+\gamma\omega)) - D'' q'_{psi})^2 > 0. \tag{B.4}$$

B.2. Regime 3

The HDS for Regime 3 is, again based on the system with full rank and eliminated state variable $S(t)$:

$$\begin{aligned} \dot{R} &= -a(\mu,\pi), \\ \dot{W} &= a(\psi,\pi) + \gamma(S_0 + R_0 + W_0 - R - W) - \gamma\omega W, \\ \dot{\mu} &= \rho\mu - \gamma\pi - D', \\ \dot{\pi} &= (\rho + \gamma\omega)\pi - D', \end{aligned} \tag{B.5}$$

and the corresponding Jacobian is

$$J_{R3} = \begin{pmatrix} 0 & 0 & -a'_\mu & -a'_\pi \\ -\gamma & -\gamma\omega & a'_\mu & a'_\pi \\ D'' & D'' & 0 & -\gamma \\ D'' & D'' & 0 & \rho + \gamma\omega \end{pmatrix}. \tag{B.6}$$

We see that the determinate of $det(J_{R3}) = -\gamma\omega(\rho+\gamma+\gamma\omega)D''a'_\mu$ and $K_{R3} = -\gamma(1+\omega)(\rho+\gamma+\gamma\omega) + D''(a'_\mu + a'_\pi)$ fulfill the conditions $det(J_{R3}) > 0$ and $K_{R3} < 0$, because $a'_\mu < 0$ and $a'_\pi < 0$. Additionally, $det(J_{R1}) < (K/2)^2$ is fulfilled, if

$$\frac{1}{4}(\gamma(1+\omega)(\rho+\gamma+\gamma\omega) - D''(a'_\mu) + a'_\pi)^2 > -\gamma\omega(\gamma+\rho+\gamma\omega)D''a'_u \tag{B.7}$$

is fulfilled in the steady state. If (B.7) is not fulfilled, the saddle path property is not affected, but the eigenvalues are complex (Tahvonen, 1989).