Liquidity Traps and Monetary Policy: Managing a Credit Crunch

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Abstract

We study a model with heterogeneous producers that face collateral and cash in advance constraints. A tightening of the collateral constraint results in a credit-crunch generated recession that reproduces several features of the financial crisis that unraveled in 2007. The model can suitable be used to study the effects on the main macroeconomic variables and of alternative policies following the credit crunch. The policy implications are in sharp contrast with the prevalent view in most Central Banks, based on the New Keynesian explanation of the liquidity trap.

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1 Introduction

In this paper, we study the effect of monetary and debt policy following a negative shock to the efficiency of the financial sector. We build a model that combines the financial frictions literature, like in Kiyotaki (1998), Moll (2014) and Buera and Moll (2015) with the monetary literature, like Lucas (1982), Lucas and Stokey (1987) and Svensson (1985). The first branch of the literature gives rise to a non-trivial financial market by imposing collateral constraints on debt contracts. The second gives rise to a money market by imposing cash-in-advance constraint in purchases. We show that the model qualitatively reproduces several aspects of the recent great recession. More importantly, we also show that a calibrated version can quantitatively match many salient features of the US experience since 2007. Finally, we use the model to learn about the effects of alternative policies.

The year 2008 will long be remembered in the macroeconomics literature. This is so not only because of the massive shock that hit global financial markets, particularly the bankruptcy of Lehman Brothers and the collapse of the interbank market that immediately followed, but also because of the unusual and extraordinary policy response that followed. The Federal Reserve doubled its balance sheet in just three months—from $900 billion on September 1 to $2.1 trillion by December 1, and it reached around $3 trillion by the end of 2012. At the same time, large fiscal deficits implied an increase in the supply of government bonds, net of the holdings by the Fed, of roughly 30% of total output in just a few years, a change never seen during peace time in the United States. Similar measures were taken in other developed economies.

Somewhat paradoxically, however, the prominent models used for policy evaluation by the main Central Banks at the time of the crisis ignored the financial sector in one hand, and the role of changes in outside liquidity on the other.[1] There were good historical reasons for both: big financial shocks have seemed to belong exclusively to emerging economies since the turbulent 1930s. In addition, monetary economics developed during the last two decades around the central bank rhetoric of exclusively emphasizing the short-term nominal interest rate, while measures of liquidity

[1] These models have been further adapted to try to address these issues (see Curdia and Woodford (2010), Christiano et al. (2011) or Werning (2011)). As we will discuss, our model captures different effects and the policy implications differ substantially.

or money were completely ignored as a stance of monetary policy.[2] But 2008 seriously challenged those generally accepted views. Consequently, we need general equilibrium models that can be used for policy evaluation during times of financial distress. The purpose of this paper is to provide one such model and to analyze the macroeconomic effects of alternative policies.

An essential role of financial markets is to reallocate capital from wealthy individuals with no profitable investment projects to individuals with profitable projects and no wealth. The efficiency of these markets determines the equilibrium allocation of physical capital across projects and therefore equilibrium intermediation and total output. The financial frictions literature, from which we build, studies models of intermediation with these properties, the key friction being an exogenous collateral constraint on investors.[3] The equilibrium allocation critically depends on the nature of the collateral constraints: The tighter the constraints, the less efficient the allocation of capital and the lower are total factor productivity and output, so a tightening of the collateral constraint creates disintermediation and a recession. We interpret this reduction in the ability of financial markets to properly allocate capital across projects as the negative shock that hit the US economy at the end of 2007.[4]

We modify this basic model by imposing a cash-in-advance constraint on households. Monetary policy has real effects, not only because of the usual well-understood distortionary effects of inflation in a cash-credit world, but, more importantly, because of the zero bound constraint on nominal interest rates that naturally arises in monetary models. Conditional on policy, the bound on the nominal interest rate may become a bound on the real interest rate. For example, imagine a policy that successfully targets a constant price level: if inflation is zero, the Fisher equation implies a zero lower bound on the real interest rate. The way in which negative real interest rates interact with the zero bound on nominal interest rates, given a target for inflation, is at the heart of the mechanism by which policy affects outcomes in the model

[2]A likely reason is that the empirical relationship between monetary aggregates, interest rates, and prices, which remained stable for most of the 20th century, broke down in the midst of the banking deregulation that started in the 1980s. For a detailed discussion, as well as a reinterpretation of the evidence that strongly favors the view of a stable money demand relationship, see Lucas and Nicolini (2015).

[3]We closely follow the work of Buera and Moll (2015) who study business cycles in the framework developed by Moll (2014) to analyze credit markets in development. See Kiyotaki (1998) for an earlier version of a related framework.

[4]As we explain in detail in Section 4.1, the behavior of the real interest rate, the variable we use to identify the shock, dates the begging of the recession in the third quarter of 2007.

discussed in the paper.

The exogenous nature of both frictions raises legitimate doubts regarding the policy invariant nature of them. Precise micro-foundations for the collateral or the cash-in-advance constraints have been hard to develop in macro models that remain tractable and general enough to provide insights on the effects of the policies we study in this paper. We thus view this as a first exploration into the the role of the above mentioned monetary and debt policies, hoping that the answers we provide, both positive and normative, can shed light into the questions we pose, as in Robert E. Lucas (2000), and Alvarez and Lippi (2009) for models with cash-in-advance constraints, and Kiyotaki (1998) or Moll (2014) for models with collateral constraints.

As we show in a simplified version of the model that can be solved analytically, if the shock to the collateral constraint that causes the recession is sufficiently large, the equilibrium real interest rate becomes negative and persistent as long as the shock is persistent. We find this property of the model particularly attractive, since a very special feature of the last years is a substantial and persistent gap between real output and its trend, together with a substantial and persistent negative real rate of interest. This feature is specific to the credit crunch: If the recession is driven by an equivalent but exogenous negative productivity shock, the real interest rate remains positive.

The reason for the drop in real interest rates is that savings must be reallocated to lower productivity entrepreneurs, but they will only be willing to do it for a lower interest rate. To put it differently, the "demand" for loans falls, which in turn pushes down the real interest rate. Several other properties of the recession generated by a tightening of the collateral constraint in the model are in line with the events that have unfolded since 2008, such as the sustained periods with an effective zero bound on nominal interest rates, and the substantial drops in investment and total factor productivity, all of these driven by a single shock. In addition, the model implies the need of a very large increases in liquidity while the zero bound binds to stabilize prices, two features present in the crisis.

A calibrated version of the full model can quantitatively account for the behavior of the real interest rate, output, capital, total factor productivity and, with somewhat less success, measures of leverage, since 2007. The one variable the model misses is labor input, that dropped substantially in the US following 2007 and is constant in the model. We find this quantitative performance remarkable, given that it is driven by a single shock. We also find it reassuring in using the model to perform policy analysis.

The paper proceeds as follows. In Section 2 we present the model. In Section 3, we define an equilibrium and characterize its properties for a particular case that, by shutting down the endogenous evolution of the wealth distribution, can be solved analytically. In Section 4 we calibrate the full model and show that, once the monetary and debt policies implemented by the US authorities are taken into account, it can explain the evolution of all relevant macro-variables, the exception being labor input as we already mentioned. In solving the model, we study the deterministic equilibrium path following the shock to intermediation. In looking at the data, we focus at the medium frequency evolution of the data, which is the frequency the model implies we should focus on, as we explain in detail. In particular, we ignore the high frequency business cycle fluctuations that are the focus of the RBC literature.

In Section 5, we perform several policy counterfactuals that help put the policies undertaken in the United States starting in 2008 into perspective. First, we solve for the equilibrium in the absence of a policy reaction. Specifically, we assume that there is no injection of liquidity on impact and that there is no further increase in the stock of government bonds (safe assets). The model implies that there will be an initial deflation, followed by an inflation rate that is higher that the steady state. These effects are the natural response of the no arbitrage condition between money and bonds. In the case that private debt contracts are indexed to the price level, the real effects are minor. On the contrary, in the more realistic case in which debt obligations are in nominal terms, the deflation strongly accentuates the recession well beyond the one generated by the credit crunch, due to a debt deflation problem. A similar result obtains with sticky wages, commonly assumed in modern monetary models. We then study active inflation-targeting policies for low values of the inflation target. In these cases, the deflation with its associated real effects can be avoided by a sufficiently large increase in the supply of government liabilities that must accommodate the credit crunch. This exercise is reminiscent of the discussion in Friedman and Schwartz, who argued that the Fed should have substantially increased its balance sheet in order to avoid the deflation during the Great Depression.[5] Was the different monetary policy

[5]In 2002, Bernanke, then a Federal Reserve Board governor, said in a speech in a conference celebrating Friedman's 90th birthday, "I would like to say to Milton and Anna: Regarding the Great Depression. You're right, we did it. We're very sorry. But thanks to you, we won't do it again" (speech published in Milton Friedman and Anna Jacobson Schwartz, *The Great Contraction, 1929-1933*) (Princeton, NJ: Princeton University Press, 2008), 227.

recently adopted the reason why the Great Contraction was much less severe than the Great Depression? Our model suggests this may well be the case.[6]

In studying inflation targeting policies, we show that the number of periods that the economy will be at the zero bound and the amount of liquidity that must be injected depend on the target for the rate of inflation. The evolution of output critically depends on the increase in liquidity. The target for inflation and the zero bound constraint on nominal rates imply a floor on how low the real interest rate can be. But for this to be an equilibrium, private savings must end up somewhere else: this is the role of the increase in government liabilities. In this heterogeneous credit-constrained agents model, outside liquidity affects equilibrium interest rates even if taxes are lump sum: Ricardian equivalence does not hold if agents discount future flows at different rates, as is the case when collateral constraints bind. As a consequence, the issuance of government liabilities (money or bonds, which are perfect substitutes at the zero bound) crowds out private investment and slows down capital accumulation. But increases in liquidity have an additional effect. In the model, a credit crunch generates a recession because total factor productivity falls. The reason, as we mentioned above, is that capital needs to be reallocated from high productivity entrepreneurs for which the collateral constraint binds to low productivity entrepreneurs for which the collateral constraint does not bind. An increase in liquidity prevents the real interest rate from falling too much, and ameliorates the drop in productivity.

In summary, a target for inflation, if low enough, ameliorates the drop in productivity (i.e., there will be less reallocation of capital to low productivity workers). But it requires a larger increase in total outside liquidity and makes the recession more prolonged (i.e., capital accumulation falls because of the crowding-out effect). The model therefore challenges the interpretation of the events following 2009 provided by a branch of the literature that, using New Keynesian models, places a strong emphasis on the interaction between the zero bound constraint on nominal interest rates and price rigidities.[7] This is also the dominant view of monetary policy at major central banks, including the Fed. According to this view, a shock—often associated with a shock to the efficiency of intermediation—drove the natural real interest rate to negative territory. The optimal monetary policy in those models is to set the

[6]Claiming that he "prevented an economic catastrophe," *Time* magazine named then-Chairman Bernanke Person of the Year on December 2009.

[7]See Krugman (1998), Eggertsson and Woodford (2003), Curdia and Eggertsson (2009), Drautzburg and Uhlig (2011), Christiano et al. (2011), Werning (2011), and Correia et al. (2013).

nominal interest rate equal to the natural real interest rate. However, because of the zero bound, that is not possible. But it is optimal, unambiguously, to keep the nominal interest rate at the zero bound, as the Fed has been doing for over seven years now. Furthermore, these models imply that it is unambiguously optimal to maintain the nominal interest rate at zero even after the negative shock reverts. This policy implication, called "forward guidance," has dominated the policy decisions in the United States since 2008 and remains the conceptual framework that justifies the "exit strategy".

On the contrary, the model we study stresses a different and novel trade-off between ameliorating the initial recession and delaying the recovery. When the central bank chooses a lower inflation target, it must inject more liquidity. As a result, the liquidity trap lasts longer and the real interest rate is constrained to be higher, ameliorating the drop in productivity. The counterpart of the milder drop in TFP is a drop in investment due to the crowding out, leading to a substantial and persistent decline in the stock of capital and a slower recovery.

Which is the optimal policy? In this heterogenous agents model, answering that question requires taking a stand on Pareto weights. We do not pursue this line, but in Section 6, we compute the distribution of welfare changes across all agents. A final Section concludes.

Related Literature We consider a monetary version of the model in Buera and Moll (2015), who apply to the study of business cycles the framework originally developed by Moll (2014) to analyze the role of credit markets in economic development. Kiyotaki (1998) is an earlier example that focuses on a two-point distribution of shocks to entrepreneurial productivity. This framework is related to a long tradition that studies the role of firms' balance sheet in business cycles and during financial crises, including Bernanke and Gertler (1989), Kiyotaki and Moore (1997), Bernanke et al. (1999), Cooley et al. (2004), Jermann and Quadrini (2012).[8]

Kiyotaki and Moore (2012) study a monetary economy where entrepreneurs face stochastic investment opportunities and friction to issue and resell equity on real assets. They also consider the aggregate effects of a shock to the ability to resell equity. In their environment, money is valuable provided that frictions to issue and

[8]See Buera and Moll (2012) for a detailed discussion of the connection of the real version of our framework with related approaches in the literature.

resell equity are tight enough. They use their model to study the effect of open market operations that consist of the exchange of money for equity. Brunnermeier and Sannikov (2013) also study a monetary economy with financial frictions, emphasizing the endogenous determination of aggregate risk and the role of macro-prudential policy. As in our model, a negative aggregate financial shock results in a deflation, although both of these papers consider environments where, for the relevant cases, the zero lower bound on the nominal interest rate is binding in every period.

Guerrieri and Lorenzoni (2011) study a model where workers face idiosyncratic labor shocks. In their model, a credit crunch leads to an increase in the demand of bonds and therefore results in negative real rates. Although our model also generates a large drop in the real interest rate, the forces underlying this result are different. In our framework, the drop in the real interest rate is the consequence of a collapse in the ability of productive entrepreneurs to supply bonds (i.e., to borrow from the unproductive entrepreneurs and workers), as opposed to an increase in the demand for bonds by these agents. In our model, a credit crunch has an opposite, negative effect on investment.

2 The Model

In this section we describe the model, which closely follows the framework in Moll (2014), modified by imposing a cash-in-advance constraint on the consumer's decision problem. As mentioned, the analysis will be restricted to a perfect foresight economy in which starting at the steady state, all agents learn at time zero that, starting next period, the collateral constraint will be tightened for several periods.

2.1 Households

All agents have identical preferences, given by

$$\sum_{t=0}^{\infty} \beta^t \left[\nu \log c_{1t}^j + (1-\nu) \log c_{2t}^j \right], \tag{1}$$

where c^j_{1t} and c^j_{2t} are consumption of the cash good and of the credit good, for agent j at time t, and $\beta < 1$. Each agent also faces a cash-in-advance constraint,

$$c^j_{1t} \leq \frac{m^j_t}{p_t}, \tag{2}$$

where m^j_t is the beginning of period money holdings and p_t is the money price of consumption at time t.

The economy is inhabited by two classes of agents, a mass L of workers and a mass 1 of entrepreneurs, which we now describe.

Entrepreneurs Entrepreneurs are heterogeneous with respect to their productivity (which is exogenous) and their wealth (which is endogenous). We assume that productivity of each entrepreneur, $z \in Z \subset \mathbb{R}^+$, is constant through their lifetime. We let $\Psi(z)$ be the measure of entrepreneurs of type z. Every period, each entrepreneur must choose whether to be active in the following period (to operate a firm as a manager) or to be passive and offer her wealth in the credit market. Thus, there are four state variables for each entrepreneur: her financial wealth (capital plus bonds), money holdings, the occupational choice (active or passive) made last period and productivity. She must decide the labor demand if active, how much to consume of each good, whether to be active in the following period, and if so, how much capital to invest in her own firm. An entrepreneur's investment is constrained by her financial wealth at the end of the period a and the amount of bonds she can sell $-b$, $k \leq -b+a$, where we assume that the amount of bonds that can be sold are limited by a simple collateral constraint of the form

$$-b^j \leq \theta k^j \tag{3}$$

for some exogenously given $\theta \in [0, 1)$. Notice that if $\theta = 1$, then all capital can be pledge and individual wealth plays no role, so this is the equivalent to not imposing any collateral constraints. In this case, the model becomes equivalent to the neoclassical growth model with a cash-in-advance constraint.

If the entrepreneur decides not to be active (i.e., to allocate zero capital to her own firm), then she invests all her non-monetary wealth to purchase bonds.

We assume that the technology available to entrepreneurs of type z is a function of capital and labor

$$y = (zk)^{\alpha} l^{1-\alpha}.$$

This technology implies that revenues of an entrepreneur net of labor payments is a linear function of the capital stock, ϱzk, where $\varrho = \alpha \left((1-\alpha)/w\right)^{(1-\alpha)/\alpha}$ is the return to the effective units of capital zk, and w denotes the real wage. Thus, the end of period investment and leverage choice of entrepreneurs with ability z and wealth a solves the following linear problem:

$$\begin{aligned} \max_{k,d} \quad & \varrho zk + (1-\delta)k + (1+r)b \\ & k \leq a - b, \\ & -b \leq \theta k, \end{aligned}$$

where r is the real interest rate. It is straightforward to show that the optimal capital and leverage choices are given by the following policy rules, with a simple threshold property[9]

$$k(z,a) = \begin{cases} a/(1-\theta), & z \geq \hat{z} \\ 0, & z < \hat{z} \end{cases}, \quad b(z,a) = \begin{cases} -(1/(1-\theta)-1)a, & z \geq \hat{z} \\ a, & z < \hat{z}, \end{cases}$$

where $\hat{z}$ solves

$$\varrho \hat{z} = r + \delta. \tag{4}$$

Given entrepreneurs' optimal investment and leverage decisions, they would face a linear return to their non-monetary wealth that is a simple function of their productivity

$$R(z) = \begin{cases} 1+r, & z < \hat{z} \\ \frac{(\varrho z - r - \delta)}{(1-\theta)} + 1 + r, & z \geq \hat{z}. \end{cases} \tag{5}$$

[9]See Buera and Moll (2015) for the details of these derivations.

Given these definitions, the budget constraint of entrepreneur j, with net worth a_t^j and productivity z^j, will be given by

$$c_{1t}^j + c_{2t}^j + a_{t+1}^j + \frac{m_{t+1}^j}{p_t} = R_t(z^j)a_t^j + \frac{m_t^j}{p_t} - T_t^e, \tag{6}$$

where we assume, for simplicity, that lump-sum taxes (transfers if negative) do not depend on the productivity of entrepreneurs.[10]

These budget constraints imply that agents choose, at t, money balances m_{t+1}^i for next period, as the cash-in-advance constraints (2) make clear. Thus, we are adopting the timing convention of Svensson (1985), in which agents buy cash goods at time t with the money holdings they acquired at the end of period $t-1$. [11] An advantage of this timing for our purposes is that it treats all asset accumulation decisions symmetrically, using the standard timing from capital theory, where production by entrepreneurs at time t is done with capital goods accumulated at the end of period $t-1$.

Workers There is a mass L of identical workers, endowed with a unit of time each period that they inelastically supply to the labor market. Thus, their budget constraints are given by

$$c_{1t}^W + c_{2t}^W + a_{t+1}^W + \frac{m_{t+1}^W}{p_t} = (1+r_t)a_t^W + w_t + \frac{m_t^W}{p_t} - T_t^W, \tag{7}$$

where a_{t+1}^W and m_{t+1}^W are real financial assets and nominal money holdings chosen at time t, and T_t^W are lump-sum taxes paid to the government. If $T_t^W < 0$, these represent transfers from the government to workers. We impose on workers a non-borrowing constraint, so $a_t^W \geq 0$ for all t.[12]

In Appendix A we provide a characterization of the problem of entrepreneurs and workers.

[10] Note that efficiency implies transferring resources from low productivity entrepreneurs to high productivity ones. However, given the exogenous nature of the collateral constraints, we do not find those exercises illuminating.

[11] This assumption implies that unexpected changes in the price level have welfare effects, since agents cannot replenish cash balances until the end of the period.

[12] This is a natural constraint to impose. It is equivalent to impose on workers the same collateral constraints entrepreneurs face, since workers will never decide to hold capital in equilibrium.

2.2 Demographics

As we will show below, active entrepreneurs will increase their wealth permanently, while inactive entrepreneurs will deplete it asymptotically. In order for the model to have a non-degenerate asymptotic distribution of wealth across productivity types, we assume that a fraction $1-\gamma$ of entrepreneurs depart for Nirvana every period, and are replaced by an equal number of new entrepreneurs. The productivity z of the new entrepreneurs is drawn from the same distribution $\Psi(z)$, i.i.d. across entrepreneurs and over time. We assume that there are no annuity markets and that each new entrepreneur inherits the assets of a randomly drawn departed entrepreneur. Agents do not care about future generations, so if we let $\hat{\beta}$ be the pure discounting factor, they discount the future with the compound factor $\beta = \hat{\beta}\gamma$, which is the one we used above.

2.3 The Government

In every period the government chooses the money supply M_{t+1}, issues one-period bonds B_{t+1}, and uses type-specific lump-sum taxes (subsidies) T_t^e and T_t^W. Government policies are constrained by a sequence of period-by-period budget constraints:

$$B_{t+1} - (1+r_t)B_t + \frac{M_{t+1}}{p_t} - \frac{M_t}{p_t} + T_t^e + LT_t^W = 0, \quad t \geq 0. \tag{8}$$

We denote by T_t the total tax receipts of the government:

$$T_t = T_t^e + LT_t^W.$$

Ricardian equivalence does not hold in this model for two related reasons. First, agents face different rates of return to their wealth. Thus, the present value of a given sequence of taxes and transfers differs across agents. Second, lump-sum taxes and transfers will redistribute wealth in general, and these redistributions do affect aggregate allocations, due to the presence of the collateral constraints. In the numerical sections, we will be explicit regarding the type of taxes and transfers we consider.

3 Equilibrium

Given policies $\{M_t, B_t, T_t^e, T_t^W\}_{t=0}^{\infty}$ and collateral constraints $\{\theta_t\}_{t=0}^{\infty}$ an equilibrium is given by prices $\{r_t, w_t, p_t\}_{t=0}^{\infty}$ and corresponding quantities such that:

- Entrepreneurs and workers maximize, taking as given prices and policies,
- The government budget constraint is satisfied, and
- Bond, labor, and money markets clear:

$$\int b_{t+1}^j dj + L b_t^W + B_{t+1} = 0, \quad \int l_t^j dj = L, \quad \int m_t^j dj + L m_t^W = M_t, \quad \text{for all } t.$$

To illustrate the mechanics of the model, we study a very special case for which we can obtain closed-form solutions. See Appendix B for a partial characterization of the dynamics of the general model.

3.1 A Simple Case with a Closed-Form Solution

An interesting feature of the model is the interaction between the credit constraints and the endogenous savings decisions. This interaction generates dynamics that imply long-run effects that are very different from the ones obtained on impact, precisely through the endogenous decisions agents make over time to save away from those constraints. A complication is that the endogenous wealth distribution becomes a relevant state variable, and it becomes impossible to obtain analytical results.

It is possible, however, to obtain closed-form solutions if we shut down that endogenous evolution of the wealth distribution. Some of the effects that the simulations of the general model exhibit are also present in this simplified version, where they are easier to understand. We now proceed to discuss that example.

Consider the case in which $\gamma \to 0$, but $\hat{\beta} \to \infty$, such that $\beta = \hat{\beta}\gamma$ is kept constant. In this limit, agents live for only a period but the saving decisions are not modified so the distribution of net-worth across types equals their share in the population. We also let z be uniform in $[0, 1]$. Then, the equilibrium condition for the credit market becomes

$$\hat{z}_{t+1} (K_{t+1} + B_{t+1}) = \frac{\theta_{t+1}}{1 - \theta_{t+1}} (1 - \hat{z}_{t+1}) (K_{t+1} + B_{t+1}) + B_{t+1}.$$

Defining $b_t = \frac{B_t}{K_t+B_t}$ and rearranging, we obtain a simple expression for the marginal entrepreneur

$$\hat{z}_{t+1} = \theta_{t+1} + (1-\theta_{t+1})\, b_{t+1}.$$

The value of aggregate TFP, which is giving by a wealth-weighted average of the productivity of active entrepreneurs,[13] equals

$$Z_t = \left(\frac{1+\theta_t+(1-\theta_t)\, b_t}{2}\right)^{\alpha}. \tag{9}$$

Finally, we normalize $L = 1$ so the law of motion for capital is given by

$$\begin{aligned} K_{t+1} \quad &= \quad \beta\left[\alpha\left(\frac{1+\theta_t+(1-\theta_t)\, b_t}{2}\right)^{\alpha} K_t^{\alpha} + (1-\delta)K_t\right] \\ &\quad +(1-\beta)\left[\int_0^{\infty}\sum_{j=1}^{\infty}\frac{T^e_{t+j}}{\prod_{s=1}^{j} R_{t+s}(z)}dz - B_{t+1}\right]. \end{aligned} \tag{10}$$

The first term on the right hand side gives the evolution of aggregate capital in an economy without taxes. In this case, aggregate capital in period $t+1$ is a linear function of aggregate output and the initial level of aggregate capital. The evolution of aggregate capital in this case is equal to the accumulation decision of a representative entrepreneur (Moll, 2014; Buera and Moll, 2015). The second term captures the effect of alternative paths for taxes, discounted using the type-specific return to their non-monetary wealth, and government debt. For instance, consider the case in which the government increases lump-sum transfers to entrepreneurs in period t, financing them with an increase in government debt and, therefore, with an increase in the present value of future lump-sum taxes. In this case, future taxes will be discounted more heavily by active entrepreneurs, implying that the second term would be negative. Thus, this policy crowds out private investment and results in lower aggregate capital next period. Note that given sequences of policies and collateral constraints, this equation fully describes the dynamics of capital.

The economy behaves as it does in Solow's growth model, except that the collateral constraint as well as the fiscal policy both matter. The collateral constraint matters because it affects aggregate TFP. Policy matters because the model, as we show in

[13] See Appendix B.1 for a more detailed discussion, and Moll (2014) for a derivation of this result.

detail below, does not exhibit Ricardian equivalence. Finally, the real interest rate is given by

$$\delta + r_{t+1} = \alpha \frac{\theta_{t+1} + (1-\theta_{t+1})\, b_{t+1}}{Z_{t+1}^{\frac{1-\alpha}{\alpha}} K_{t+1}^{1-\alpha}}.$$

In order to gain understanding of some of the effects on equilibrium outcomes of changes in the collateral constraint and on the effects of monetary and fiscal policy, we now solve several simple exercises. In the first two exercises, we solve for the real economy, where $\nu = 0$ and focus the discussion on the evolution of real variables. We first set all transfers to zero and study the effect of a credit crunch: an anticipated drop in θ_t that lasts several periods and then goes back to its steady state value. We show that total factor productivity, output, and capital accumulation drop, so the effect of output is persistent. We also show that if the credit crunch is large enough, the real interest rate becomes negative. We then keep θ_t constant and study the effect of debt-financed transfers to show the effect of an increase in the outside supply of bonds in the equilibrium. We show that debt issuance crowds out private investment but increases total factor productivity, so the effect on output is ambiguous. In addition, debt issuance increases the real interest rate. Finally, in Section 3.1.3 we discuss the behavior of the price level following a credit crunch where the real interest rate becomes negative and the zero bound on the nominal interest rate becomes binding. If the central bank does not change the nominal quantity of money, a deflation follows.

3.1.1 The Effect of a Credit Crunch

To isolate the effect of a credit crunch, we set $b_t = 0$ and $T_t^e = T_t^w = 0$. In this case,

$$\hat{z}_t = \theta_t \text{ and } Z_t = \left(\frac{1+\theta_t}{2}\right)^{\alpha}.$$

Given the level of capital, the interest rate is given by

$$\delta + r_t = \frac{\theta_t}{(1+\theta_t)^{1-\alpha}} \frac{2^{1-\alpha}\alpha}{K_t^{1-\alpha}}, \tag{11}$$

which implies that the real interest rate falls with θ_t. This drop in the real interest rate is what provides the incentives to the less efficient entrepreneurs to enter until the credit market clears, thereby reducing TFP. The drop in the real interest rate as the collateral constraint falls enough is a general feature of the model, which does not depend on the particular simplifying assumptions used in this example. In general, $r_t = \hat{z}_t / \mathbf{E}[z|z \geq \hat{z}_t]^{1-\alpha} K_t^{\alpha-1} - \delta$, which tends to $-\delta$ as θ_t, and therefore $\hat{z}_t$, converges to zero.[14] We find this feature of the model particularly attractive in studying the great recession: A single shock can explain both the gap between output and trend and the negative real interest rates. We will further discuss this issue in the calibration section.

The law of motion for capital is given by

$$K_{t+1} = \beta \left[\alpha \left(\frac{1+\theta_t}{2} \right)^{\alpha} K_t^{\alpha} + (1-\delta) K_t \right]. \tag{12}$$

We model a temporary credit crunch as

$$\begin{aligned} \theta_0 &= \theta^{ss} \\ \theta_t &= \theta_l < \theta^{ss} \text{ for } t = 1, 2, ..., T \\ \theta_t &= \theta^{ss} \text{ for for } t > T \end{aligned}$$

and assume that all agents have perfect foresight. The effect on capital is identical to a temporary drop in TFP in Solow's model: capital does not change on impact, but starts going down until T. Then, it starts going up to the steady state. The interest rate drops on impact, since the ratio $\frac{\theta_{t+1}}{(1+\theta_{t+1})^{1-\alpha}}$ is increasing on θ_{t+1}. Note that

$$\delta + r_1 = \frac{\theta_l}{(1+\theta_l)^{1-\alpha}} \frac{2^{1-\alpha}\alpha}{K_{ss}^{1-\alpha}},$$

so the real interest rate will be negative if θ_l is low enough. The evolution of the interest rate depends on the evolution of capital and on the collateral constraint parameter.

[14] One can also show that the (negative) effect of a credit crunch on the real interest rate would be larger than that of a (negative) TFP shock provided $\partial E[z|z \geq \hat{z}] / \partial \hat{z} \, (\hat{z}/E[z|z \geq \hat{z}]) < 1$, a condition satisfied by a large class of commonly used distributions.

3.1.2 The Effect of Policy

We consider the case in which there are no taxes or transfers to workers (so $T_t^w = 0$).[15] First, note that (9) trivially implies that total factor productivity is increasing on the ratio of debt to total assets. We show now that it also crowds out investment.

Assume that the economy starts at the steady state and consider the policy

$$B_0 = 0, \quad B_1 = -T_0 = B > 0 \ \text{ and } \ (1 + r_1)\, T_0 + T_1 = 0,$$

so $B_t = 0$ for $t \geq 2$. Thus, at time o there is a deficit (a transfer to all entrepreneurs) financed by issuing debt. Then, at time 1, there is a surplus (tax to all entrepreneurs) that is enough to payoff all bonds issued at time zero.

Given this policy, the law of motion for capital (10) becomes

$$K_1 = K_{ss} - (1 - \beta) B \left[1 - \int_0^1 \frac{(1 + r_1)}{R_1 (z)} dz \right]. \tag{13}$$

Note that if we eliminate the collateral constraint and set $\theta = 1$, $R_1 (z) = (1 + r_1)$ for all z (and $\hat{z} = 1$). Therefore,

$$\int_0^1 \frac{(1 + r_1)}{R_1 (z)} dz = 1$$

which implies $K_1 = K_{ss}$ and Ricardian equivalence holds. But when $\theta \in (0, 1)$, from (5) it follows that $R_1(z) > (1 + r_1)$ for $z > \hat{z}$ (and $\hat{z} \in (0, 1)$), then

$$0 < \int_0^1 \frac{(1 + r_1)}{R_1 (z)} dz < 1.$$

Thus, the level of K_1 is lower than the steady state for any positive level of debt.[16]

Thus, starting at $B = 0$, as debt increases, total factor productivity goes up as seen from (9), but capital goes down, so the net effect on output is ambiguous.[17]

[15] We solve the case in which workers are also taxed in Appendix C, where, to keep analytical tractability, we also assume that workers are hand to mouth. In our simulations we solve for the general case in which workers can hold bonds.

[16] The opposite is true if the government lends such that $B < 0$.

[17] The relationship between government debt and aggregate capital is non-monotonic. In particular, one can show that as $B \to \infty$, aggregate capital converges to the steady state value in an economy with $\theta = 1$.

Finally, note that the interest rate on period 1 is given by

$$\delta + r_1 = \frac{\theta + b_1(1-\theta)}{(1+\theta+b_1(1-\theta))^{1-\alpha}} \frac{\alpha 2^{1-\alpha}}{K_1^{1-\alpha}},$$

where the first term is increasing on b_1, so the interest rate will be higher than in the steady state.

To summarize, a credit crunch and an increase in debt have opposite effects on total factor productivity and on the real interest rate: while the credit crunch reduces both, the increases in debt increase both. On the contrary, both the credit crunch and the debt increase reinforce each other in that they reduce capital accumulation. As increases in outside liquidity (bonds plus money at the zero bound) dampen the drop in the real interest rate, they will be effective at achieving a target for inflation when the zero bound binds. Doing so also implies a lower drop in TFP (a higher threshold) but a larger drop in capital. The net effect on output is in general ambiguous, but it can be shown to be positive in the neighborhood of $B = 0$.[18]

This trade-off will be present in our simulations of the general model that allows for rich dynamics of the wealth distribution and uses more realistic functional forms for the distribution of z.

3.1.3 Deflation Follows Passive Policy

In the Appendix B.2.1 we consider the cashless limit of the monetary version of the model and study the behavior of the price level following a credit crunch that drives the real interest rate into negative territory. In particular, we provide sufficient conditions so that the credit crunch drives the real interest rate below zero to the point at which the zero bound is reached and a deflation results if policy does not respond. Intuitively, at this point, there is an excess demand for money as a "store of value." That excess demand is, of course, real rather than nominal. As the nominal quantity is fixed by policy, the demand pressure results in deflation. The excess demand for money as a store of value will be positive until future inflation is high enough such that the return on money is as negative as the return on bonds. The initial deflation allows for future inflation along the path, required for the arbitrage condition to hold, with a zero "long-run" inflation. This zero long run inflation is the

[18] In the case in which workers are the only individuals being taxed and subsidized, the net effect on output is negative. The analysis of the net effect on output is presented in Appendix C.

natural consequence of a constant nominal money supply.

4 Calibration and Evaluation of the model

In this Section we first calibrate the model to the United States economy and discuss how we take the model to the data. We discuss in detail the frequency we want to focus on and the way we de-trend the data. Second, we numerically solve the model and compare it to the data. Finally, we perform several policy experiments to illustrate the role of policy during the Great Recession, and to consider the impact of alternative policy responses.

4.1 Calibration

The model has very few parameters. We first set the capital share be one third and the (annual) depreciation of capital to be seven percent, which are standard values. For simplicity we consider the cashless limit in which $\nu \to 0$ and we also let the initial stock of money go to zero at the same rate, so the price level can be determined. The zero bound constraints on nominal interest rates still must hold. We set the distribution of abilities to be log-normal, $z \sim \ln \mathcal{N}(0, \sigma_z)$, and choose the standard deviation σ_z so that the log dispersion of productivity among entrepreneurs in the model matches that among manufacturing establishments in the United States, as reported by Hsieh and Klenow (2009).[19]. We choose the rate at which entrepreneurs exit $1 - \gamma = 0.10$ to match to the average exit rate of US establishments from the Business Dynamic Statistics (BDS). The initial parameter of the collateral constraint, $\theta = 0.56$, is chosen to match the ratio of liabilities to capital for the US non-corporate sector in the second quarter of 2007.[20] Given the previous parameters, we set the discount factor β equal to 0.981, so the real interest rate is two percent. Table 4.1 summarizes the parameter values we use.[21]

[19]If there are diminishing returns to scale all entrepreneurs will be active in equilibrium. Therefore, by matching the log dispersion of productivity among all entrepreneurs, active and inactive, the calibration is robust to the inclusion of arbitrary small diminishing returns to scale.

[20]We measure the ratio of liabilities to capital by dividing the ratio of liabilities to gross value added of the US non-corporate sector from the flow of funds by the aggregate capital to output ratio.

[21]We also need to specify the relative number of workers and entrepreneurs in the economy. We assume that workers are 25% of the population, $L/(1+L) = 1/4$. We choose a low share of workers, who in our model choose to be against their borrowing constraint in a steady state, to limit the

Parameters	Targets
$\alpha = 1/3$, $1-(1-\delta)^4 = 0.07$	Standard Values
$z \sim \ln N(0, 3.36)$	Log dispersion of estab., US manuf.
$1-\gamma^4 = 0.10$	avg. exit rate of US establishments
$B_0/(4Y_0)$=0.62	Total Public, Federal Debt in the US, 2007:Q2
$\beta = 0.981$	2% real interest rate
$\theta_0 = 0.56$	Liabilities to capital, US non-corporate sector 2007:Q2

Table 1: Calibration, Initial Steady State

With this calibrated version, we simulate the model, choosing the value of the collateral constraint so as to reproduce the evolution of the real interest rate in the US since the financial crisis. Specifically, we assume that starting at that steady state, all agents learn that the collateral constraint will tighten for several periods and that the Fed and the central government will substantially increase their liabilities, i.e., their supply of liquid assets. We chose the sequence $\{\theta_t\}_{t=0}^{\infty}$ so that the model broadly matches the evolution of the real interest rate, given the path for government liabilities. The inputed evolution of government liabilities must be associated with a specific path of taxes and transfers. We assume that transfers are given solely to entrepreneurs, active and inactive, while taxes are lump-sum. In Section 5.2 we consider the case with lump-sum transfers. This second case exhibits larger departures from Ricardian equivalence, as we discuss in detail below.

We chose to focus on the behavior of the real interest rate in the United States following the financial crisis to calibrate the sequence $\{\theta_t\}_{t=0}^{\infty}$ for a couple of reasons. First, as discussed in detail in the last section, it is the reduction in θ_t that drives down the real interest rate. The direct theoretical relationship between the unobservable shock and the real rate makes it an attractive target for the calibration.

Second, the very long period of negative real interest rates, depicted in the left panel of Figure 1, is one of the most remarkable features of the events of the last decade and speaks to the persistence of the shock that drove the real rates below zero. The series labeled "real realized" is just the difference between the short term nominal interest rate between quarter t,and quarter $t+1$ and the observed inflation

non-Ricardian elements in the model. This number is consistent with the fraction of households with zero net liquid assets, which was 23% in the US in 2001 (Kaplan and Violante, 2014).

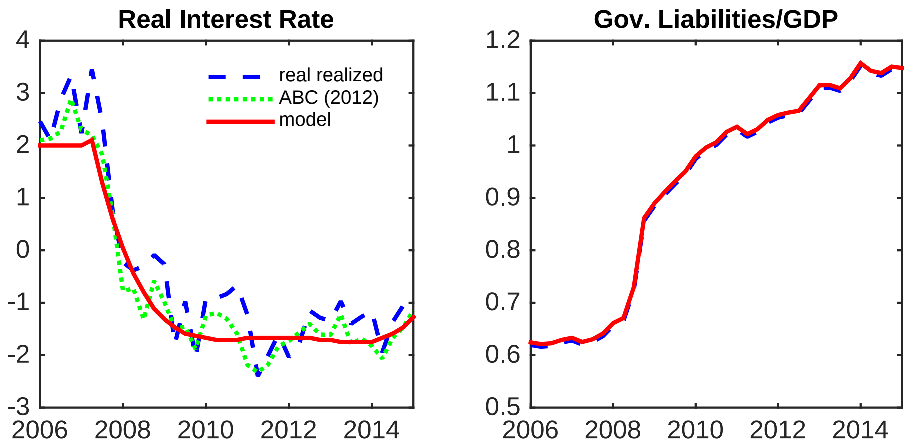

Figure 1: Real Interest Rate, Credit Growth and Public Liquidity, Data and Model Calibrated Paths

rate, using the consumer price index, also between t and $t+1$. The series labeled "ABC 2014" is obtained from Ajello et al. (2012), which compute it using a no-arbitrage model that jointly explains the dynamics of consumer prices as well as the nominal and real term structure of risk-free rates. Both deliver the same picture. The series label "model" is a smooth version of the data reported by Ajello et al. (2012). This is our calibration target.

We use the behavior of this real rate to identify the timing, the severity and the persistency of the shock. As it can be seen in the left panel of Figure 1, the drop in the rate is in the third quarter of 2007, so we choose this quarter as the onset of the crisis. We then chose the sequence θ_t such that the simulated series for the real interest rate matches the solid line in the left Panel of Figure 1. The simulation starts at the steady state - taken to be the third quarter of 2007 - and assumes an unanticipated but perfectly forecastable path for θ_t. As mentioned before, the equilibrium does depend on the injection of total liquidity. Thus, in the simulation we also assume the total supply of outside liquidity to be the one observed in the data starting in 2007, depicted in the second panel of Figure 1. This series is the sum of the total public federal debt and the balance sheet of the Federal Reserve Banks net of their holdings of Treasury bonds. An important part of the policy response to the crisis by the Fed was to increase the supply of liquidity in term of bank reserves in exchange for Mortgage Back Securities. In addition, there were large transfer and tax programs that resulted in an unprecedented increase in the level of the public debt. Furthermore, we assume a path for the money supply that is consistent with an inflation of 2% per year in periods in which the nominal rate is strictly positive. When the nominal rate is at the zero lower bound, the inflation rate equals minus the real interest rate, i.e., the observed path of inflation. According to the model, these liquidity injections have real effects. We will discuss exactly how in Section 4.3, below.

4.2 Evaluation of the model

We now compare the simulations of the model with the US data since the third quarter of 2007 - the date identified as the beginning of the crisis by the real interest rate - till the first quarter of 2015, the end of our sample. In looking at variables such as output, capital, labor or productivity, we face a difficulty that does not arise when looking at real interest rates: the trend in the data. Our model is stationary, but it can be modified to incorporate exogenous productivity growth, the same way it is done in the Solow model. To the extent that the exogenous component of productivity grows at a constant rate, allowing for this exogenous component is equivalent to removing a linear trend to the natural logarithm of the data. That is the strategy we pursue in comparing the data to the model.

In the next Table we show the quarterly growth rate of the linear trend for the natural logarithm of output, capital, hours and productivity using quarterly data.[22] We report results for three different starting periods, 1947, 1960 and 1980. In all cases, the last period was the third quarter of 2007, the period where the crisis started according to our calibration.

	Initial Period		
	1947:Q3	1960:Q1	1980:Q1
Output	0.0088	0.0081	0.0080
Hours	0.0033	0.0042	0.0039
Capital	0.0092	0.0086	0.0077
Productivity	0.0027	0.0023	0.0028

Table 2: Linear Trends of (log) GDP, Hours, Capital and TFP

[22]The linear trend has been computed by ordinary least squares regressions of the log of the corresponding variable on time.

The results are surprisingly robust to the initial date used for output and hours. It is less so for capital. However, in the case of capital, the data shows that a linear trend does not adjust well for the first two samples: The data cuts the trend only twice for the first two periods, suggesting a slowdown of the trend over time. However, for the sample that starts in 1980, the trend cuts the data several times. Thus, we do choose the value for the sample that starts in 1980. The trend for productivity seems also relatively stable, but that hides the fact that it grew very rapidly from 1947 to 1973 (a slope of 0.0043) and then it remained essentially constant till 1983. It then grew at 0.0028 the rate reported for the sample that starts in 1980.

Given this discussion, we chose the values for the trends to be the ones that result from the period starting in 1980, the ones reported in the last column of Table 2.

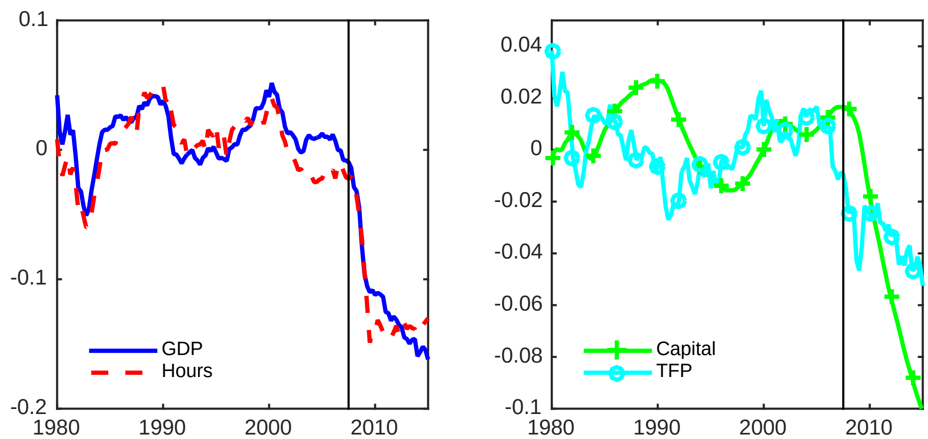

Figure 2: Detrended GDP, Hours, Capital Stock and TFP, 1980 to 2015

To show the effect of de-trending in a way that makes clear the long lasting effect of the crisis that started in 2007, in Figure 2 we depict the difference between the natural logarithm of the data and its trend (computed from 1980 to 2007) for output and hours in the left panel, and for capital and productivity in the right panel. As it can be seen, there are fluctuations around trend that, till 2007 never go beyond 5% (3%) in absolute value for any of the series in the left (right) panel. However, after 2007, the deviations are all negative and way larger than anything that has been seen before.

We now argue that, except for hours, a large fraction of these changes can be accounted for by the single shock we model, and that we calibrated to match the evolution of the real interest rate.

The deviations from trend exhibit in Figure 2 are the ones we compare to the simulation of the calibrated model. To begin with, we should notice we assumed the labor supply to be constant, so the model will be unable to replicate the very large drop in hours since 2007. This would not be different if we had leisure in the utility function: a shock to the collateral constraint, as we model it, does not have a direct effect on the labor market.[23] If the reader was hoping to learn something meaningful regarding the relationship between credit constraints and the labor participation rate, she should stop reading now.

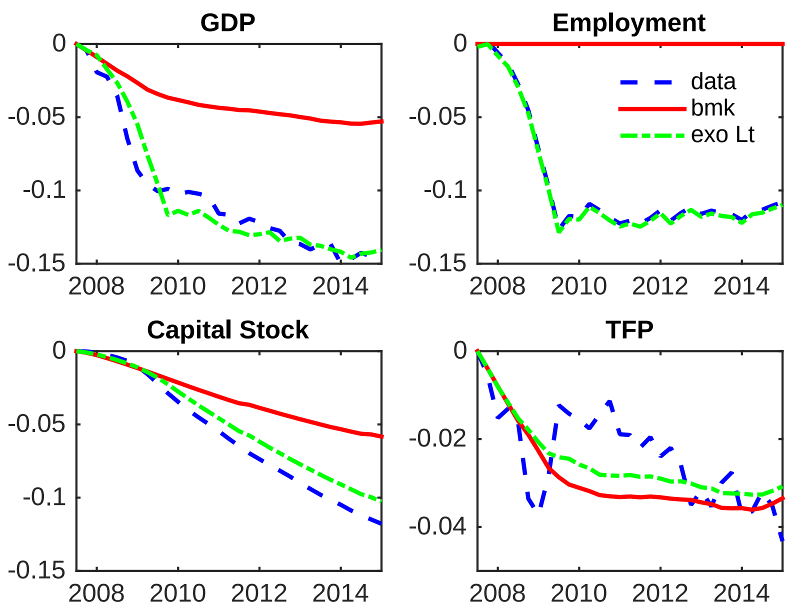

Figure 3: Great Recession in the Benchmark Model and Model with Exogenous Hours

In Figure 3 we compare the de-trended data for output, productivity, capital, and hours, with the simulation of the model. The solid line is the simulation of the model. The dashed line is the data. The two panels in the left show that the model captures the direction and the persistency of the drops in capital and output - relative to trend - but misses the magnitudes: it only explains around one third of the drop in output and half of that of capital. The lower right panel shows that the model does a very good job at tracking the behavior of productivity, missing the high frequency movements. It is important to highlight that no parameter has been chose to fit this curve - or any of the other pictures in this figure! The lower panel in the left

[23] As we show below, adding sticky wages a la Calvo cannot explain it either for parameter values used in the literature. The effects in the model lasts less than three years.

shows that the model, with constant labor, misses the behavior of hours. One could conjecture, therefore, that part of the reason why the model misses the magnitudes in explaining output and capital is related to the failure of explaining labor. One way to evaluate this conjecture, given that in the model labor supply is exogenous, is to simply impose in the simulation the behavior of hours that we saw in the data. The result is depicted in the Figure with the dashed-dotted line. Once we feed the model with the observed value for hours, the match of the model is remarkably good.

Figure 4: Credit Growth in the Benchmark Model vs. The Non-Corporate Sector

The solid line in Figure 4 shows the path for the growth rate of credit in the calibrated model. We also show the growth rate of credit to the non-financial, non-corporate sector from the flow of funds, normalized to be zero at the beginning of the crisis (dashed line). While there is substantial correlation between the data and the model, the simulation over predicts the speed with which credit drops in the data, and correspondingly, under predicts the persistence of the contraction.

A natural caveat regarding the behavior of credit growth in the model is that debt contracts have one period maturity, so the speed at which firms are force to deleverage is very high. In the data, debt contracts last for many periods, and they take a few periods to get processed, approved and executed. Thus, the comparison between the model that abstracts from all these complications and the data is trickier than what it appears at first sight.[24] All in all, it can be seen that indeed, a big change occurs

[24]We would like to highlight though, that the maturity structure should not matter for the the steady state, so we feel comfortable with the use of the credit to value added to calibrate θ in the steady state - see Table 1.

in 2007 Q3, the quarter the real interest rate identifies as the beginning of the crisis.

Notice that the growth rate of credit almost fully recovers by the end of our sample. Would this indicate that maybe the crisis is reverting (in the model credit to capital starts growing when the collateral parameter starts growing) and the main variables will now begin converging to its trend, putting an end to the "secular stagnation"? This model certainly implies so, with the exception of labor input, with its corresponding impact on output and capital.

Taken all together, these exercises provide evidence that the mechanism discussed in the model captures many of the relevant features of the post 2007 events, the big exception is the behavior of hours. Something else ought to explain why hours behaved as they did.

The interpretation provided by the model then, is that the credit crunch lasted at least 8 years, with some weak indication that some reversal may be taking place: The real interest rate seems to be trending upwards and credit growth, while very small, became positive at the end of 2014. The two facts are consistent with the gradual unwinding of the financial shock.

To summarize, the model does, in our view, a very decent job in replicating the recent events, once the current policy is taken into account. With this acceptable background, we now use the model to study the effect of policy.

5 Alternative Policy Responses

Was the policy response to the financial crisis, i.e., the large increase in the supply of liquidity, instrumental in avoiding an even larger, Great Depression like recession? What would have been the consequence of an even more aggressive response? To explore these question we use our calibrated model to analyze alternative policy responses.

For all the experiments we consider we proceed as before: we start the economy at the steady state and assume that in the first period, agents learn that there will be a deterministic credit crunch as calibrated in the previous section. All remaining parameters are also kept at the calibrated values. We then consider different scenarios for monetary and debt policy.

First, we illustrate the predicted evolution for the economy as a result purely of the credit crunch, in the absence of any policy response, so we maintain total liquidity at its initial value which is 62% of initial GDP. This case allows us to identify the pure effect of policy in the model. For this case, we also consider extensions with nominal debt and sticky wages. Second, we assume that monetary and fiscal policies are such that inflation is kept low and constant at alternative targeted values that are consistent with the typical mandates of central banks. To achieve the desired target, monetary and fiscal policy must be active, and the equilibrium outcome will depend on the accompanying debt, tax, and transfer policies. Thus, we consider alternative lump-sum tax and subsidy schemes. The benchmark model discussed in the previous section is a particular case, in which we interpret the observed policy response as implementing the actual path of inflation.

5.1 Nonresponsive Policy

We now assume that the quantity of money and bonds remain fixed all time. Since there is no change in policy, we only need to adjust lump-sum transfers to reflect the changes in the interest rate, so $T_t^e = T_t^w = r_t B_0$ for all t. Note that although we focus on money rules, in an equilibrium, given a money rule, we obtain a unique sequence of interest rates. One could therefore think of policies as setting those same interest rates.[25]

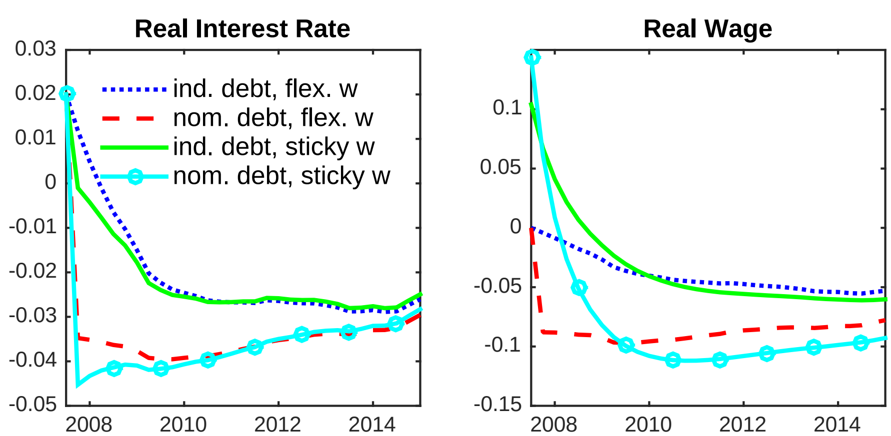

Figure 5: Nonresponsive Policy, Alternative Nominal Frictions

[25]If one were to think of policy as setting a sequence of interest rates, the issue of price level determination should be addressed. The literature has adopted two alternative routes: the Taylor principle or the fiscal theory of the price level. We abstract from those implementation issues in this paper.

In figures 5 and 6 we present results for the benchmark model as discussed so far, and for extensions that allow for both nominal debt and/or sticky wages. The results for the benchmark case with indexed debt and flexible wages (blue, dotted line) are consistent with the theoretical results of the special case model discussed in Section 3.3. A credit crunch results in a large decline in the real interest rate which bottoms at minus three percent (left panel of Figure 5), which is over 100 basis points larger than the drop in the benchmark model with the observed policy response which bottoms at minus two percent (left panel of Figure 1).

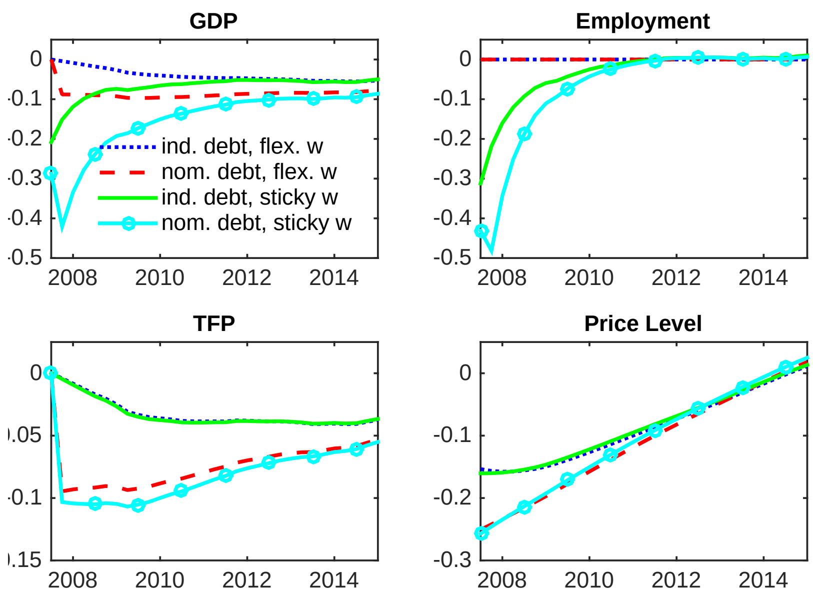

Figure 6: Nonresponsive Policy, Alternative Nominal Frictions

As discussed in Section 3.1.3 and illustrated in the lower right panel of Figure 6, there is a large deflation on impact and positive inflation afterwards, guaranteeing that agents are indifferent between holding real balances and bonds paying a low return. The excess demand for store of value at the zero bound leads to a desire to hoard real money balances. Since the supply of liquidity is held fixed in this exercise, the price level must go down.[26]

In the context of the benchmark model, the deflation has little effect. But the

[26]In this perfect foresight equilibrium, the deflation occurs on impact. This feature would be different if, for instance, every period there is a constant probability of exiting the credit crunch.

results suggests that a potential problem may arise if debt instruments are nominal obligations or if there is downward wage rigidity as the New Keynesian models assume.[27] We now discuss these two extensions.

Nominal Bonds We next consider the case in which entrepreneurs and the government issue nominal bonds only. As before, the real value of bond issuance is restricted by the collateral constraint in (3). The results, which are substantially different, are given by the red-dashed line in Figures 5 and 6. The recession is deeper and more persistent, driven mainly by a sharper decline in TFP (lower, left panel of Figure 6). The intuition for the large negative effect of the debt deflation is simple: the initial deflation implies a large redistribution from high productivity, leveraged entrepreneurs toward bondholders, who are inactive, unproductive entrepreneurs. The ability of productive entrepreneurs to invest is now hampered by both the tightening of collateral constraints and the decline of their net worth. As a consequence, there needs to be a larger decline in the real interest rate so that in equilibrium more capital is reallocated from productive to unproductive entrepreneurs (left panel of Figure 5), which results in a larger deflation and a nominal interest rate that remains at zero for longer (lower, right panel of Figure 6).

This example shows that the initial deflation can be very costly in terms of output and could provide motivation for policy interventions to stabilize the price level and output. An alternative motivation is given by the existence of nominal rigidities.

Sticky Wages We consider now the model with restrictions in the setting of nominal wage, following the New Keynesian tradition. In particular, we consider workers that are grouped into households with a continuum of members supplying differentiated labor inputs. Each member of the household is monopolistically competitive and gets to revise the wage in any given period with a constant probability as in Calvo (1983). A detailed description of this extension and the calibration, which are totally standard in the literature, is provided in Appendix D.

The solid, green line in Figures 5 and 6 shows the evolution of the economy for the case with rigid wages. As in the first example, we assume that private bonds are indexed to the price level and the supply of money and bonds remain constant. With

[27]This "debt deflation" problem has been mentioned as one of the possible costs of deflations before, particularly in reference to the Great Depression (Fisher, 1933).

rigid wages, the initial deflation causes an increase in the real wage (right panel of Figure 5) and a sharp decline in the labor input (top right panel of Figure 6). This results in a substantially more severe recession. As in the previous examples in this figure, the real interest rate becomes negative and the nominal interest rate is at the zero lower bound for various periods. Unlike the case with nominal debt contracts, frictions in the setting of wages does not result in important effects on TFP. [28]

Interactions of Nominal Frictions Is there an interesting interaction between these two nominal frictions, nominal debt contracts and sticky nominal wages? The dramatic dynamics illustrated by the solid lines with circle markers, which corresponds to that of an economy with nominal debt contracts and sticky wages, provides a loud answer to this question. As discussed, with nominal debt contracts the credit crunch results in a large redistribution of wealth from debtors (productive entrepreneurs) towards creditors (unproductive entrepreneurs). This implies a lower real interest rate, and a larger initial deflation. With sticky wages, initially the real wage is larger, and therefore, there is a substantially larger drop in employment. This feedbacks into a lower profitability of productive entrepreneurs, and lower TFP.

The previous discussion suggests that the initial deflation can be very costly in terms of output. An obvious question, then, is what can monetary and fiscal policy do, if anything, to stabilize the price level and output? As we hinted when discussing the calibration of the benchmark model, the unprecedented policy response was key to avoid the deflation and generate the moderate and stable inflation observed during the crisis. We next consider cases where the government implements alternative inflation targets to clarify the role of the observed policy response, and the merits of alternative policies.

5.2 Inflation Targeting

We now consider the cases in which the government implements alternative inflation targets, which for simplicity we assume constant and denote by π. We compare these cases with the benchmark economy given by the solid line in Figure 3, which is the particular example in which the inflation target is the path of inflation in the data.

[28]Note that for this parametrization, which follows the literature, the effect of sticky wages on the evolution of total hours last about two years and a half. Thus, while the size of the drop in labor is almost as in the data, it reverts way to fast in the model.

As long as the zero bound does not bind, so $r_t > -\frac{\pi}{1+\pi}$, inflation is determined by standard monetary policy. However, if given the target, the natural interest rate is inconsistent with the zero bound, the government needs to increase real money balances M_{t+1}/p_t and/or government bonds B_{t+1} in order to satisfy the excess demand for real assets. In order to do so, it will also need to implement a particular scheme of taxes and transfers. Because of the collateral constraints, redistributions may have significant effects on the economy, so the way in which the taxes and transfers are executed matters, an issue we will address.

It is important to emphasize that once the economy is at the liquidity trap, real money and bonds are perfect substitutes and the only thing that matters is the sum of the two, so there is an indeterminacy in the composition of total outside liquidity. We will further discuss the subtle distinction between monetary and debt policy at the zero bound, but in order to focus on the effects of policy (monetary and/or fiscal) and without loss of generality, we assume that the government sets the quantity of money to be equal to the money required by individuals to finance their purchases of cash goods in every period.[29] Then, public debt is given by

$$B_{t+1} = \begin{cases} B_t & \text{if } r_{t+1} > -\frac{\pi}{1+\pi} \\ \int_0^{\hat{z}_{t+1}} \Phi_{t+1}(dz) - \frac{\theta_{t+1}}{1-\theta_{t+1}} \int_{\hat{z}_{t+1}}^{\infty} \Phi_{t+1}(dz) & \text{if } r_{t+1} = -\frac{\pi}{1+\pi}. \end{cases} \tag{14}$$

Obviously, lump-sum taxes (subsidies) must be adjusted accordingly to satisfy the government budget constraint in (8).

These conditions fully determine the evolution of the money supply, government bonds, and the aggregate level of taxes (transfers), but they leave unspecified how taxes (transfers) are distributed across entrepreneurs and workers. As in the benchmark calibration, we consider the case in which taxes are purely lump sum for all periods. But in periods when the government increases the supply of bonds, we consider two cases: first, we assume that the proceeds from the sale of bonds, net of interest payments and the adjustment of the supply of real balances, are transferred only to the entrepreneurs so $T_t^W = 0$ whenever $T_t^e < 0$. This case captures a scenario in which the government responds to a credit crunch by bailing out productive en-

[29] An expression for the transactional demand for money, m_{t+1}^T, is given by equation (18) in Appendix A.

trepreneurs and bondholders. We refer to this as the "bailout" case.[30] The second case that we consider is one in which transfers are also purely lump sum, that is, $T_t^W = T_t^e$ for all t, z. We refer to this as the "lump-sum" case.

We now address the question, can the government mitigate the consequences of the credit crunch by choosing alternative inflation targets? In particular, is it desirable that the government chooses a sufficiently high inflation target in order to avoid the zero lower bound? We explore these questions in Figure 7. There we present the evolution of three economies relative to the benchmark where the target for inflation is the one observed in the US since 2007 (around 2%), the solid line depicted in Figure 3. In all cases, the variables in the Figure represent log deviations from the benchmark economy.

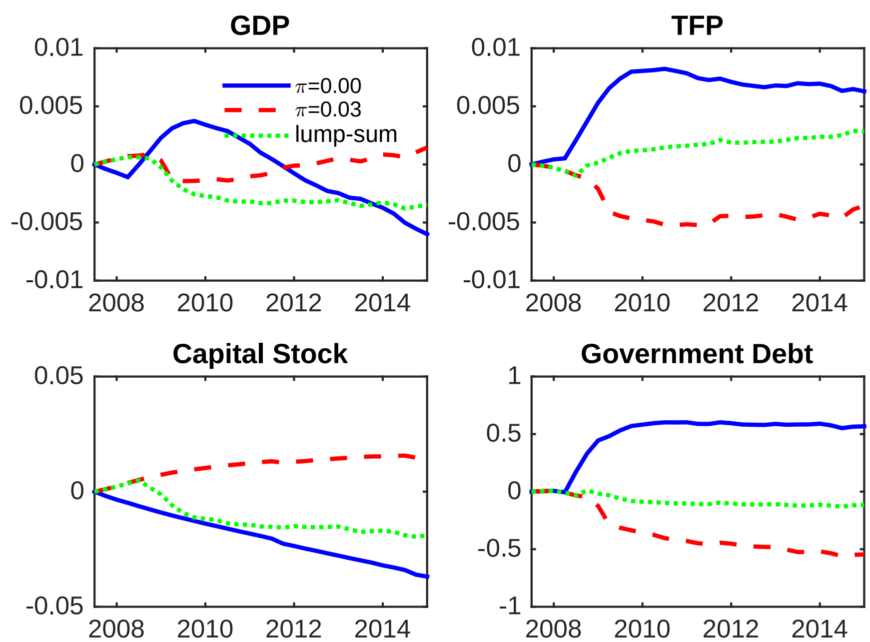

Figure 7: Alternative Inflation Targets and Transfer Scheme. Log deviations from the benchmark case (solid line in Figure 3).

We now explain each alternative economy in detail.

The solid line in Figure 7 represents an economy like the benchmark, but with a low inflation target, $\pi = 0$. As it can be seen in the upper-left panel, where

[30]The transfer to bondholders is consistent with the evidence presented by Veronesi and Zingales (2010) for the bailout of the financial sector in 2008.

we plot GDP, this tight inflation target would have implied a less severe recession between 2009 and 2011 (the solid line is above zero) in this counterfactual. However, starting in 2012, the value of GDP would have been lower than in the benchmark. There are two forces that have opposite effects. As the upper-right panel shows, a tighter inflation target would have implied a less pronounced drop in TFP for the whole period. By preventing the real interest rate from falling too much, there is less reallocation from high productivity entrepreneurs to low productivity ones. But at the same time, it would have implied a larger collapse in investment (lower-left panel), that slowly reduces the capital stock. In order to avoid a drop in the real rate, the government must increase its liabilities, which crowds out private investment. Due to the persistent nature of capital, the TFP effect dominates in the short run, while capital accumulation dominates in the long run. Overall, the simulations imply that a zero inflation target would have implied a less severe recession in the aftermath of the crisis, at the cost of a loss of output of almost 1% by 2014. To accomplish that, as the lower-right panel shows, would have implied a substantially larger increase in nominal liabilities: the total stock should had been around 65% higher than what it is today.

The dashed line in Figure 7 represents an economy with a higher inflation target ($\pi = 3\%$) than the benchmark. The effects are exactly the opposite: TFP is lower, but investment does not drop so much. Overall, the net effect is also the opposite, with a larger initial recession but a faster recovery. At the same time, the injection of liquidity does not need to increase as much, the total stock would had been, according to the model, about 50% of what they are today. Note that the total differences in output with respect to the benchmark are smaller than in the case of the tight inflation target. The reason is that for this inflation rate the zero bound is not binding anymore, so inflation is determined by standard monetary forces, known for not having big real effects, and not by the behavior of the real interest rate.

In order to study the effects of different transfer schemes, the dotted line in Figure 7 represents the difference between a counterfactual economy where the government implements the same inflation target as in the benchmark case, but using purely lump-sum transfers. As it can be seen from the left-bottom panel, the magnitude of the crowding out is higher than in the benchmark case in which the government makes transfers only to the entrepreneurs. The reason is that now part of the transfers go to workers, who in equilibrium have a large marginal propensity to consume, since they will be against their borrowing constraint in finite time.[31] In summary, the recovery is faster when the government rebates the proceeds from the increase in the debt solely to entrepreneurs.[32]

5.3 Monetary or Fiscal Policy?

At the zero bound, real money and bonds are perfect substitutes. Thus, standard open market operations in which the central bank exchanges money for short-term bonds have no impact on the economy. What is needed is an effective increase in the supply of government liabilities, which at the zero bound can be money or bonds. How can these policies be executed? Clearly, one way to do it is through bonds, taxes, and transfers. But another way is through a process described long ago: helicopter drops, whereby increases of money are directly transferred to agents. Sure enough, to satisfy the government budget constraint and maintain price stability these helicopter drops need to be compensated with future"vacuums" (negative helicopter drops) or future open market operations once the nominal interest becomes positive.

Although the distinction between a central bank or the Treasury making direct transfers to agents may be of varying relevance in different countries because of alternative legal constraints, there is little conceptual difference in the theory. To fully control inflation during a severe credit crunch, the sum of real money plus bonds must go up at the zero bound. Otherwise, there will be an initial deflation, followed by an inflation rate that will be determined by the negative of the real interest rate. If these policies are understood as being outside the realm of central banks, then central banks should not be given tight inflation target mandates: inflation is out of their control during a severe credit crunch.

[31] In a steady state, the interest rate is strictly lower than the rate of time preferences, $(1+r_\infty)\beta < 1$. Therefore, workers, who earn a flow of labor income each period, will choose to be against their borrowing constraint in finite time.

[32] See Appendix C for analytical comparative static results of the effect of changes in government debt in the neighborhood of $B_1 = 0$ for the simple example introduced in Section 3.1. There we consider the case in which all taxes and transfers are made to entrepreneurs, and the polar case in which workers are the only agents that are taxed and receive transfers.

6 Distribution of Welfare Impacts

In the previous section, we focused on the impact of policies on aggregate outcomes and factor prices. The aggregate figures suggest a relatively simple trade-off at the aggregate level. These dynamics, though, hide very disparate effects of a credit crunch and alternative policies among different agents. Although workers are hurt by the drop in wages, the profitability of active entrepreneurs and their welfare can increase as a result of lower factor prices. Similarly, unproductive entrepreneurs are bondholders in equilibrium, and therefore their welfare depends on the behavior of the real interest rate.

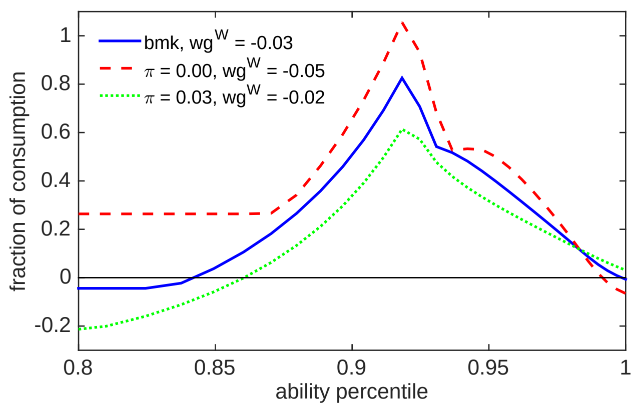

Figure 8: Distribution of Welfare Gains

Figure 8 presents the impact of a credit crunch on the welfare of workers and on entrepreneurs of different ability under alternative inflation targets for the bailout case. We measure the welfare impact of a credit crunch in terms of the fraction of consumption that an individual is willing to permanently forgo in order to experience a credit crunch.[33] If positive (negative), we refer to this measure as the welfare gains (losses) from a credit crunch and alternative policy responses.

The dotted line shows the welfare gains for entrepreneurs from a policy that implements a 3% inflation rate as a function of the percentile of their ability distribution.

[33]For entrepreneurs, we consider the welfare of individuals that at the time of the shock have wealth equal to the average wealth of its type. For workers, their welfare is calculated assuming, as is true in the steady state of the model, that they own no wealth when the credit crunch is announced.

This level of inflation is implemented with a negligible increase in the net supply of outside liquidity, and therefore, the economy in the long-run returns to the initial steady state. Unproductive entrepreneurs are clearly hurt by a credit crunch, since the return on the bonds they hold becomes negative for over 31 quarters and only gradually returns to the original steady state. Their losses amount to over 20% of permanent consumption. On the contrary, entrepreneurs who become active as the credit crunch lowers factor prices, and who increase their profitability, benefit the most. The same effect increases welfare for previously active entrepreneurs, but they are hurt by the tightening of collateral constraints, which limit their ability to leverage their high productivity. The welfare losses for workers are shown by the legend of each curve. Clearly, workers are hurt by experiencing a credit crunch, since the wages drop for a number of periods. The credit crunch amounts to a permanent drop of two percentage point in their consumption, $wg^W = -0.02$.

The other two curves show the welfare consequences of lower inflation targets. The solid line corresponds to the benchmark economy, where the inflation is closed to 2%, and the dashed line is an economy with no inflation. The lower the inflation target, the higher the real interest rate, both during the credit crunch and in the new steady state.[34] Unproductive entrepreneurs benefit from the highest interest rate. Similarly, productive entrepreneurs benefit from the lowest wages associated with the lowest capital during the transition and in the new steady state.[35] Although individual entrepreneurs do not internalize it, collectively they benefit from the lower wages associated with a lower aggregate stock of capital. The lower the inflation target, the lower the capital stock and the lower the wages, so the welfare of workers goes down when the target goes down.

[34]Given the debt policy equation (14), the government debt in the new steady state will be higher the lower the inflation target is. In the model, a higher level of government debt implies a lower level of capital in the new steady state.

[35]The non-monotonic nature of the welfare effects is related to the heterogeneous impact due to the changing nature of the occupational choice of agents during the transition. For example, the entrepreneur that benefits the most is the most productive inactive entrepreneur in the steady state. As the real rate goes down, that agent becomes an entrepreneur and starts borrowing to profit from the difference between his productivity and the now low interest rate and also from the lower equilibrium wage. On the other hand, the most productive entrepreneur also benefits from the low input prices, but is hurt by the reduction in ability to borrow. Thus, although she gets a higher margin per unit of capital, she can only manage a lower amount of capital.

7 Conclusions

A contraction in credit due to a tightening of collateral constraints leads to a recession and a drop in the return of safe assets. In a monetary economy, the nominal return on safe assets cannot be negative, so the negative of the rate of inflation is a lower bound on its real return. We showed that if the contraction in credit is large enough, then this constraint becomes binding and the economy enters a liquidity trap. In this case, a deflation occurs if policy is passive. This deflation may interact with collateral constraints, creating debt deflation and worsening the recession if debt obligations are in nominal terms. In addition, it creates a large drop in employment if wages are sticky.

We characterize a policy that avoids that costly deflation. That policy resembles the one followed by the Federal Reserve as a reaction to the 2008 crisis and is in line with Friedman and Schwartz's explanation of the severity of the Great Depression.

The policies that avoid the deflation involve a large increase in money or bonds, which are perfect substitutes at the zero bound. These policies do stabilize prices and output. There is a side effect of these policies, though: they generate a slow recovery. We argue that many of the features of the model capture the characteristics of the last financial crisis that hit the United States starting in 2008. It also sheds new light into the events that unravelled in Japan in the early 1990s and continue till today.

The interpretation of the crisis provided by the model in this paper is in contrast to the dominant view in most central banks and is supported by a literature that emphasizes price frictions. According to that literature, it is unambiguously optimal to maintain the economy at the zero bound even after the shock that drove real interest rates to negative values reverts. The model of this paper implies that avoiding the zero bound or not implies nontrivial trade-offs: ameliorating the drop in output at the cost of a slower recovery. The policy trade-offs are even more subtle when the heterogeneous effects across agents are taken into account.

Our model rationalizes the notion that the inflation determination mechanisms differ substantially when the policy authority decides to be at the zero bound. Away from the zero bound, it depends on standard monetary mechanisms. But at the zero bound, it is total outside liabilities that matter: inflation can be controlled only by managing the real interest rate so it does not become too negative.

References

Ajello, Andrea, Luca Benzoni, and Olena Chyruk, "Core and 'Crust': Consumer prices and the Term Structure of Interest Rates," Manuscript, Research Department, Federal Reserve Bank of Chicago 2012.

Alvarez, Fernando and Francesco Lippi, "Financial Innovation and the Transactions Demand for Cash," *Econometrica*, 03 2009, *77* (2), 363–402.

Bernanke, Ben and Mark Gertler, "Agency Costs, Net Worth, and Business Fluctuations," *American Economic Review*, March 1989, *79* (1), 14–31.

Bernanke, Ben S., Mark Gertler, and Simon Gilchrist, "The financial accelerator in a quantitative business cycle framework," in J. B. Taylor and M. Woodford, eds., *Handbook of Macroeconomics*, Vol. 1 of *Handbook of Macroeconomics*, Elsevier, 1999, chapter 21, pp. 1341–1393.

Brunnermeier, Markus K. and Yuliy Sannikov, "The I Theory of Money," Manuscript, Princeton University 2013.

Buera, Francisco J. and Benjamin Moll, "Aggregate Implications of a Credit Crunch," Working Paper 17775, National Bureau of Economic Research January 2012.

_ and _ , "Aggregate Implications of a Credit Crunch: The Importance of Heterogeneity," *American Economic Journal: Macroeconomics*, July 2015, *7* (3), 1–42.

Calvo, Guillermo A., "Staggered Prices in a Utility-Maximizing Framework," *Journal of Monetary Economics*, September 1983, *12* (3), 383–398.

Christiano, Lawrence, Martin Eichenbaum, and Sergio Rebelo, "When Is the Government Spending Multiplier Large?," *Journal of Political Economy*, 2011, *119* (1), 78–121.

Cooley, Thomas, Ramon Marimon, and Vincenzo Quadrini, "Aggregate Consequences of Limited Contract Enforceability," *Journal of Political Economy*, August 2004, *112* (4), 817–847.

Correia, Isabel, Emmanuel Farhi, Juan Pablo Nicolini, and Pedro Teles, "Unconventional Fiscal Policy at the Zero Bound," *American Economic Review*, June 2013, *103* (4), 1172–1211.

Curdia, Vasco and Gauti Eggertsson, "What Caused the Great Depression?," Manuscript, Federal Reserve Bank of New York 2009.

_ **and Michael Woodford**, "Credit Spreads and Monetary Policy," *Journal of Money, Credit and Banking*, 09 2010, *42* (s1), 3–35.

Drautzburg, Thorsten and Harald Uhlig, "Fiscal Stimulus and Distortionary Taxation," Working Paper 17111, National Bureau of Economic Research June 2011.

E, Jr Lucas Robert and Nancy L Stokey, "Money and Interest in a Cash-in-Advance Economy," *Econometrica*, May 1987, *55* (3), 491–513.

Eggertsson, Gauti B. and Michael Woodford, "The Zero Bound on Interest Rates and Optimal Monetary Policy," *Brookings Papers on Economic Activity*, 2003, *34* (1), 139–235.

Fisher, Irving, "The Debt-Deflation Theory of Great Depressions," *Econometrica*, 1933, *1* (4), 337–357.

Guerrieri, Veronica and Guido Lorenzoni, "Credit Crises, Precautionary Savings, and the Liquidity Trap," Working Paper 17583, National Bureau of Economic Research November 2011.

Jermann, Urban and Vincenzo Quadrini, "Macroeconomic Effects of Financial Shocks," *American Economic Review*, February 2012, *102* (1), 238–71.

Kaplan, Greg and Giovanni L. Violante, "A Model of the Consumption Response to Fiscal Stimulus Payments," *Econometrica*, 04 2014, *82* (4), 1199–1239.

Kiyotaki, Nobuhiro, "Credit and Business Cycles," *Japanese Economic Review*, March 1998, *49* (1), 18–35.

_ **and John Moore**, "Credit Cycles," *Journal of Political Economy*, April 1997, *105* (2), 211–48.

_ **and** _ , "Liquidity, Business Cycles, and Monetary Policy," Working Paper 17934, National Bureau of Economic Research March 2012.

Krugman, Paul R., "It's Baaack: Japan's Slump and the Return of the Liquidity Trap," *Brookings Papers on Economic Activity*, 1998, *29* (2), 137–206.

Lucas, Jr. Robert E., "Inflation and Welfare," *Econometrica*, March 2000, *68* (2), 247–274.

Lucas, Robert E. and Juan Pablo Nicolini, "On the stability of money demand," *Journal of Monetary Economics*, 2015, *73* (C), 48–65.

Lucas, Robert Jr., "Interest rates and currency prices in a two-country world," *Journal of Monetary Economics*, 1982, *10* (3), 335–359.

Moll, Benjamin, "Productivity Losses from Financial Frictions: Can Self-Financing Undo Capital Misallocation?," *American Economic Review*, October 2014, *104* (10), 3186–3221.

Svensson, Lars, "Money and Asset Prices in a Cash-in-Advance Economy," *Journal of Political Economy*, October 1985, *93* (5), 919–944.

Veronesi, Pietro and Luigi Zingales, "Paulson's Gift," *Journal of Financial Economics*, September 2010, *97* (3), 339–368.

Werning, Ivan, "Managing a Liquidity Trap: Monetary and Fiscal Policy," Working Paper 17344, National Bureau of Economic Research August 2011.

Appendix (For Online Publication)

A Optimality Conditions

The optimal problem of agents is to maximize (1) subject to (2) and (6) for entrepreneurs or (7) for workers. Note that the only difference between the two budget constraints is that entrepreneurs have no labor income. For workers, as for inactive entrepreneurs, the real return to their non-monetary wealth equals $1 + r_t$. In what follows, to save on notation, we drop the index for individual entrepreneurs j unless strictly necessary. Since this is a key aspect of the model, we first briefly explain the zero bound equilibrium restriction on the nominal interest rate that arises from the agent's optimization problem.[36] Then, we discuss the other first-order conditions.

In this economy, gross savings (demand for bonds) come from inactive entrepreneurs and, potentially, from workers. Note that the return on holding financial assets for these agents is $R_t(z) = (1 + r_t)$, while the return on holding money—ignoring the liquidity services—is given by p_t/p_{t+1}. Thus, if there is intermediation in equilibrium, the return on holding money cannot be higher than the return on holding financial assets. If we define the nominal return as $(1 + r_t) \frac{p_t}{p_{t-1}}$, then for intermediation to be nonzero in equilibrium, the zero bound constraint

$$(1 + r_t) \frac{p_t}{p_{t-1}} - 1 \geq 0 \tag{15}$$

must hold for all t.

The first-order conditions of the household's problem imply the standard Euler equation and intratemporal optimality condition between cash and credit goods:

$$\frac{1}{\beta} \frac{c_{2t+1}(z)}{c_{2t}(z)} = R_{t+1}(z), \quad t \geq 0, \tag{16}$$

$$\frac{\nu}{1 - \nu} \frac{c_{2t+1}(z)}{c_{1t+1}(z)} = R_{t+1}(z) \frac{p_{t+1}}{p_t}, \quad t \geq 1. \tag{17}$$

Solving forward the period budget constraint (6), using the optimal conditions (16) and (17) for all periods, and assuming that the cash-in-advance constraint is binding at the beginning of period $t = 0$, we obtain the solutions for consumption of the credit good and financial assets for agents that face a strictly positive opportunity

[36] Formal details are available from the authors upon request.

cost of money in period $t+1$,[37]

$$\begin{aligned} c_{2t}(z) &= \frac{(1-\nu)(1-\beta)}{1-\nu(1-\beta)}\left[R_t(z)a_t - \sum_{j=0}^{\infty}\frac{T_{t+j}^e}{\prod_{s=1}^{j}R_{t+s}(z)}\right] \\ a_{t+1}(z) &= \beta\left[R_t(z)a_t - \sum_{j=0}^{\infty}\frac{T_{t+j}^e}{\prod_{s=1}^{j}R_{t+s}(z)}\right] + \sum_{j=1}^{\infty}\frac{T_{t+j}^e}{\prod_{s=1}^{j}R_{t+s}(z)}. \end{aligned}$$

These equations always characterize the solution for active entrepreneurs even when nominal interest rates are zero. The reason is that for them, the opportunity cost of holding money is given by $R_t(z)p_{t+1}/p_t > (1+r_t)\,p_{t+1}/p_t \geq 1$, where the last inequality follows form (15). The solution also characterizes the optimal behavior of inactive entrepreneurs, as long as $(1+r_t)\,p_{t+1}/p_t - 1 > 0$.

The solution for inactive entrepreneurs in periods in which the nominal interest rate is zero, $(1+r_t)\,p_{t+1}/p_t - 1 = 0$, is

$$a_{t+1}(z) + \frac{m_{t+1}(z)}{p_t} - \frac{m_{t+1}^T(z)}{p_t} = \beta\left[R_t(z)a_t - \sum_{j=0}^{\infty}\frac{T_{t+j}^e}{\prod_{s=1}^{j}R_{t+s}(z)}\right] + \sum_{j=1}^{\infty}\frac{T_{t+j}^e}{\prod_{s=1}^{j}R_{t+s}(z)},$$

where

$$\frac{m_{t+1}^T(z)}{p_t} = \frac{\nu(1-\beta)\beta}{1-\nu(1-\beta)}\left[R_t(z)a_t - \sum_{j=0}^{\infty}\frac{T_{t+j}^e}{\prod_{s=1}^{j}R_{t+s}(z)}\right] \tag{18}$$

are the real money balances that will be used for transaction purposes in period $t+1$. Thus, $m_{t+1}/p_t - m_{t+1}^T/p_t \geq 0$ are the excess real money balances, hoarded from period t to $t+1$.

The optimal plan for workers is slightly more involved, since their income is non-homogeneous in their net worth and they will tend to face binding borrowing constraints in finite time. In particular, as long as the $(1+r_\infty)\beta < 1$, as will be the case in the equilibria we will discuss, where r_∞ is the real interest rate in the steady state, workers drive their wealth to zero in finite time and are effectively hand-to-mouth consumers in the long run. That is, for sufficiently large t,

$$c_{2,t}^W = \frac{(1-\nu)\left(w_t - T_t^W\right)}{1-\nu(1-\beta)} \text{ and } c_{1,t+1}^W = \frac{m_{t+1}^W}{p_{t+1}} = \frac{\nu\left(w_t - T_t^W\right)}{1-\nu(1-\beta)}\frac{\beta p_t}{p_{t+1}}.$$

[37] Note that it could be possible that initial money holdings are so large for an active entrepreneur that the cash-in-advance constraint will not be binding in the first period. This case will not be relevant provided initial real cash balances are not too big.

Along a transition, workers may accumulate assets for a finite number of periods. This would typically be the case if they expect a future drop in their wages—as in the credit crunch we consider—or they receive a temporarily large transfer, $T_t^W < 0$.

B Partial Characterization of the Equilibrium Dynamics

To illustrate the mechanics of the model, we first provide a partial characterization of the equilibrium dynamics of the economy for the case in which the zero lower bound is never binding, $1 + r_{t+1} > p_t/p_{t+1}$ for all t, workers are hand-to-mouth, $a_t^W = 0$ for all t, and the share of cash goods is arbitrarily small, $\nu \approx 0$. Second, we discuss some properties of the model when the zero bound constraint binds.

B.1 Equilibrium Away from the Zero Bound

Let $\Phi_t(z)$ be the measure of wealth held by entrepreneurs of productivity z at time t. Integrating the production function of all active entrepreneurs, equilibrium output is given by a Cobb-Douglas function of aggregate capital K_t, aggregate labor L, and aggregate productivity Z_t,

$$Y_t = Z_t K_t^{\alpha} L^{1-\alpha}, \tag{19}$$

where aggregate productivity is given by the wealth-weighted average of the productivity of active entrepreneurs, $z \geq \hat{z}_t$,

$$Z_t = \left(\frac{\int_{\hat{z}_t}^{\infty} z \Phi_t(dz)}{\int_{\hat{z}_t}^{\infty} \Phi_t(dz)} \right)^{\alpha}. \tag{20}$$

Note that Z_t is an increasing function of the cutoff $\hat{z}_t$ and a function of the wealth measure $\Phi_t(z)$. In turn, given the capital stock at $t+1$, which we discuss below, the evolution of the wealth measure is given by

$$\begin{aligned} \Phi_{t+1}(z) \;\; = \;\; & \gamma \left[\beta \left[R_t(z)\, \Phi_t(z) - \sum_{j=0}^{\infty} \frac{T_{t+j}^e \Psi(z)}{\prod_{s=1}^{j} R_{t+s}(z)} \right] + \sum_{j=1}^{\infty} \frac{T_{t+j}^e \Psi(z)}{\prod_{s=1}^{j} R_{t+s}(z)} \right] \\ & + (1-\gamma)\, \Psi(z) \left(K_{t+1} + B_{t+1} \right), \end{aligned} \tag{21}$$

where the first term on the right-hand side reflects the decision rules of the γ fraction of entrepreneurs that remain alive, and the second reflects the exogenous allocation of assets of departed entrepreneurs among the new generation.

Then, given the (exogenous) value for θ_{t+1} and the wealth measure $\Phi_{t+1}(z)$, the cutoff for next period is determined by the bond market clearing condition

$$\int_0^{\hat{z}_{t+1}} \Phi_{t+1}(dz) = \frac{\theta_{t+1}}{1-\theta_{t+1}} \int_{\hat{z}_{t+1}}^{\infty} \Phi_{t+1}(dz) + B_{t+1}. \tag{22}$$

To obtain the evolution of aggregate capital, we integrate over the individual decisions and use the market clearing conditions. It results in a linear function of aggregate output, the initial capital stock, and the aggregate of the (individual-specific) present value of taxes,

$$\begin{aligned} K_{t+1} + B_{t+1} &= \beta\left[\alpha Y_t + (1-\delta)K_t + (1+r_t)B_t\right] - \beta \int_0^{\infty} \sum_{j=0}^{\infty} \frac{T^e_{t+j}}{\prod_{s=1}^{j} R_{t+s}(z)} \Psi(dz) \\ &\quad + \int_0^{\infty} \sum_{j=1}^{\infty} \frac{T^e_{t+j}}{\prod_{s=1}^{j} R_{t+s}(z)} (dz) . \end{aligned} \tag{23}$$

Solving forward the government budget constraint (8), using that $\nu \approx 0$, and substituting into (23),

$$\begin{aligned} K_{t+1} &= \beta\left[\alpha Y_t + (1-\delta)K_t\right] + (1-\beta) \int_0^{\infty} \sum_{j=1}^{\infty} \frac{T^e_{t+j}}{\prod_{s=1}^{j} R_{t+s}(z)} \Psi(dz) \\ &\quad -(1-\beta) \int_0^{\infty} \sum_{j=1}^{\infty} \frac{T^e_{t+j}}{\prod_{s=1}^{j}(1+r_{t+s})} \Psi(dz) \\ &\quad +\beta L T^W_t - (1-\beta) L \sum_{j=1}^{\infty} \frac{T^W_{t+j}}{\prod_{s=1}^{j}(1+r_{t+s})}. \end{aligned} \tag{24}$$

The first term gives the evolution of aggregate capital in an economy without taxes. In this case, aggregate capital in period $t+1$ is a linear function of aggregate output and the initial level of aggregate capital. The evolution of aggregate capital in this case is equal to the accumulation decision of a representative entrepreneur (Moll, 2014; Buera and Moll, 2015). The second term captures the effect of alternative paths for taxes, discounted using the type-specific return to their non-monetary wealth, while the last term is the present value of taxes from the perspective of the government. For instance, consider the case in which the government increases lump-sum transfers to entrepreneurs in period t, financing them with an increase in government debt and, therefore, with an increase in the present value of future lump-sum taxes. In this case, future taxes will be discounted more heavily by active entrepreneurs, implying that the last term is bigger than the second. Thus, this policy crowds out private investment and results in lower aggregate capital next period.

Finally, we describe the determination of the price level. In the previous derivations, in particular, to obtain (24), we have used that $\nu \approx 0$, and therefore, the money market clearing condition is not necessarily well defined.[38] More generally, given monetary and fiscal policy, the price level is given by the equilibrium condition in the money market,

$$\frac{M_{t+1}}{p_t} = \frac{\nu(1-\beta)\beta}{1-\nu(1-\beta)}\Bigg[\alpha Y_t + (1-\delta)K_t + (1+r_t)B_t - \int_0^\infty \sum_{j=0}^{\infty} \frac{T^e_{t+j}}{\prod_{s=1}^{j} R_{t+s}(z)} \Psi(dz)\Bigg]. \tag{25}$$

The nominal interest rate is obtained from the intertemporal condition of inactive entrepreneurs,

$$\frac{1}{\beta}\frac{c_{2t+1}}{c_{2t}} = \frac{1+i_{t+1}}{\frac{p_{t+1}}{p_t}} = 1 + r_{t+1}. \tag{26}$$

Note that, except for the well-known Sargent-Wallace initial price level indeterminacy result, we can think of monetary policy as sequences of money supplies, $\{M_t\}_{t=0}^\infty$, or sequences of nominal interest rates, $\{i_t\}_{t=0}^\infty$. We will think of policy as determining exogenously one of the two sequences, abstracting from the implementability problem.[39]

There are two important margins in this economy. The first is the allocation of capital across entrepreneurs, which is dictated by the collateral constraints and which determines measured TFP (see (20)). The second is the evolution of aggregate capital over time, which, in the absence of taxes, behaves as in Solow's model (see (24) and set $T^e_{t+j} = T^W_{t+j} = 0$). Clearly, fiscal policy has aggregate implications: the net supply of bonds affects (22) and taxes affect (24). However, monetary policy does not, since

[38] To determine the price level in the cashless limit, we need to assume that as $M_{t+1}, \nu \to 0$, $M_{t+1}/\nu \to \tilde{M}_{t+1} > 0$. See details in Section 3.3.3.

[39] Because we use log utility, there is a unique solution for prices, given the sequence $\{M_t\}_{t=0}^\infty$.

none of those equations depend on nominal variables. Monetary policy does have effects, since it distorts the margin between cash and credit goods, but in a fashion that resembles the effects of monetary policy in a representative agent economy. This is the case only if, as assumed above, the zero bound does not bind.

B.2 Equilibrium at the Zero Bound

In periods in which the zero bound binds, successful inflation-targeting policies will affect equilibrium quantities if the target for inflation is tight enough. The reason is that, by successfully controlling inflation, monetary policy, together with the zero bound on nominal interest rates, can impose a bound on real interest rates. To see this, use (26) and the zero bound to write

$$1 + i_t = \frac{p_t}{p_{t-1}} (1 + r_t) \geq 1, \text{ so } r_t \geq \frac{p_{t-1} - p_t}{p_t} = -\left(\frac{\pi_t}{1 + \pi_t}\right), \tag{27}$$

where π_t is the inflation rate.

Imagine now an economy with zero net supply of bonds that enters a credit crunch, generated by a drop in maximum leverage, θ_t. Equation (22) implies that the threshold $\hat{z}_{t+1}$ has to go down, to reduce the left-hand side and increase the right-hand side so as to restore the equilibrium. This drop in the gross supply of private bonds will reduce the real interest rate, so the marginal entrepreneurs that were lending capital now start borrowing until market equilibrium is restored (i.e., the net supply of bonds is zero). If the credit crunch is large enough, the equilibrium real interest rate may become negative. If inflation is not high enough, the bound (27) may be binding. Imagine, for instance, the case of inflation targeting with a target equal to zero. Then, the real interest rate cannot become negative.

What will an equilibrium look like? In order to support the zero inflation policy, the government needs to inject enough liquidity so that the net supply of money (or bonds, since they are perfect substitutes at the zero bound) goes up to the point where conditions (22) and (27) are jointly satisfied. This policy will have implications for the equilibrium cutoff $\hat{z}_{t+1}$. In addition, as can be seen in (23), the injection of liquidity (increases in B_{t+1}) affects capital accumulation. Thus, at the zero bound, the level of inflation chosen by the central bank, if low enough, can affect the two relevant margins in the economy. To further explore these implications in this general model, we need to solve it numerically. But before doing that, we now present a particular (very special) case that can be analytically solved and analyzed, which we find very useful in isolating and understanding some of the mechanisms of the model.

B.2.1 Deflation Follows Passive Policy

We want to discuss the behavior of the price level following a credit crunch that drives the real interest rate into negative territory and such that the zero bound constraint binds during at least one period. We do this in the simple example introduced in Section 3.1. The characterization is even simpler in the case in which the zero bound is binding for only one period. As it turns out, if the parameters satisfy certain properties, this will indeed be the case. Thus, as we explain in the example, we will make two assumptions that parameters must satisfy for the equilibrium to be such that the zero bound binds only in one period. Under these conditions, we then explain why deflation will be the result of a credit crunch if policy does not respond.

The cashless limit We consider the limiting case of the cashless economy (i.e., $\nu \to 0$). In this case, the distortions associated with a positive nominal interest rate do not affect the real allocation.[40] In taking the limit, though, we also let nominal money balances shrink at the same rate so we can still meaningfully determine the equilibrium price level. The details follow.

When the cash-in-advance constraint is binding, the first-order condition is

$$p_t c_t^1 = \frac{\nu}{1-\nu} c_t^2 p_{t-1} \frac{1}{R_t(z)}.$$

We define $\overline{m}_t = \frac{m_t}{\nu}$, so $p_t c_t^1 = m_t = \overline{m}_t \nu$. Replacing above and taking the limit when $\nu \to 0$, we obtain

$$\overline{m}_t = c_t^2 p_{t-1} \frac{1}{R_t(z)}.$$

Finally, using the optimal rule for the credit good specialized for the limiting case

$$c_t^2 = (1-\beta) R_t(z) k_t$$

and aggregating over all agents, we obtain

$$\overline{M}_t = (1-\beta) K_t p_{t-1}. \tag{28}$$

Because of the cashless limit and since debt and transfers are all zero, the real variables

[40] In the general case, non-negligible money balances crowd out capital and ameliorate the drop in the real interest rate and in total factor productivity.

follow the solution described in (12) and (11), irrespectively of the evolution of the price level. However, the price level does depend on the behavior of real variables, so it is useful to obtain some explicit solutions.

If we let $\beta \equiv (1+\rho)^{-1}, \rho > 0$, the steady state is given by

$$K_{ss} = \left(\frac{\alpha}{\rho+\delta}\right)^{\frac{1}{1-\alpha}} \left(\frac{1+\theta_{ss}}{2}\right)^{\frac{\alpha}{1-\alpha}} \tag{29}$$

and

$$\left(\frac{2\theta_{ss}}{1+\theta_{ss}}\right)(\rho+\delta) = r_{ss} + \delta. \tag{30}$$

We assume that

$$\frac{2\theta_{ss}}{(1+\theta_{ss})} > \beta, \tag{31}$$

which implies that the real interest rate is positive in the steady state.[41] During the credit crunch, at $t = 1$ the real interest rate is

$$\delta + r_1 = (\rho+\delta)\frac{2\theta_l}{(1+\theta_l)}\left(\frac{1+\theta_l}{1+\theta_{ss}}\right)^{\alpha},$$

which is negative as long as

$$\frac{2\theta_l}{(1+\theta_l)}\left(\frac{1+\theta_l}{1+\theta_{ss}}\right)^{\alpha} < \frac{\delta}{(\rho+\delta)}. \tag{32}$$

Clearly, there exists a value for $\theta_l \in (0, \theta_{ss})$ such that this constraint is satisfied.

The conditions that determine the price level As we assume policy is pasive, we let $\overline{M}_{t+1} = \overline{M}$ and assume that in equilibrium $i_t > 0$ for $t \geq 2$. Then, using (28) we obtain that for all $t \geq 2$

$$1 + i_t = (1+r_t)\frac{p_t}{p_{t-1}} = (1+r_t)\frac{K_t}{K_{t+1}}.$$

[41] The necessary condition for positive interest rates in the steady state, $\frac{2\theta_{ss}}{(1+\theta_{ss})} > \frac{\delta}{\rho+\delta}$, is weaker. The stronger condition that we assume will also imply that the zero bound on nominal interest rates binds one period at most and simplifies the example.

Note that the real interest rate is positive, but there is deflation. It is possible to show, however, that under assumption (31) the deflation is not enough to make the nominal interest rate negative from time 2 on.

Lemma 1: *Given assumption* (31), $i_t > 0$ for $t \geq 2$.

Proof: Use the solutions for the real interest rate and capital (12) and (11) to write

$$1+i_t = \left[\alpha\theta_{ss}\left(\frac{1+\theta_{ss}}{2}K_t\right)^{\alpha-1} + (1-\delta)\right]\frac{K_t}{K_{t+1}} = \frac{\left[\alpha\theta_{ss}\left(\frac{1+\theta_{ss}}{2}\right)^{\alpha-1}K_t^\alpha + (1-\delta)K_t\right]}{\beta\left[\alpha\left(\frac{1+\theta_{ss}}{2}\right)^{\alpha}K_t^\alpha + (1-\delta)K_t\right]}.$$

Assume now, toward a contradiction, that

$$1+i_t = \frac{\left[\alpha\theta_{ss}\left(\frac{1+\theta_{ss}}{2}\right)^{\alpha-1}K_t^\alpha + (1-\delta)K_t\right]}{\beta\left[\alpha\left(\frac{1+\theta_{ss}}{2}\right)^{\alpha}K_t^\alpha + (1-\delta)K_t\right]} < 1.$$

Then,

$$\alpha\theta_{ss}\left(\frac{1+\theta_{ss}}{2}\right)^{\alpha-1}K_t^\alpha + (1-\delta)K_t < \beta\left[\alpha\left(\frac{1+\theta_{ss}}{2}\right)^{\alpha}K_t^\alpha + (1-\delta)K_t\right],$$

which can be written as

$$\alpha\left(\frac{1+\theta_{ss}}{2}\right)^{\alpha}K_t^\alpha\left(\frac{2\theta_{ss}}{1+\theta_{ss}} - \beta\right) + (1-\delta)K_t(1-\beta) < 0.$$

Assumption (31) implies that the first term on the left-hand side is positive. As δ and $\beta \in (0,1)$, this is a contradiction. □

Since all cash-in-advance constraints are binding from $t \geq 2$,

$$p_t = \frac{\overline{M}}{(1-\beta)K_{t+1}} \text{ for } t \geq 1.$$

We now show that under certain conditions on the parameters, the zero bound is binding at $t = 1$.

Lemma 2: *If* $\delta > \rho$*, then there exists a* $\tilde{\theta}_l > 0$ *such that* $i_1 = 0$ *for all* $\theta_l \in (0, \tilde{\theta}_l]$.

Proof: Assume, toward a contradiction, that $i_1 > 0$. Then

$$\overline{M} = (1-\beta) K_1 p_0$$

so

$$\frac{p_1}{p_0} = \frac{K_1}{K_2},$$

and the solution for the nominal interest rate is given by

$$1 + i_1 = (1 + r_1) \frac{p_1}{p_0} = \left[\frac{2\theta_l}{(1+\theta_l)} \left(\frac{1+\theta_l}{1+\theta_{ss}} \right)^{\alpha} (\rho + \delta) + (1-\delta) \right] \frac{K_1}{K_2},$$

but

$$\frac{K_1}{K_2} = \frac{\alpha \left(\frac{1+\theta_{ss}}{2}\right)^{\alpha} K_{ss}^{\alpha} + (1-\delta) K_{ss}}{\alpha \left(\frac{1+\theta_l}{2}\right)^{\alpha} K_{ss}^{\alpha} + (1-\delta) K_{ss}} = \frac{\alpha \left(\frac{1+\theta_{ss}}{2}\right)^{\alpha} + (1-\delta) K_{ss}^{1-\alpha}}{\alpha \left(\frac{1+\theta_l}{2}\right)^{\alpha} + (1-\delta) K_{ss}^{1-\alpha}}.$$

Replacing the solution for K_{ss},we obtain

$$\frac{K_1}{K_2} = \frac{\alpha \left(\frac{1+\theta_{ss}}{2}\right)^{\alpha} + (1-\delta) \frac{\alpha}{\frac{1}{\beta}-1+\delta} \left(\frac{1+\theta_{ss}}{2}\right)^{\alpha}}{\alpha \left(\frac{1+\theta_l}{2}\right)^{\alpha} + (1-\delta) \frac{\alpha}{\frac{1}{\beta}-1+\delta} \left(\frac{1+\theta_{ss}}{2}\right)^{\alpha}} = \frac{1/\beta}{\left(\frac{1+\theta_l}{1+\theta_{ss}}\right)^{\alpha} (\rho+\delta) + (1-\delta)}.$$

Then

$$1 + i_1 = \frac{1}{\beta} \frac{\frac{2\theta_l}{(1+\theta_l)} \left(\frac{1+\theta_l}{1+\theta_{ss}}\right)^{\alpha} (\rho+\delta) + (1-\delta)}{\left(\frac{1+\theta_l}{1+\theta_{ss}}\right)^{\alpha} (\rho+\delta) + (1-\delta)}. \tag{33}$$

Thus, for the lower bound on the nominal interest rate to be binding, we need

$$\frac{1}{\beta} \frac{\frac{2\theta_l}{(1+\theta_l)} \left(\frac{1+\theta_l}{1+\theta_{ss}}\right)^{\alpha} (\rho+\delta) + (1-\delta)}{\left(\frac{1+\theta_l}{1+\theta_{ss}}\right)^{\alpha} (\rho+\delta) + (1-\delta)} < 1,$$

which implies that

$$\frac{2\theta_l}{(1+\theta_l)} \left(\frac{1+\theta_l}{1+\theta_{ss}} \right)^{\alpha} (\rho+\delta) + (1-\delta) < \beta \left(\frac{1+\theta_l}{1+\theta_{ss}} \right)^{\alpha} (\rho+\delta) + \beta (1-\delta)$$

or

$$(1-\delta)(1-\beta) < \left(\frac{1+\theta_l}{1+\theta_{ss}}\right)^{\alpha}(\rho+\delta)\left[\beta - \frac{2\theta_l}{(1+\theta_l)}\right].$$

We now briefly characterize the function

$$f(\theta_l) = \left(\frac{1+\theta_l}{1+\theta_{ss}}\right)^{\alpha}(\rho+\delta)\left[\beta - \frac{2\theta_l}{(1+\theta_l)}\right].$$

Equation (31) implies that $f(\theta_{ss}) = 0$. On the other hand,

$$f(0) = \left(\frac{1}{1+\theta_{ss}}\right)^{\alpha}\frac{(\rho+\delta)}{1+\rho}.$$

As $\delta > \rho$,

$$\frac{\delta}{1-\delta} > (1-\beta) = \frac{\rho}{1+\rho}$$

so

$$\theta_{ss} < 1 < \frac{\delta}{1-\delta}\frac{1+\rho}{\rho}$$

and

$$1+\theta_{ss} < \frac{\delta+\rho}{(1-\delta)\,\rho}.$$

Thus,

$$(1+\theta_{ss})^{\alpha} < 1+\theta_{ss} < \frac{\delta+\rho}{(1-\delta)\,\rho}$$

and

$$(1+\theta_{ss})^{\alpha}(1-\delta)\,\rho < \delta+\rho$$

or

$$\frac{\rho}{1+\rho}(1-\delta) = (1-\beta)(1-\delta) < \left(\frac{1}{1+\theta_{ss}}\right)^{\alpha}\frac{\delta+\rho}{1+\rho}.$$

Thus, by the intermediate value theorem, there exists a $\widetilde{\theta}_l \in (0, \theta_{ss})$ such that

$$(1-\delta)(1-\beta) = \left(\frac{1+\widetilde{\theta}_l}{1+\theta_{ss}}\right)^{\alpha} (\rho+\delta) \left[\beta - \frac{2\widetilde{\theta}_l}{\left(1+\widetilde{\theta}_l\right)}\right].$$

Since $f(\theta_l)$ is decreasing, the zero bound will bind for all $\theta_l \in (0, \widetilde{\theta}_l]$. □

Finally, we show that if the economy starts at the steady state, at time zero, when agents learn there is a credit crunch, the equilibrium price level must be strictly below its steady state value.

Lemma 3: *Under the assumptions of Lemmas 1 and 2,* $p_0 < p_{ss}$ *for all* $\theta_l \in (0, \tilde{\theta}_l)$.

Proof: The ratio of the price level at $t = 0$ to the price level in the steady state p_{ss} is given by

$$\begin{aligned}
\frac{p_0}{p_{ss}} =& (1+r_1)\frac{p_1}{p_{ss}} \\
=& \frac{1}{\beta} \frac{\frac{2\theta_l}{(1+\theta_l)} \left(\frac{1+\theta_l}{1+\theta_{ss}}\right)^{\alpha} (\rho+\delta) + (1-\delta)}{\left(\frac{1+\theta_l}{1+\theta_{ss}}\right)^{\alpha} (\rho+\delta) + (1-\delta)},
\end{aligned}$$

which is equal to the right-hand side of (33) and, therefore, it is strictly less than one provided $\theta_l \in (0, \tilde{\theta}_l)$. □

The credit crunch drives the real interest rate below zero to the point at which the zero bound is reached. At this point, there is an excess demand for money as a "store of value." That excess demand is, of course, real rather than nominal. As the nominal quantity is fixed by policy, the demand pressure results in deflation. The excess demand for money as a store of value will be positive until future inflation is high enough such that the return on money is as negative as the return on bonds. The initial deflation allows for future inflation along the path, required for the arbitrage condition to hold, with a zero "long-run" inflation. This zero long run inflation is the natural consequence of a constant nominal money supply.

C The Effect of Public Debt Around $B = 0$

In this appendix we characterize the effect of public debt on GDP for two limiting cases. First, we consider the example presented in Section 3.1.2, where only entrepreneurs pay taxes and receive subsidies associated with the temporary one-period increase in government debt. For this case, we show that GDP tends to be an increasing function of the level of public debt in the neighborhood of $B = 0$. Second, we consider the polar case in which only workers pay taxes and receive subsidies associated with the temporary one-period increase in government debt. In this case, we show that GDP is a decreasing function of the level of public debt in the neighborhood of $B = 0$. These examples illustrate that the net effect of government debt on aggregate output depends on the particular implementation of the debt policy and on the relative size of workers and entrepreneurs in the population.

C.1 Taxing/Subsidizing Only Entrepreneurs

Differentiating (13) around $B_1 = 0$,

$$\left.\frac{\partial K_1}{\partial B_1}\right|_{B_1=0} = -(1-\beta)\left[1 - \int \frac{(1+r_{ss})}{R_{ss}(z)} dz\right]. \tag{34}$$

Similarly, differentiating (9) around $B_1 = 0$,

$$\left.\frac{\partial Z_1}{\partial B_1}\right|_{B_1=0} = \alpha Z_{ss} K_{ss}^{-1} \frac{1-\theta}{1+\theta}. \tag{35}$$

Thus, the net effect on GDP around $B_1 = 0$ is as follows:

$$\begin{aligned}
\left.\frac{\partial Y_1}{\partial B_1}\right|_{B_1=0} &= \alpha Z_{ss} K_{ss}^{\alpha-1} \frac{1-\theta}{1+\theta} - \alpha Z_{ss} K_{ss}^{\alpha-1}(1-\beta)\left[1 - \int \frac{(1+r_{ss})}{R_{ss}(z)} dz\right] \\
&= \alpha Z_{ss} K_{ss}^{\alpha-1}\left[\frac{1-\theta}{1+\theta} - (1-\beta)\left[1 - \int \frac{(1+r_{ss})}{R_{ss}(z)} dz\right]\right].
\end{aligned}$$

Finally, using the expressions for $R_1(z)$ and solving the integral, we have

$$\left.\frac{\partial Y_1}{\partial B_1}\right|_{B_1=0} = \alpha Z_{ss} K_{ss}^{\alpha-1}(1-\theta)\left[\frac{1}{1+\theta} - (1-\beta)\left[1 - \frac{1+r_{ss}}{r_{ss}+\delta}\theta \log\left(\frac{r_{ss}+\delta}{1+r_{ss}}\frac{1}{\theta} + 1\right)\right]\right],$$

where around $B_1 = 0$ the real interest rate $r_{ss} = (\rho+\delta)2\theta/(1+\theta) - \delta$. It is straightforward to show that this expression is positive for β close to 1 or θ close to 0.

C.2 Taxing/Subsidizing Only Workers

In this case,

$$\left.\frac{\partial K_1}{\partial B_1}\right|_{B_1=0} = -1, \tag{36}$$

and the effect on TFP is also given by (35). Thus,

$$\left.\frac{\partial Y_1}{\partial B_1}\right|_{B_1=0} = -\alpha Z_{ss} K_{ss}^{\alpha-1} \frac{2\theta}{1+\theta} < 0.$$

D Environment with Sticky Wages

In this appendix we describe the extension with rigid wages that is solved in Section 5.1. In order to allow for sticky wages, we now consider the case in which workers are grouped into households with a continuum of members indexed by $h \in [0, 1]$, each supplying a differentiated labor input l_{ht}. Each member is endowed with a unit of time. Preferences of the household are described by

$$\sum_{t=0}^{\infty} \beta^t \left[\zeta\nu \log c_{1t}^W + \zeta(1-\nu)\log c_{2t}^W + (1-\zeta)\log(N_t)\right],$$

where leisure is

$$N_t = 1 - \int_0^1 l_{ht} dh. \tag{37}$$

The differentiated labor varieties aggregate up to the labor input L_t, used in production by individual entrepreneurs, according to the Dixit-Stiglitz aggregator

$$L_t = \left[\int_0^1 l_{ht}{}^{\frac{\eta-1}{\eta}} dh\right]^{\frac{\eta}{\eta-1}}, \eta > 1. \tag{38}$$

Each member of the household, which supplies a differentiated labor variety, behaves under monopolistic competition. They set wages as in Calvo (1983), with the probability of being able to revise the wage $1-\alpha^w$. This lottery is also *i.i.d.* across workers and over time. The workers that are not able to set wages in period 0 all share the same wage w_{-1}. Other prices are taken as given.

There is a representative firm that produces homogeneous labor to be used in production by the entrepreneurs using the production function (38). The representative firm minimizes $\int_0^1 w_{ht} l_{ht} dh$, where w_{ht} is the wage of the h-labor, for a given aggregate L_t, subject to (38). The demand for n_{ht} is

$$l_{ht} = \left(\frac{w_{ht}}{w_t} \right)^{-\eta} L_t, \tag{39}$$

where W_t is the aggregate wage level, given by

$$w_t = \left[\int_0^1 {w_{ht}}^{1-\eta} dh \right]^{\frac{1}{1-\eta}}. \tag{40}$$

It follows that $\int_0^1 w_{ht} n_{ht} dh = w_t L_t$. In order to simplify the analysis, we also assume that workers are hand to mouth. In this case, the representative worker maximizes

$$E_0 \sum_{t=0}^{\infty} \beta^t \left[\zeta \nu \log c_{1t}^W + \zeta (1-\nu) \log c_{2t}^W + (1-\zeta) \log (N_t) \right]$$

subject to

$$c_{1t}^W + c_{2t}^W + \frac{m_{t+1}^W}{p_t} = \frac{1}{p_t} \int_0^1 w_{ht} l_{ht} dh + \frac{m_t^W}{p_t} - T_t^W,$$

$$l_{ht} = \left(\frac{w_{ht}}{w_t} \right)^{-\eta} L_t,$$

and

$$c_{1t}^W \leq \frac{m_t^W}{p_t}.$$

Note that although consumption and total labor will not be stochastic, each particular w_{ht} will be a random variable. From the first-order conditions of representative workers, we obtain

$$w_{ht} = \tilde{w}_t = \frac{\eta}{\eta - 1} \sum_{j=0}^{\infty} \xi_{t+j} \frac{1-\zeta}{\zeta(1-\nu)} \frac{p_{t+j} c_{2t+j}^W}{N_{t+j}},$$

where

$$\xi_{t+j} = \frac{(\beta\alpha^w)^j \frac{\zeta(1-\nu)}{c^W_{2t+j}} \frac{1}{p_{t+j}} w^\eta_{t+j} L_{t+j}}{\sum_{j=0}^{\infty} (\alpha^w\beta)^j \frac{\zeta(1-\nu)}{c^W_{2t+j}} \frac{1}{p_{t+j}} w^\eta_{t+j} L_{t+j}}$$

and

$$\sum_{j=0}^{\infty} \xi_{t+j} = 1.$$

The evolution of the cost of a composite unit of labor is

$$w_t = \left[(1-\alpha^w)\,\tilde{w}_t^{1-\theta^w} + \alpha^w w_{t-1}^{1-\theta^w}\right]^{\frac{1}{1-\theta^w}},$$

and

$$L_t = \left[\alpha^w \left(\frac{w_{t-1}}{w_t}\right)^{-\theta^w} \frac{1-N_{t-1}}{L_{t-1}} + (1-\alpha^w)\left(\frac{\tilde{w}_t}{w_t}\right)^{-\theta^w}\right]^{-1} (1-N_t)$$

solves for the aggregate composite labor input given aggregate leisure.

To implement this extension, we follow Correia et al. (2013) and calibrate $\zeta = 0.3$, $\eta = 3$, and $\alpha^w = 0.85$. To simplify the calculations, we consider the cashless limit. The other parameter values are set as in the other numerical examples.