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Convergence revisited

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Abstract

The conventional approach to testing whether economies converge examines the cross-sectional relationship between the growth rate of per capita output over some time period and the initial level of per capita output. This paper shows that this approach is valid only if economies have identical first-order autoregressive dynamic structures and all permanent cross-economy differences are completely controlled for, conditions that are grossly violated for the data sets considered here. The paper also develops an alternative approach that is valid under much less restrictive conditions. Strong evidence is found for conditional convergence of the 48 contiguous U.S. states and a group of 54 countries. Our evidence does not support theories in which trend growth rates differ across economies endogenously.

Key words: Neoclassical growth models; Endogenous growth models; Unit roots; Common trends; Monte Carlo simulation

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1. Introduction

Both the theoretical foundations and the empirical implications of the neoclassical growth model developed by Robert Solow (1956) and extended by

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David Cass (1965) have been widely investigated recently. On the theoretical front, Paul Romer (1986), Robert Lucas (1988), and Sergio Rebelo (1991) *inter alia* have examined whether alternative models may better explain observed growth experiences. Typically, these endogenous growth models relax the neoclassical assumption of diminishing returns to reproducible factors of production. Partly in response to this trend, several other studies have investigated whether economies converge as predicted by neoclassical models.[1] Baumol (1986), Dowrick and Nguyen (1989), Wolff (1991), Barro (1991), Barro and Xavier Sala-i-Martin (1991, 1992), and Mankiw, Romer, and Weil (1992) conclude that economies do indeed converge. All of these studies reach their conclusions by examining the cross-sectional relationship between the growth rate of output per capita (or per worker) over some time period and the initial level of per capita output. In this study, we show that this approach, which we term the conventional approach, is valid only under incredible assumptions. Specifically, the economies must have identical first-order autoregressive dynamic structures and all permanent cross-economy differences must be completely controlled for. We also develop an alternative approach that is valid under much less restrictive conditions.[2] We then examine annual data for the 48 contiguous U.S. states over the period 1929–1991 and for 54 countries over the period 1950–1990. As in the literature, we find that the conventional approach provides what appears to be solid evidence that economies converge. This evidence is illusory, however, since impressive evidence exists that the economies do not have identical first-order autoregressive dynamic structures, and hence that the inferences produced by the conventional approach are invalid. Using our alternative approach, we find emphatic evidence that the 48 U.S. states and the 54 countries do in fact converge. We also find strong evidence that convergence is conditional rather than absolute for both samples. Our evidence therefore supports one of the basic implications of neoclassical growth models, but also indicates that a common assumption in these models, that economies are identical except for initial conditions and stochastic disturbances, is seriously deficient. By coincidence, our findings are similar to those of the conventional approach, but this fact in no way implies that this approach is valid. The alternative presented here should be preferred because in other samples the conventional approach can easily produce misleading conclusions.

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[1] We should acknowledge that some endogenous growth models also predict convergence, e.g., Tamura (1991) and Kelly (1992). Furthermore, growth may be endogenous globally but individual economies may still experience diminishing social returns to their own accumulation of reproducible factors, e.g., Parente and Prescott (1994). For expositional convenience, however, we label models in which economies converge as neoclassical and models in which trend growth rates differ as endogenous growth models.

[2] Using a modification of the conventional approach, Durlauf and Johnson (1992) find evidence against global convergence but consistent with multiple regimes. See Section 2.4 for discussion of studies that employ other alternatives to the conventional approach.

The rest of the paper is organized as follows. Section 2 formulates a statistical model in order to evaluate the conventional approach and to develop an alternative approach. Section 3 describes our data, conducts conventional tests, shows that they are invalid, and proceeds with the alternative approach. Section 4 concludes.

2. Econometric discussion

2.1. Definition

Consider a collection of economies 1, 2, ... , N that have eventual access to the same body of technical knowledge. For any one of these economies, non-stochastic neoclassical growth models typically imply that a unique balanced growth path exists, that any deviation of the state variables from their values along the balanced growth path are eventually eliminated, and hence that initial values of state variables have no long-run effect on their levels. The assumption of a common body of technical knowledge further implies that the balanced growth parths of the N economies are parallel. As a result, the state variables of the economies eventually differ by constant amounts. Let y_{nt} be the logarithm of per capita output for economy n during period t, valued at constant and common international prices. The analysis above indicates that for each economy n,

$$\lim_{i \to \infty} (y_{n,t+i} - a_{t+i}) = \mu_n, \tag{2.1.1}$$

where a_t is the common trend followed by the economies and μ_n is a parameter. The series a_t can be thought of as the logarithm of an index of Harrod-neutral technology available to economies 1, 2, ... , N. The parameter μ_n determines the level of economy n's parallel balanced growth path. Unless all economies have identical structures, the μs should typically be nonzero. By contrast, endogenous growth models imply that the initial values of the state variables affect their steady-state so that $\lim_{i\to\infty}(y_{n,t+i} - a_{t+i})$ moves with $y_{nt} - a_t$.

Extending the analysis above to a stochastic world is straightforward. Economies 1, 2, ... , N are said to converge if, and only if, a common trend a_t and finite parameters $\mu_1, \mu_2, \ldots, \mu_N$ exist such that

$$\lim_{i \to \infty} \mathrm{E}_t(y_{n,t+i} - a_{t+i}) = \mu_n \tag{2.1.2}$$

for $n = 1, 2, \ldots, N$. Because a_t is unobservable, Eq. (2.1.2) is not useful as it stands. Averaging its members over the N economies generates

$$\lim_{i \to \infty} \mathrm{E}_t(\bar{y}_{t+i} - a_{t+i}) = \frac{1}{N} \sum_{n=1}^{N} \mu_n, \tag{2.1.3}$$

where $\bar{y}_t \equiv \sum_{n=1}^{N} y_{nt}/N$. We measure the level of the common trend a_t so that the left-hand member of Eq. (2.1.3) is zero. Subtracting each member of Eq. (2.1.3) from the corresponding member of Eq. (2.1.2),

$$\lim_{i\to\infty} \mathrm{E}_t(y_{n,t+i} - \bar{y}_{t+i}) = \mu_n. \tag{2.1.4}$$

According to Eq. (2.1.4), the deviations of $y_{1,t+i}, y_{2,t+i}, \ldots, y_{N,t+i}$ from their cross-economy average $\bar{y}_t$ can be expected, conditional on current information, to approach constant values as i approaches infinity.[3] However, Eq. (2.1.4) holds if, and only if, $y_{nt} - \bar{y}_t$ is stationary with an unconditional mean vector μ_n for $n = 1, 2, \ldots, N$. Therefore, economies $1, 2, \ldots, N$ are said to converge if, and only if, every y_{nt} is nonstationary but every $y_{nt} - \bar{y}_t$ is stationary. We define convergence to be absolute or conditional depending on whether $\mu_n = 0$ for all n or $\mu_n \neq 0$ for some n. The economies are said to diverge if, and only if, $y_{nt} - \bar{y}_t$ is nonstationary for all n.[4]

2.2. The conventional approach

The conventional approach attempts to infer whether and how economies converge by applying ordinary least squares to

$$g_n = \alpha + \beta y_{n0} + \gamma' x_n + v_n, \tag{2.2.1}$$

where $g_n \equiv (y_{nT} - y_{n0})/T$ is the average growth rate of per capita output for economy n between periods 0 and T, x_n is a vector of variables that control for permanent cross-economy differences in either levels or growth rates of per capita output, α and β are parameters, γ is a parameter vector, and v_n is an error term with a zero mean and finite variance. According to Eq. (2.2.1) with $\beta < 0$, economies that are initially rich after controlling for the permanent differences associated with x_n and with any economy-specific effects in v_n grow more slowly than economies that are initially poor on the same basis. The economies therefore converge as defined in the previous section if $\beta < 0$, and the convergence is absolute only if $\gamma = 0$ and conditional if $\gamma \neq 0$. By contrast, if $\beta = 0$, Eq. (2.2.1) implies that divergence takes place since the output differences observed in period 0 are expected to persist intact through period T.

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[3] In other words, convergence implies that the cross-economy distribution of $\lim_{i\to\infty}\mathrm{E}_t(y_{n,t+i} - a_{t+i} - \mu_n)$ has all of its probability density at zero. Because the world is stochastic, this degeneracy in no way implies that the cross-economy distribution of $y_{n,t+i} - a_{t+i} - \mu_n$ becomes degenerate as i approaches infinity. See Friedman (1992) and Quah (1993b) for related discussions.

[4] One might also wish to allow some of the economies to converge while the others diverge and to determine which converge and which diverge. This statistical problem, however, is difficult and well beyond the scope of this paper. (We thank G.S. Maddala for explaining just how difficult the problem is.) Fortunately, the issue does not arise for the data sets analyzed in this paper since all of the economies in each data set are found to converge.

Unfortunately, the estimators $\hat{\beta}$ and $\hat{\gamma}$ obtained by applying ordinary least squares to Eq. (2.2.1) are unlikely to be useful for inferring what β and γ are since v_n is uncorrelated with y_{n0} only under incredible conditions.[5] Evans (1996) has shown that v_n is uncorrelated with y_{n0} if, and only if, $y_{nt} - \bar{y}_t$ is generated by the process

$$y_{nt} - \bar{y}_t = \delta_n + \lambda(y_{n,t-1} - \bar{y}_{t-1}) + u_{nt}, \qquad (2.2.2)$$

with

$$\delta_n = \xi' x_n, \qquad (2.2.3)$$

where $\lambda \equiv (1 + \beta T)^{1/T}$, $\xi \equiv (\lambda - 1)\gamma/\beta$, and u_{nt} is a serially uncorrelated error term with a zero mean and finite and constant variance σ_n^2. Eq. (2.2.2) implies that economies 1, 2, ..., N converge (that is, every $y_{nt} - \bar{y}_t$ is stationary) if, and only if, $\lambda < 1 (\beta < 0)$[6] and diverges if $\lambda = 1$ ($\beta = 0$). Furthermore, if u_{nt} becomes uncorrelated across countries as N approaches infinity, inferences based on $\tau(\hat{\beta})$, the heteroskedasticity-consistent t-ratio of β, and $\Phi(\hat{\gamma})$, the heteroskedasticity-consistent F-ratio of γ, are valid.[7] By contrast, if the data-generating process does not take the form (2.2.2), inferences based on $\tau(\hat{\beta})$ and $\Phi(\hat{\gamma})$ are invalid.

Using Cass's model with identical CES utility and Cobb–Douglas technology, Barro and Sala-i-Martin (1992) show that Eq. (2.2.2) holds approximately near the balanced growth paths for economies 1, 2, ..., N. Mankiw, Romer, and Weil establish a similar result for Solow's model with Cobb–Douglas technology.[8] One might therefore consider Eq. (2.2.2) to be a good approximation *a priori*. Before doing so, one should consider several empirically important phenomena from which these authors abstract.

The parameters of technology differ widely across economies. According to *National Accounts*, the share of output paid to capital in the OECD over the period 1960–1988 ranged from 0.199 for Luxembourg to 0.720 for Turkey. Moreover, the share of gross state product paid to capital also varied widely across U.S. states over the period 1970–1986; see our 1996 paper for evidence.

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[5] Throughout our analysis, we assume that x_n is uncorrelated with v_n. This assumption is problematic only if the restriction (2.2.3) below is violated.

[6] I take for granted that $\lambda \geq 0$ and hence that $\beta \geq -1/T$.

[7] Friedman (1992) and Quah (1993a) seem to argue that the conventional approach produces invalid inferences under all circumstances. Such a position is untenable. We do concur with them, however, that the conventional approach is unlikely to produce valid inferences in practice and should therefore be abandoned.

[8] Strictly speaking, their models imply that $\sigma_n = 0$. The data-generating processes implied by stochastic versions of their models are unknown. For expositional convenience, we assume that Eq. (2.2.2) with $\sigma_n > 0$ would also be an adequate approximation in this case.

With this feature incorporated, the models discussed above imply the data-generating process

$$\lambda_n(L)(y_{nt} - \bar{y}_t) = \delta_n + u_{nt}, \tag{2.2.4}$$

where $\lambda_n(L) \equiv 1 - \lambda_{n1} L$, L is the lag operator, and the λ_{n1}s are parameters that lie on the interval [0, 1] and differ appreciably across economies.

The number of distinct and empirically important state variables in actual economies exceeds one because capital can be disaggregated into components like equipment and structures with very different depreciation rates; investment generates adjustment costs; and temporary, but persistent, idiosyncratic shocks to technology and preferences may occur. Moreover, y_{nt} may be contaminated with stationary measurement error. With these features incorporated, the models imply that the degree of the polynomial $\lambda_n(L)$ exceeds one.

Finally, *x*s of modest dimension are highly unlikely to be able to control completely for all permanent cross-economy differences in per capita output as well as for the nonzero means of any measurement error that may contaminate the *y*s.

2.3. An alternative approach

Except by happenstance, the conventional approach produces invalid inferences unless the economies have identical first-order autoregressive dynamic structures and all permanent cross-economy differences in their per capita outputs are perfectly controlled for.[9] This subsection shows how to avoid making these highly implausible assumptions while potentially enhancing efficiency since the conventional approach throws away much of the time-series variation in the *y*s.

Beginning with the data-generating process (2.2.4), the appendix shows that

$$\Delta(y_{nt} - \bar{y}_t) = \delta_n + \rho_n(y_{n,t-1} - \bar{y}_{t-1}) + \sum_{i=1}^{p} \varphi_{ni}\Delta(y_{n,t-i} - \bar{y}_{t-i}) + u_{nt}, \tag{2.3.1}$$

where ρ_n is negative if the economies converge and zero if they diverge,[10] δ_n is a parameter, and the φs are parameters such that all roots of $\sum_i \varphi_{ni} L^i$ lie outside

[9] Durlauf and Johnson make the similar point that the conventional approach is unlikely to produce valid inferences unless it is applied to a homogeneous sample of countries. They also provide empirical evidence that the large samples used in the literature are too heterogeneous for the conventional approach to yield valid inferences.

[10] This parameter is zero even if only some of the economies diverge. We do not exploit this fact in this paper.

the unit circle. We assume that *u*s become uncorrelated across economies as N approaches infinity.[11]

Levin and Lin (1993) have formulated a procedure for testing the null hypothesis that $\rho_n = 0$ and $\delta_n = 0$ for all n against the alternative hypothesis that $\rho_n < 0$ for all n and δ_n may be nonzero for some n. Endogenous growth models, however, predict not only that $\rho_n = 0$ for all n but also that $\delta_n \neq 0$ for all n since differences in technology, preferences, government policy, and market structure generate differences in trend growth rates. For this reason, we have modified their procedure. The modified procedure entails the following four steps.

(1) Apply ordinary least squares to Eq. (2.3.1) to obtain $\hat{\sigma}_n$, the standard error of estimate. Then calculate the normalized series $\hat{z}_{nt} \equiv (y_{nt} - \bar{y}_t)/\hat{\sigma}_n$ for each n.

(2) Using ordinary least squares, obtain the parameter estimate $\hat{\rho}$ and its t-ratio $\tau(\hat{\rho})$ by estimating

$$\Delta\hat{z}_{nt} = \hat{\delta}_n + \rho\hat{z}_{n,t-1} + \sum_{i=1}^{p} \varphi_{ni}\Delta\hat{z}_{n,t-i} + \hat{u}_{nt} \tag{2.3.2}$$

as a panel for $n = 1,2, \ldots, N$ and $t = 1,2, \ldots, T$, where $\hat{\delta}_n \equiv \delta_n/\hat{\sigma}_n$ and $\hat{u}_{nt} \equiv u_{nt}/\hat{\sigma}_n$.

(3) If $\tau(\hat{\rho})$ exceeds an appropriately chosen critical value, reject H_0: $\forall_n \rho_n = 0$ in favor of H_1: $\forall_n \rho_n < 0$. If not, H_0 may hold.

(4) If H_0 can be rejected, calculate the F-ratio

$$\Phi(\hat{\delta}) = \frac{1}{N-1}\sum_{n=1}^{N} [\tau(\hat{\delta}_n)]^2, \tag{2.3.3}$$

where $\tau(\hat{\delta}_n)$ is the t-ratio of the estimator of δ_n obtained by applying ordinary least squares to Eq. (2.3.1) for economy n. If $\Phi(\hat{\delta})$ exceeds an appropriately chosen critical value, infer that convergence is conditional. If not, convergence may be absolute.

Under the null hypothesis H_0, $\tau(\hat{\rho})$ converges in distribution to standard normal as T and N approach infinity while N/T approaches zero; see the Appendix for proof.

Furthermore, as T approaches infinity while N and p remain fixed, the F-ratio $\Phi(\hat{\delta})$ converges in distribution to $F[N-1,(N-1)(T-p-2)]$. Unfortunately, the asymptotic distributions of $\tau(\hat{\rho})$ and $\Phi(\hat{\delta})$ do not accurately approximate the

[11] Dispensing with this restriction by estimating the system (2.3.1) as seemingly unrelated regression (SUR) is impractical for two reasons. First, if $\rho_n = 0$ for all n, the asymptotic distribution of the SUR estimator is unknown to the best of our knowledge. Second, if $\rho_n < 0$ for all n, the SUR estimator is known to have low power unless N is appreciably less than T. Indeed, it does not even exist if $N \geq T$.

distributions for the samples that we consider. For that reason, we employ Monte Carlo simulations to provide approximate distributions for inference. See the appendix for the details.

2.4. Other studies that use time-series tests

Bernard and Durlauf (1991) employ standard univariate methods for testing whether the differences $y_{nt} - y_{jt}$ are stationary for all (j, n) pairs. They typically fail to reject the null hypothesis of no convergence even for pairs of OECD countries. Our approach, which treats the data as a panel, should have much greater power than theirs. To illustrate this possibility, we performed Monte Carlo simulations with 10,000 iterations in order to estimate the power of the procedure of Levin and Lin to reject the null hypothesis $\check{H}_0$: $\forall_n(\rho_n = 0) \cap (\delta_n = 0)$ in favor of H_1: $\forall_n \rho_n < 0$ when in fact

$$\Delta(y_{nt} - \bar{y}_t) = -0.05(y_{n,t-i} - \bar{y}_{t-1}) + u_{nt}, \quad (2.4.1)$$

where u_{nt} is normally, independently, and identically distributed over time and across economies with a zero mean and unit variance, and $y_{n0} - \bar{y}_0 = 0$ for all n. Using the standard method with $N = 1$ and $T = 37$ and a size of 0.05, we estimate that $\check{H}_0$ is rejected in only a fraction 0.0634 of the simulated samples. This fraction rises to 0.8246 with $N = 54$. We believe that our procedure yields similar gains in power.[12]

Bernard and Durlauf (1995) use the method of Johansen (1991) to investigate whether the *ys* for various groups of countries are cointegrated with each other, finding little evidence of cointegration for most groups of countries. However, they could not consider large groups because the number of parameters that must be estimated for any given group is proportional to the square of its size.

Like us, Quah (1990) treats the data as a panel. Unfortunately, his tests are flawed because his alternative hypothesis is absolute convergence. The next section provides compelling evidence that if convergence does take place, it is in fact conditional. Hsiao (1986) shows that in this case, the estimate of ρ is biased upward if no fixed effects are included in the regression. Quah's method is thus biased against rejecting divergence.[13]

[12] A direct comparison of our approach with theirs is difficult because the distribution of $\tau(\hat{\rho})$ depends strongly on the nuisance parameters $\delta_1/\sigma_1, \delta_2/\sigma_2, \ldots, \delta_N/\sigma_N$ for $T = 37$.

[13] In addition, Quah (1993a, 1994) constructs short-run and long-run Markov-chain transition matrices for output relative to a world average for a sample of countries, finding evidence in favour of a 'two-camp world, divided between haves and have-nots'.

3. Empirical results

In this section, we investigate empirically whether and how economies converge. After describing our data, we present results obtained using the conventional approach and our alternative approach.

3.1. The data

We consider two data sets. The first is constructed by dividing per capita personal income for each of the 48 contiguous U.S. states by the GNP deflator for the entire United States.[14] The resulting series for real per capita personal income are annual and span the period 1929–1991. We consider this data set, which we refer to as STATES, because high factor mobility within the United States makes convergence likely.

We obtained the second data set from the Penn World Tables of Robert Summers and Alan Heston (1991) as updated in 1993. It consists of annual data on real GDP per worker (RGDPW) for 54 countries over the period 1950–1990. The sample includes every country for which the data are available for the entire sample period except those that are centrally planned economies or major oil exporters.[15] Each series is measured in terms of a common international basket of goods. We refer to this data set as SUMHES.

3.2. Empirical analysis using the conventional approach

The first and second columns of the first row of Table 1 report our estimate of β in Eq. (2.2.1) for STATES together with its standard error and t-ratio and the marginal significance level of the t-ratio. This estimate is negative and highly significant, apparently providing strong evidence that the contiguous U.S. states converge. In order to test whether convergence is absolute, we constructed the following four dummy variables, which are one for the indicated states and zero for all other states: the six New England states; the three Midatlantic states, Delaware, and Maryland; the five Great Lakes states; the eleven states of the

[14] The sources for the data are *State Personal Income, 1929–1987*; *National Income and Product Accounts, 1929–1982*; *Business Statistics, 1961–1990*; and the *Survey of Current Business* for July and August 1992.

[15] The countries included in the samples are Argentina, Australia, Austria, Belgium, Bolivia, Brazil, Canada, Chile, Colombia, Costa Rica, Cyprus, Denmark, Dominican Republic, Ecuador, Egypt, El Salvador, Finland, France, Germany, Greece, Guatemala, Guyana, Honduras, Iceland, India, Ireland, Italy, Japan, Kenya, Luxembourg, Mauritius, Mexico, Morocco, Netherlands, New Zealand, Nigeria, Norway, Pakistan, Panama, Paraguay, Peru, Philippines, Portugal, South Africa, Spain, Sweden, Switzerland, Thailand, Trinidad and Tobago, Turkey, United Kingdom, United States, Uruguay, and Venezuela.

Table 1
Estimates, standard errors, t-ratios, and marginal significance levels for the conventional approach

Sample	$\hat{\beta}$	$\tau(\hat{\beta})$	$\Phi(\hat{\gamma})$
STATES	− 0.0110 (0.0008)	− 13.8 [0.0000]	3.02 [0.0167]
SUMHES	− 0.0122 (0.0018)	− 6.62 [0.0000]	29.2 [0.0000]

Estimation was by ordinary least squares. The figures in parentheses are standard errors that are robust to heteroskedasticity, and the figures in brackets are marginal significance levels.

South plus West Virginia, Kentucky, and Oklahoma; and the eleven Rocky Mountain and Pacific states. The third column of the first row reports the F-ratio for the estimated coefficients on these four control variables in the regression equation (2.2.1). Because it is statistically significant at the 0.05 level, the convergence of the contiguous states appears not to be absolute as Barro and Sala-i-Martin have reported.

The first and second column of the second row of Table 1 reports our estimate of β for SUMHES together with its standard error and t-ratio and the marginal significance level of the t-ratio. This estimate is negative and highly significant, apparently providing strong evidence that the countries of SUMHES also converge. We used the following five control variables in the regression: the average ratios of real gross domestic investment, real government consumption, real exports plus imports to real gross domestic product, the average growth rate of the labor force, and the average fraction of the population aged 12–17 enrolled in secondary schools.[16] The third column of the second row reveals that the estimated coefficients on these variables are highly significant. Absolute convergence can thus be resoundingly rejected in favor of conditional convergence. Our finding of conditional convergence for countries confirms those of Barro and Mankiw, Romer, and Weil *inter alia*.

3.3. Empirical analysis using the alternative approach

Pretesting revealed that three years is a reasonable choice for the lag length p for both STATES and SUMHES. With that choice in Eq. (2.3.1), the restriction that the ρs are identical across the economies and all of the φs are zero can be rejected at significance levels less than 10^{-8} for both STATES and SUMHES.

[16] We constructed the first four variables from series in the Penn World Tables and the fifth variable from data generously provided by Robert Barro. The former are averages of annual data for 1950–1990, and the latter is the average of data for 1950, 1960, 1970, and 1985.

Table 2
Estimates, standard errors, t-ratios, and marginal significance levels for the alternative approach

Sample	$\hat{\rho}$	$\tau(\hat{\rho})$	$\Phi(\hat{\delta})$	$\Phi(\hat{\eta})$
STATES	− 0.0826	− 15.2	2.70	6.15
	(0.0054)	[0.0000]	[0.0436]	[0.0000]
SUMHES	− 0.0430	− 7.74	3.46	13.8
	(0.0056)	[0.0291]	[0.0872]	[0.0000]

Estimation was by ordinary least squares. The figures in parentheses are standard errors, and the figures in brackets are marginal significance levels. The marginal significance levels for $\tau(\hat{\rho})$ and $\Phi(\hat{\delta})$ come from Monte Carlo simulations. $\Phi(\hat{\eta})$ is robust to heteroskedasticity.

Therefore, the data-generating process is not (2.2.2) and (2.2.3), the estimates reported in the previous subsection are inconsistent, and the inferences based on them are invalid, possibly meaningless, and relatively uninformative in any case. This subsection uses our alternative approach for making more informative inferences.

The first and second columns of Table 2 report our estimates of ρ in Eq. (2.3.2) for STATES and SUMHES together with standard errors, t-ratios, and marginal significance levels. The t-ratios provide overwhelming evidence that the U.S. states converge and strong evidence that the countries of SUMHES converge. Conditional on convergence, the third column provides fairly strong evidence against absolute convergence for STATES[17] but only weak evidence against absolute convergence for SUMHES.

Our failure to reject absolute convergence for the countries of SUMHES may stem from low power for tests based on $\Phi(\hat{\delta})$. Fortunately, a potentially more powerful test is available. Eq. (2.3.1) implies that $-\delta_n/\rho_n$ is the unconditional mean of $y_{nt} - \bar{y}_t$ if $\rho_n < 0$. Consequently, absolute convergence can be tested against conditional convergence by comparing $\Phi(\hat{\eta})$, the heteroskedasticity-consistent F-ratio obtained by applying ordinary least squares to

$$-\hat{\delta}_n/\hat{\rho}_n = v + \eta' x_n + w_n, \qquad n = 1, 2, \ldots, N, \tag{3.3.1}$$

to an appropriately chosen critical value from the $F(K, N - K - 1)$ distribution. In Eq. (3.3.1), $\hat{\delta}_n$ and $\hat{\rho}_n$ are the estimators of δ_n and ρ_n obtained by applying ordinary least squares to Eq. (2.3.1), x_n is a $K \times 1$ vector of variables describing economy n, v is a parameter, η is a $K \times 1$ parameter vector, and w_n is an error term.

[17] Using data on total factor productivity, capital's share in gross state product, and rental rates, our 1996 paper produces much more compelling evidence that the convergence of STATES is conditional rather than absolute.

The fourth column of Table 2 reports $\Phi(\hat{\eta})$ and its marginal significance level for STATES and SUMHES. The entries of x_n in Eq. (3.3.1) consist of the variables described in the previous subsection. The evidence against absolute convergence is incisive for both the U.S. states and the countries of SUMHES.

4. Conclusions

The conventional approach and our alternative approach lead to the same conclusion: the 48 contiguous U.S. states and our group of 54 countries converge conditionally. A natural question to ask is, then: 'Why use the alternative approach when the conventional approach produces the same conclusion?' The answer, of course, is that the inferences of the conventional approach are invalid for the data that we analyzed and no doubt for virtually all data sets encountered in practice. As a result, good luck contributed substantially to the 'right' inference produced by the conventional approach. In other applications, it could easily produce 'wrong', as well as invalid, inferences since luck is not always good. The alternative approach is not appreciably more difficult to implement and produces valid inferences under much less restrictive assumptions.

Our empirical findings are consistent with neoclassical growth models, which predict convergence, and inconsistent with most endogenous growth models, which predict divergence. Furthermore, our evidence shows that a common assumption of both classes of models, that economies differ only in initial conditions and stochastic disturbances, may be seriously deficient.

Although our evidence is inconsistent with most endogenous growth models, it is not inconsistent with endogenous growth of the world economy. How endogenous the trend growth rate of the world is remains an open question, which merits future study.

Appendix

A.1. Derivation of Eq. (2.3.1)

The polynomial $\lambda_n(L)$ can be written in the form $\pi_n(L)D(L)$, where $D(L)$ is either 1 or Δ depending on whether the economies converge or diverge and $\pi_n(L)$ has all of its roots outside the unit circle. Eq. (2.2.4) can be rewritten as

$$\pi_n(L)D(L)(y_{nt} - \bar{y}_t) = \mu_n + u_{nt}. \tag{A.1.1}$$

A finite natural number q exists such that for all n, $\pi_n(L)$ can approximated arbitrarily well as a qth-degree polynomial. As a result, Eq. (A.1.1) can be rewritten in the form (2.3.1) with $\rho_n = 0$, $\varphi_{ni} = \pi_{ni}$ for $i = 1, 2, \dots, p$ and $p = q$ if the economies diverge $[D(L) = \Delta]$ and with $\rho_n = \sum_{i=1}^{p+1} \pi_{ni} - 1$,

$\varphi_{ni} \equiv -\sum_{j=i+1}^{p+1} \pi_{nj}$ for $i = 1, 2, \ldots, p$ and $p = q - 1$ if the economies converge $[D(L) = 1]$. In the latter case, $\rho_n < 0$ since all of the roots of $\pi_n(L)$ lie outside the unit circle.

A.2. Derivation of some properties of $\hat{\rho}$ under H_0

Without essential loss of generality, I assume that $p = 0$ in Eq. (2.3.1), which can then be rewritten in the form

$$\Delta z_{nt} = \delta_n/\sigma_n + \rho_n z_{n,t-1} + \varepsilon_{nt}, \tag{A.2.1}$$

where $z_{nt} \equiv (y_{nt} - \bar{y}_t)/\sigma_n$ and $\varepsilon_{nt} \equiv u_{nt}/\sigma_n$. Applying ordinary least squares to Eq. (A.2.1) while imposing the restriction $\rho_1 = \rho_2 = \cdots = \rho_N \equiv \rho$ yields the estimator

$$\tilde{\rho} \equiv \sum_{n=1}^{N} \sum_{t=1}^{T} (z_{n,t-1} - \bar{z}_n)\Delta z_{nt} \Bigg/ \sum_{n=1}^{N} \sum_{t=1}^{T} (z_{n,t-1} - \bar{z}_n)^2, \tag{A.2.2}$$

where

$$\bar{z}_n \equiv T^{-1} \sum_{t=0}^{T-1} z_{nt}. \tag{A.2.3}$$

Substituting Eq. (A.2.1) into Eq. (A.2.2), noting that by construction $\sum_{t=1}^{T} (z_{n,t-1} - \bar{z}_n)(\delta_n/\sigma_n) = 0$ for every n, and rearranging produces

$$\tilde{\rho} = \sum_{n=1}^{N} b_n \rho_n + \sum_{n=1}^{N} \sum_{t=1}^{T} (z_{n,t-1} - \bar{z}_n)\varepsilon_{nt} \Bigg/ \sum_{n=1}^{N} \sum_{t=1}^{T} (z_{n,t-1} - \bar{z}_n)^2, \tag{A.2.4}$$

where

$$b_n \equiv \sum_{t=1}^{T} (z_{n,t-1} - \bar{z}_n)^2 \Bigg/ \sum_{n=1}^{N} \sum_{t=1}^{T} (z_{m,t-1} - \bar{z}_m)^2. \tag{A.2.5}$$

Under H_0, Eq. (A.2.1) can be solved backward from period t to period 1 to obtain

$$z_{nt} = z_{n0} + (\delta_n/\sigma_n)t + \sum_{i=1}^{t} \varepsilon_{ni}. \tag{A.2.6}$$

Eq. (A.2.6) implies that z_{nt} is eventually dominated by the term $(\delta_n/\sigma_n)t$. Hence, assuming that $\omega^2 \equiv \lim_{n\to\infty} (1/N)\sum_{n=1}^{N} (\delta_n/\sigma_n)^2$ exists and is positive, we have

$$\begin{aligned} &\operatorname*{plim}_{N,T\to\infty} N^{-1}T^{-3} \sum_{n=1}^{N} \sum_{t=1}^{T} (z_{n,t-1} - \bar{z}_n)^2 \\ &= \lim_{N,T\to\infty} N^{-1} \sum_{n=1}^{N} T^{-3} \sum_{t=1}^{T} \left\{(\delta_n/\sigma_n)\left[(t-1) - T^{-1}\sum_{i=0}^{T-1} i\right]\right\}^2 \end{aligned}$$

$$= \lim_{N\to\infty} N^{-1} \sum_{n=1}^{N} (\delta_n/\sigma_n)^2 \left[\lim_{T\to\infty} \sum_{t=1}^{T} \left(\frac{t}{T} - \frac{1}{2} - \frac{1}{2T} \right)^2 \left(\frac{1}{T} \right) \right]$$

$$= \lim_{N\to\infty} N^{-1} \sum_{n=1}^{N} (\delta_n/\sigma_n)^2 \int_0^1 (r - \tfrac{1}{2})^2 \, \mathrm{d}r = \omega^2/12 \qquad \text{(A.2.7)}$$

and

$$N^{-1/2} T^{-3/2} \sum_{n=1}^{N} \sum_{t=1}^{T} (z_{n,t-1} - \bar{z}_n)\varepsilon_{nt}$$

$$\xrightarrow{d} N^{-1/2} \sum_{n=1}^{N} T^{-3/2} \sum_{t=1}^{T} \left\{ (\delta_n/\sigma_n) \left[(t-1) - T^{-1} \sum_{i=0}^{T-1} i \right\} \varepsilon_{nt} \right.$$

$$= N^{-1/2} \sum_{n=1}^{N} (\delta_n/\sigma_n) \sum_{t=1}^{T} \left(\frac{t}{T} - \frac{1}{2} - \frac{1}{2T} \right) (T^{-1/2} \varepsilon_{nt})$$

$$\xrightarrow{d} N^{-1/2} \sum_{n=1}^{N} (\delta_n/\sigma_n) \int_0^1 (r - \tfrac{1}{2}) \, \mathrm{d}W_n(r)$$

$$= N^{-1/2} \sum_{n=1}^{N} (\delta_n/\sigma_n) \mathrm{N}(0, \tfrac{1}{12})$$

$$= N^{-1/2} \sum_{n=1}^{N} \mathrm{N}[0, \tfrac{1}{12}(\delta_n/\sigma_n)^2]$$

$$\xrightarrow{d} \mathrm{N}\left\{ 0, \frac{1}{12} \left[\lim_{N\to\infty} \frac{1}{N} \sum_{n=1}^{N} (\delta_n/\sigma_n)^2 \right] \right\}$$

$$= \mathrm{N}(0, \omega^2/12), \qquad \text{(A.2.8)}$$

as T and N approach infinity. In Eq. (A.2.8), $W_n(r)$ is the Wiener process associated with the error term ε_{nt} and r is a variable of integration associated with t/T. The derivation of Eq. (A.2.8) uses three basic properties of Wiener processes, which hold for all real numbers r and s lying on the interval (0, 1): (i) $\mathrm{d}W_n(r)$ is normally distributed with a zero mean and variance $\mathrm{d}r$; (ii) $\mathrm{Ed}W_m(r)\mathrm{d}W_n(s) = 0$ if $r \neq s$; and (iii) $\lim_{N\to\infty}\mathrm{Ed}W_m(r)\mathrm{d}W_n(r) = 0$ if $m \neq n$ since ε_{mt} and ε_{nt} become uncorrelated with each other for every $m \neq n$ as N approaches infinity. The fifth line of Eq. (A.2.8) follows from the fourth line because properties (i) and (ii) imply that $\int_0^1(r - \frac{1}{2})\,\mathrm{d}W_n(r)$ is normally distributed with a mean $\int_0^1(r - \frac{1}{2})\,\mathrm{Ed}W_n(r) = 0$ and a variance $\int_0^1\int_0^1 (r - \frac{1}{2})(s - \frac{1}{2})\,\mathrm{d}W_n(r)\,\mathrm{d}W_n(s) = \int_0^1(r - \frac{1}{2})^2\mathrm{d}r = 1/12$. The seventh line follows from property (iii) and the central limit theorem for triangular arrays; see Levin and Chien-Fu Lin. Hence, Eqs. (A.2.4), (A.2.5), (A.2.7), and (A.2.8) and the continuous mapping theorem imply that under H_0,

$$N^{1/2} T^{3/2} \tilde{\rho} \xrightarrow{d} (12/\omega^2)\mathrm{N}(0, \omega^2/12) = \mathrm{N}(0, 12/\omega^2), \qquad \text{(A.2.9)}$$

as T and N approach infinity. Using the result (A.2.9) and applying the continuous mapping theorem again then yields

$$\tau(\hat{\rho}) \equiv \tilde{\rho} \Big/ \left\{ 1 \cdot \left[\sum_{n=1}^{N} \sum_{t=1}^{T} (z_{n,t-1} - \bar{z}_n)^2 \right]^{-1} \right\}^{1/2}$$

$$= \left[N^{-1} T^{-3} \sum_{n=1}^{N} \sum_{t=1}^{T} (z_{n,t-1} - \bar{z}_n)^2 \right]^{1/2} (N^{1/2} T^{3/2} \tilde{\rho})$$

$$\xrightarrow{d} (\omega^2/12)^{1/2} \mathrm{N}(0, 12/\omega^2) = \mathrm{N}(0, 1). \qquad \text{(A.2.10)}$$

The factor 1 in braces in the first line of (A.2.10) is the known variance of ε_{nt}.

Under H_1, Eq.(A.2.1) implies that the unconditional variance of z_{nt} is $1/[1 - (1 + \rho_n)^2]$, which equals $\rho_n^{-1}(2 + \rho_n)^{-1}$. Eqs. (A.2.4) and (A.2.5) then imply that

$$\operatorname*{plim}_{N,T \to \infty} \tilde{\rho} = \lim_{N \to \infty} N^{-1} \sum_{n=1}^{N} (2 + \rho_n)^{-1} \Big/ \lim_{N \to \infty} N^{-1} \sum_{n=1}^{N} \rho_n^{-1}(2 + \rho_n)^{-1} < 0, \qquad \text{(A.2.11)}$$

on the assumption that the limits exist. As a result, $\tau(\tilde{\rho})$, $N^{1/2} T^{1/2} \tilde{\rho}$, and *a fortiori* $N^{1/2} T^{3/2} \tilde{\rho}$ diverge toward negative infinity as N and T approach infinity.

Now consider the estimator $\hat{\rho}$. Because $\hat{\sigma}_n$, the standard error of estimate obtained by applying ordinary least squares to Eq. (2.3.1), is consistent, $\hat{\rho}$ is asymptotically equivalent to $\tilde{\rho}$. As a result, all of the properties derived for $\tilde{\rho}$ above also hold for $\hat{\rho}$.

A.3. Derivation of the asymptotic distribution of $\Phi(\hat{\delta})$

Suppose that Eq. (A.2.1) is estimated. Because ε_{nt} is serially uncorrelated with a zero mean and unit variance, becomes uncorrelated across economies as N approaches infinity, and is uncorrelated with $z_{n,t-1}$, it follows immediately that as T approaches infinity while N and p remain fixed,

$$\left[\sum_{n=1}^{N} (\hat{\delta}_n/\sigma_n) m_n (\hat{\delta}_n/\sigma_n)/(N-1) \right] \Big/ 1 \xrightarrow{d} F[N-1, (N-1)(T-1)], \qquad \text{(A.3.1)}$$

under the hypothesis $\forall_n(\rho_n < 0) \cap (\delta_n = 0)$, where m_n is the (1, 1) entry of the matrix $\{[\iota, \vec{z}_n]'[\iota, \vec{z}]\}^{-1}$ and $\vec{z}_n' \equiv [z_{n,T-1}, z_{n,T-2}, \ldots, z_{n0}]$. Because $\tau(\hat{\delta}_n) \equiv \hat{\delta}_n \sqrt{m_n}/\hat{\sigma}_n$ and because $\hat{\sigma}_n$ converges in probability to σ_n under H_1, $\Phi(\hat{\delta})$ defined by Eq. (2.3.3) is asymptotically equivalent to the statistic (A.3.1).

A.4. Monte Carlo simulations

We used the following procedure for estimating the marginal significance levels of $\tau(\hat{\rho})$ and $\Phi(\hat{\delta})$ reported in Table 2. First, we used ordinary least squares to estimate the parameters of the two null models

$$\Delta(y_{nt} - \bar{y}_t) = \delta_n + \sum_{i=1}^{p} \varphi_{ni}\Delta(y_{n,t-i} - \bar{y}_{t-i}) + u_{nt}, \tag{A.4.1}$$

$$\Delta(y_{nt} - \bar{y}_t) = \rho_n(y_{n,t-1} - \bar{y}_t) + \sum_{i=1}^{p} \varphi_{ni}\Delta(y_{n,t-i} - \bar{y}_{t-i}) + u_{nt}, \tag{A.4.2}$$

for each of the two data sets. Second, we used a normal random number generator to generate 10,000 data sets for each of the four fitted null models. Third, for each of the generated data sets, we used the procedures outlined in Section 2.3 to estimate the corresponding alternative model and obtained the desired test statistic. Four samples of 10,000 test statistics resulted. Fourth, we sorted each sample of simulated t-ratios (F-ratios) into ascending (descending) order and added a zeroth observation of $-\infty$ ($+\infty$) and a 10,001st observation of 0. Fifth, we estimated the marginal significance level of each $\tau(\hat{\rho})$ [$\Phi(\hat{\delta})$] to be $s/10000$, where s is the integer such that $\tau(\hat{\rho})$ [$\Phi(\hat{\delta})$] lies between the sth and $(s+1)$th smallest (largest) simulated test statistics in the corresponding sample.

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