Calibration and evaluation of DSGE Models

Fabio Canova
EUI and CEPR
September 2014

Outline

- Basic philosophy.
- Choice of parameters.
- Evaluation of the fit.
- Sensitivity of the measurement.
- Criticisms.

References

Ascari G. (2011) Better horses for tougher courses, *Manchester Journal*, supplement "The future of Macroeconomics", 17-20

Baxter, M. and Crucini, M. (1993) "Explaining Saving-Investment Correlations", *American Economic Review*, 83, 416-436.

Canova, F. (1994) " Statistical Inference in Calibrated Models", *Journal of Applied Econometrics*, 9, S123-S144.

Canova, F. (1995) " Sensitivity Analysis and Model Evaluation in Simulated Dynamic General Equilibrium Economies", *International Economic Review*, 36, 477-501.

Canova, F., Finn, M. and Pagan, A. (1994), "Evaluating a Real Business Cycle Model", in C. Hargreaves (ed.), *Nonstationary Time Series Analyses and Cointegration*, Oxford, UK: Oxford University Press.

Chari, V., Kehoe, P. and McGratten, E. (2009) "New Keynesian models: not yet useful for policy analysis, *American Economic Journal: Macroeconomics*, 1, 242-266.

Christiano, L. and M. Eichenbaum (1992), " Current Business Cycle Theories and Aggregate Labor Market Fluctuations", *American Economic Review*, 82, 430-450.

DeJong, D., Ingram, B. and Whiteman, C., (1996), "Beyond Calibration", *Journal of Business and Economic Statistics*, 14, 1-10.

Gregory, A. and Smith, G. (1993), " Calibration in Macroeconomics", in Maddala, G.S. (ed.), *Handbook of Statistics*, vol. 11, Amsterdam, North Holland.

Hansen, L. and Heckman, J. (1996) "The Empirical Foundations of Calibration", *Journal of Economic Perspective*, 10, 87-104.

Kim, K. and Pagan, A. (1994) "The Econometric Analysis of Calibrated Macroeconomic Models", in Pesaran, H. and M. Wickens (eds.), *Handbook of Applied Econometrics*, Vol.I, London: Blackwell Press.

Kydland, F. and Prescott, E. (1982), “Time To Build and Aggregate Fluctuations”, *Econometrica*, 50, 1345-1370 .

Kydland, F. and Prescott, E. (1996) "The Computational Experiment: An Econometric Tool", *Journal of Economic Perspective*, 10, 69-85.

Leeper, E., Traum, N., Walker, T. (2011) Clearing up the Fiscal Morass, Indiana University working paper.

Pagan, A. and Shannon, (1985), “Sensitivity Analysis for Linearized Computable General Equilibrium Models”, in J.Piggott and J. Whalley (eds.) *New Developments in Applied General Equilibrium Analysis*, Cambridge: Cambridge University Press.

Showen, J. and Whalley, J. (1992), Applying General Equilibrium, New York, Cambridge University Press.

Sims, C (1996) "Macroeconomics and Methodology", *Journal of Economic Perspectives*, 10, 105-120.

1 Introduction

- Should we calibrate or estimate?

- If one estimates should it be Bayesian or Classical?

- What is exactly calibration?

- Why is calibration popular?

- How do you recognize a good from a bad calibration?

2 Philosophy: Kydland-Prescott (1991,1996)

- Choose a question and select a model capable of answering the question.

- Solve the model.

- Choose the inputs (exogenous stochastic processes and parameters).

- Evaluate the quality of the model (compare the output to the data).

- Perform the experiment/answer the question.

A 2 minute history of econometrics

- Frish (1933): Econometrics is "quantitative economic theory".

- Cowles Commission (1948): Econometrics is testing hypotheses.

- Beginning of 1980s: calibration.

- Now some calibration and some estimation.

Why switching back and forth?

Why quantitative experiments make sense?

3 Uncontroversial steps

- Choice of question: Typically quantitative:

- How much of the variance of x_i is due to shock e_j? (Kydland and Prescott (1982))

- Can the addition of a shock v reconcile the discrepancy we have previously found between the model and the data? (Christiano and Eichenbaum (1992)).

- How useful is the introduction of a feature z in reproducing the dynamics of the data? (Wei (1998)).

Sometimes the question is qualitative: can the model reproduce the humped shaped response of output to monetary shocks? What inertial features may help in doing this?

• Choice of model: $y_t = f(\epsilon_t, \theta)$; y_t $m \times 1, \theta$ $k \times 1$, ϵ_t $l \times 1$ vectors , $l \leq m$.

e_t exogenous variables, θ parameters.

- Must have some relationship with the question asked. Could be RBC/New Keynesian; Competitive/non-competitive labor market; open/closed economy.

- Should it be realistic? (i.e. should it be complicated?)

- Should it be data congruent? (i.e. should it explain the data well?)

A model is a false description of the data, meaning:

- There will always be dimensions along which it is unrealistic (will the representative agent assumption be ever realistic?)

- There will always be some variable which is not explained well.

- What are the properties of the error (discrepancy) between model and data?

In standard linear econometric models $y_t = x_t\beta + u_t$ one assumes $E_t(u_t) = 0$, $corr(u_t, u_{t-\tau}) = 0$, i.e. the model $(X_t\beta)$ is correct on average and explains the correlation structure in the data.

Can we make these assumptions for false economic models? No, in general.

- Should we add ad-hoc features to get a good fit? Mechanisms/features should be theoretically sound (e.g. incomplete markets or different type of preferences vs. habits or informational restrictions). Something will always be left unexplained. A model is NOT reality (see e.g. Ascari (2011) for a recent reiteration).

• Solution: f unknown; choose $h(\epsilon_t, \alpha)$, where $\alpha = h^*(\theta)$ such that:

$$\min ||f(\epsilon_t, \theta) - h(\epsilon_t, \alpha)|| < \iota \tag{1}$$

$||.||$ is some distance metric (could be local or global).

- (Log) linear approximations: local.

- Second or higher order approximations: local.

- Discretization of the state space, projection methods: global.

- Choice of the process for ϵ_t.

-Use tractable processes, such as AR(1) or ARMA(1,1).

- Choose the ARMA parameters to match as close as possible the data.

(see next few slides)

4 Controversial steps

4.1 Parameter Selection

How do scientists conduct experiments in biology/physics?

Are economic experiments similar to biology/physics experiments?

- Want to measure the temperature of boiling water at 4000 meters the mountains.

- Instrument = Thermometer.

- Graduate the instrument to some observations (e.g. make sure that 0 correspond to freezing water and 100 to boiling water at see level). Interpolate values in the middle.

- Do the measurement, report result, comment.

- Measurement is a number, i.e. water boils at 86C.

Many potential sources of uncertainty in the experiment:

• Uncertainty in the measurement due to say, atmospheric conditions, unexpected heating problems, etc.. Can be accounted for if the experiment is repeated are result averaged.

• Uncertainty about the quality of the instrument. Repeat experiment using different instruments, eliminate extreme measurements, average.

• Uncertainty about the calibration? Could water freeze at different degrees in summer or winter? Could observations be dependent on unobserved forces e.g. location dependent? Maybe. Calibrate the thermometer in different conditions, check if the measurement change.

Can we draw an analogy between experiments in natural sciences and economics? In part yes.

- Instrument = economic model.

- Graduation = want to make sure that the model reproduces some important facts (point estimates of some statistics).

i) A RBC model is an extension of a growth model. Make sure it reproduces long run averages of the data (steady states). Then, use it to ask cyclical questions.

ii) A New-Keynesian model has distortions. Want the model to reproduce equilibrium without distortions (flexible price equilibrium). Then, use the model to evaluate second best policies.

• Problem: the graduation procedure applied to economic experiments leaves (many) free parameters.

- Economic models are highly parametrized.

- In experimental science this is typically not the case.

- What do we do with the remaining parameters?

Let the actual data be y_t, let M_1 be graduation statistics $\theta = (\theta_1, \theta_2, \theta_3)$

- θ_3 are parameters which do not appear in M_1.

- θ_2 are parameters that can not be simultaneously determined by M_1.

• θ_2, θ_3 are thus free parameters. How do you choose them?

- Given some $\theta_2 = \bar{\theta}_2$, and some $\epsilon_t = \bar{\epsilon}_t$, it is possible to implicitly solve θ_1 from $0 = M_1(y_t, \bar{\epsilon}_t, \theta_1, \bar{\theta}_2) \equiv \tilde{M}_1(\theta_1, \bar{\theta}_2)$, where M_1 are, typically, steady states.

To choose θ_2 one has three options:

i) Refer to other studies. Problems: selectivity bias, estimation approach used to obtain them could be incoherent with the model. Advantages: discipline profession.

ii) Fix it arbitrary; need to perform sensitivity analysis.

iii) Use additional observations and some theoretical relation (e.g. $r = \beta^{-1} - 1$ and data on the real rate).

To choose θ_3 (the parameters not entering M_1):

iii) Estimate them using e.g. $\min_{\theta_3} ||M_2(y_t) - M_2(\bar{\epsilon}_t, \bar{\theta}_2, \hat{\theta}_1, \theta_3)||$, where $M_2 \neq M_1$ and $\hat{\theta}_1$ an estimate of θ_1.

What are $M_2(y_t)$? Typically, second moments, auto and cross covariances, spectra, but could also be impulse responses, VAR coefficients, Gini coefficients, wealth distributions,etc.

What kind of estimation approach can you use? Informal method of moment (grid search), formal methods (GMM, SMM, ML, etc.)

Limited information are typically preferred to full information methods.

Example 4.1

$$\begin{aligned}
\max_{(c_t, K_{t+1}, N_t)} \quad & E_0 \sum_t \beta^t \frac{(c_t^{\vartheta}(1-N_t)^{1-\vartheta})^{1-\varphi}}{1-\varphi} && (2) \\
G_t + c_t + K_{t+1} &= GDP_t + (1-\delta)K_t && (3) \\
\ln \zeta_t &= \bar{\zeta} + \rho_z \ln \zeta_{t-1} + \epsilon_{1t} \quad \epsilon_{1t} \sim (0, \sigma_z^2) && (4) \\
\ln G_t &= \bar{G} + \rho_g \ln G_{t-1} + \epsilon_{4t} \quad \epsilon_{4t} \sim (0, \sigma_g^2) && (5) \\
GDP_t &= \zeta_t K_t^{1-\eta} N_t^{\eta} && (6)
\end{aligned}$$

K_0 are given, c_t is consumption, N_t is hours, K_t is the capital stock. Let G_t be financed with lump sum taxes and λ_t the Lagrangian on (3).

The FOC are ((10) and (11) equate factor prices and marginal products)

$$\begin{aligned}
\lambda_t &= \vartheta c_t^{\vartheta(1-\varphi)-1}(1-N_t)^{(1-\vartheta)(1-\varphi)} && (7)\\
\lambda_t \eta \zeta_t k_t^{1-\eta} N_t^{\eta-1} &= -(1-\vartheta) c_t^{\vartheta(1-\varphi)}(1-N_t)^{(1-\vartheta)(1-\varphi)-1} && (8)\\
\lambda_t &= E_t \beta \lambda_{t+1}[(1-\eta)\zeta_{t+1} K_{t+1}^{-\eta} N_{t+1}^{\eta} + (1-\delta)] && (9)\\
w_t &= \eta \frac{GDP_t}{N_t} && (10)\\
r_t &= (1-\eta)\frac{GDP_t}{K_t} && (11)
\end{aligned}$$

Using (7)-(8) we have:

$$-\frac{1-\vartheta}{\vartheta}\frac{c_t}{1-N_t} = \eta \frac{GDP_t}{N_t} \qquad (12)$$

- Here K_t is the state, (ζ_t, G_t) the shocks, $(\lambda_t, c_t, N_t, GDP_t, w_t, r_t)$ the controls (the endogenous variables).

- Seven equations (3)-(6)-(8)-(9)-(10)-(11)-(12)) and seven unknowns $((\zeta_t, G_t)$ are exogenous): a solution exist.

Log linearizing the equilibrium conditions

$$
\begin{aligned}
\hat{\lambda}_t - (\vartheta(1-\varphi)-1)\hat{c}_t + (1-\vartheta)(1-\varphi)\frac{N^{ss}}{1-N^{ss}}\hat{N}_t &= 0 \qquad (13)\\
\hat{\lambda}_{t+1} + \frac{(1-\eta)(GDP/K)^{ss}}{(1-\eta)(GDP/K)^{ss}+(1-\delta))}(\widehat{GDP}_{t+1} - \hat{K}_{t+1}) &= \hat{\lambda}_t \qquad (14)\\
\frac{1}{1-N^{ss}}\hat{N}_t + \hat{c}_t - \widehat{gdp}_t &= 0 \qquad (15)\\
\hat{w}_t - \widehat{GDP}_t + \hat{n}_t &= 0 \qquad (16)\\
\hat{r}_t - \widehat{GDP}_t + \hat{k}_t &= 0 \qquad (17)\\
\widehat{GDP}_t - \hat{\zeta}_t - (1-\eta)\hat{K}_t - \eta\hat{N}_t &= 0 \qquad (18)\\
(\frac{g}{GDP})^{ss}\hat{g}_t + (\frac{c}{GDP})^{ss}\hat{c}_t + (\frac{K}{GDP})^{ss}(\hat{K}_{t+1} - (1-\delta)\hat{K}_t) - \widehat{GDP}_t &= 0 \qquad (19)
\end{aligned}
$$

(18) and (19) are the production function and resource constraint.

Four types of parameters appear in the log-linearized conditions:

i.) Technological parameters (η, δ).

ii) Preference parameters $(\beta, \varphi, \vartheta)$.

iii) Steady state parameters $(N^{ss}, (\frac{c}{GDP})^{ss}, (\frac{K}{GDP})^{ss}, (\frac{g}{GDP})^{ss})$.

iv) Parameters of the driving process $(\rho_g, \rho_z, \sigma_z^2, \sigma_g^2)$.

Question: How do we obtain values for these 13 parameters?

The steady state of the model (using (9)-(12)-(3)) is:

$$\frac{1-\vartheta}{\vartheta}(\frac{c}{GDP})^{ss} = \eta\frac{1-N^{ss}}{N^{ss}} \quad (20)$$

$$\beta[(1-\eta)(\frac{GDP}{K})^{ss}+(1-\delta)] = 1 \quad (21)$$

$$(\frac{g}{GDP})^{ss}+(\frac{c}{GDP})^{ss}+\delta(\frac{K}{GDP})^{ss} = 1 \quad (22)$$

$$\frac{GDP}{wc} = \eta \quad (23)$$

$$\frac{K}{i} = \delta \quad (24)$$

Five equations in 8<13 parameters!! Can't calibrate all parameters using the steady states. Need to choose.

For example: (20)-(22) determine $(N^{ss}, (\frac{c}{GDP})^{ss}, (\frac{K}{GDP})^{ss}, \eta, \delta)$ *given* $((\frac{g}{GDP})^{ss}, \beta, \vartheta)$*.*

Here $\theta_2 = [(\frac{g}{GDP})^{ss}, \beta, \vartheta]$ *and* $\theta_1 = [N^{ss}, (\frac{c}{GDP})^{ss}, (\frac{K}{GDP})^{ss}, \eta, \delta]$ *and* M_1 *are steady state relationships.*

How do we set the parameters in θ_2*? Use external information!*

- $(\frac{g}{GDP})^{ss}$ *it is typically chosen to be the average G/Y in the data.*

- $\beta = (1 + r)^{-1}$ *so typically set it so that* $r^{ss} = [0.0075, 0.0150]$ *per quarter.*

- ϑ *is related to Frish elasticity of labor supply: use micro study estimates. Which one? Controversial.*

The other 5 parameters of the model do not enter the steady state and correspond to θ_3. How do we choose them?

- The parameters of the exogenous process. Rule:

a) If the shock is observable: estimate the free parameters from the data. Occasionally, shock is observable (government expenditure) or observable conditional on some parameters (e.g. Solow residuals).

i) ρ_g, σ_g^2 backed out from government expenditure data.

ii) ρ_z, σ_z^2 backed out from Solow residual i.e. estimate the variance and the AR(1) of $\hat{z} = \ln GDP_t - (1-\eta)K_t - \eta N_t$, once η is chosen.

b) If ϵ_t unobservable: estimate the free parameters so that statistics of simulated data match statistics of actual data, i.e. choose the variance of, e.g. preference shocks, so that $var(GDP_t) = var(GDP_t^M)$.

- For φ: coefficient of relative risk aversion (RRA) is $1 - \vartheta(1 - \varphi)$. Then

(a) appeal to existing estimates of RRA and, given ϑ, find φ.

(b) fix φ arbitrarily;

(c) use (7) or (9) to estimate it by e.g. GMM (see later on);

(d) select it by simulation so that, e.g., $var(c_t) = var(c_t^M)$.

• Problem: estimates θ_3 are typically conditional on $\bar{\theta}_2, \hat{\theta}_1$.

Distribution of θ_3 distorted if $\bar{\theta}_2, \hat{\theta}_1$ are inconsistent estimators of θ_2, θ_1.

Example 4.2

$$\begin{aligned} y_t &= x_t\beta + u_t & (25) \\ u_t &= \rho u_{t-1} + \epsilon_t \qquad \epsilon_t \sim iid(0, \sigma_\epsilon^2) & (26) \end{aligned}$$

The GLS estimator for (β, ρ)*: use* $y_t - \rho y_{t-1} = \beta(x_t - \rho x_{t-1}) + \epsilon_t$.

$(\hat{\beta}|\rho) = (\tilde{x}_t'\tilde{x}_t)^{-1}(\tilde{x}_t'\tilde{y}_t)$ *where* $\tilde{x} = x_t - \rho x_{t-1}$ *and* $\tilde{y} = y_t - \rho y_{t-1}$.

When is $\beta_{GLS} = \hat{\beta}|\hat{\rho}$ *obtained e.g. with a two step approach?*

If $\hat{\rho}$ *consistent:* $(\hat{\beta}|\hat{\rho}) \xrightarrow{P} (\hat{\beta}|\rho) = \int(\beta, \rho)d\rho$ *as* $T \to \infty \equiv \beta_{GLS}$. *Otherwise problems.*

Example 4.3 - *Set T=1000, replications =1000.*

- *True values:* $\beta = 0.5, \rho = 0.9$

Distribution of $\beta|\hat{\rho}$

	25th percentile	*mean*	*75th percentile*
$\rho = 0$	*0.396*	*0.478*	*0.599*
$\rho = 0.4$	*0.443*	*0.492*	*0.553*
$\rho = 0.9$	*0.479*	*0.501*	*0.531*

- Moments of the distribution of β depend on $\hat{\rho}$.

General Problems

- Potentially many estimates of θ_2, which one to choose?

- Potentially many estimates of θ_3, depending on the estimation approach, which one to choose?

- Different parameters obtained with different objective functions. Consistent inference is difficult.

Alternative approach

- Treat θ_2, θ_3 as joint free parameters. Setup a prior (range) consistent with available evidence or with the prior of researchers; use steady states to back out range of values for θ_1.

a) Objective range (Canova (1994), Canova (1995)).

b) Subjective range (DeJong, Ingram, Whiteman (1996); Geweke (1999)).

Example 4.4 *a) Most existing estimates of φ (obtained with different estimation approaches) are around 2. Some have used values up to 10 in calibration (Merha and Prescott (1985)). Hence, the a-priori distribution for φ can be approximated with a $\chi^2(4)$, which has the mode at 2 and 5% of probability above 6.*

b) $\ln \varphi \sim N(3, 1)$.

- This way we have ranges for θ_1 which are consistent with the steady states.

Advantages:

- Avoids selection bias for θ_2 and/or θ_3.

- Uses all available information about parameters and model structure.

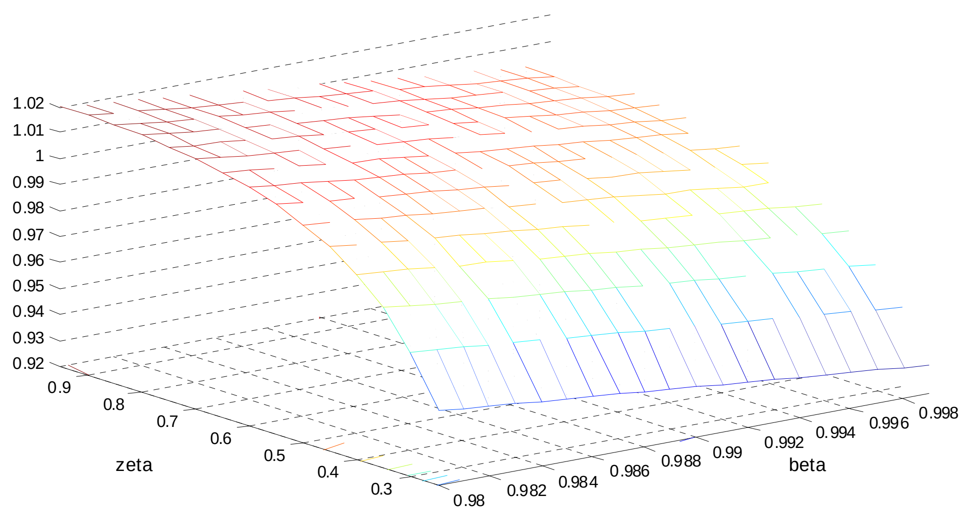
- Can be used to check what is the potential range of outcomes of the model when parameters are within some interval (prior-predictive analysis, see e.g. Leeper et al. (2011)).

- Provides automatic sensitivity analysis within a range of values.

Example 4.5 *New Keynesian Phillips curve.*

$$\pi_t = \beta\pi_{t+1} + \frac{(1-\zeta_p)(1-\zeta_p\beta)}{\zeta_p} mc_t + e_t \tag{27}$$

e_t is an expectation error. Use the output gap as observable proxy for marginal costs. What are the prior range for AR(1) of inflation, given uniform ranges for (ζ_p, β)?.



- Steady state calibration is equivalent to first moment matching.

- Steady state calibration, plus estimation of θ_3 is equivalent to first and second moment matching.

- Calibration uses economically interesting moments to find parameters. Standard econometric approaches use orthogonality or statistical conditions - less relevant from an economic point of view.

4.2 Evaluation of a calibrated model

Is the model appropriate to conduct the experiment? Need to make sure it is the case.

• If calibrated to the steady state, certain data are automatically matched on average. Need something more.

• If the model is false, can't use standard statistics to do additional checks (e.g. check if there is no serial correlation in the discrepancy between model and data). Moreover, statistical fit and economic trust are different.

• Typically, construct stylized facts (second moments, turning points, impulse responses, etc.) in the model and the data and compare them.

• **The stylized facts used to evaluate the model should be different than the statistics M_1 and M_2 used to calibrate the parameters.**

Preliminary data transformation are needed to compute stylized facts. Detrending vs. filtering.

- Detrending: some variables may have unit roots. Second moments may not be defined.

- Filtering 1: the second order implication of the model are not comparable with the data (there is much more in the data). Filter actual data.

- Filtering 2: the model and the data have many implications. For comparison focus only on the cyclical components (filter both actual data and model simulated data).

Under certain assumptions filtering detrends the data, i.e. it makes the data stationary.

Problem:

- Detrend or filter?

- If detrend, which method do we use? Typically LT, Segm-LT, FOD

- If filter, which filter do we use? Typically, HP, BP.

Can we construct stylized facts without filtering or detrending? (Harding and Pagan (2002, 2006))

Branch to other set of notes.

How do you compare a model and to the data? Five approaches:

- Informal comparison using moments.

- Formal/informal comparison using the time series representation of the decision rules.

- Informal examination of the properties of estimated shocks.

- Probabilistic comparison.

- In-sample and out-of-sample regression methods.

4.2.1 Informal comparisons

Assume you have found a reasonable way to compute stylized facts. These are numbers (e.g. the variability of cyclical output is 1.76)

- Model output is a number as well (without uncertainty in either θ or ϵ or both, variability of output in the model is a number, say 1.45).

- How do you compare 1.76 and 1.45? Eyeball econometrics!!

- Could allow sampling uncertainty in ϵ (e.g. simulate the calibrated model 10 times using the same θ but different shocks). If averages are computed, still compare a number to a number.

- Weaker interpretation (we do not want the model to grossly violate some basic statistics). Difficult to say what "grossly" means.

- Fancier business cycle accounting: Chari, Kehoe, McGrattan (2007).

In a RBC model the intratemporal condition is

$$-\frac{1-\vartheta}{\vartheta}\frac{c_t}{1-N_t} = \eta\frac{GDP_t}{N_t} \tag{28}$$

Calibrate ϑ, η. Use data on c_t, N_t, GDP_t. Call the difference between LHS and RHS of (28) a " labor wedge".

- If theory is correct and calibration is right, the wedge should be zero for every possible t. If it is not:

- Study what factors can account for size, serial correlation and other properties of the "wedge".

- Compare the time series properties of the wedge with (estimated) structural shocks. Study what kind of frictions can induce the "wedge"?

Reverse engineering approach: checks what is needed to make sure that the first order condition of the model satisfy the data counterpart. Can be used also for estimated models.

Example 4.6 *(Chari, Kehoe, McGrattan (2009)) In Smets and Wouters (2003, 2007) wage markup dominant source of fluctuations in output, hours and inflation (in the long run above 50 percent).*

Smets and Wouters: $l_t = [\int_0^1 l_{it}^{1/1+\lambda_t} di]^{1+\lambda_t}$ *so elasticity of substitution is* $\frac{1+\lambda_t}{\lambda_t}$*. Intratemporal condition is* $w_t = (1-\lambda_t)\frac{U_{lt}}{U_{ct}}$.

From BC accounting: $z_t F_{lt}(1-\tau_t) = \frac{U_{lt}}{U_{ct}}$*. If firms do not have monopoly power and prices not sticky* $w_t = z_t F_{lt}$*. Hence* $(1-\tau_t) = \frac{1}{1-\lambda_t}$

Markup shocks are labor wedge shocks (this is why they are important!).

i) Are they reasonable? Not really: standard deviation of markup shocks is 25.87. Fluctuations in elasticity too large; composition of l_t can't change so much!! Careful: metric of comparison here is HP filtered data.

ii) Are they structural? i.e. are they invariant to e.g. (monetary) policy intervention? Are they bad/good shocks? Potential stories: 1) fluctuations in the bargaining power of unions due to variations in policies toward unions (bad shock, not invariant to policy interventions); 2) fluctuations in the value of leisure (good shock, invariant).

iii) Do they give an appealing view of BC fluctuations? If 1) business cycles due to greed of workers (bid up wages when bargaining power change); if 2) 1970s recession due to attack of workers laziness. Both unappealing.

Conclusion: A model where the main source of fluctuation is a reduced form shock (lacking interpretation) and where the economics is hard to accept, can not be used to do computation experiments.

4.2.2 Comparison using the decision rules I: VARs

Example 4.7 *The log-linearized conditions of the RBC model are:*

$$\hat{\lambda}_t - (\vartheta(1-\varphi)-1)\hat{c}_t + (1-\vartheta)(1-\varphi)\frac{N^{ss}}{1-N^{ss}}\hat{N}_t = 0 \quad (29)$$

$$\hat{\lambda}_{t+1} + \frac{(1-\eta)(GDP/K)^{ss}}{(1-\eta)(GDP/K)^{ss}+(1-\delta))}(\widehat{GDP}_{t+1} - \hat{K}_{t+1}) = \hat{\lambda}_t \quad (30)$$

$$\frac{1}{1-N^{ss}}\hat{N}_t + \hat{c}_t - \widehat{gdp}_t = 0 \quad (31)$$

$$\hat{w}_t - \widehat{GDP}_t + \hat{n}_t = 0 \quad (32)$$

$$\hat{r}_t - \widehat{GDP}_t + \hat{k}_t = 0 \quad (33)$$

$$\widehat{GDP}_t - \hat{\zeta}_t - (1-\eta)\hat{K}_t - \eta\hat{N}_t = 0 \quad (34)$$

$$(\frac{g}{GDP})^{ss}\hat{g}_t + (\frac{c}{GDP})^{ss}\hat{c}_t + (\frac{K}{GDP})^{ss}(\hat{K}_{t+1} - (1-\delta)\hat{K}_t) - \widehat{GDP}_t = 0 \quad (35)$$

Letting $y_{2t} = (\hat{\lambda}_t,\ \hat{K}_t),\ y_{1t} = (\hat{c}_t,\ \hat{N}_t,\ \widehat{GDP}_t,\ \hat{w}_t,\ \hat{r}_t)$*,* $y_{3t} = [\hat{\zeta}_t, \hat{g}_t]'$*,* $y_t = [y_{1t}, y_{2t}]'$*:*

$$\mathcal{A}_0 y_{t+1} = \mathcal{A}_1 y_t + \mathcal{A}_2 y_{3t}$$

$$\mathcal{A}_0 = \begin{bmatrix} 1 & -\frac{(1-\eta)(GDP/K)^{ss}}{(1-\eta)(GDP/K)^{ss}+(1-\delta)} & 0 & 0 & \frac{(1-\eta)(GDP/K)^{ss}}{(1-\eta)(GDP/k)^{ss}+(1-\delta)} & 0 & 0 \\ & 1/(\frac{GDP}{K})^{ss} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\mathcal{A}_2 = \begin{bmatrix} 0 & 0 \\ 0 & -(g/GDP)^{ss} \\ 0 & 0 \\ 0 & 0 \\ -1 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$$\mathcal{A}_1 = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{(1-\delta}{(\frac{GDP}{K})^{ss}} & -(\frac{c}{GDP})^{ss} & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & \frac{1}{1-N^{ss}} & -1 & 0 & 0 \\ -1 & 0 & \vartheta(1-\varphi)-1 & -\frac{(1-\vartheta)(1-\varphi)N^{ss}}{1-N^{ss}} & 0 & 0 & 0 \\ 0 & 1-\eta & 0 & \eta & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 & -1 & 1 & 0 \\ 0 & 1 & 0 & 0 & -1 & 0 & 1 \end{bmatrix}$$

Solution:

$$B_0(\theta)y_t \;=\; B_1(\theta)y_{t-1} + B_2(\theta)y_{3t} \qquad (36)$$

Log-linearized solution of a DSGE is a restricted VAR(1): compare vector autoregressive representations of the model and of the data.

i) Via lag restrictions.

Example 4.8 *Decision rules of a basic RBC model imply that a VAR(1) should be enough to represent the dynamics of the data. Is it true? i.e. if we run a VAR on the data, would the coefficients on* $t-2$*,* $t-3$ *be equal to zero? Generally no.*

ii) Via exclusion restrictions.

Example 4.9 *Decision rules of a basic RBC model imply that in a VAR(1) with capital, consumption lags should not enter the empirical model for consumption. Is it true in the data? Generally no.*

iii) Via unit root/cointegration checks (Canova, Finn and Pagan (1994)).

Example 4.10 *If the technology shock ζ_t has a unit root, all the variables in y_t but hours and the real rate must have a unit root. Further since the trend is common and $\log c_t - \log y_t$ should be stationary. Is it true?*

• **If there are non-observable variables, can't compare format of decision rules to a VAR.**

4.2.3 Comparing using decision rules II: ARMA

- The log-linear solution of a DSGE model for the observable variables is not VAR(1) but an ARMA.

Result If

$$\left[\begin{array}{cc} \delta_{11}(\ell) & \delta_{12}(\ell) \\ \delta_{21}(\ell) & \delta_{22}(\ell) \end{array}\right]\left[\begin{array}{c} y_{1t} \\ y_{2t} \end{array}\right]=\left[\begin{array}{c} \epsilon_{1t} \\ \epsilon_{2t} \end{array}\right]$$

The univariate representation for y_{1t} is

$$[\delta_{11}(\ell)-\delta_{12}(\ell)\delta_{22}(\ell)^{-1}\delta_{21}(\ell)]y_{1t} = \epsilon_{1t}-\delta_{12}(\ell)\delta_{22}(\ell)^{-1}\epsilon_{2t} \quad (37)$$

Example 4.11 *In an RBC model with labor/leisure choice the solution for $\hat{N}_t$ is ARMA(∞, ∞).*

Comparison 1: Check if ACF of hours is the same in model and data.

Comparison 2: What kind of ARMA model can we fit to the actual data. Do estimated coefficients match theoretical ones?

Let $u(c_t, N_t) = \log(c_t) + \vartheta_N(1 - N_t)$, $G_t = 0$ *and* $\beta = 0.99, \eta = 0.36, \vartheta_N = 2.6, \delta = 0.025, \rho_z = 0.95, \sigma_z = 0.07$. *US S.A. data: Average Weekly Hours of Private Nonagricultural Establishments.*

ACF of hours, sample 1964;1-2003:1

	standard deviation	*corr*(h_t, h_{t-1})	*corr*(h_t, h_{t-2})	*corr*(h_t, h_{t-3})
actual data	*0.517*	*0.958*	*0.923*	*0.896*
simulated data	*0.473*	*0.848*	*0.704*	*0.570*
	ARMA(2,2) for actual hours			
	AR(1)	*AR(2)*	*MA(1)*	*MA(2)*
actual data	*1.05(4.54)*	*-0.07 (-0.33)*	*-0.12 (-0.49)*	*-0.05(-0.64)*

Conclusions:

1) Standard deviation similar, ACF of the model less persistent.

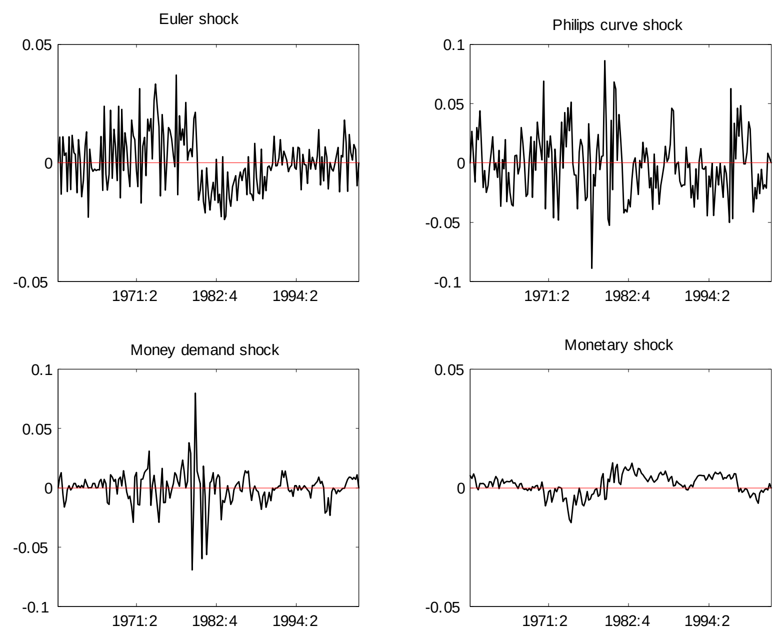
2) Data wants AR(1) for hours. First theoretical coefficient $= 1.57$, larger than the first estimated coefficient$=1.05$.

4.2.4 Shock analysis

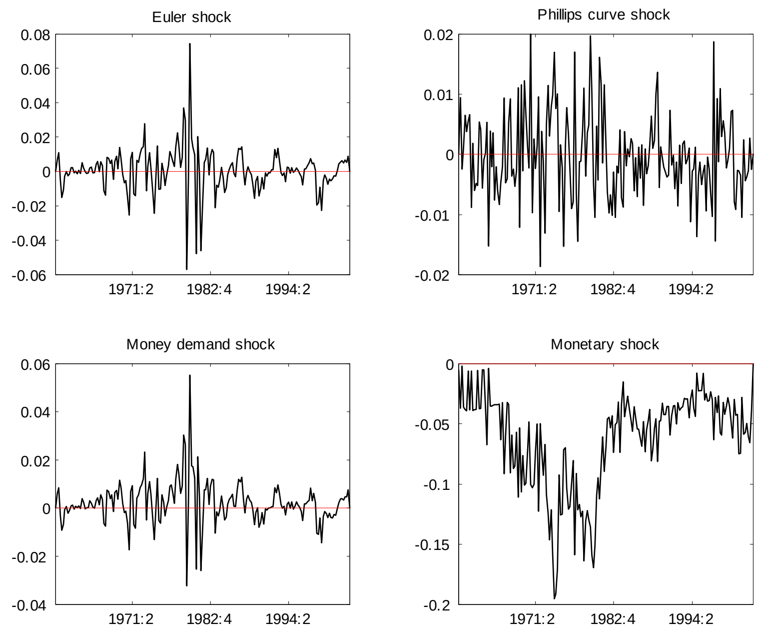
• Back out shocks needed to match the data. Are they reasonable? Do they display features which match what we know about them?

$$B_0(\theta)y_t \quad = \quad B_1(\theta)y_{t-1} + B_2(\theta)y_{3t} \tag{38}$$

- Given θ and y_0 find $y_{3t},\ t = 1, 2, \ldots.$



Estimated shocks. NK model with backward looking policy rule



Estimated Shocks. NK model with contemporaneous policy rule

4.2.5 Probabilistic evaluation methods

a) Using simulation variability

Suppose your are interested in $\rho_{y_1,y_2}(\tau),\ \tau = 0, 1, 2, \ldots$. Fix θ

1) Draw $(\{\epsilon_t\}_{t=1}^{T})^l$, solve the model, and compute $\rho_{y_1,y_2}(\tau)^l$.

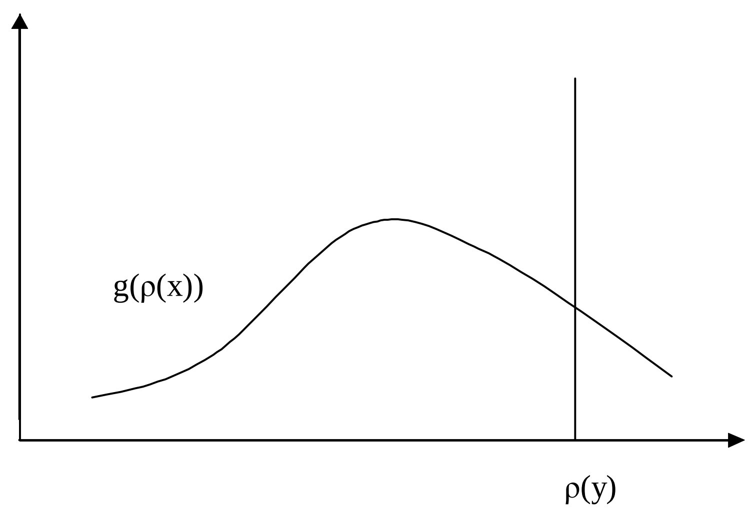
2) Repeat step 1) L times.

3) Order simulations outcomes.

4) Calculate, e.g.,

- the number of replications for which $\rho_{y_1,y_2}(\tau)^l < \rho_{y_1,y_2}(\tau)$ (i.e. calculate the percentile of the simulated distribution whether the actual value is).

- whether the actual value is inside a 68% (95%) simulation interval.



Distribution in the model and value in the actual data

Since: $\hat{\rho}_{y_1,y_2}(\tau) \xrightarrow{D} N(\rho_{y_1,y_2}(\tau), V_\rho(\tau))$ where $V_\rho(\tau) = \frac{1}{2T}(1-|\rho_{y_1,y_2}(\tau)|)^2$ and $\lim_{T\to\infty} T^{0.5}\frac{(\rho_{y_1,y_2}(\tau)-\hat{\rho}_{y_1,y_2}(\tau))}{V_\rho(\tau))^{0.5}} \xrightarrow{D} N(0,1)$ each τ.

Could also:

- Compare $T^{0.5}\frac{(\rho_{y_1,y_2}(\tau)^l-\rho_{y_1,y_2}(\tau))}{(V_\rho^A(\tau))^{0.5}}$ to a $\mathbb{N}(0,1)$ and record rejection rate (say at a 5% level) or p-values.

- Repeat L times. Construct the empirical distribution of p-values or the percentages of times the model is rejected.

Example 4.12 *Compare correlation hours-wage in the RBC and in the data*

Cross correlation hours/wage

	corr(h_t, w_{t-1})	corr(h_t, w_t)	corr(h_t, w_{t+1})
Size (% below actual)	*0.40*	*0.27*	*0.32*
Normality (% rejection)	*0.59*	*0.72*	*0.66*
68% Bands	*[0.39, 0.65]*	*[0.45, 0.70]*	*[0.38, 0.64]*
actual correlations	*0.517*	*0.522*	*0.488*

Can add parameter uncertainty to the simulations:

- Jointly draw θ^l from some prior distribution and $(\{\epsilon_t\}_{t=1}^T))^l$ from a given distribution.

- Repeat exercises as before. How much would parameter uncertainty add to the properties of the model?

b) Using sampling variability

Suppose interest in some statistics and let $\mathfrak{S}_h$ be the statistics of the model and $\mathfrak{S}_y$ the statistics of the data.

- For fixed ϵ_t, θ, $\mathfrak{S}_h$ can be computed without error (either analytically, from the VAR representation of the solution or by simulating very long time series).

- To compute the error in measuring $\mathfrak{S}_y$ in actual data:

i) Specify a time series model of y_t. Estimate $\bar{y}$ and σ_y^2.

ii) Draw ϵ^l from a $N(0, \sigma_y^2)$ or from the empirical distribution of $y_t - \bar{y}$. Construct $y_t^l = \bar{y} + \epsilon^l$. Compute $\mathfrak{S}_y^l$

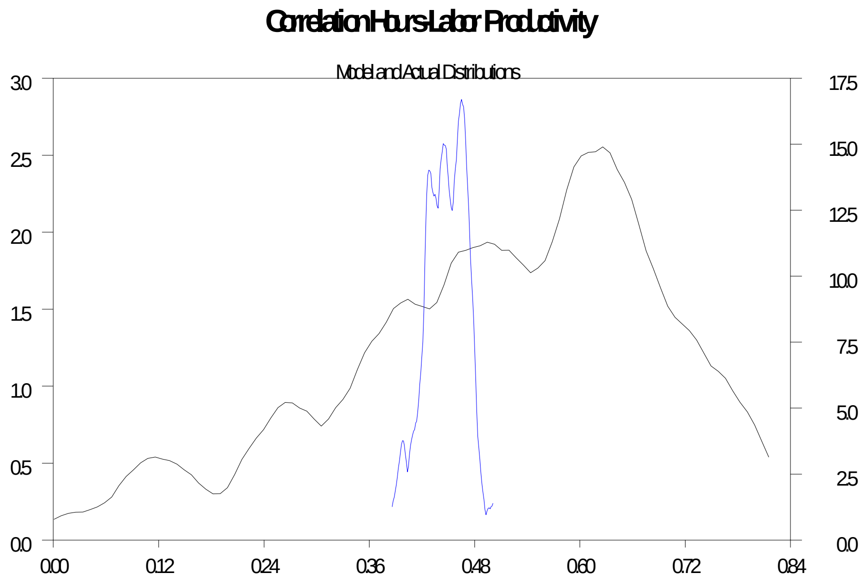
iii) Repeat i) and ii) L times.

iv) Order simulated $\mathfrak{S}_y^l$, construct percentiles and confidence intervals. Examine where simulated value lies.

• To draw from the empirical distribution need uncorrelated and homoschedastic ϵ_t.

c) Using sampling and simulation variability

Example 4.13 *Continuing with hours-wage correlation exercise*



- 95% simulated interval of actual correlation (0.44,0.52) is inside the 68% interval for the model correlation (0.35, 0.75).

- Only central 25% mass of the distribution for the model correlation is inside the 68% interval of the distribution for the actual one.

4.2.6 Regression methods: in-sample checks

- Compute RMSE/MAD for basic model and competitors.

Let y_t^1 and y_t^2 the predicted value of y_t from models m_1 and m_2. Estimate jointly

$$y_t - y_t^1 = \mu + \epsilon_t^1 \tag{39}$$

$$y_t - y_t^2 = \mu + \epsilon_t^2 \tag{40}$$

where $\epsilon_t^1, \epsilon_t^2$ have the same variance, σ^2. Estimate the mean and the variance of each model separately. Use a $\chi^2(2)$ test to verify if the restrictions hold (if they do than $RMSE = (\mu^2 + \sigma^2)^{0.5}$ is the same for the two specifications).

- Compute unbiasedness regressions

$$y_t = a + by_t^* + u_t \tag{41}$$

y_t^* is the predicted value. Ideally $a = 0, b = 1$.

- Compute predictive regressions

$$y_t = ay_t^b + by_t^* + e_t \tag{42}$$

y_t^b is the predicted value, say, from a baseline time series model. Look for $b \neq 0$, i.e. does the structural model adds information to the time series model?

• Add predicted values in a VAR and test significance (similar in spirit to predictive regressions - looks at lagged info)

$$y_t = A(\ell)y_{t-1} + B(\ell)y^*_{t-1} + u_t \tag{43}$$

Test $B(\ell) = 0$, jointly or separately for each equation.

• Case studies: how the model performs in particular episodes, i.e. a recession or an expansion; a period of high or low inflation, etc.

• If predictive analysis is performed can look at the distribution of prediction errors. Compare, for example, overlap of the distributions of different models (structural vs. time series) to see if there is additional information in the structural model.

Example 4.14 *Candidate models: i) ARMA(1,1), ii) BVAR-TVC with output, inflation and interest rate and a iii) New-Keynesian model. Evaluation based on the fit of the inflation equation. Sample 1955:1-2002:4.*

1) ARMA:$\pi_t = \rho_1 \pi_{t-1} + e_t + \rho_2 e_{t-1}$

2) TVC-BVAR:
$y_t = a_t + b_t(\ell) y_{t-1} + e_t$
$\theta_t = \rho \theta_{t-1} + (1 - \rho)\theta_0 + u_t$
$\theta_t \sim N(0, \Omega)$, $\theta = vec(a_t, b_t(\ell))$.

3) NK model:

$IS : x_t = E_t x_{t+1} - \frac{1}{\phi}(r_t - E_t \pi_{t+1}) + g_t$

$PC : \pi_t = \beta E_t \pi_{t+1} + \frac{\phi(1-\zeta)(1-\beta\zeta)}{\zeta} x_t + u_t$

$Taylor - Rule : r_t = \psi_r r_t - 1 + (1 - \psi_r)(\phi_x x_{t-1} + \phi_p \pi_{t-1}) + e_t$

with $v_t = (g_t, u_t) = \rho v_{t-1} + \eta_t;\ \eta_t\ iid\ N(0, \sigma^2)$.

Pick parameter from estimates in Canova (2009), $\beta = 0.983,\ \phi = 3.04, \zeta = 0.7709$.

In-sample Inflation RMSE, percentage points

Model	ARIMA	BVAR-TVC	NK
	1.88	1.04	1.33

In-sample, inflation correlations: actual and predicted

Model	-1	0	1
ARIMA	0.67	0.88	0.76
BVAR-TVC	0.77	0.89	0.72
NK	0.56	0.68	0.51

Unbiaseness regressions

Model	a	b	p-value $a = 0, b = 1$
ARIMA	0.159 (2.01)	0.79 (1.88)	0.03
BVAR-TVC	0.109 (1.56)	0.67 (2.06)	0.02
NK	0.035 (0.99)	0.56 (1.71)	0.01

Predictive regressions

Model	a	b
ARIMA-NK	0.82 (2.17)	0.23 (1.65)
BVAR-NK	0.73 (1.96)	0.35 (2.00)

Case study: Peak inflation, late 1970s

Model	date	68% range
ARIMA	1979:2	[1978:4, 1980:2]
BVAR	1979:4	[1979:1, 1980:4]
NK	1981:2	[1979:4, 1982:2]
Actual	1980:1	

4.2.7 Forecasting with calibrated models

Recall: log linearized decision rule of a DSGE model is of the form:

$$y_{2t} = A_{22}(\theta)y_{2t-1} + A_{21}(\theta)y_{3t} \quad (44)$$

$$y_{1t} = A_1(\theta)y_{2t} = A_{11}(\theta)y_{2t-1} + A_{12}(\theta)y_{3t} \quad (45)$$

$y_{2t} =$ states and the driving forces, $y_{1t} =$ controls, y_{3t} shocks. $\mathcal{A}_{ij}(\theta), i, j = 1, 2$ are time invariant matrices which depend on θ, the structural parameters. There are cross equation restrictions since $\theta_i, i = 1, \ldots, n$ appears in more than one entry of these matrices.

- (45) is a state space or a restricted VAR(1) model

- Assume $\theta = \bar{\theta}$ (calibrated).

• Unconditional forecast: $y_{3t+\tau} = 0, \forall \tau > 0$, let the system run. With a VAR(1) representation: let $y_t = (y_{1t}, y_{2t})$. Then $y_{t+\tau} = A^{\tau} y_t$ and $y_{2t+\tau} = S\hat{A}^{\tau}$, where $A = A(\bar{\theta})$ and S is a selection matrix, picking up the second set of elements from A.

To calculate uncertainty around point forecasts:

1. Draw θ^l from some (prior) distribution, compute A^l and$y^l_{t+\tau}, \quad l = 1, 2, \ldots, L$, each horizon τ.

2. Order $y^l_{t+\tau}$ over l, each τ and extract 16-84 or 2.5-97.5 percentiles.

• Conditional forecast 1: Manipulating shocks.

This is the same as computing impulse responses, i.e. need to orthogonalize the disturbances if they are not orthogonal. Only difference is that the impulse may last more than one period. Choose $y_{3t+\tau} = \bar{y}_{3t+\tau}$, $\tau = 0, 1, 2, \ldots, \bar{\tau}$. Given A find $y_{2t+\tau} = A_{22}(\theta) y_{2t+\tau-1} + A_{21}(\theta) y_{3t+\tau}$ and $y_{1t+\tau} = A_1(\theta) y_{2t+\tau}$.

To calculate uncertainty around the forecasted path, use same algorithm employed for unconditional forecasts (i.e. draw θ's from some (prior) distributions).

• Conditional Forecast 2: Manipulating endogenous states

Back out the shocks needed to produce the path $\bar{y}_{2t+\tau}$, $\tau = 0, 1, 2, \ldots$. Use the first equation of (45) to do this. Then $y_{1t+\tau} = A_1(\theta)\bar{y}_{2t+\tau}$, $\tau = 1, 2, \ldots$. Same as above to compute uncertainty around the forecasted path.

Identification problem: there may be different elements of y_{3t} which may induce the require path for $y_{2t+\tau}$.

Example 4.15 *What is the range of paths for consumption from next quarter up to 10 years if the capital stock is higher by ten percent in all these periods? Question: how do we increase the capital stock? Via technology shocks? Via labor supply shocks?*

• Conditional Forecast 3: Manipulating endogenous controls. Separate $y_{1t} = [y^A_{1t}, y^B_{1t}]$ and $y^A_{1t+\tau} = \bar{y}^A_{1t+\tau}$, $\tau = 0, 1, 2, ...$. Back out the path of $y_{2t+\tau}$ needed to produce $\bar{y}^A_{1t+\tau}$. With this path compute $y^B_{1t+\tau}$. Same identification problems as above; less problematic.

Example 4.16 *Suppose that interest rates are (discretionarily) kept 50 basis point higher than the endogenous Taylor rule would imply. What is the effect on inflation?*

4.2.8 Regression methods: out-of-sample checks.

Out-of-sample RMSE, percentage points, inflation forecasts

Model	1 quarter	4 quarters	8 quarters
ARIMA	1.43	2.16	2.92
BVAR-TVC	1.21	1.72	1.89
NK	1.33	1.58	1.87

Out-of-sample Predictive regressions, estimates of b

Model	1 quarter	4 quarters	8 quarters
ARIMA-NK	0.35 (1.71)	0.42 (1.97)	0.34 (2.00)
BVAR-NK	0.17 (1.66)	0.35 (1.89)	0.44 (2.06)

5 Sensitivity of the measurement

Suppose we are happy with the model and perform a measurement.

Do we trust the measurement? To evaluate need to repeat experiment taking into account

a) data uncertainties (measurement error, possibly due to different filtering approaches).

b) model uncertainties (parameters and shocks)

• With probabilistic methods, automatically calculate measurement uncertainty (see e.g. example 4.4)

Is there a simple way to summarize uncertainty in the measurement?

- Elasticity measures (uncertainty due to parameters): $-\theta\frac{\mathfrak{S}''(\theta)}{\mathfrak{S}'_y(\theta)} \approx -\theta\frac{((\mathfrak{S}_y(\theta+\iota)/\iota)+\upsilon)/\upsilon}{\mathfrak{S}_y(\theta+\iota)/\iota}$. How much does $\mathfrak{S}_h$ changes when we change θ?.

- Elasticity measures (uncertainty due to data): $\mathfrak{S}_y$ changes when we change y_t? (can do by bootstrapping, by taking regional or international data, etc.).

6 An example

Question: Can an RBC model generate the high saving-investment correlation observed in OECD countries?

Two country RBC model, single consumption good, labor is immobile.

$$E_0 \sum_{t=0}^{\infty} \frac{\beta^t}{1-\varphi}[C_{it}^{\vartheta}(1-N_{it})^{(1-\vartheta)}]^{1-\varphi}$$

$$GDP_{it} = \zeta_{it}(K_{it})^{1-\eta}(X_{it}N_{it})^{\eta} \quad i = 1,2$$

$$K_{it+1} = (1-\delta_i)K_{it} + \frac{b}{2}(\frac{K_{it+1}}{K_{it}} - 1)^2 K_{it}, \quad \imath = 1,2$$

where C_{it} is consumption, $1 - N_{it}$ is leisure, β is the discount factor, $1 - \vartheta(1-\varphi)$ the coefficient of relative risk aversion, ϑ the share of consumption in utility, K_t is capital, η is the share of labor in GDP, $X_{it} = gnX_{it-1}\ \forall i$, $gn \geq 1$, b is a parameter. Assume:

$$\begin{bmatrix} \ln \zeta_{1t} \\ \ln \zeta_{2t} \end{bmatrix} = \begin{bmatrix} \bar{\zeta}_1 \\ \bar{\zeta}_2 \end{bmatrix} + \begin{bmatrix} \rho_1 & \rho_2 \\ \rho_2 & \rho_1 \end{bmatrix} \begin{bmatrix} \ln \zeta_{1t-1} \\ \ln \zeta_{2t-1} \end{bmatrix} + \begin{bmatrix} \epsilon_{1t} \\ \epsilon_{2t} \end{bmatrix}$$

where $\epsilon_t = [\epsilon_{1t}\ \ \epsilon_{2t}]' \sim \mathbb{N}(0, \begin{bmatrix} \sigma_\epsilon^2 & \sigma_{12} \\ \sigma_{12} & \sigma_\epsilon^2 \end{bmatrix})$ and $[\bar{\zeta}_1, \bar{\zeta}_2]'$ is a vector of constants. Here σ_{12} controls the contemporaneous and ρ_2 the lagged spillover of the shocks. Government budget constraint:

$$G_i = T_{it} + T_i^y GDP_{it}$$

The resource constraint is:

$$\Psi(GDP_{1t} - G_{1t} - C_{1t} - K_{1t+1} + K_{it}) + (1 - \Psi)(GDP_{2t} - G_{2t} - C_{2t} - K_{2t+1} + K_{it}) \geq 0 \quad (46)$$

where Ψ is the fraction of world population living in country 1.

Actual saving are computed as $Sa_t = GDP_t - C_t - G_t$.

Data refers to the period 1970:1-1993:3 for US and for Europe after linear trend elimination.

With the model, generate samples of T=95 and replicate 500 times.

Evaluate the model using the diagonal elements of the 4×4 spectral density matrix of the data (savings and investment for the two countries) and the coherence between saving and investment in the two countries.

Parameters selection

Parameter	Basic	Empirical Density	Subjective Density
Share of consumption (ϑ)	0.5	Uniform [0.3,0.7]	Normal (0.5, 0.02)
Steady State hours (N^{ss})	0.20	Uniform[0.2, 0.35]	Normal (0.2, 0.02)
Discount Factor (β)	0.9875	Trunc. N [0.9855, 1.002]	Normal(0.9875, 0.01)
Utility Power (φ)	2.00	Trunc. $\chi^2(2)[0, 10]$	Normal(2, 1)
Share of Labor (η)	0.58	Uniform[0.50, 0.75]	Normal(0.58, 0.05)
Growth rate (gn)	1.004	Normal(1.004, 0.001)	1.004
Depreciation Rate (δ)	0.025	Uniform[0.02, 0.03]	Normal(0.025, 0.01)
Persistence (ρ_1)	0.93	Normal(0.93, 0.02)	Normal(0.93, 0.025)
Lagged Spillover (ρ_2)	0.05	Normal(0.05, 0.03)	Normal(0.05, 0.02)
Standard Deviation of technology (σ_ϵ)	0.00852	Trunc. $\chi^2(1)$ [0, 0.0202]	Normal(0.00852, 0.004)
Immediate Spillover (σ_{12})	0.40	Normal(0.35, 0.03)	Normal(0.4, 0.02)
Country Size (Ψ)	0.50	Uniform[0.10, 0.50]	0.5
Adjustment cost (b)	1.0	1.0	1.0
Tax Rate (T^y)	0.0	0.0	0.0

The Fit of the Model

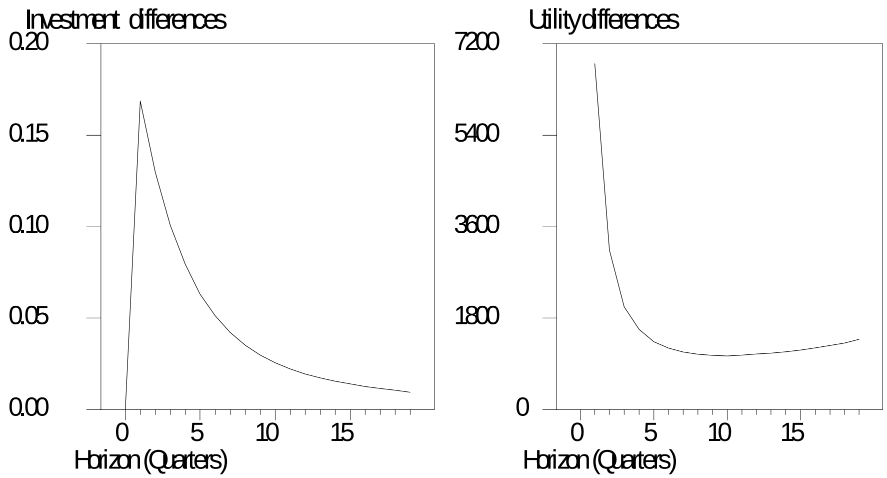
	US Spectra		Europe Spectra		US Coherence	Europe Coherence
	Sa	Inv	Sa	Inv	Sa-Inv	Sa-Inv
Actual data	0.75	0.88	0.68	0.49	85.41	93.14
Simulated data (fixed parameters)	0.36	0.18	0.35	0.18	94.04	93.00
Covering						
Fixed parameters	46.46	8.63	55.71	43.57	98.99	92.91
Subjective density	35.30	23.40	32.89	37.00	98.17	90.34
Empirical density	19.63	18.60	21.11	20.20	94.71	95.69
Critical Value						
Fixed parameters	90.80	99.89	82.16	93.91	15.60	49.04
Subjective density	71.80	89.90	66.00	76.60	19.80	51.89
Empirical density	62.50	79.70	73.30	74.60	33.46	29.60
Error						
Fixed parameters	0.25	0.55	0.30	0.28	-9.17	0.37
Subjective density	0.19	0.56	0.29	0.28	-9.01	0.81
Normal density	0.13	0.58	0.42	0.35	-6.07	-2.86

Covering = how many times on average, at business cycle frequencies, the diagonal elements of the spectral density matrix and the coherences of model generated data lie within a 95% confidence band for the corresponding statistics of actual data.

Critical Value = percentile of the simulated distribution of the spectral density matrix of saving and investment in the two countries where the value of the spectral density matrix of the actual data (taken here to be estimated without an error) lies, on average, at business cycle frequencies.

Error = median error (across simulations) needed to match actual spectral density with model.

Experiment: What is the effects of tax cuts from 0.20 to 0.00?



Compensating variations: 0.11 each period, 14% of C^{ss}.

For $\vartheta \in [0.3, 0.7]$ and $\varphi \in [1, 4]$, average compensating variation is [0.09,0.12] each period.

7 Criticisms to calibration exercises

- Choice of parameters is often arbitrary/incoherent (Canova, 1994, Hansen and Heckman, 1996).

- Incoherent use of loss functions (Hansen and Heckman,1996). Use different loss functions to choose different parameters.

- Models with same input but different specification of the primitives (e.g. CES utility vs non-expected utility) may produce different measurements.

- Straight-jacket (Pesaran and Smith, 2011) or discipline provider (Kydland and Prescott 1996)?

8 Exercises

Exercise 1 *Consider the RBC model*

$$\max E_0 \sum_t \beta^t \frac{c_t^{1-\varphi}}{1-\varphi} + \log(1 - N_t) \tag{47}$$

$$c_t + k_{t+1} - (1-\delta)k_t = N_t^{\eta} k_t^{1-\eta} \zeta_t \tag{48}$$

$E\zeta_t = \zeta^{ss}$; $\hat{\zeta} \equiv (\zeta_t - \zeta^{ss})/\zeta^{ss} = \rho\hat{\zeta}_{t-1} + e_t,\ e_t \sim (0, \sigma^2)$.

a) Log linearize the equilibrium conditions.

b) Derive the steady states and appropriately calibrate the parameters of the model. Describe how you choose the free parameters.

c) Find the decision rules (the matrices of the solution). Describe at least two set of restrictions that the model imposes on the data.

d) Test these implications using data from your favorite country.

Exercise 2 *Consider the RBC model*

$$\max E_0 \sum_t \beta^t \frac{c_t^{1-\varphi}}{1-\varphi} + \log(1 - N_t) \tag{49}$$

$$c_t + k_{t+1} - (1-\delta)k_t = N_t^{\eta} k_t^{1-\eta} \zeta_t \tag{50}$$

$E\zeta_t = \zeta^{ss}$; $\hat{\zeta} \equiv (\zeta_t - \zeta^{ss})/\zeta^{ss} = \rho\hat{\zeta}_{t-1} + e_t, \; e_t \sim (0, \sigma^2)$.

a) Log linearize the equilibrium conditions.

b) Choose a range for the parameters of the model. Trace out how $var(c)/var(y)$ varies as we change φ, σ^2. Compute $var(c)/var(y)$ in the data of your county. Does the model fit well?

c) Study what would be the effect of a 10 percent increase in the volatility of technology shocks on $var(c)/var(y)$.

Exercise 3 *Consider the model with capacity utilization:*

$$\max E_0 \sum_t \beta^t [\ln c_t + \vartheta_l (1 - N_t)] \tag{51}$$

$$c_t + i_t = \zeta_t (K_t k u_t)^{1-\eta} N_t^{\eta} \tag{52}$$

$E\zeta_t = \zeta^{ss}$; $\hat{\zeta} \equiv (\zeta_t - \zeta^{ss})/\zeta^{ss} = \rho \hat{\zeta}_{t-1} + e_t,\ e_t \sim (0, \sigma^2)$, *where* ku_t *is capacity utilization (and is a choice variable), capital accumulates according to* $K_{t+1} = (1 - \delta(ku_t))K_t + i_t$ *and* $\delta(ku_t) = \delta_1 k u_t^{\delta_2}$ *where* δ_1, δ_2 *are parameters.*

a) Log linearize the equilibrium conditions.

b) Derive the steady state and calibrate the parameters of the model. Choose δ_1, δ_2 *so that utilization is 1 in the steady state. Describe how you choose the free parameters.*

c) Find the decision rules (the matrices of the solution). Construct one and two steps ahead forecasts for output assuming $k_0 = k^{ss} + 0.01$*.*

d) Simulate output data using $y_t = y_{t-1} + e_t,\ e_t \sim N(0, 0.05)$ *and* $y_0 = 100$*. How do the forecast of the model compare to the simulated data? Calculate the mean square error.*

Exercise 4 *Consider the model with capacity utilization:*

$$\max E_0 \sum_t \beta^t [\ln c_t + \vartheta_l (1 - N_t)] \tag{53}$$

$$c_t + i_t = \zeta_t (K_t k u_t)^{1-\eta} N_t^\eta \tag{54}$$

$E\zeta_t = \zeta^{ss}$; $\hat{\zeta} \equiv (\zeta_t - \zeta^{ss})/\zeta^{ss} = \rho\hat{\zeta}_{t-1} + e_t,\ e_t \sim (0, \sigma^2)$, *where* ku_t *is capacity utilization (and is a choice variable), capital accumulates according to* $K_{t+1} = (1 - \delta(ku_t))K_t + i_t$ *and* $\delta(ku_t) = \delta_1 ku_t^{\delta_2}$ *where* δ_1, δ_2 *are parameters.*

a) Log linearize the equilibrium conditions.

b) Choose a range for the parameters of the model. Trace out how the first autoregressive coefficient of simulated of output changes as we change δ_2, η. *Compute the first order autoregressive coefficient of output in the data of your favorite country. Does the model fit?*

c) Study what would be the effect a 10 percent drop in the persistence of technology shocks (ρ) *on the autoregressive coefficient of simulated output.*