Money, Financial Stability and Efficiency*

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Abstract

Most analyses of banking crises assume that banks use real contracts. However, in practice contracts are nominal and this is what is assumed here. We consider a standard banking model with aggregate return risk, aggregate liquidity risk and idiosyncratic liquidity shocks. We show that, with non-contingent nominal deposit contracts, the first-best efficient allocation can be achieved in a decentralized banking system. What is required is that the central bank accommodates the demands of the private sector for fiat money. Variations in the price level allow full sharing of aggregate risks. An interbank market allows the sharing of idiosyncratic liquidity risk. In contrast, idiosyncratic (bank-specific) return risks cannot be shared using monetary policy alone; real transfers are needed.

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1 Introduction

Most models in the banking literature (e.g., Diamond and Dybvig, 1983; Chari and Jagannathan, 1988; Jacklin and Bhattacharya, 1988; Calomiris and Kahn, 1991; Allen and Gale, 1998, 2000; Diamond and Rajan, 2001, 2005) treat banking as a real activity with no role for fiat money. Following Diamond and Dybvig (1983), consumers' liquidity preference is modeled as uncertainty about their time preference for consumption. Liquid assets are modeled as a storage technology. A deposit contract promises a depositor a fixed amount of consumption depending on the date of withdrawal. Thus, a crisis can arise when a large number of consumers decide to withdraw their deposits from the banking system, because the demand for goods is greater than the banks' limited stock of liquid assets.

While "real" models have provided valuable insights into the nature of financial fragility, they do not capture important aspects of reality, such as the role of fiat money in the financial system. In practice, financial contracts are almost always written in terms of money. This fact has important consequences for the theory. Because the central bank can costlessly create fiat money in a crisis, there is no reason why the banking system should find itself unable to meet its commitments to depositors. As Willem Buiter (2007) has argued,

> "Liquidity is a public good. It can be managed privately (by hoarding inherently liquid assets), but it would be socially inefficient for private banks and other financial institutions to hold liquid assets on their balance sheets in amounts sufficient to tide them over when markets become disorderly. They are meant to intermediate short maturity liabilities into long maturity assets and (normally) liquid liabilities into illiquid assets. Since central banks can create unquestioned money at the drop of a hat, in any amount and at zero cost, they should be the money providers of last resort both as lender of last resort and as market maker of last resort...."

In this paper, we develop a model, based on Allen, Carletti and Gale (2009), henceforth ACG, in which fiat money is issued by the central bank. Deposit contracts and loan contracts are denominated in terms of money and money is used in transactions. In other words, money

is both a unit of account and a medium of exchange.[1] In contrast to most of the banking literature, which is reviewed in detail below, we show that the combination of nominal contracts and a central bank policy of accommodating commercial banks' demand for money leads to first best efficiency. This result holds when there are aggregate liquidity and asset return shocks and also when there are idiosyncratic (bank specific) liquidity shocks.

Time is represented by a sequence of three dates and, at each date, there is a single good that can be used for consumption or investment. Assets are represented by constant returns to scale technologies that allow the the consumers' initial endowment of the good to be transformed into consumption at the second and third dates. The short-term asset is represented by a storage technology: one unit of the good invested in this technology yields one unit of the good at the next date. The long-term asset is represented by a technology that requires an investment at the initial date and yields a random return at the final date. The expected return of the long-term asset is greater than the return of the short-term asset.

There is a large number of ex ante identical consumers, each of whom is endowed with one unit of the good at the initial date. At the beginning of the second date, each consumer receives a time-preference shock that makes him either an early consumer, who wants to consume only at the second date, or a late consumer, who wants to consume only at the third date. The proportion of early and late consumers is itself random, an important source of aggregate uncertainty.

We characterize the first best allocation as the solution to a planner's problem. The planner invests the consumers' endowments in a portfolio of short- and long-term assets and then distributes the returns to these assets to the early and late consumers. The portfolio is chosen before the realization of the aggregate state, that is, the fraction of early consumers and the return on the risky asset. The consumption allocation is determined after the realization of the aggregate state and is therefore state contingent. We then show how this allocation can be implemented using a simple institutional structure and non-contingent nominal contracts.

In the decentralized economy, there are three types of institutions, a central bank, commercial banks and firms. At the initial date, the central bank makes money available to

[1] In what follows, "money" refers to fiat money issued by the central bank.

the commercial banks on an intraday basis at a zero interest rate. The banks make loans to the firms and the firms in turn use the money to buy the consumers' endowments and invest them in the short- and long-term assets. At the intermediate and final dates, the central bank again makes intraday loans to the banks. The banks use this money to pay for depositors' withdrawals. The depositors in turn use the money to purchase goods from the firms. Then the firms use the same money to repay their loans to their banks and the banks use it to repay the central bank. The central bank's policy is passive: at each date it supplies the amount of money demanded by the commercial banks. Commercial banks and firms are assumed to be profit maximizing but in a competitive equilibrium they earn zero profit. Consumers are expected utility maximizers, but in equilibrium their decision problem is simple: they deposit the money received in exchange for the sale of their endowments at the first date and withdraw and spend all their money at the second or third date, depending on whether they are early or late consumers.

The main features of our model are the following:

- A competitive equilibrium implements the same state-contingent allocation as the planner's problem, even though deposit contracts represent a fixed claim (in terms of money) on the banks.

In spite of the debt-like nature of the deposit contract, it is possible to implement a state-contingent allocation because deposit contracts are written in terms of money. Regardless of the liquidity and asset return shocks, banks are able to meet their commitments as long as the central bank supplies them with sufficient amounts of fiat money. The price level adjusts in response to aggregate shocks in order to clear markets. When the number of early consumers is high, the amount of money withdrawn from the banks is also high and this increases the price level. When the returns on the long asset are low, the supply of goods is also low and this increases the price level. The adjustments in the price level ensure that early and late consumers' receive the efficient, state-contingent levels of consumption.

- A central bank policy of passively accommodating the demands of the commercial banks for money is sufficient to eliminate financial crises and achieve the first best.

The role of the central bank is simply to provide the necessary money so that each bank can meet withdrawals by its depositors. Price level adjustments lead to the of optimal level of real balances and the optimal allocation of consumption at each date.

- The quantity theory of money holds in equilibrium: the price level at each date is proportional to the supply of money extended to the commercial banks by the central bank.

This result follows from the market-clearing condition in the goods market at each date. More surprisingly, money is super-neutral in this model.

- The central bank can control the nominal interest rate and the expected inflation rate, but it has no effect on the equilibrium allocation of goods.

It is crucial that fiat money only circulates *within* the trading day. No one holds fiat money *between* dates. Instead, consumers hold bank deposits and banks hold loans. Deposits and loans are denominated in terms of money, but they are also interest bearing, so any change in the expected inflation rate is compensated by a change in the nominal interest rate. Thus, money is not merely neutral, it is super-neutral.

The baseline model can be extended in a number of ways. We can introduce idiosyncratic (bank-specific) liquidity shocks without upsetting the efficiency results. The interbank market allows banks to reshuffle money between banks that receive high and low liquidity shocks at the second date so that each bank can meet the required level of withdrawal by its depositors, without being subject to distress. The process is reversed at the third date, so that banks with a large proportion of late consumers can meet the higher number of withdrawals then. We can also extend the efficiency result to a multi-period setting.

- First best efficiency can be achieved by monetary policy alone when the model is extended to allow for idiosyncratic (bank-specific) liquidity risk and multiple periods.

Accommodative monetary policy alone is not always sufficient to achieve efficiency, however.

- Monetary policy alone is not sufficient to allow the sharing of idiosyncratic (bank-specific) asset return risk.

If the banks' asset-specific returns are observable, the government could introduce an insurance scheme. Alternatively, a private scheme could achieve the same end by securitizing the assets and allowing banks to hold a diversified portfolio of asset backed securities. Such schemes are vulnerable to moral hazard if there is asymmetric information about asset returns. Insuring low returns gives banks an incentive to engage in asset substitution and to misrepresent the realized returns of the assets. Clearly, pooling idiosyncratic return risks is more difficult than implementing an accommodative monetary policy.

The results obtained from our monetary model of banking stand in stark contrast to those obtained from models with real contracts. ACG is a case in point. In their model, banks face uncertain liquidity demands from their customers at the second date. If this uncertainty is idiosyncratic and there is no uncertainty about aggregate liquidity preference, the interbank market efficiently redistributes liquid assets among banks, allowing each bank to meet the needs of its customers. If there is uncertainty about aggregate liquidity preference, on the other hand, it is not sufficient merely to re-shuffle the existing stock of the liquid asset. The long term-asset will have to be sold to obtain additional liquidity and this may require changes in interest rates and asset prices. Some banks will suffer capital losses and may not be able to meet their commitments to their customers. It is critical here that markets are assumed to be incomplete, so that there is no way the banks can hedge their liquidity shocks.

ACG show that the introduction of a central bank may solve this problem. The central bank engages in open market operations to fix the price of the long asset at the end of the first period (or equivalently fix the short term interest rate). This intervention removes the inefficiency associated with a lack of hedging opportunities and allows the banks to implement the constrained efficient allocation provided there is no bankruptcy. This result is in line with the argument of Goodfriend and King (1988) that open market operations are sufficient to address pure liquidity risk on the interbank market.

The ACG result is surprising because it suggests that open market operations aimed at providing adequate aggregate liquidity can, at the same time, implement constrained-efficient risk sharing in the presence of aggregate uncertainty about the timing of consumption. In fact, the intervention of the central bank seems to go beyond the normal scope of monetary policy. In addition to providing money by buying and selling government bonds, the central

bank is responsible for actions that normally fall within the purview of the Treasury, such as varying the size of the national debt and levying lump sum taxes to pay for the debt. As Kiyotaki (2009) points out, this kind of "heavy" intervention in the economy makes the efficiency result less surprising and perhaps less likely to be implemented by the central bank.

The rest of the paper proceeds as follows. The remainder of this section considers the related literature in detail. Section 2 describes the primitives of the real economy. The efficient allocation is derived in Section 3, where we describe and solve the appropriate planner's problem. In Section 4, we introduce a financial system with money as a medium of exchange and define the equilibrium of this economy. The main results are found in Section 5, where we show that the efficient allocation can be decentralized as an equilibrium. A number of simple extensions are considered. In Section 6.1, the results of Section 4 are extended to allow for arbitrary nominal interest rates. In Section 6.2, we allow for idiosyncratic liquidity shocks to individual banks and show that the interbank market allows the efficient allocation to be decentralized in this case too. Section 6.3 considers the case of idiosyncratic return risk. The multi-period case is considered in Section 6.4. Finally, Section 7 contains some concluding remarks. Some of the longer proofs are relegated to the appendixes in Sections 8 and 9.

1.1 Related literature

As we have noted, most of the literature on banking crises has assumed contracts are written in real terms. The papers that have considered fiat money and banking crises can be divided into two strands. The first introduces banks into models of fiat currency. Many of these models seek to explain historical crises that occurred at a time when fiat currency played an important role in the financial system. Skeie (2008) points out that in modern financial systems, fiat currency no longer plays a very significant role. The vast majority of transactions involve the transfer of money from one bank account to another. The second strand of the literature considers this type of financial system without introducing fiat currency.

An important contribution to the first strand is Champ, Smith and Williamson (1996). They address the issue of why Canada had no banking crises in the late nineteenth and early twentieth centuries while the U.S. had many. Their explanation is that Canada

allowed the amount of money in circulation to expand to meet demand during harvest time while the U.S. financial system was such that this could not happen. The effect of this difference was that in Canada liquidity shocks could be easily absorbed but in the U.S. they led to banking panics. Since currency played an important role during this period, they use an overlapping generations model with two-period lived consumers to justify the use of currency. The consumers live in two different locations. Instead of random preference shocks as in Diamond and Dybvig (1983), consumers are subject to relocation shocks. Each period a random proportion of young consumers in each location is forced to move to the other location. These shocks are symmetric so that the population in each place remains constant. Banks make risk-free loans, hold reserves of currency, issue bank notes and write deposit contracts that are contingent on the proportion of the consumers that relocate. When young consumers relocate they can transport currency or the notes issued by the banks with them but nothing else. The authors show that if the banks are allowed to vary their issuance of notes to accommodate the different levels of relocation shocks then there exists a stationary Pareto optimal equilibrium. In this equilibrium, currency and bank notes are perfect substitutes and the nominal interest rate is zero. However, if the bank note issuance is fixed so the random relocation demand cannot be accommodated, there will be a banking crisis if the shock is large enough to exhaust the banks' currency reserves. The authors interpret these two possibilities as being consistent with the Canadian and U.S. experiences from 1880-1910, respectively.

Antinolfi, Huybens and Keister (2001) build on the model of Champ, Smith and Williamson (1996) by replacing the private issue of bank notes with a lender of last resort that is willing to lend freely at a zero nominal interest rate. A stationary Pareto optimal equilibrium again exists but in addition there is a continuum of nonoptimal inflationary equilibria. Antinolfi, Huybens and Keister are able to show that these can be eliminated if the lender of last resort places an appropriately chosen upper bound on the amount that each individual bank can borrow or is willing to lend freely at a zero real interest rate. Smith (2002) considers a similar model but without elastic money supply. He shows that the lower the inflation rate and nominal interest rate, the lower is the probability of a banking crisis. Reducing the inflation rate to zero in line with the Friedman rule eliminates banking crises. However, this

is inefficient as it leads banks to hold excessive cash reserves at the expense of investment in higher yielding assets.

Cooper and Corbae (2002) consider a model with increasing returns to scale in the intermediation process between savers and entrepreneurs. This leads to multiple equilibria that are interpreted as different levels of confidence. A calibrated version of the model with low confidence levels is able to match many features of the Great Depression.

Diamond and Rajan (2001) develop a model where banks have special skills to ensure that loans are repaid. By issuing real demand deposits, banks can precommit to recoup their loans. This allows long term projects to be funded and depositors to consume when they have liquidity needs. However, this arrangement leads to the possibility of a liquidity shortage in which banks curtail credit when there is a real shock. Diamond and Rajan (2006) introduce money and nominal deposit contracts into this model to investigate whether monetary policy can help alleviate this problem. They assume there are two sources of value for money. The first arises from the fact that money can be used to pay taxes (the fiscal value). The second is that money facilitates transactions (the transactions demand). They show that the use of money can improve risk sharing since price adjustments introduce a form of state contingency to contracts. However, this is not the only effect. Variations in the transactions value of money can lead to bank failures. Monetary intervention can help to ease this problem. If the central bank buys bonds with money, this changes liquidity conditions in the market and allows banks to fund more long-term projects than would be possible in the absence of intervention.

Allen and Gale (1998) develop a model of banking crises caused by asset return uncertainty with three dates, early and late consumers as in Diamond and Dybvig (1983), and initially, real contracts. Building on the empirical work of Gorton (1988), it is assumed that at the intermediate date investors receive a signal concerning the return of the banks' long term assets. If the signal indicates returns are sufficiently low, the late consumers will withdraw their deposits along with the early consumers and there will be a banking crisis. Allen and Gale go on to show that if contracts are written in nominal terms and a central bank can supply money to commercial banks then the incentive-efficient allocation can be implemented. The central bank gives money to the banks and they then pay this out to-

gether with goods to depositors. The early depositors use their money to buy goods from early withdrawing late consumers who then hold money until the final date. Variations in the price level allow risk sharing.

Cao and Illing (2011) develop a model where banks can invest in a liquid asset with a return one period away or an illiquid asset with a higher return but where the date of payoff is random. It could be one or two periods away. They consider the case where a central bank can create money as in Allen and Gale (1998) and prevent a banking crisis when payout from the illiquid asset is delayed. The problem is that this policy creates a moral hazard. The fact that banks know that the central bank will provide liquidity when there is a shortage makes them more willing to invest in the illiquid asset. In the equilibrium with intervention, they overinvest relative to the efficient allocation. Imposing equity requirements does not solve this problem. However, a policy of imposing ex ante liquidity requirements and having the central bank create money ex post does allow the optimal second best contract to be implemented.

The second strand of papers starts with Skeie (2008), who develops a standard banking model with nominal contracts and fiat money within the banking system. Depositors are subject to preference shocks in the usual way. There is no aggregate liquidity risk or return uncertainty. In contrast to Diamond and Dybvig (1983) he shows that there is a unique equilibrium and it is efficient. If deposits are withdrawn by late consumers at the intermediate date the price of the consumption good adjusts and this discourages such withdrawals. In order for there to be runs on banks there must be some other friction such as problems in the interbank market. Freixas, Martin and Skeie (2009) develop a model with aggregate liquidity risk, which like ACG also has idiosyncratic liquidity shocks to banks. The main part of their paper undertakes a real analysis where they show that there can be multiple equilibria. The central bank can determine the interest rate to implement the equilibrium with the efficient allocation. In an appendix, they show that money can be introduced along the lines of Skeie (2008) and the same results hold.

The current paper belongs in this second strand of literature. All payments are made with fiat money and money can be created costlessly by the central bank. In contrast to the other papers surveyed here, it is shown that first best efficiency, rather than just

incentive or constrained efficiency, can be achieved in a wide range of situations. This includes aggregate and idiosyncratic liquidity risk as well as aggregate asset return risk. An accommodative monetary policy can prevent crises and implement an efficient allocation of resources. Idiosyncratic asset return shocks are more difficult to deal with. They require more intensive intervention by the government or private risk-sharing institutions. The model provides a benchmark for studying more realistic models with market imperfections and the interventions required to correct them.

2 The real economy

In this section we describe the primitives of the real economy. The model is based on ACG. There are three dates $t = 0, 1, 2$ and a single good that can be used for consumption or investment at each date.

There are two assets, a short-term asset that we refer to as the *short asset* and a long-term asset that we refer to as the *long asset*. The short asset is represented by a riskless storage technology, where one unit of the good invested at date t produces one unit of the good at date $t + 1$, for $t = 0, 1$. The long asset is a constant-returns-to-scale investment technology that takes two periods to mature: one unit of the good invested in the long asset at date 0 produces a random return equal to R units of the good at date 2.

There is a large number (strictly, a continuum with unit measure) of identical consumers. All consumers have an endowment of one unit of the good at date 0 and nothing at dates 1 and 2. Consumers are uncertain about their future time preferences. With probability λ they are *early consumers*, who only value the good at date 1, and with probability $1 - \lambda$ they are *late consumers*, who only value the good at date 2. The fraction of early consumers λ is a random variable. The utility of consumption is denoted by $u(c)$ where $u(\cdot)$ is a von Neumann Morgenstern utility function with the usual properties.

We assume that the random variables λ and R have a joint cumulative distribution function F. We assume that, the support of F is the interval $[0, 1] \times [0, R^{\max}]$. The mean of R is denoted by $\bar{R} > 1$ and the mean of λ is denoted by $0 < \bar{\lambda} < 1$. Since all consumers are symmetric, $\bar{\lambda}$ is also the probability that a typical consumer is an early consumer.

Uncertainty about time preferences generates a preference for liquidity and a role for intermediaries as providers of liquidity insurance. The expected utility of a consumption profile (c_1, c_2) is given by

$$\bar{\lambda} u(c_1) + \left(1 - \bar{\lambda}\right) u(c_2),$$

where $c_t \geq 0$ denotes consumption at date $t = 1, 2$.

All uncertainty is resolved at the beginning of date 1. In particular, the state (λ, R) is revealed and depositors learn whether they are early or late consumers. While each depositor's realization of liquidity demand is private information, the state (λ, R) is publicly observed.

3 The efficient allocation

Suppose that a central planner were to make all the investment and consumption decisions in order to maximize the expected utility of the representative consumer. At the first date, the planner would invest the representative consumer's endowment of 1 unit of the good in a portfolio consisting of $0 \leq y \leq 1$ units of the short asset and $1 - y$ units of the long asset. Then, at the second date, once the aggregate state of nature (λ, R) is known, the planner would assign $c_1(\lambda, R)$ units of the good to the representative early consumer and $c_2(\lambda, R)$ units of the good to the representative late consumer. The total amount of consumption available at date 1 is given by y, the amount invested in the short asset. Since the fraction of early consumers is λ, the planner's allocation will be feasible at date 1 if and only if

$$\lambda c_1(\lambda, R) \leq y, \tag{1}$$

for every aggregate state (λ, R). The left hand side of (1) is the total amount consumed at date 1 and the right hand side is the total supply of goods. If the amount consumed, $\lambda c_1(\lambda, R)$, is less than the total supply, y, the difference

$$S(\lambda, y) = y - \lambda c_1(\lambda, R),$$

is stored until the last period. At date 2, the fraction of late consumers is $1-\lambda$ so the planner's allocation will be feasible if and only if

$$\begin{aligned}(1-\lambda)\,c_2(\lambda,R) &= (1-y)\,R+S(\lambda,R) \\ &= (1-y)\,R+y-\lambda c_1(\lambda,R)\,,\end{aligned} \tag{2}$$

for every aggregate state (λ, R). The left hand side of (2) is total consumption at date 2 and the right hand side is the total supply of the good. We assume the two sides are equal since all of the good must be used up at the last date. Re-arranging the terms in the equation above, we can re-write this condition in terms of total consumption at dates 1 and 2 and the total returns of the two assets:

$$\lambda c_1(\lambda,R)+(1-\lambda)\,c_2(\lambda,R)=y+(1-y)\,R. \tag{3}$$

The planner's task is to maximize the expected utility of the representative consumer subject to the feasibility constraints (1) and (3). A necessary condition for maximizing the expected utility of the representative consumer is that, given the portfolio y chosen at the first date, the expected utility of the representative consumer is maximized in each aggregate state (λ, R). In other words, for a given value of y and a given state (λ, R), the allocation must maximize the representative consumer's expected utility subject to the feasibility conditions (1) and (3). This problem can be written as

$$\begin{array}{ll}\max & \lambda u(c_1)+(1-\lambda)\,u(c_2) \\ \text{s.t.} & \lambda c_1\leq y,\ \lambda c_1+(1-\lambda)\,c_2=y+(1-y)\,R.\end{array} \tag{4}$$

This problem has a very simple yet elegant solution. Either there is no storage, in which case $\lambda c_1 = y$ and $(1-\lambda)\,c_2=(1-y)\,R$, or there is positive storage between the two dates, in which case $c_1=c_2=y+(1-y)\,R$. This solution can be summarized by the following two "consumption functions,"

$$c_1(\lambda,R) = \min\left\{\frac{y}{\lambda}, y+(1-y)\,R\right\}, \tag{5}$$

$$c_2(\lambda,R) = \max\left\{\frac{(1-y)\,R}{1-\lambda}, y+(1-y)\,R\right\}. \tag{6}$$

These consumption functions are illustrated in Figure 1 below.

— Figure 1 about here —

The left hand panel illustrates the relationship between consumption and R, holding λ constant. For very small values of R, the late consumers would receive less than the early consumers if there were no storage. This cannot be optimal, so some of the returns of the short asset will be re-invested up to the point where consumption is equalized between early and late consumers. At some critical value of R, the long asset provides just enough to equalize the consumption of early and late consumers without storage. For higher values of R, early consumers consume the output of the short asset (i.e., there is no storage). Late consumers consume the entire output of the long asset and their per capita consumption is increasing in R.

The right hand panel illustrates the relationship between consumption and λ, holding R constant. For small values of λ, the short asset provides more consumption than is needed by early consumers, so some is stored and given to late consumers. At the margin, the rate of exchange between early and late consumption is one for one, so optimality requires that early and late consumers receive the same consumption. For some critical value of λ, there is just enough of the short asset to provide early consumers the same amount of consumption as late consumers. That is, $\frac{y}{\lambda} = \frac{(1-y)R}{1-\lambda}$. For higher values of λ, early consumers continue to consume the entire output of the short asset but their per capita consumption is declining in λ. The late consumers by contrast, receive the entire output of the long asset and their per capita consumption is increasing as λ increases.

Note that the consumption functions in (5) and (6) are determined by the choice of y and the exogenous shocks (λ, R), so the planner's problem can be reduced to maximizing the expected utility of the representative consumer with respect to y. The optimal portfolio choice problem is:

$$\max_{y} \qquad E\left[\lambda u\left(c_1\left(\lambda, R\right)\right) + (1-\lambda) u\left(c_2\left(\lambda, R\right)\right)\right], \tag{7}$$

where $c_1(\lambda, R)$ and $c_2(\lambda, R)$ are defined in (5) and (6). The solution to the planner's problem is summarized in the following proposition.

Proposition 1 *The unique solution to the planner's problem consists of a portfolio choice*

y^* and a pair of consumption functions $c_1^*(\lambda, R)$ and $c_2^*(\lambda, R)$ such that y^* solves the portfolio choice problem (7) and $c_1^*(\lambda, R)$ and $c_2^*(\lambda, R)$ satisfy (5) and (6), respectively.

Proof. See Appendix A in Section 8. ■

4 Money and exchange

In this section we describe a decentralized economy consisting of four groups of actors, a central bank that provides fiat money to the private sector; a banking sector that borrows from the central bank, makes loans and takes deposits; a productive sector that borrows from the banking sector in order to invest in the short and long assets; and a consumption sector that sells its initial endowment to firms and deposits the proceeds in the banking sector to provide for future consumption.

The central bank's only function is to provide money that the private sector needs to facilitate transactions. It lends to banks on an intraday basis and charges zero interest. The central bank's policy is passive in the sense that it provides whatever amounts of money the banks demand.

There is free entry to the commercial banking sector. Banks compete for deposits by offering contracts that offer consumers future payments in exchange for current deposits. Consumers respond by choosing the most attractive of the contracts offered. Free entry ensures that banks offer deposit contracts that maximize consumers' welfare and earn zero profits in equilibrium. Otherwise, a bank could enter and make a positive profit by offering a more attractive contract. There is no loss of generality in assuming that consumers deposit all their money in a bank at date 0 since the bank can do anything the consumers can do. The bank promises the consumer D_1 units of money if he withdraws at date 1 and D_2 units of money if he withdraws at date 2.

There is free entry to the productive sector, which ensures that in equilibrium firms earn zero profits. Firms take out one period loans from banks in the first period and use the money to purchase goods from the consumers. These goods are invested in the two assets. Some of the returns from these assets are sold at date 1 and used to repay part of the firm's debt. The rest is rolled over and repaid at date 2 using the proceeds from selling the

remaining asset returns at date 2.

As described earlier, consumers have an initial endowment of goods which they sell in exchange for money at the first date. This money deposited in the consumers' bank accounts and provides income that can be used for consumption in future periods. Consumers have Diamond-Dybvig preferences and maximize expected utility.

We assume that all transactions are mediated by money. Money is exchanged for goods and goods for money, loans are made and repaid in terms of money. Only banks have access to loans from the central bank and only firms have access to loans from the banks. Consumers can save using deposit accounts at the banks. Given the timing of consumption, there is no need for consumers to borrow.[2] Consumers have the option to purchase goods and store them. Banks can lend to one another on an interbank market, but for the moment there is no need for this activity. There are no forward markets. These assumptions give rise to a particular flow of funds at each date, which we describe next.

Date 0 The transactions occur in the following order:

1. Banks borrow funds from the central bank.
2. Firms borrow from the banks.
3. Firms purchase goods from the consumers.
4. Consumers deposit the proceeds from the sale of goods in their bank accounts.
5. Banks repay their intraday loans to the central bank.

The flow of funds at date 0 is illustrated in Figure 2 below. We see that the money supply M_0 created by the central bank follows a circuit from the central bank to banks to firms to consumers to banks and, finally, back to the central bank. At each stage the same amount of money changes hands so that the net demand for money is zero at the end of the period.

— Figure 2 here —

[2] As Cone (1983) and Jacklin (1987) showed, consumers must be excluded from the market for borrowing and lending at date 1, otherwise they will undermine the ability of the banks to provide them with liquidity insurance.

Dates 1 and 2 At the beginning of the second date, all uncertainty is resolved. Each consumer learns whether he is an early or late consumer and the aggregate state (λ, R) is realized. Transactions occur in the same order at date 1 and date 2:

1. Banks borrow funds from the central bank.
2. Consumers withdraw their savings from their bank accounts.
3. Consumers use these funds to purchase goods from the firms.
4. Firms repay part of their loans to the banks.
5. Banks repay their intraday loans to the central bank.

The flow of funds is illustrated for date 1 in Figure 3. The pattern is the same for date 2. As before, the net demand for money at the end of the period is zero.

— Figure 3 here —

4.1 Market clearing and the price level

We assume that all trades in the economy are mediated by money. Fiat money is provided to commercial banks by the central bank. The central bank does not charge interest on intraday balances. At date 0, the banks lend money to the firms to allow them to purchase goods from the consumers who have the money transferred to their accounts in the banks. The banks use these deposits to repay the intraday loan to the central bank. At date 1, the banks again borrow money from the central bank and give it to the early consumers who choose to purchase goods from the firms at the second date. The firms then use the money to repay part of their loans to the bank. The banks return the money to the central bank. At date 2, this process is repeated: the banks borrow money from the central bank to pay the late consumers who spend the money on goods. The firms use the money to repay the balance of their loans and the banks return the money to the central bank.

The (nominal) interest rate on loans between periods t and $t+1$ is denoted by r_{t+1}. That is, one dollar borrowed at date t requires a repayment of $1 + r_{t+1}$ dollars at date $t+1$.

Without essential loss of generality we can set interest rates to zero: $r_1 = r_2 = 0$ (this assumption is relaxed in Section 6.1).

To describe an allocation of money and goods we need the following notation:

M_0 = money supply at date 0;

P_0 = price level at date 0;

y = investment in the short asset at date 0;

$M_t(\lambda, R)$ = money supply at date $t = 1, 2$ in state (λ, R);

$P_t(\lambda, R)$ = price level at date $t = 1, 2$ in state (λ, R);

$c_t(\lambda, R)$ = consumption at date $t = 1, 2$ in state (λ, R);

D_t = money value of deposit at date $t = 1, 2$.

The standard homogeneity property of excess demands with respect to prices allows us to normalize the price level at date 0 to unity:

$$P_0 = 1.$$

At date 0 the demand for money comes from firms, who need the money in order to buy goods from consumers. Since there is one unit of the good (per capita), firms will borrow one unit of money from the banks in order to purchase the goods. The banks demand this amount of money from the central bank, which therefore must supply the amount

$$M_0 = P_0 = 1 \tag{8}$$

to meet the banks' demand.

At date 1, early consumers withdraw their deposit D_1 from the bank and supply it inelastically in exchange for consumption goods. The amount needed by banks is therefore λD_1 and this is the amount supplied by the central bank:

$$M_1(\lambda, R) = \lambda D_1. \tag{9}$$

The firms supply either y, if $P_1(\lambda, R) > P_2(\lambda, R)$, or an amount less than or equal to y, if $P_1(\lambda, R) = P_2(\lambda, R)$. In the latter case, firms are indifferent about whether to sell or store the goods so, in equilibrium, they supply the amount demanded by consumers. Thus, the goods market clears if

$$\lambda c_1(\lambda, R) \leq y. \tag{10}$$

The firms return their revenue to the banks in partial payment of their debts and the remaining debt is rolled over.

At date 2, late consumers use their deposit D_2 in the bank and supply it inelastically in exchange for consumption goods. The amount needed by banks is therefore $(1-\lambda) D_2$ and this is the amount supplied by the central bank:

$$M_2(\lambda, R) = (1-\lambda) D_2. \tag{11}$$

The firms supply all their goods inelastically, that is the return from the long asset, $(1-y) R$, *plus* the amount stored from the previous period, $y - \lambda c_1(\lambda, R)$. Thus, the goods market clears if

$$\lambda c_1(\lambda, R) + (1-\lambda) c_2(\lambda, R) = (1-y) R + y. \tag{12}$$

The firms use the proceeds from their sales of the consumption good to repay their remaining debt to the banks.

4.2 The bank's decision

The representative bank's decision problem is quite simple. At the first date, the bank lends money to firms and accepts the money as deposits from consumers. In order to satisfy its budget constraint, the outflow of loans must equal the inflow of deposits. Without loss of generality, consider the case of a bank that makes loans of one dollar and receives an equal amount of deposits. Since the nominal interest rate has been normalized to zero, the repayment of the loan will yield a stream of payments equal to one dollar spread across the last two dates. The bank offers a deposit contract that promises D_1 dollars if the depositor withdraws at date 1 and D_2 dollars if he withdraws at date 2. This is feasible for the bank if $\lambda D_1 + (1-\lambda) D_2 \leq 1$ for every (λ, R). In case the repayment of loans does not coincide

with the withdrawal of deposits, the bank will plan to use the interbank market to obtain money as needed. In equilibrium, the two flows will be perfectly matched. Competition among banks will cause them to offer depositors the most attractive deposit contracts. This implies that

$$\lambda D_1 + (1 - \lambda) D_2 = 1.$$

Assuming a non-degenerate distribution of λ, this condition must be satisfied for multiple values of λ and this is only possible if D_1 and D_2 are equal and, hence, equal to 1. The bank will earn zero profits and there is no possibility of doing better.

4.3 The firm's decision

Now consider the representative firm's decision problem. Since the firm's technology exhibits constant returns to scale, there is no loss of generality in restricting attention to a firm that borrows one unit of money at date 0. The firm can obtain one unit of the good with the money it has borrowed, since $P_0 = 1$. Suppose it invests y units in the short asset and $1 - y$ units in the long asset. This will produce y units of the good at date 1 and $(1 - y) R$ units of the good at date 1. In equilibrium, it must be optimal to hold the long asset between dates 1 and 2 in every state (λ, R). It may be also optimal to store the good between dates 1 and 2. These conditions require that $P_1(\lambda, R) \geq P_2(\lambda, R)$, otherwise the short asset would dominate the long asset at date 1, and $P_1(\lambda, R) = P_2(\lambda, R)$ in any state in which the good is stored between dates 1 and 2. Then, in any case, it will be optimal for the firm to set storage equal to zero in calculating the optimal profit. Since the nominal interest rate is zero, the firm's total revenue is $P_1(\lambda, R) y + P_2(\lambda, R)(1 - y) R$ in state (λ, R). Then the firm's budget constraint requires that

$$P_1(\lambda, R) y + P_2(\lambda, R)(1 - y) R \geq 1, \ \forall \ (\lambda, R),$$

and the profit will be zero in equilibrium if and only if the equality holds as an equation for every value of (λ, R).[3]

[3] The zero-profit condition at date 0 implies that the firm must earn zero profits in a set of states that occurs with probability one. Since the price functions are continuous in (λ, R), the continuity of prices in (λ, R) implies that the zero-profit condition holds for every state in the support of the distribution.

To sum up, the firm's choice of y^* is optimal if it yields zero profit in every state and there is no alternative plan that yields non-negative profit everywhere and positive profit with positive probability. More formally, the zero-profit condition for y^* can be written

$$P_1(\lambda, R)\, y^* + P_2(\lambda, R)\,(1 - y^*)\, R = 1, \ \forall \ (\lambda, R),$$

and the requirement that no feasible y yields positive profit can be written as follows: if there exists a state (λ, R),

$$P_1(\lambda, R)\, y + P_2(\lambda, R)\,(1 - y)\, R > 1,$$

then there exists a state (λ', R') such that

$$P_1(\lambda', R')\, y + P_2(\lambda', R')\,(1 - y)\, R < 1.$$

In other words, a production plan y' that produces positive profits in some state must produce negative profits in another state.

The assumption that firms must satisfy their budget constraints with probability one is obviously restrictive. This kind of assumption is standard in general equilibrium theory. One interpretation is that the bank making the loan imposes covenants that prevent the firm from undertaking any production plan that carries a risk of default. In practice, banks have limited information about the actions chosen by firms. It is well known that asymmetric information gives rise to moral hazard and the possibility of default and there is a vast literature dealing with these problems. We ignore these issues in order to provide a set of sufficient conditions in which monetary policy can achieve the first best. This has to be regarded as a benchmark model.

4.4 The consumer's decision

The consumer's decision is straightforward. Consumers deposit the proceeds from selling their endowment of goods to firms. If they turn out to be early consumers they use the withdrawals from their bank accounts to purchase consumption goods at date 1. If they are late consumers they will keep their funds in the bank at date 1 provided

$$c_1(\lambda, R) \leq c_2(\lambda, R).$$

At date 2 they will use their savings to purchase goods from the firms.

4.5 Equilibrium

An equilibrium consists of the price functions $(P_0^*, P_1^*(\cdot), P_2^*(\cdot))$, the money supply functions $(M_0^*, M_1^*(\cdot), M_2^*(\cdot))$, the portfolio choice y^*, the consumption functions $(c_1^*(\cdot), c_2^*(\cdot))$ and the deposit contract (D_1^*, D_2^*) such that the following conditions are satisfied.

Market clearing The market clearing conditions (8) through (12) are satisfied.

Optimal bank behavior The representative bank lends to firms and accepts deposits at the first date. It offers a deposit contract $(D_1^*, D_2^*) = (1, 1)$ to depositors.

Optimal firm behavior The representative firm buys one unit of the good at date 0 and chooses a portfolio y^* such that $P_1^*(\lambda, R) y^* + P_2^*(\lambda, R)(1 - y^*) R = 1$ for every (λ, R). This is optimal for the firm if, for any y,

$$P_1^*(\lambda, R) y + P_2^*(\lambda, R)(1 - y) R > 1, \ \exists \ (\lambda, R)$$
$$\text{implies } P_1^*(\lambda', R') y + P_2^*(\lambda', R')(1 - y) R' < 1, \ \exists \ (\lambda', R').$$

Optimal consumer behavior Each consumer supplies his endowment inelastically at date 0 and has the money he receives in exchange deposited in his bank account. He uses this one unit of money at date 1 if he is an early consumer to purchase

$$c_1^*(\lambda, R) = \frac{1}{P_1^*(\lambda, R)}$$

units of the good. Similarly, if he is a late consumer, he uses the one unit of money at date 2 to enable him to consume

$$c_2^*(\lambda, R) = \frac{1}{P_2^*(\lambda, R)}$$

units of the good.

It is interesting to note that the equilibrium defined above satisfies the Quantity Theory of Money. If the total income (equals total expenditure) at date $t = 1, 2$ is denoted by $Y_t(\lambda, R)$ and defined by

$$Y_1(\lambda, R) = \lambda c_1(\lambda, R) \text{ and } Y_2(\lambda, R) = (1 - \lambda) c_2(\lambda, R),$$

then the market-clearing conditions imply that

$$M_t(\lambda, R) = P_t(\lambda, R) Y_t(\lambda, R),$$

for every state (λ, R) and each date $t = 1, 2$. The Quantity Theory of Money is satisfied in the sense that the price level at each date is proportional to the amount of money supplied by the central bank.

5 Decentralization

In this section we show the existence of an efficient equilibrium. Our approach is constructive. We assume that the equilibrium allocation is efficient, that is, the amount invested in the short asset, y^*, and the consumption functions, $(c_1^*(\cdot), c_2^*(\cdot))$, are taken from the solution to the planner's problem discussed in Section 3. Then the goods-market-clearing conditions, (10) and (12), are satisfied by construction. It remains to show that the money supply, prices, and deposit contracts can be defined to satisfy the equilibrium conditions.

We set the deposit contracts $(D_1^*, D_2^*) = (1, 1)$ and then use the consumers' budget constraints to define the price functions $P_t^*(\lambda, R)$ for $t = 1$,

$$P_1^*(\lambda, R) = \frac{1}{c_1^*(\lambda, R)} \tag{13}$$

and for $t = 2$,

$$P_2^*(\lambda, R) = \frac{1}{c_2^*(\lambda, R)}. \tag{14}$$

The money supply by the central bank responds passively to the commercial banks' demand at each date so we can use the money-market-clearing equations (9) and (11) to define the

central bank's money supply functions:

$$M_1^*(\lambda, R) = \lambda$$

and

$$M_2^*(\lambda, R) = 1 - \lambda.$$

The banks' total liabilities (deposits) at date 0 are equal to their assets (loans). They lend $P_0^* = 1$ to firms and receive deposits of $P_0^* = 1$. In state (λ, R) at date 1, withdrawals equal λ and repayments by firms also equal λ. In state (λ, R) at date 2, withdrawals equal $1 - \lambda$. Since interest rates are zero, the total repayment of the loans will equal the original loan amount and the bank makes zero profits on the loan. Similarly, the withdrawals equal the original deposit amount and the bank makes zero profits on the deposits.

Finally, consider the firms' problem. As we have shown, the firm will make zero profits since the amount of money it receives for its output, $\lambda + 1 - \lambda = 1$, is equal to the amount of money it originally borrows from the bank. It is feasible for the firm to supply the optimal levels of consumption, $\lambda c_1^*(\lambda, R)$ and $(1 - \lambda) c_2^*(\lambda, R)$, at dates 1 and 2 respectively. To see that this is optimal, we have to check that it is optimal for the firm to store the good in states where $\lambda c_1^*(\lambda, R) < y^*$. But from the planner's problem, we know that $\lambda c_1^*(\lambda, R) < y^*$ implies that $c_1^*(\lambda, R) = c_2^*(\lambda, R)$, in which case the definition of price functions in equations (13) and (14) implies that $P_1^*(\lambda, R) = P_2^*(\lambda, R)$. Thus, storage is optimal.

To complete our demonstration of the optimality of the firm's behavior, we have to show that the firm cannot profitably deviate from the specified production plan without being unable to repay its loan in some states. Without loss of generality, we can assume the firm borrows one unit of cash from a bank at date 0. The firm must choose a value of y so that it can repay this debt in every state (λ, R). Let (λ_0, R_0) be a state satisfying $\lambda_0 = y^*$ and $R_0 > 1$. Then

$$\frac{y^*}{\lambda_0} = 1 < \frac{1 - y^*}{1 - \lambda_0} R_0,$$

which implies that $c_1^*(\lambda_0, R_0) < c_2^*(\lambda_0, R_0)$. In fact, the continuity of the feasibility conditions implies that $c_1^*(\lambda, R) < c_2^*(\lambda, R)$ for any state (λ, R) sufficiently close to (λ_0, R_0). The firms'

total revenue TR in state (λ, R) is

$$TR = P_1^*(\lambda, R)\, y + P_2^*(\lambda, R)\,(1-y)\, R = \frac{y}{c_1^*(\lambda, R)} + \frac{(1-y)\, R}{c_2^*(\lambda, R)} \tag{15}$$

and, for all states sufficiently close to (λ_0, R_0), this simplifies to

$$TR = \frac{\lambda}{y^*} y + \frac{(1-\lambda)}{(1-y^*)}(1-y).$$

so

$$\frac{dTR}{dy} = \frac{(\lambda - y^*)}{y^*(1-y^*)}.$$

Note that $0 < y^* < 1$ since $u'(c) \to \infty$ as $c \to 0$ and $\bar{R} > 1$. Thus, $\frac{dTR}{dy} > 0$ for $\lambda > y^*$ and $\frac{dTR}{dy} < 0$ for $\lambda < y^*$. Since $TR = 1$ for all values of λ when $y = y^*$, it follows that if $y < y^*$ then $TR < 1$ for some $\lambda > y^*$ sufficiently close to λ_0 and, similarly, if $y > y^*$ then $TR < 1$ for some $\lambda < y^*$ sufficiently close to λ_0. Hence the firm cannot deviate from $y = y^*$ and still repay its loan for all (λ, R).

We have the following result.

Proposition 2 *The unique solution to the planner's problem can be supported as an equilibrium* $e = (P_0^*, P_1^*(\cdot), P_2^*(\cdot), M_0^*, M_1^*(\cdot), M_2^*(\cdot), c_1^*(\cdot), c_2^*(\cdot), y^*, D_1^*, D_2^*)$.

6 Extensions

6.1 Nominal interest rates

We have claimed that we can set nominal interest rates equal to zero without loss of generality. This is because the real rates of interest, which are all that matter when money is not held as a store of value outside the banking system between periods, are independent of the nominal rate as long as the price levels are adjusted appropriately. Suppose that

$$e = (P_0^*, P_1^*(\cdot), P_2^*(\cdot), M_0^*, M_1^*(\cdot), M_2^*(\cdot), c_1^*(\cdot), c_2^*(\cdot), y^*, D_1^*, D_2^*, r_0^*, r_1^*)$$

is an equilibrium with interest rates normalized to $r_0^* = r_1^* = 0$ and suppose that we choose some arbitrary nominal interest rates $r_0^{**} > 0$ and $r_1^{**} > 0$. Then we claim that there exists

an equilibrium

$$e' = \left(P_0^{**}, P_1^{**}(\cdot), P_2^{**}(\cdot), M_0^{**}, M_1^{**}(\cdot), M_2^{**}(\cdot), c_1^*(\cdot), c_2^*(\cdot), y^*, D_1^{**}, D_2^{**}, r_0^{**}, r_1^{**}\right),$$

with the same allocation $(c_1^*(\cdot), c_2^*(\cdot), y^*)$, where

$$P_0^{**} = 1, P_1^{**}(\lambda, R) = \frac{(1+r_0^{**})}{c_1^*(\lambda, R)}, \text{ and } P_2^{**}(\lambda, R) = \frac{(1+r_0^{**})(1+r_1^{**})}{c_2^*(\lambda, R)},$$

$$M_0^{**} = 1, M_1^{**}(\lambda, R) = \lambda(1+r_0^{**}), \text{ and } M_2^{**}(\lambda, R) = (1+r_0^{**})(1+r_1^{**})$$

and

$$D_1^{**} = (1+r_0^{**}) \text{ and } D_2^{**} = (1+r_0^{**})(1+r_1^{**}).$$

Clearly, the money-market-clearing conditions (9) and (11) and the goods-market-clearing conditions (10) and (12) are satisfied. The banks continue to earn zero profits since

$$1 = \frac{\lambda(1+r_0^{**})}{(1+r_0^{**})} + \frac{(1-\lambda)(1+r_0^{**})(1+r_1^{**})}{(1+r_0^{**})(1+r_1^{**})}.$$

This equation says that the present value of repayments (respectively, withdrawals) equals the value of the initial loan amount (respectively, the initial deposit amount). It is equally easy to see that the bank cannot profitably deviate from this strategy.

For firms, the zero-profit condition follows immediately from the fact that (a) expenditures at date 0 equal the loan at date 0 and (b) the revenues at date $t = 1, 2$ equal the repayments at dates $t = 1, 2$, respectively. The banks' zero-profit condition implies that the present value of the firm's repayments equal the value of the loan, so the firm makes zero profit on borrowing and lending. The optimality of storage in states where $\lambda c_1^*(\lambda, R) < y^*$ follows from the fact that $c_1^*(\lambda, R) = c_2^*(\lambda, R)$ implies that $(1+r_1^{**}) P_1^{**}(\lambda, R) = P_2^{**}(\lambda, R)$. The argument given in Section 9 can be used to show that there is no profitable deviation for firms from the efficient production plan.

6.2 Idiosyncratic liquidity shocks and the interbank market

The preceding analysis can easily be extended to deal with heterogeneity in the liquidity shocks received by individual banks. Suppose that banks are identified with points on the unit interval and let $\lambda_i = \theta_i \lambda$ be the fraction of early consumers among bank i's depositors,

where $\{\theta_i\}$ are i.i.d. random variables with $E[\theta_i] = 1$ for all i. The efficient allocation is the same as before and the market-clearing prices will also be the same as before since the shocks $\{\theta_i\}$ are idiosyncratic and do not affect the aggregate proportion of early consumers. Since in equilibrium the banks offer deposit contracts that satisfy $D_1 = D_2 = D^* = 1$, the shock θ_i has no effect on the bank's ability to meet the withdrawals of its depositors. More precisely,

$$\begin{aligned} P_1^*(\lambda, R)\, y^* + P_2(\lambda, R)(1 - y^*) R &= 1 \\ &= \theta_i \lambda + (1 - \theta_i \lambda) \\ &= \theta_i \lambda P_1^*(\lambda, R)\, c_1^*(\lambda, R) + (1 - \theta_i \lambda) P_2^*(\lambda, R)\, c_2^*(\lambda, R), \end{aligned}$$

since $P_1^*(\lambda, R)\, c_1^*(\lambda, R) = P_2^*(\lambda, R)\, c_2^*(\lambda, R) = D^* = 1$.

The banks with $\theta_i > 1$ borrow from banks with $\theta_i < 1$ at date 1 and repay the loan at date 2 when the number of late consumers will be correspondingly lower. The interbank market clears because the Law of Large Numbers implies that

$$\int_0^1 \theta_i di = 1.$$

Let $B_1(\theta_i, \lambda, R)$ denote the net interbank borrowing at date 1 by a bank with shock θ_i and let $B_2(\theta_i, \lambda, R)$ denote the repayment at date 2. Then, for every (λ, R) and θ_i,

$$\begin{aligned} B_1(\theta_i, \lambda, R) &= \theta_i \lambda D^* - P_1^*(\lambda, R)\, \lambda c_1^*(\lambda, R) \\ &= \theta_i \lambda D^* - \frac{\lambda D^*}{\lambda c_1^*(\lambda, R)} \lambda c_1^*(\lambda, R) \\ &= (\theta_i - 1)\, \lambda D^* = (\theta_i - 1)\, \lambda, \end{aligned}$$

for every (λ, R) and θ_i. Thus,

$$\int_0^1 B_1(\theta_i, \lambda, R)\, di = \int_0^1 (\theta_i - 1)\, \lambda di = 0,$$

for every (λ, R). Similarly, at date 2,

$$\begin{aligned} B_2(\theta_i, \lambda, R) &= (1-\theta_i\lambda) D^* - P_2^*(\lambda, R)(1-\lambda) c_2^*(\lambda, R) \\ &= (1-\theta_i\lambda) D^* - \frac{(1-\lambda) D^*}{(1-\lambda) c_2^*(\lambda, R)} (1-\lambda) c_2^*(\lambda, R) \\ &= (\lambda - \theta_i\lambda) D^* = (1-\theta_i)\lambda, \end{aligned}$$

for every (λ, R) and θ_i. Then

$$\int_0^1 B_2(\theta_i, \lambda, R)\, di = \int_0^1 (1-\theta_i)\lambda di = 0,$$

for every (λ, R).

6.3 Idiosyncratic return risk

One source of uncertainty that cannot be dealt with by monetary policy alone is idiosyncratic or bank-specific asset return risk. In order to achieve efficient sharing of idiosyncratic asset return risk, it is necessary to introduce new markets or institutions. To analyze this case, we assume without loss of generality that there is no aggregate uncertainty and that there are no idiosyncratic liquidity shocks. These risks can be shared efficiently through the interbank market and adjustments in the price level. We assume that the probability of being an early consumer is a constant $\bar{\lambda}$ and that the expected return on the long asset is a constant $\bar{R} > 1$, but that each bank receives a random return $\tilde{R}_i$ on its holding of the long asset. The returns are i.i.d. across banks and the mean return is $E\left[\tilde{R}_i\right] = \bar{R}$. Then the Law of Large Numbers implies that

$$\int_0^1 \tilde{R}_i di = \bar{R}$$

with probability one. In other respects the model remains the same as before.

To simplify the analysis further, we can assume that banks hold the long and short assets directly, thus eliminating any reference to firms and their need to borrow.

From the point of view of the central planner, idiosyncratic risk is irrelevant. Since the planner can redistribute returns in any way he pleases, only the aggregate (mean) return matters. Since the mean return on the long asset is a constant, $\bar{R}$, the central planner's

problem is essentially a decision problem under certainty: [4]

$$\begin{array}{ll} \max & \bar{\lambda} u(c_1) + \left(1-\bar{\lambda}\right) u(c_2) \\ \text{s.t.} & \bar{\lambda} c_1 \leq y \\ & \bar{\lambda} c_1 + \left(1-\bar{\lambda}\right) c_2 = y + (1-y)\bar{R}. \end{array}$$

The short asset is used to satisfy the demands of early consumers and the long asset is used to satisfy the demands of the late consumers. The optimum is characterized by the first-order condition

$$u'(c_1) = \bar{R} u'(c_2),$$

where

$$c_1 = \frac{y}{\bar{\lambda}},$$
$$c_2 = \frac{(1-y)\bar{R}}{1-\bar{\lambda}}.$$

Now suppose that y^* is the efficient portfolio and (c_1^*, c_2^*) is the efficient consumption profile. How can we implement this outcome? At date 0, we can assume without loss of generality that the price of goods is $P_0^* = 1$ and the money supply is $M_0^* = 1$. The central bank supplies money passively to the commercial banks that use it to buy goods from consumers. The banks invest the goods in the short and long asset in the same proportions, y^* and $1 - y^*$, as the efficient allocation. The consumers then deposit their money in the banks in exchange for a demand deposit contract promising them D_t units of money if they withdraw at date $t = 1, 2$. We can set $D_1^* = D_2^* = 1$ on the assumption that the nominal interest rates r_0^* and r_1^* are both zero.

At dates 1 and 2, in order to satisfy the market clearing condition, the price levels must be

$$P_1^* = \frac{1}{c_1^*} \text{ and } P_2^* = \frac{1}{c_2^*}.$$

At these prices, the average bank (i.e., one that earns a return of $\bar{R}$ on the long asset) will be just solvent: the average bank requires one unit of money to repay its depositors and its

[4] In Section 3, we solved the planner's problem in two stages. First, we maximized the consumer's utility at date 1 with respect to (c_1, c_2) for given values of y and (λ, R); then we maximized the consumer's expected utility with respect to y at date 0, taking the consumption functions $c_1(\lambda, R)$ and $c_2(\lambda, R)$ as given. Here, the certain values of $\bar{\lambda}$ and $\bar{R}$ are known at date 0, so we can optimize with respect to y, c_1 and c_2 at date 0.

portfolio will yield

$$\begin{aligned} P_1^* y^* + P_2^* (1 - y^*) \bar{R} &= \frac{1}{c_1^*} y^* + \frac{1}{c_2^*} (1 - y^*) \bar{R} \\ &= \bar{\lambda} + \left(1 - \bar{\lambda}\right) = 1. \end{aligned}$$

However, any bank that makes a return $R < \bar{R}$ on the long asset will have too little money to meet the withdrawals of its depositors at dates 1 and 2 and any bank that makes a return $R > \bar{R}$ will have more than enough money to meet its withdrawals.

This problem can be solved by providing banks with deposit insurance of the following form: each bank that has a deficit $\left(R - \bar{R}\right)(1 - y^*) < 0$ is paid an amount of money equal to $\left(\bar{R} - R\right)(1 - y^*) > 0$, whereas any bank that has a surplus $\left(R - \bar{R}\right)(1 - y^*) > 0$ must pay a tax equal to that amount. Then every bank has exactly the right nominal value of assets to pay back its depositors and by construction the money market and goods markets will clear. More precisely, if we set $M_1^* = \lambda$, then banks will have enough cash to pay the early consumers one dollar each at date 0, the consumers will have enough cash to purchase the efficient amount of good, since $P_1^* c_1^* = 1$, and the goods market will clear because $\lambda c_1^* = y^*$. Similarly, if we set $M_2^* = 1 - \lambda$, the banks will have just enough money to pay the late consumers one dollar each at date 2, the consumers will have enough cash to purchase the efficient amount of the good, since $P_2^* c_2^* = 1$, and the goods market will clear because $\left(1 + \bar{\lambda}\right) c_2^* = (1 - y^*) \bar{R}$.

We have not specified the timing of the taxes and insurance payments very precisely because, thanks to the interbank markets, the timing is not important. For concreteness, we can assume that insurance payments are made and taxes collected at date 2. If banks are short of money at date 1, they can simply borrow from the central bank or from other commercial banks in order to obtain the necessary amounts to pay their depositors. Since they are solvent in present value terms (taking into account insurance payments and taxes) they can certainly repay these loans.

This brief sketch suggests how the efficient allocation might be implemented. To show that these prices and quantities constitute an equilibrium, we need to check that banks are maximizing consumers' expected utility subject to the zero profit condition.

There are, of course, many other ways to achieve the same end. A market solution would require banks to pool their assets to get rid of the idiosyncratic risk. We could think of this as a form of securitization in which the individual banks "originate loans" (i.e., make investments in risky long assets) and then sell then sell these loans to an investment trust in exchange for asset backed securities, i.e., bonds promising payments equal to the mean return on the long asset. Another form of implementation would be to have an explicit insurance contract where the asset returns are insured and the risk is pooled.

It is clear that none of these methods can avoid the necessity of making real transfers among banks and that these transfers cannot be achieved through price level adjustments alone. In practice such transfers may be difficult to accomplish. Banks with high returns will have an incentive to hide them from both government and private schemes.

The reference to securitization also brings to mind the risks of moral hazard when banks are effectively being offered insurance against idiosyncratic asset return shocks. For simplicity, we have assumed here that asset returns are exogenous and that the returns on each bank's assets are observable (and verifiable). In practice, a bank's portfolio is endogenous and opaque. Even when the bank's choice is restricted to short and long assets, as here, moral hazard issues can arise if the bank's choice of portfolio cannot be observed (Bhattacharya and Gale, 1987). More generally, banks choose the riskiness of the assets they hold and insuring the asset returns will give the bank an incentive to hold even riskier assets. So the problem of achieving efficient risk sharing when banks are exposed to idiosyncratic and endogenous asset return risk is much more challenging than our simple model suggests.

To sum up, monetary policy alone is not sufficient to eliminate idiosyncratic return risk and, in the presence of moral hazard and asymmetric information, it may not be possible to eliminate all idiosyncratic return risk in an incentive-compatible way. The incentive efficient allocation of risk may still be subject to individual defaults and the possibility of contagion. What we have provided here is a benchmark in the form of sufficient (but restrictive) conditions for efficiency.

6.4 The multi-period case

Suppose that, instead of three dates, we have a finite sequence of dates $t = 0, 1, ..., T$. As usual, all uncertainty is resolved at date 1, when the random vectors $\boldsymbol{\lambda} = (\lambda_1, ..., \lambda_T)$ and $\mathbf{R} = (R_1, ..., R_T)$ are realized. The random variable λ_t is the fraction of consumers that wish to consume only at date t. There is a long asset for each date $t = 1, ..., T$. The random variable R_t is the return on one unit of the good invested at date 0 in the asset that pays off at date t. We also allow for investment in the short asset at each date $t = 1, ..., T$. We assume that the support of $\boldsymbol{\lambda}$ is $\left\{\boldsymbol{\lambda} \geq \mathbf{0} : \sum_{t=1}^{T} \lambda_t = 1\right\}$ and the support of $\mathbf{R}$ is $[1, R_{\max}]^T$, for some $1 < R_{\max} < \infty$. Since the long asset dominates the short asset, there will be no investment in the short asset at date 0. However, the short asset may be used at subsequent dates to smooth consumption intertemporally, for some realizations of $(\boldsymbol{\lambda}, \mathbf{R})$. For example, when the realization of R_t is high, some of the output can be carried forward to offset high liquidity shocks or low asset returns in subsequent periods.

Let $c_t(\boldsymbol{\lambda}, \mathbf{R})$ denote the consumption at date t in state $(\boldsymbol{\lambda}, \mathbf{R})$ and let $\mathbf{x} = (x_1, ..., x_T)$ denote the portfolio of long assets. The planner will choose a portfolio $\mathbf{x}$ and a sequence of consumption functions $\{c_t(\boldsymbol{\lambda}, \mathbf{R})\}$ in order to maximize

$$\mathbf{E}\left[\sum_{t=1}^{T} \lambda_t u\left(c_t(\boldsymbol{\lambda}, \mathbf{R})\right)\right]$$

subject to

$$\begin{aligned} \sum_{t=1}^{T} x_t &= 1 \\ \sum_{s=1}^{t} \lambda_s c_t(\boldsymbol{\lambda}, \mathbf{R}) &\leq \sum_{s=1}^{t} R_s x_s, \ \forall t, \forall(\boldsymbol{\lambda}, \mathbf{R}). \end{aligned}$$

The first constraint ensures that the investments in the long assets exhaust the endowment. The second constraint ensures that, at every date, the cumulative consumption at that date is less than or equal to the cumulative output at that date. These constraints incorporate the possibility of storage. Let $(\mathbf{x}^*, c_1^*(\cdot), ..., c_T^*(\cdot))$ denote the solution to this problem.

This solution has a simple form. If consumption could be carried forward and backward through time without restriction, it would be optimal to equate per capita consumption each

period. However, consumption can only be carried forward through time. When R_t is high or λ_t is low, it is optimal to carry forward output using the short term asset until there is another high R_t or low λ_t. Thus the dates $t = 1, ..., T$ can be partitioned into n intervals $\{1, ..., t_1\}$, $\{t_1 + 1, ..., t_2\}$, ... , $\{t_n + 1, ..., T\}$, where $t_0 = 0$, $t_{n+1} = T$ and $i = 0, ..., n$. Each of these intervals $\{t_i + 1, ..., t_{i+1}\}$ corresponds to a sequence of dates where a positive amount of the good is being stored at each date $t_i + 1, ..., t_{i+1} - 1$ and none of the good is stored at the last date t_{i+1}. Then the first-order conditions for the planner's problem imply that consumption is equalized across every date in the interval $\{t_i + 1, ..., t_{i+1}\}$ and the feasibility conditions hold exactly at the end dates $t_1, ..., t_{n+1}$, that is,

$$\sum_{s=1}^{t_i} \lambda_s c_s^* (\boldsymbol{\lambda}, \mathbf{R}) = \sum_{s=1}^{t_i} R_s x_s^*, \text{ for } i = 1, ..., n.$$

These feasibility conditions clearly imply that

$$\sum_{s=t_i+1}^{t_{i+1}} \lambda_s c_s^* (\boldsymbol{\lambda}, \mathbf{R}) = \sum_{s=t_i+1}^{t_{i+1}} R_s x_s^*, \text{for } i = 0, ..., n.$$

Using the fact that consumption is constant throughout the interval $\{t_i + 1, ..., t_{i+1}\}$, we can solve this equation for the value of $c_t^* (\boldsymbol{\lambda}, \mathbf{R})$ for any $t \in \{t_i + 1, ..., t_{i+1}\}$ and the result is

$$c_t^* (\boldsymbol{\lambda}, \mathbf{R}) = \frac{\sum_{s=t_i+1}^{t_{i+1}} x_s^* R_s}{\sum_{s=t_i+1}^{t_{i+1}} \lambda_s}. \tag{16}$$

We next show that the optimal solution to the planner's problem can be decentralized as an equilibrium. At date 0, we normalize $M_0^* = P_0^* = 1$. Firms borrow one unit of money, purchase the consumers' endowments, and invest them in a portfolio $\mathbf{x}^*$ of the long assets. We assume that the nominal interest rate on loans to the firms is zero. Consumers deposit the money in the bank in exchange for a deposit contract that will offer $D_t = 1$ unit of money to any consumer who withdraws at date t.

To ensure the goods market clears, we set

$$P_t^* (\boldsymbol{\lambda}, \mathbf{R}) = \frac{1}{c_t^* (\boldsymbol{\lambda}, \mathbf{R})},$$

for every date $t = 1, ..., T$ and every state $(\boldsymbol{\lambda}, \mathbf{R})$. To ensure that the demand for money

equals the supply, we define the money supply functions recursively by setting

$$M_t(\boldsymbol{\lambda}, \mathbf{R}) = \lambda_t D_t = \lambda_t,$$

for every date $t = 1, ..., T$ and every state $(\boldsymbol{\lambda}, \mathbf{R})$.

In the usual way, it can be shown that firms make zero profits and can repay their loans and that banks break even on their loans and deposits. At each date t, the amount of money the representative bank borrows from the central bank, $M_t(\boldsymbol{\lambda}, \mathbf{R}) = \lambda_t$, is returned to it as the consumers pay $\lambda_t P_t^*(\boldsymbol{\lambda}, \mathbf{R}) c_t^*(\boldsymbol{\lambda}, \mathbf{R}) = \lambda_t$ to the firms in exchange for consumption and the firms use this revenue to repay (part of) their debt to the banks. Over the course of the dates $t = 1, ..., T$, the representative bank is paid one unit of money, the amount of the initial loan. Note that it is optimal for the banks to choose $D_t = 1$. They can't afford to pay $D_t > 1$ and competition ensures $D_t < 1$ will not attract any customers. Since loans make zero profits, the banks cannot increase profits by changing the amount of loans.

The analysis of the firm's problem is contained in Appendix B in Section 9, where we show that firms make zero profits if they choose the portfolio $\mathbf{x}^*$ and that they make losses with positive probability if they choose any feasible $\mathbf{x} \neq \mathbf{x}^*$.

7 Conclusion

This paper has developed a model of banking with nominal contracts and money. We introduce a wide range of different types of uncertainty, including aggregate return uncertainty, aggregate liquidity shocks, and idiosyncratic (bank-specific) liquidity shocks. With deposit contracts specified in real terms, as most of the literature assumes, these risks would lead to banking crises. We have shown, however, that with nominal contracts and a central bank, it is possible to eliminate financial instability. More importantly, it is possible to achieve the first best allocation. This does not require heavy intervention by the central bank or the government. All that is required is that the central bank accommodates the commercial banks' liquidity needs. Moreover, because the central bank can set the nominal interest rate, they can also control the expected rate of inflation. The one type of risk that cannot easily be dealt with is idiosyncratic return shocks. This requires that the a government or

private institution make transfers between banks with high and low returns to achieve the first best. Implementing this type of scheme is problematic as it creates moral hazard and other incentive problems.

8 Appendix A

To characterize the efficient allocation, we assume that a planner invests in a portfolio of the short and long assets and distributes the proceeds directly to the early and late consumers. The portfolio of investments, expressed in per capita terms, consists of y units of the short asset and $1-y$ units of the long asset. The allocation of consumption will depend on the random vector (λ, R), so we write the consumption profile of the typical consumer as $(c_1(\lambda, R), c_2(\lambda, R))$. Then the planner's problem is to

$$\begin{array}{ll} \max_{(c_1,c_2,y)} & E\left[\lambda u(c_1(\lambda,R)) + (1-\lambda)u(c_2(\lambda,R))\right] \\ \text{s.t.} & \lambda c_1(\lambda,R) \leq y \text{ and } \lambda c_1(\lambda,R) + (1-\lambda)\, c_2(\lambda,R) \leq y + (1-y)\, R. \end{array} \tag{17}$$

Note that the problem contains an infinite number of constraints, one pair for each value of (λ, R). The first constraint says that the total consumption given to the early consumers must not exceed the supply of the short asset at date 1. The second constraint says that total consumption summed over the two dates cannot exceed the total returns of the two assets. The constraints are expressed this way to take account of the possibility of storage between date 1 and date 2.

For given values of y and (λ, R), the consumption profile $(c_1(\lambda, R), c_2(\lambda, R))$ must maximize $\lambda u(c_1(\lambda, R))+(1-\lambda)u(c_2(\lambda, R)$ subject to the two feasibility constraints. The first-order conditions for this problem, which are necessary and sufficient, can be written as

$$u'(c_1(\lambda,R)) - u'(c_2(\lambda,R)) \geq 0$$

with the complementary slackness condition

$$\left[u'(c_1(\lambda,R)) - u'(c_2(\lambda,R))\right]\left[y - \lambda c_1(\lambda,R)\right] = 0.$$

Note that $u'(c_1(\lambda, R)) = u'(c_2(\lambda, R))$ implies that $c_1(\lambda, R) \leq c_2(\lambda, R)$ so the incentive constraint is automatically satisfied. In other words, the first best (Pareto-efficient) allocation

is the same as the second best (incentive-efficient) allocation.

Since $u'(c_1(\lambda, R)) = u'(c_2(\lambda, R))$ implies that

$$c_1(\lambda, R) = c_2(\lambda, R) = y + (1-y)R,$$

the optimal consumption functions can be written as

$$c_1^*(\lambda, R) = \min\left\{\frac{y}{\lambda}, y + (1-y)R\right\} \text{ and } c_2^*(\lambda, R) = \max\left\{\frac{(1-y)R}{1-\lambda}, y + (1-y)R\right\}.$$

Substituting these values for $c_1^*(\lambda, R)$ and $c_2^*(\lambda, R)$ into the planner's problem, we obtain the optimal portfolio choice problem:

$$\max_{0 \le y \le 1} \quad \mathbf{E}\left[\lambda u(\min\left\{\frac{y}{\lambda}, y + (1-y)R\right\}) + (1-\lambda)u(\max\left\{\frac{(1-y)R}{1-\lambda}, y + (1-y)R\right\})\right].$$

The objective function is continuous in y and hence attains a maximum. Since the function u is strictly concave, the maximizer y^* is unique.

9 Appendix B

The total revenue of a representative firm is

$$\sum_{t=1}^{T} P_t^*(\boldsymbol{\lambda}, \mathbf{R}) x_t^* R_t = \sum_{t=1}^{T} \frac{x_t^* R_t}{c_t^*(\boldsymbol{\lambda}, \mathbf{R})}.$$

Now consider the expression for the firm's revenue during the interval $\{t_i + 1, ..., t_{i+1}\}$. Since there is no storage in the last period of each interval, (16) implies that

$$\begin{aligned}
\sum_{t=t_i+1}^{t_{i+1}} \frac{x_t^* R_t}{c_t^*(\boldsymbol{\lambda}, \mathbf{R})} &= \sum_{t=t_i+1}^{t_{i+1}} \frac{x_t^* R_t}{\left(\frac{\sum_{s=t_i+1}^{t_{i+1}} x_s^* R_s}{\sum_{s=t_i+1}^{t_{i+1}} \lambda_s}\right)} \\
&= \left(\frac{\sum_{s=t_i+1}^{t_{i+1}} \lambda_s}{\sum_{s=t_i+1}^{t_{i+1}} x_s^* R_s}\right) \sum_{t=t_i+1}^{t_{i+1}} x_t^* R_t \\
&= \sum_{s=t_i+1}^{t_{i+1}} \lambda_s
\end{aligned}$$

and hence

$$\sum_{t=1}^{T} \frac{x_t^* R_t}{c_t^*(\boldsymbol{\lambda}, \mathbf{R})} = \sum_{i=0}^{n} \sum_{s=t_i+1}^{t_{i+1}} \lambda_s = 1.$$

This proves that each firm earns zero profits and can repay its debt to the bank.

Can the firm make a positive profit by deviating from the equilibrium portfolio $\mathbf{x}^*$? Under the assumptions we have made, we can show that it is not possible for the firm to deviate at all without violating its budget constraint in some non-negligible set of states. The proof is by contradiction. Suppose that $\mathbf{x} \neq \mathbf{x}^*$ is a feasible portfolio, i.e., $\mathbf{x} \geq \mathbf{0}$ and $\sum x_t = 1$, and satisfies the budget constraint

$$\sum_{t=1}^{T} P_t^*(\boldsymbol{\lambda}, \mathbf{R}) x_t R_t \geq 1$$

for every $(\boldsymbol{\lambda}, \mathbf{R})$. Note that we do not have to assume that $\mathbf{x}$ is profitable in any state, just that it at least breaks even. Now fix some $\mathbf{R}$ and suppose that $\boldsymbol{\lambda}^0 = (0, ..., 0, 1, 0, ..., 0)$, where the 1 is in the τ-th place. Recall that by assumption $\boldsymbol{\lambda}^0$ belongs to the support of $\boldsymbol{\lambda}$ and hence is the limit of a sequence of random vectors $\boldsymbol{\lambda}^q \to \boldsymbol{\lambda}^0$ that have a small but positive measure of consumers at each date $s = 1, ..., \tau$. Then it is clear that, in the limit, assuming the consumption functions are continuous in $\boldsymbol{\lambda}$ and $\mathbf{R}$, consumption will have to be equalized at the dates $1, ..., \tau$ and will become unboundedly large at the dates $\tau + 1, ..., T$. Thus,

$$c_s^*(\boldsymbol{\lambda}, \mathbf{R}) = \begin{cases} c_\tau^*(\boldsymbol{\lambda}, \mathbf{R}) & \text{for } s = 1, ..., \tau \\ \infty & \text{for } s = \tau + 1, ..., T \end{cases}$$

and, therefore,

$$\begin{aligned} \sum_{t=1}^{T} P_t^*(\boldsymbol{\lambda}, \mathbf{R}) x_t R_t &= \sum_{s=1}^{\tau} \frac{x_s R_s}{c_\tau^*(\boldsymbol{\lambda}, \mathbf{R})} \\ &\geq \sum_{t=1}^{T} P_t^*(\boldsymbol{\lambda}, \mathbf{R}) x_t^* R_t \\ &= \sum_{s=1}^{\tau} \frac{x_s^* R_s}{c_\tau^*(\boldsymbol{\lambda}, \mathbf{R})}. \end{aligned}$$

Thus, for any $\tau = 1, ..., T$ and any $\mathbf{R}$,

$$\sum_{s=1}^{\tau} x_s R_s \geq \sum_{s=1}^{\tau} x_s^* R_s.$$

This means that $\mathbf{x}$ provides at least as much output as $\mathbf{x}^*$ at every date and, since $\mathbf{x} \neq \mathbf{x}^*$, there must be some date at which the inequality is strict. In other words, $\mathbf{x}$ produces strictly more than $\mathbf{x}^*$ at some dates. But this contradicts the optimality of $\mathbf{x}^*$. Thus, the only optimal choice of portfolio is $\mathbf{x} = \mathbf{x}^*$.

References

[1] Allen, F., E. Carletti and D. Gale (2009). "Interbank Market Liquidity and Central Bank Intervention," *Journal of Monetary Economics* 56, 639-652.

[2] Allen, F. and D. Gale (1998). "Optimal Financial Crises," *Journal of Finance* 53, 1245-1284.

[3] Allen, F. and D. Gale (2000). "Financial Contagion," *Journal of Political Economy* 108, 1-33.

[4] Antinolfi, G., E. Huybens, and T. Keister (2001). "Monetary Stability and Liquidity Crises: The Role of the Lender of Last Resort," *Journal of Economic Theory* 99, 187-219.

[5] Bhattacharya, S. and D. Gale (1987). "Preference Shocks, Liquidity and Central Bank Policy," in W. Barnett and K. Singleton (eds.), *New Approaches to Monetary Economics*, Cambridge: Cambridge University Press, 69-88.

[6] Buiter, Willem (2007) "Maverecon," *Financial Times* December 13, 2007, as quoted in Cao and Illing (2009).

[7] Calomiris, C. and C. Kahn (1991). "The Role of Demandable Debt in Structuring Optimal Banking Arrangements," *American Economic Review* 81, 497-513.

[8] Cao, J. and G. Illing (2011): "Endogenous Systemic Liquidity Risk," *International Journal of Central Banking* (forthcoming).

[9] Champ, B., B. Smith and S. Williamson (1996). "Currency Elasticity and Banking Panics: Theory and Evidence," *Canadian Journal of Economics* 29, 828-864.

[10] Chari, V. and R. Jagannathan (1988). "Banking Panics, Information, and Rational Expectations Equilibrium," *Journal of Finance* 43, 749-60.

[11] Cone, K. (1983). *Regulation of Depository Institutions*. Stanford University Ph.D. dissertation.

[12] Cooper, R. and D. Corbae, (2002). "Financial Collapse: A Lesson from the Great Depression," *Journal of Economic Theory*

107, 159-190.

[13] Diamond, D. and P. Dybvig (1983). "Bank Runs, Deposit Insurance, and Liquidity," *Journal of Political Economy* 91, 401-419.

[14] Diamond, D. and R. Rajan (2001). Liquidity Risk, Liquidity Creation and Financial Fragility: A Theory of Banking," *Journal of Political Economy* 109, 2431-2465.

[15] Diamond, D. and R. Rajan (2005). "Liquidity Shortages and Banking Crises," *Journal of Finance* 60, 615-647.

[16] Diamond, D. and R. Rajan (2006). "Money in a Theory of Banking," *American Economic Review* 96, 30-53.

[17] Freixas, X., A. Martin, D. Skeie (2009). "Bank Liquidity, Interbank Markets and Monetary Policy," working paper, Federal Reserve Bank of New York.

[18] Goodfriend, M. and R. King (1988). "Financial Deregulation, Monetary Policy and Central Banking," *Federal Reserve Bank of Richmond Economic Review* 74, 3-22.

[19] Gorton, G. (1988). "Banking Panics and Business Cycles," *Oxford Economic Papers* 40, 751-781.

[20] Jacklin, C. (1987). "Demand Deposits, Trading Restrictions, and Risk Sharing," in E. Prescott and N. Wallace, eds., *Contractual Arrangements for Intertemporal Trade,* University of Minnesota Press, Minneapolis, MN, 26-47.

[21] Jacklin, C., and S. Bhattacharya (1988). "Distinguishing Panics and Information-Based Bank Runs: Welfare and Policy Implications," *Journal of Political Economy* 96, 568-592.

[22] Kiyotaki, N. (2009)."Comment on Interbank Market Liquidity and Central Bank Intervention," *Journal of Monetary Economics* 56, 653-656.

[23] Skeie, D. (2008). "Banking with Nominal Deposits and Inside Money," *Journal of Financial Intermediation* 17, 562-584.

[24] Smith, B. (2002). "Monetary Policy, Banking Crises, and the Friedman Rule," *American Economic Review* 92, 128-134.

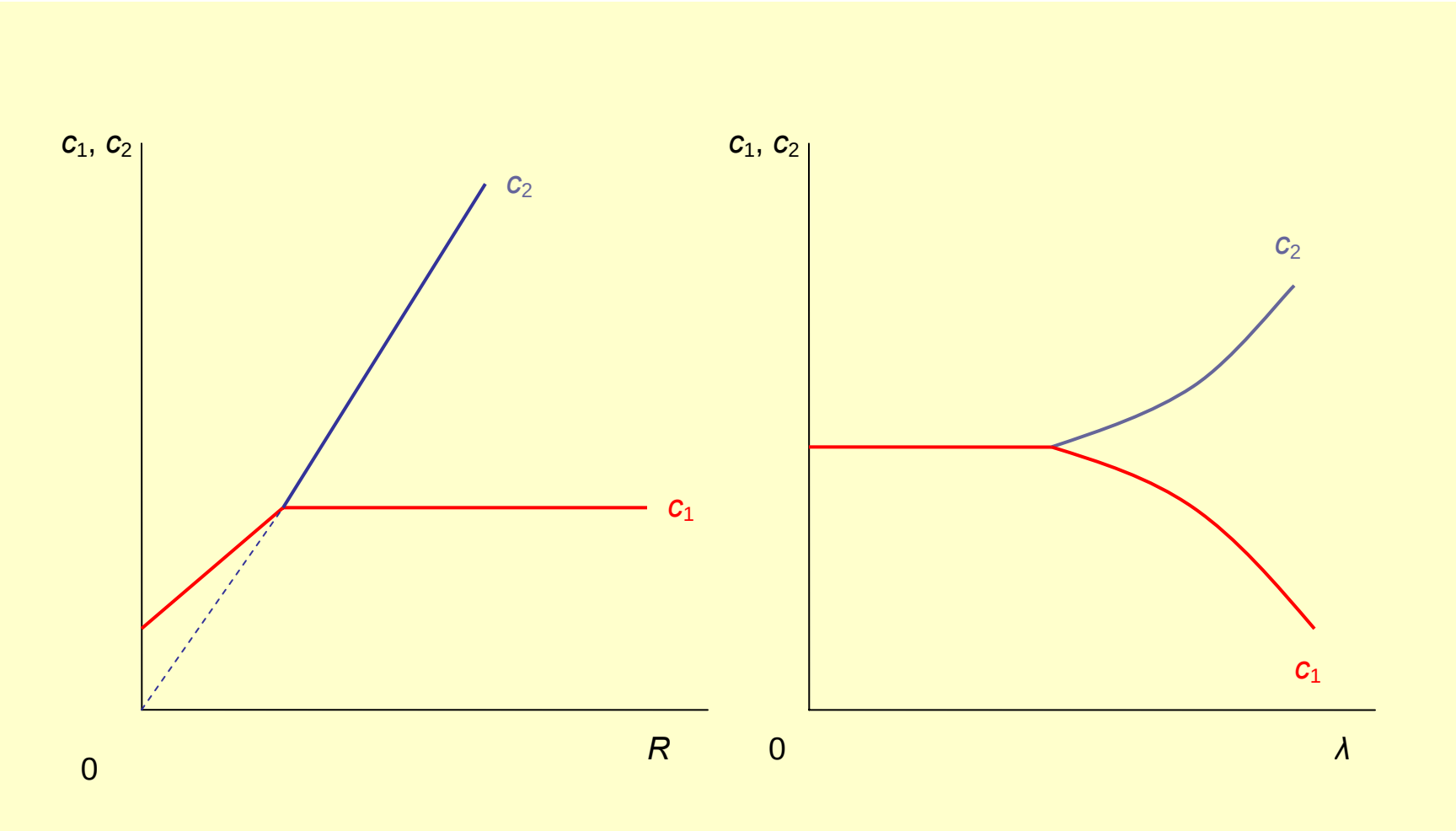

Figure 1: Consumption Functions at Dates 1 and 2

The left hand panel shows the consumption of an individual at each date as a function of R holding λ constant. The right hand panel shows the consumption of an individual at each date as a function of λ holding R constant.

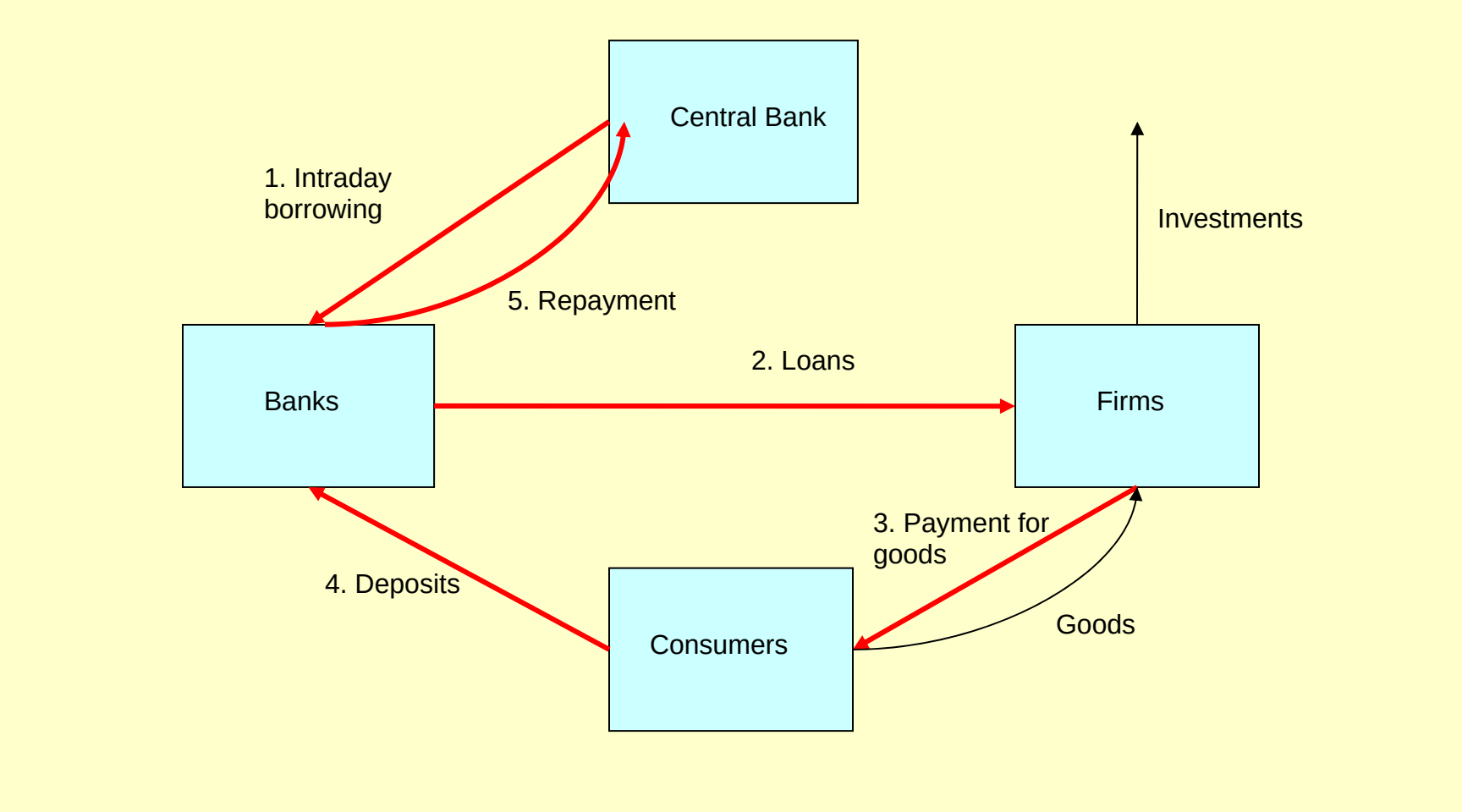

Figure 2: Flow of Funds at Date 0

1. Banks borrow cash from the central bank. 2. Firms borrow cash from the banks. 3. Firms purchase goods from the consumers. 4. Consumers deposit cash with the banks. 5. Banks repay their intraday loans to the central bank.

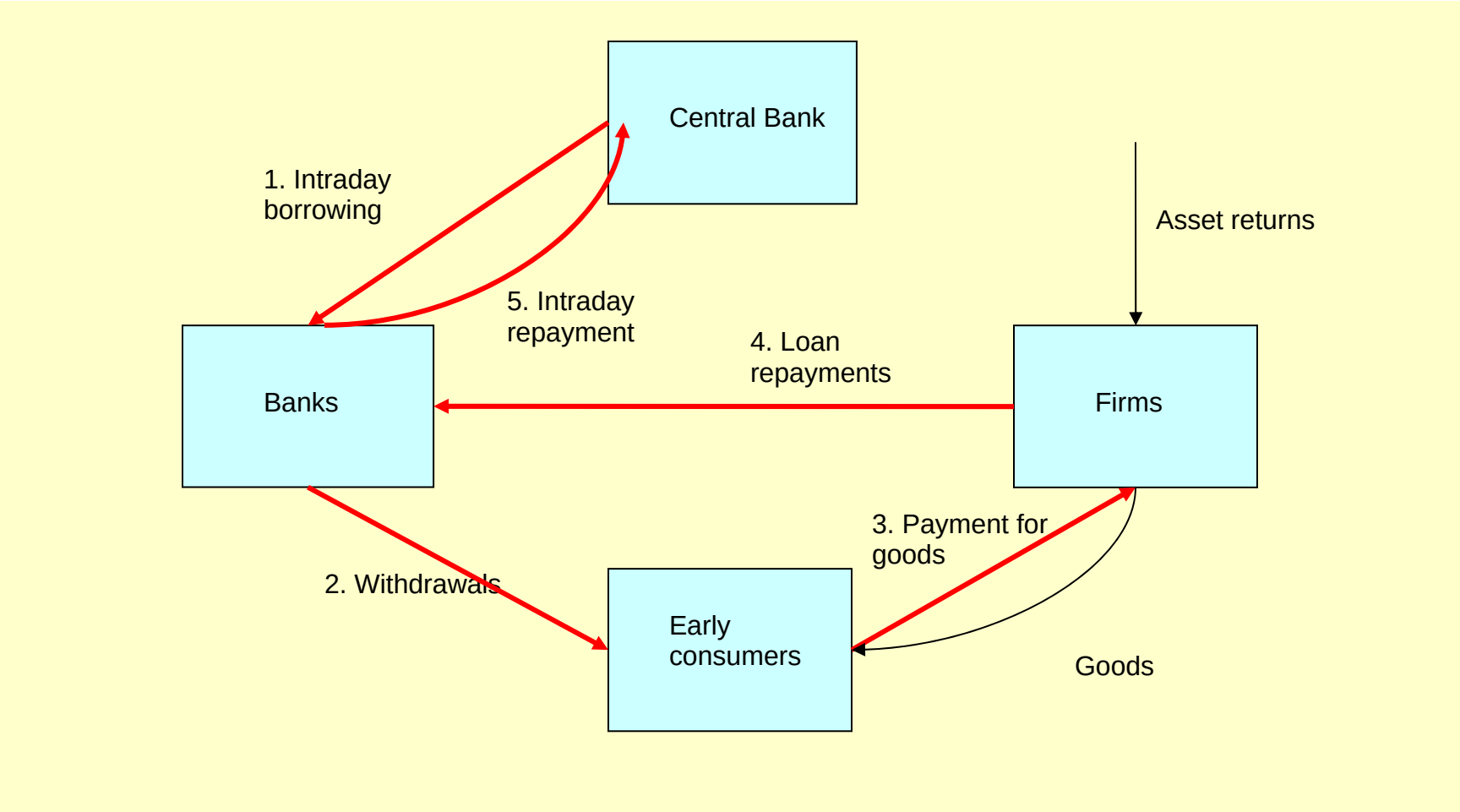

Figure 3: Flow of Funds at Dates 1 and 2

1. Banks borrow cash from the central bank. 2. Early consumers withdraw cash from the banks. 3. Consumers purchase goods from the firms. 4. Firms repay part of their loans to the banks. 5. Banks repay their intraday loans to the central bank.