

# Applied Contract Theory

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## **Topics**

1. Adverse Selection and Insurance
2. Moral Hazard and Personnel Economics
3. Matching, Contracts, and Markets

## **Course Objectives**

Build upon IECT to explore important applications of main theories

Learn how to build simple applied theory models

Assess the relevance of empirical evidence and match with theoretical predictions

Draw lessons for how one can bring together theoretical and empirical work

## **Readings (books)**

Botlon and Dewatripont. Contract Theory. The MIT Press. 2004

Salanie, B. The Economics of Contracts: A Primer. MIT Press. 1997

Mas Colell, Whinston, Green. Microeconomic Theory. Oxford University Press (Chapter 13-14). 1995

Jean-Jacques Laffont and David Martimort. The Theory of Incentives: The Principal-Agent Model. Princeton University Press. 2002

Roth, Alvin and Marilda Sotomayor. Two-Sided Matching: A Study in Game-Theoretic Modeling and Analysis, Cambridge, Cambridge University Press. 1990

## Lecture 0: Course Introduction

Stiglitz, Joseph E. “The Contribution of the Economics of Information to Twentieth Century Economics”. *Quarterly Journal of Economics*, Nov. 2000, 1441-1478.

Chiappori, Pierre-Andre; Salanie, Bernard. “Testing Contract Theory: a Survey of Some Recent Work ”, in *Advances in Economics and Econometrics - Theory and Applications*, Eighth World Congress, M. Dewatripont, L. Hansen and P. Turnovsky, ed., *Econometric Society Monographs*, Cambridge University Press, Cambridge, 2003, 115-149.

## **Empirical Tests of Information Economics: Methodological Issues**

Do the departing assumptions of information economics hold? Moral Hazard: Agents respond to incentives, but do they? Adverse Selection: Some agents are privately informed, but are they?

Are contracts designed optimally? Do principals design contracts consistent with incentive theory? Predictions on how contracts should vary as a function of the environment?

Equilibrium and welfare predictions? Tests regarding market outcomes and efficiency properties

## **Tests of contract theory require information on:**

Contracts offered in equilibrium

Information available to all parties: what is privately and publicly observed?

Behavioral choice variables: action/effort taken, contract selected...

Outcomes: payments and compensation

In addition, contracts need to be somewhat standardized to apply econometric tools

## Methodological Issues

Necessity to control for unobserved heterogeneity

**Example:** Choice of risk-sharing in a tenant-landlord relationship. Theoretical prediction on trade-off between incentives and risk sharing

**Prediction:** Under the assumption that risk is exogenous (e.g. weather), more likely to use sharecropping in less risky environments

This prediction is correct “everything else equal”. Assume, however, that tenants vary in risk attitude. Risk neutral agents will choose to work in risky environments and will prefer high powered incentives (less risk sharing). Would reject the theory!

**Conclusion:** Sometimes difficult to draw inference on causality from correlation between contracts and environment

## Methodological Issues

**Identification problem 1:** Contracts are endogenous. Need to control for all exogenous variables that influence contract choice (see above sharecropping example)

**Identification problem 2:** Different theories may be consistent with the evidence. Say that you find that people who face more comprehensive coverage exhibit a larger accident probability. This is consistent both with pure adverse selection and pure moral hazard or a combination of both!

## **Main Themes of Course**

- (a) Distinguish different sources of asymmetric information (AS vs. MH)
- (b) Identifying asymmetric information using dynamic data
- (c) Role of sorting in markets in explaining contract heterogeneity

## Policy Implications

**Example 1:** In cross sectional data, one finds that better health coverage is correlated with higher expenditure level (e.g. Rand Health Insurance Experiment). If this is driven by moral hazard then the policy implication would be to introduce deductible to reduce over-expenditure. If this is driven by adverse selection then introducing deductible would reduce welfare as it would increase risk without any effect on expenditures

**Example 2:** Conducting a before-after estimation of the switch from fixed salary to piece rate, one finds that a dramatic increase in productivity (e.g. Lazear, 2000). If this is driven by incentive responses, then the introduction of piece rates may be welfare improving. If this is driven by self-selection, then the introduction of piece rates is a zero-sum game between employers

# Lecture 1: Testing for Asymmetric Information

Review of literature testing for asymmetric information

Introduction to positive correlation hypothesis

General tests of asymmetric information

Application to insurance markets

Rothchild and Stiglitz, "Equilibrium in Competitive Insurance Markets: An Essay on the Economics of Imperfect information," QJE, (November 1976):629-650.

Chiappori, Pierre-Andre; Salanie, Bernard. "Testing for Asymmetric Information in Insurance Markets" *Journal of Political Economy*. 108, 1, 2000, 56-78.

Chiappori, Pierre-Andre, Bruno Jullien, Bernard Salanie, and Francois Salanie, "Asymmetric Information in Insurance: General Testable Implications," *Rand Journal of Economics* (forthcoming).

Amy Finkelstein and Kathleen McGarry, "Multiple Dimensions of Private Information: Evidence from Long-Term Care Insurance Market" (*AER*, forthcoming)

Dean Karlan and Jonathan Zinman, "Observing Unobservables: Identifying Information Asymmetries with a Consumer Credit Field Experiment"

## **Testing for Asymmetric Information: General Issues**

Adverse selection in insurance theory assumes that policy holders know more about accident probability and/or distribution of losses in the event of accident than insurers (Rothchild-Stiglitz, 1976)

**Test for existence of asymmetric information:** Do consumers know more about payoff relevant variables? Care only about common value information (riskiness) rather than private values (risk aversion)

**Test prediction of model:** Do consumers' private information influence market outcomes?

Remark: Follow literature in chronological order. Focus first on plain vanilla test of RS model but will then consider additional issues that may also matter: (a) Insurance

purchaser may also have private information on preferences (risk aversion). (b) Moral hazard also plays a role in insurance markets. (c) Market for insurance may not be competitive

## Testing for Asymmetric Information: Plan

Step 1: Positive correlation hypothesis (RS)

Step 2: Literature review suggest that this hypothesis is (often?) rejected

Step 3: Derive the positive correlation hypothesis under general assumptions on consumer preferences and firm behavior. Derive finer tests to identify what assumptions of the model are violated (CJSS, 2004). Conclude that two assumptions in Rothchild Stiglitz are violated in application to car insurance market

Step 4: Consider alternative approach to identify which assumptions may be violated (FM, 2006)

Step 5: Separating adverse selection from moral hazard issues (KZ, 2005)

## Step 1: RS Model (BD 13.1.2)

Similar to MGW 13-D but application to insurance. Policy holders are privately informed about their risk. There is competition in insurance provision. Contracts are exclusive

Consumers may have an accident in which case their wealth  $w$  is decreased by  $L$ . Type is risk or probability to have an accident.  $p_i$  is the probability that type  $i$  has an accident with  $p_1 < p_2$  and the fraction of type  $i$  in the population is  $\alpha_i$ . There are two competing insurers

RS consider the following game. (1) In the first stage, insurers offer contracts  $(I_i, D_i)$  where  $I_i$  is the insurance premium and  $D_i$  is the deductible. Individual  $i$  utility from contract  $i'$  is  $p_i u(w - D_{i'} - I_{i'}) + (1 - p_i) u(w - I_{i'})$ . (2) In stage 2,

individuals choose the best possible contract or no insurance (outside option). (3)  
Finally, nature determine accident outcomes and contracts are executed

**Remark:** Single crossing condition (the indifference curves of two different types do not cross more than once) holds in insurance markets. High risk types are willing to pay more for a marginal reduction in risk (i.e. higher coverage or lower deductible). It should be possible to separate high and low risks: offer high risks greater coverage at a higher premium so that low risks do not want to imitate high risks

**Conclusion from RS** (The analysis follows MGW13-D)

No pooling equilibrium

Complete insurance to high risk,  $D_2 = 0$ , and zero profit implies  $I_2 = p_2L$

Deductible to low risk that maximizes low risk 's utility  $p_1 u(w - D_1 - I_1) + (1 - p_1) u(w - I_1)$  subject to separation  $u(w - I_2) \geq p_2 u(w - D_1 - I_1) + (1 - p_2) u(w - I_1)$  and non-negative profits  $I_1 \geq p_1(L - D_1)$

IC(2,1) and non-negatif profit constraints bind. The low risk contract is given by  $u(w - I_2) = p_2 u(w - (1 - p_1)D_1 - p_1 L) + (1 - p_2) u(w - I_1)$  and  $I_1 = p_1(L - D_1)$

No equilibrium with few high risks (pooling deviation)

## Testing for Asymmetric Information Positive Correlation Hypothesis

Different levels of coverage are offered in equilibrium

Contracts with more comprehensive coverage are chosen by agents with higher expected risk of accident. One should find a positive correlation between risk ( $p_i$ ) and coverage ( $-D_i$ ) for policy holders that are identical to the insurance company (positive correlation hypothesis)

Weak hypothesis that should hold under the existence of asymmetric information. Rejecting that hypothesis implies that there is no asymmetric information or that other assumptions of the model are violated.

Accepting the hypothesis does not mean that there is necessarily adverse selection. Could be evidence of moral hazard. Consumers could pick insurance contract for

exogenous reason (e.g. pick company down the corner). Those who end up with a contract which offers more coverage are less cautious. (More on this kind of inverted causality later.)

## **Step 2: Basic positive correlation test**

Chiappori-Salanie (CS, JPE 00)

High risk agents should everything else equal buy more insurance (that is, low deductible contracts) and should be more likely to file claims

Need to control for the information available to the insurance company. Distinction between comparisons across risk-class and within risk-class. Omitted variable would create a spurious bias between contract choice and risk

Focus on beginners: For senior drivers, need to control for past driving records because this information is available to the insurer. For example, drivers could learn (non-privately) their types over time. Then, drivers with poor records take more insurance and have more accidents even though there is no adverse selection

**Findings:** Reject the hypothesis for young drivers: No evidence of adverse selection

**Additional hypothesis:** drivers could learn about their type over time (endogenous adverse selection). More senior drivers who have not had an accident may know more about their risk. Again, no evidence of adverse selection

Conclude that there is no evidence of asymmetric information in car insurance market

## **Comments**

Exclusive versus non-exclusive contracts. Non-exclusivity restricts the set of possible contracts: for example, convex price schedules cannot be implemented and neither can insurance rationing

Why would agents within the same risk-class chose different contracts? (a) Adverse selection: privately informed about riskiness (common values) (b) Preference: heterogeneity in risk aversion (private values) (c) Random reasons. In any event, the test is not a test of why there are different contracts offered in equilibrium

## **Testing for Asymmetric Information: Literature review**

### **(a) Correlation test**

Absence of evidence of asymmetric information: Automobile insurance (CS, 00), Life insurance (Cawley and Philipson, 97)

Evidence of asymmetric information: Mortgage related unemployment insurance (Bach, 99)

**(b) Unravelling prediction:** If it is not possible to screen, then Akerlof's prediction may hold, high type consumers drop from the market (unravelling), and the equilibrium market price should exceed the competitive price that would obtain for the entire population in the absence of AI

Application: Annuities and life insurance  $\Rightarrow$  Can compute fair price from distributions of life expectancies. Although annuity buyers do not live longer, those who buy more backloaded annuities do (Friedman and Warshawsky (90), Bruggiavini (93), Pertoba (00))

### **Step 3: Test of Asymmetric Information CJSS, 2004**

Need a deeper understanding of data generating process

**Demand side:** (a) AI: Asymmetric information on riskiness (common value), (b) RA: Asymmetric information on risk aversion (private value), (c) MA: Moral hazard

**Supply side:** (a) Market structure, (b) Profit outcomes (0-profit assumption)

**Issue:** Which combination of demand/supply generates the positive correlation test?

## Simple Model

Assume there are two possible events, no-loss and loss  $\{0, L\}$ . An insurance contract is a pair  $C_i = (P_i, R_i)$  where  $P_i$  is the premium and  $R_i$  the reimbursement. Assume the researcher observes two contracts such that  $R_2 > R_1$  and the probability of a claim on contract  $C_i$  is  $q_i$

**Demand:** (A1) Consumers care about distribution of wealth with preference ordering with respect to first order stochastic dominance, (A2) Risk aversion with respect to mean preserving spread, (A3) Realistic expectations meaning that agents correctly assess loss distribution

**Supply:** Zero profit  $P_i - q_i R_i = 0$

**Definition:** Assume  $y$  is a compounded lottery such that  $y = x + z$  has distribution  $G(\cdot)$  where  $x$  has distribution  $F(\cdot)$  and  $z$  has distribution  $H_x(\cdot)$  with zero mean. We say that  $G(\cdot)$  is a mean preserving spread of  $F(\cdot)$

Example: If  $z$  is equal to  $+1$  or  $-1$  with equal probability (independent of  $x$ ),  $y = x + z$  is a mean preserving spread of  $x$

## Three Steps Approach

If some consumers choose contract  $C_1$ , then it must be true that  $P_2 \geq P_1$ . Proof by contradiction that the opposite inequality would violate first order stochastic dominance preference. A stronger claim holds under mean preserving spread preferences.

**Corollary 1:** Assume consumers have preferences (A1-A3) and some agents prefer  $C_1$  over  $C_2$  then  $P_2 - P_1 \geq q_1(R_2 - R_1)$

This prediction holds under AI/MH/RA. Revealed preference argument. It assumes only consistency of consumer choice

**Corollary 2:** Assume consumers have preferences (A1-A3) and 0-profit, then  $q_2 \geq q_1$

Under either AI, MH, RA or a combination of these, we would expect a positive correlation to hold if firms earn zero profit

**Remark:** Corollary 2 also holds under the weaker assumption that profits do not increase with coverage ( $R_2 \geq R_1$  implies  $\pi(C_2) \leq \pi(C_1)$ )

**Corollary 3:** Assume consumers have preferences (A1-A3) and RA is publicly observed, then  $q_2 \geq q_1$

If RA is publicly observed, then the positive correlation prediction holds even if firms do not earn zero profits

## **CJSS: Findings and conclusions**

Accept hypothesis of Corollary 1 but reject hypothesis of Corollary 2 and 3 (positive correlation test)!

The first finding implies that cannot reject A1-A3. Taken together with the first finding, the second finding suggests that (a) 0-profit assumption is violated and (b) if there is heterogeneity in risk aversion, it has to be privately observed

Need new models of insurance market!

## **Step 4: Finkelstein and McGarry 2006 (Long Term Health Care)**

Private long term health insurance covers risks (i.e. nursing home) that are only partially taken into account by private insurance policies or by public insurance for the elderly

Unique data set: (a) Individual subjective assessment on chance to enter a nursing home, (b) Company assessment of risk, (c) Individual investment in risk reducing behavior (flue shot, health check... and other measures of self insurance)

No evidence of positive correlation between coverage and risk after controlling for company risk classification. Those with more long term insurance do not use more nursing home care

Look for direct evidence of private information. After controlling for company risk classification, the individual's belief about nursing home use predicts actual use, that is, individuals have information about the likelihood of risk occurrence that the insurance company doesn't

Existence of unobserved heterogeneity More cautious individuals (not observed by company) are more likely to own long-term care insurance and less likely to use long-term care

## Step 5: Separating AS and MH (Karlan and Zinman, 2005)

Use an experimental design for pre-approved credit cards to identify adverse selection and moral hazard

**Asymmetric information in credit market:** A credit card issuer sends pre-approved credit cards with fixed credit limit but different interest rates for carry over balance. How should the default rate depend on the interest rate? Under pure AS, it is possible that poor risk individuals, who cannot find better credit otherwise, are more likely to accept the high interest offer and more likely to default. Under pure MH, identical individuals take the offers but those who face high interest rate have less incentive to invest in effort to reduce the likelihood of default

The key insight of paper is to demonstrate that it is possible to identify the two dimensions of AI by distinguishing the offer rate  $r^O$ , which is the rate that is initially

proposed in the mail offer, and the contract rate  $r^c$ , which is announce upon acceptance as the actual rate used to compute interest payments, and by randomly varying both rates

## Model

Assume the (final date) expected utility of a borrower is

$$U = p(q, e)[R(q) - (1 + r)B] - e,$$

where  $q$  is represents the borrower's riskiness type distributed with density  $f$  and s.t.  $p_q < 0$  and  $R_q \begin{matrix} > \\ \equiv \\ < \end{matrix} 0$ ,  $B$  is the principal of the loan,  $r$  is the interest rate, and  $e$  is the repayment effort such that  $p_e > 0$  and  $p_{ee} < 0$  and associated cost  $c(e) = e$ . The borrower's outside option in the event they refuse a loan is 0

The lender's (final date) expected profits is  $p(q, e)(1 + r)B - (1 + r^*)B$  where  $r^*$  is the refinancing cost

**Step 1:** Adverse selection alone (no effort  $e$ )

All types  $q$  such that  $U = p(q)[R(q) - (1 + r)B] > 0$  accept contract  $r$ . Define  $q^*$  the type such that  $U = 0$  (assume well-defined)

**Case 1:** Adverse selection, (Stiglitz-Weiss, AER 81).  $p(q)R(q)$  constant with  $q$ , or  $p_q R + R p_q = 0$

This implies that risky types accept the contract since  $\frac{dU}{dq} = -p_q(1 + r)B > 0$  and riskiness increases with the level of interest rate since

$$\frac{dq^*}{dr} = -\frac{p}{(1 + r)p_q} > 0$$

**Remark:** Since riskiness increases with the interest rate, credit rationing may result (in an equilibrium model) even when there is competition because firms do not want to increase the interest rate (or the number of loans) when there is excess demand as this would decrease the average quality and result in negative profits (this was one of the original contributions of SW)

**Case 2:** Positive selection, (deMesa-Webb, QJE 87).  $R(q)$  constant with  $q$ , or  $R_q = 0$

This implies that riskless types accept the contract since  $\frac{dU}{dq} = p_q(R - (1+r)B) < 0$  and riskiness decreases with the level of interest rate since

$$\frac{dq^*}{dr} = -\frac{pB}{R - (1+r)B} < 0$$

Under positive selection, applicant average quality increases with the interest rate. Assumptions on covariations across  $(p, R)$  matter!

**Step 2:** Moral hazard alone (no selection  $q$ )

The FOC of the borrower's problem gives  $p_e(R - (1 + r)B) = 1$ . Taking full derivative w.r.t.  $r$  gives

$$\frac{de}{dr} = \frac{p_e B}{p_{ee}(R - (1 + r)B)} < 0$$

Effort and repayment probability ( $\frac{dp}{dr} = \frac{dp}{de} \frac{de}{dr}$ ) decrease with the interest rate (debt-overhang effect)

**Step 3:** Responses to offer rate  $r^o$  and contract rate  $r^c$ . The effort level  $e$  depends only on the contract rate  $r^c$  and not on the offer rate  $r^o$  because only the former

determines the repayment burden. The marginal type  $q$  depends only on the contract rate  $r^c$  and not on the offer rate  $r^o$  because consumers know only the former when they decide to accept the offer

**Conclusions:** Denote the expected default rate  $Ep = \int_A [1 - p(q, e)] \frac{f(q)}{\int_A f(x) dx} dq$  where  $A$  is the subset of consumers who accept the offer. (a) Evidence of MH if  $\frac{dEp}{dr^o} < 0$ . (b) Evidence of AS (SW) if  $\frac{dEp}{dr^c} > 0$  and (MW) if  $\frac{dEp}{dr^c} < 0$ .

**Experimental design:** Time line of decisions with distinction between offer rate  $r^o$  and contract rate  $r^c$

**Findings:** (a) MH among male borrowers and AS among female borrowers. (b) AI problems decrease with length of relationship with lender. (c) AI explain about 20% of default in sample

**Contribution:** This paper contributes to three literatures:

(a) Test of information economics: It is possible to design a random experiment to separate adverse selection from moral hazard

(b) Policy issue: Can lenders perfectly predict default risks and price loans accordingly? Or are they asymmetrically informed? Does this have an impact on market outcomes (e.g. credit rationing)? Understand potential breakdown in market for small loan (micro finance) that may be more sensitive to AI due to lack of collateral and reputation (poor credit rating)

(c) Try to tell apart different models of credit default: adverse selection (SW), positive selection (DW)

## **Asymmetric information in credit market (review)**

Credit markets could be plagued by asymmetric information AI: adverse selection AS because borrowers are privately informed about their capacity to pay back loans (firms informed about profitability of projects); moral hazard MH because borrowers can take actions to improve chances to pay back debt (firm can select less risky projects and take actions to reduce risk). How prevalent are AI problems in credit markets? Is it possible to identify different sources of AI? Can AI explain phenomenon of credit rationing (some borrowers are willing to pay market rate but are denied a loan)?

**Credit rationing:** (Stiglitz and Weiss, 1987) Under MH or AS, the expected return of a loan may not increase monotonically with the interest rate charged. Profit maximizing interest rates may be kept at levels that do not clear markets. Banks do

not increase rates because this would attract worse borrowers. At equilibrium interest rate there is excess demand for credit (credit rationing). Among observationally identical groups of borrowers, some may obtain loans while others do not (or obtain only partial loans)

**Literature:** Survey asking whether consumers have been denied a loan (Japelli, QJE 1990). Estimate marginal propensity to consume out of exogenous change in liquidity. Take advantage of exogeneity in the timing rule used by credit card issuers to change credit limit changes (Gross and Souleles, QJE 2000)

# Lecture 2: Moral Hazard and Personnel Economics

## Linear-Normal-Exponential Moral Hazard Model

### Basic Setup (Holmstrom and Milgrom 1991, BD 4.2)

Agent chooses effort  $e \geq 0$  at cost  $c(e)$  such that  $c(e) = \frac{ce^2}{2}$ . The agent has utility over effort  $e$  and wage  $w$ ,  $U(e, w) = -\exp[-r(w - c(e))]$  and reservation utility  $\underline{U}$ . (The coefficient of absolute risk aversion, CARA,  $r = -\frac{u''}{u'}$ , is constant)

The principal observes performance outcome  $s = e + \epsilon_s$ , where  $\epsilon_s \sim N(0, \sigma_s^2)$

The principal uses linear compensation contracts (Holmström and Milgrom 1987), i.e.  $w = \beta_0 + \beta s$ , where  $\beta_0$  is the agent's fixed salary and  $\beta$  is the "piece rate" on signal  $s$

The Principal is risk neutral and maximizes expected profits, assumed to be equal to the effort expended by the agent, minus compensation,  $E(e - w)$

**Lemma:** If variable  $x$  is normal,  $x \sim N(\mu, \sigma^2)$ , then  $E[\exp(x)] = \exp\left(\mu + \frac{\sigma^2}{2}\right)$

**Issues:**

1. Compute the agent's effort choice given incentive scheme  $(\beta_0, \beta)$
2. Compute the optimal incentive scheme  $(\beta_0, \beta)$
3. Comparative static: how does the piece rate  $\beta$  vary with  $r$ ,  $c$ , and  $\sigma_s^2$ ?

## The Agent's Problem

$$Max_e E \left\{ - \exp \left[ -r \left( \beta_0 + \beta(e + \epsilon_s) - \frac{ce^2}{2} \right) \right] \right\}$$

The above Lemma simplifies this expression to,

$$Max_e - \exp \left[ -r \left( \beta_0 + \beta e - \frac{ce^2}{2} \right) + \frac{(r\beta\sigma_s)^2}{2} \right]$$

The expression in brackets (minus) is called the certainty equivalent. This corresponds to the level of riskless compensation that gives the agent the same level utility as under the risky compensation contract  $(\beta_0, \beta)$ .  $\frac{(r\beta\sigma_s)^2}{2}$  is the risk premium the principal has to give the agent to face risk. The agent maximizes her certainty equivalent

$$Max_e \left\{ r \left( \beta_0 + \beta e - \frac{ce^2}{2} \right) - \frac{(r\beta\sigma_s)^2}{2} \right\}$$

taking first order condition and solving for  $e$  gives

$$e = \frac{\beta}{c}$$

## The Principal's Problem

$$\begin{aligned} & \underset{\beta_0, \beta}{\text{Max}} E[e - (\beta_0 + \beta s)] \\ \text{s.t. } & E(U) \geq \underline{U} \quad PC \\ & e = \frac{\beta}{c} \quad IC \end{aligned}$$

The principal's objective simplifies to  $e - (\beta_0 + \beta e)$ . Assuming that the participation constraint binds, this constraint can be written as

$$-\exp \left[ -r \left( \beta_0 + \beta e - \frac{ce^2}{2} \right) + \frac{(r\beta\sigma_s)^2}{2} \right] = \underline{U}$$

$$-r \left( \beta_0 + \beta e - \frac{ce^2}{2} \right) + \frac{(r\beta\sigma_s)^2}{2} = \log -\underline{U}$$

$$\beta_0 + \beta e = \frac{ce^2}{2} + \frac{r(\beta\sigma_s)^2}{2} + k$$

where  $k$  is a constant. Plugging both constraints into the principal's maximization problem gives

$$\underset{\beta_0, \beta}{Max} E \left[ \frac{\beta}{c} - \left( \frac{c \left( \frac{\beta}{c} \right)^2}{2} + \frac{r(\beta\sigma_s)^2}{2} + k \right) \right] = \frac{\beta}{c} - \frac{\beta^2}{2c} - \frac{r(\beta\sigma_s)^2}{2} - k$$

The first order condition with respect to  $\beta$  gives

$$\beta = \frac{1}{1 + rc\sigma_s^2}$$

**Interpretation**

- $0 < \beta \leq 1$
- $\beta = 1$  if  $r = 0$  or  $\sigma_s^2 = 0$
- $\beta < 1$  if  $r > 0$  and  $\sigma_s^2 > 0$
- $\frac{d\beta}{dr} < 0$ , more risk averse agents face less powerful incentives
- $\frac{d\beta}{d\sigma} < 0$ , less powerful incentive in more risky environments
- $\frac{d\beta}{dc} < 0$ , less powerful incentive when greater disutility of effort

## **Applications**

Because the linear-normal-exponential model is flexible, it has been extended in several directions to capture different organizational problems. One can consider multiple performance measures, multiple effort dimensions or tasks, multiple agents, and multiple principals

Aggregation of multiple performance measures in incentive contracts (BD 4.6.1)

Relative performance evaluation (BD 8.1.3, Prendergast 2.1.2)

Multi-tasking (HM 91, Baker 92, BD 6.2.1)

Cooperation and collusion amongst agents (BD 8.2.2, HM 90)

## Multi-principal (Dixit, AER 97)

The solution approach for these applications follows the three steps taken before. First solve the agent(s) choice of effort taking the incentive weights as given. In the single agent case, one has to solve an optimization problem while if there are multiple agents one has to solve for the non-cooperative Nash equilibrium. Given the agent(s) effort response(s), the next step is to compute the principal(s) profits. Finally, the analysis is completed by solving for the principal's optimization problem in the single principal case. Otherwise, one has to solve the non-cooperative Nash equilibrium if there are multiple principals

## Relative weight of performance measure in incentive contracts

BD 4.6.1 and see also Prendergast 2.1.2

Agent takes privately observed effort  $a$  and principal observes two performance measures

Profits  $q = a + \varepsilon_q$ , where  $\varepsilon_q \sim N(0, \sigma_q^2)$

Stock price  $P = a + \varepsilon_P$ , where  $\varepsilon_P \sim N(0, \sigma_P^2)$

Covariance between  $q$  and  $P$  is  $\sigma_{qP}$

Payment  $w = t + sq + fP$

$$\text{Utility } U(w, a) = -\exp\left[-\eta\left(w - \frac{1}{2}ca^2\right)\right]$$

$$\text{Optimal agent action } a = \frac{s+f}{c}$$

The optimal contract minimizes the risk premium for a given action and trades off risk and incentives. Let  $\Omega = \frac{\sigma_q^2\sigma_P^2 - \sigma_{qP}}{\sigma_q^2 + 2\sigma_{qP} + \sigma_P^2}$ . The optimal contract is

$$s = \frac{\sigma_P^2 - \sigma_{qP}}{\sigma_q^2 + 2\sigma_{qP} + \sigma_P^2} \frac{1}{1 + \eta c \Omega}$$

$$f = \frac{\sigma_q^2 - \sigma_{qP}}{\sigma_q^2 + 2\sigma_{qP} + \sigma_P^2} \frac{1}{1 + \eta c \Omega}$$

To illustrate, consider the case  $\sigma_{qP} = 0$ . Then  $s = \frac{\sigma_P^2}{\sigma_q^2 + \sigma_P^2 + \eta c \sigma_q^2 \sigma_P^2}$  and  $f = \frac{\sigma_q^2}{\sigma_q^2 + \sigma_P^2 + \eta c \sigma_q^2 \sigma_P^2}$  and the relative importance of a performance measure depends on its relative variance  $\frac{s}{f} = \frac{\sigma_P^2}{\sigma_q^2}$ . More generally  $\frac{s}{f} = \frac{\sigma_P^2 - \sigma_{qP}}{\sigma_q^2 - \sigma_{qP}}$

Similarly, if stock price is a noisy measure of profits  $\varepsilon_P = \varepsilon_q + \varepsilon$  where  $\varepsilon$  is a normally distributed, independent, such that  $\sigma_P^2 = \sigma_q^2 + \sigma^2$  and  $\sigma_{qP} = \sigma_q^2$ , then,  $f = 0$  and  $s = \frac{1}{1 + \eta c \sigma_q^2}$

Information principle: Any signal that lowers variance of “risk” should be included. For example, the shocks could be  $\varepsilon_P = \varepsilon + \varepsilon_{industry}$  and  $\varepsilon_q = \varepsilon' + \varepsilon_{industry}$  where  $\varepsilon_{industry}$  a common industry risk. Or  $\varepsilon_P = \varepsilon + \varepsilon_{exchangerate}$  where the firm’s profits depends on the exchange rate at which oversea profits will be valued

## Relative performance evaluation

BD 8.1.3 and see also Prendergast 2.3

Assume a single principal observes performance from two agents according to  $q_i = a_i + \epsilon_i + \alpha\epsilon_{-i}$

Note that the two agents' incentive problems can be solved independently.

The optimal contract, then is

$$w_1 = t_1 + \frac{1 + \alpha^2}{1 + \alpha^2 + \eta c \sigma^2 (1 - \alpha^2)^2} q_1 - \frac{2\alpha}{1 + \alpha^2} \frac{1 + \alpha^2}{1 + \alpha^2 + \eta c \sigma^2 (1 - \alpha^2)^2} q_2$$

When  $\alpha = 0$  we get the same formula as before. Relative performance evaluation should be used when agents face common risk. When  $\alpha$  is close to  $+/- 1$ , the

principal can filter out all risk by subtracting (or adding) both performance measures and the first best is approximated

## Multi-tasking

HM91, Baker 92, BD 6.2.1 and see also Prendergast 2.2.1

Assume two tasks with cost function  $\psi(a_1, a_2) = \frac{1}{2}(c_1 a_1^2 + c_2 a_2^2) + \delta a_1 a_2$  where the two tasks are substitute  $\sqrt{c_1 c_2} \geq \delta \geq 0$

Performance measures  $q_i = a_i + \epsilon_i$ , where  $\epsilon_i \sim N(0, \sigma_i^2)$  and covariance  $\sigma_{12}$

Payment  $w = t + s_1 q_1 + s_2 q_2$  and the principal's objective function is  $a_1 + a_2 - w$

Optimal actions  $a_i = \frac{s_i c_i - \delta s_j}{c_i c_j - \delta^2}$

The effort put in task  $i$  decreases as the weight on task  $j$  increases and this is because tasks are substitutes  $\delta \geq 0$

Say task  $j$  captures outside activity (e.g. independent sales person). As the return on outside activity increases ( $s_j$  increases), the firm has to increase the incentives it gives the agent if it wants to hold the level of effort constant. It may be optimal at some point to prevent the agent from engaging in outside activities (theory to task allocation)

The optimal contract sets the weight on task 1

$$s_1 = \frac{1 + (c_2 - \delta)\eta\sigma_2^2}{1 + \eta c_2 \sigma_2^2 + \eta c_1 \sigma_1^2 + \eta^2 \sigma_1^2 \sigma_2^2 (c_1 c_2 - \delta^2)}$$

and the symmetric formula holds for  $s_2$ . When the tasks are independent,  $\delta = 0$ , we have as before  $s_1 = \frac{1}{1 + \eta c_1 \sigma_1^2}$

As the quality of measure 2 worsens ( $\sigma_2$  increases), both weights decrease (notice that  $\text{sgn}[\frac{d}{d\sigma_2}s_1] = \text{sgn}[(\delta - c_1)\eta\sigma_1^2 - 1] \leq 0$ ). This is because effort substitution implies that performance weights are complement

If one performance measure becomes very noisy ( $\sigma_i \rightarrow \infty$ , that is, the task outcome is 'unobservable') the incentive effect on the other task may be so small that it becomes optimal to use alternative methods to induce effort or to separate tasks

## Cooperation and collusion amongst agents

BD 8.2.2 and see also Prendergast 2.4

The principal hires two agents. Performance outcomes are generated according to  $q_i = a_i + \epsilon_i$  with the  $\epsilon_i$  correlated

The principal sets the compensation for each agent as a function of both agents' performance outcomes. Subsequently, agents select effort simultaneously after having agreed on a side contract. In contrast with the relative performance evaluation case, the agents may be able to agree on their effort levels and also possibly on redistribution (side payments) among themselves

## Multi-principal (Dixit, AER 97 and Final 05)

Consider the basic framework of Holmstrom and Milgrom (1987) but with 2 principals and cross effects in the cost function

The agent chooses effort  $e_1 \geq 0$  and  $e_2 \geq 0$  at cost  $c(e_1, e_2) = \frac{1}{2}(e_1^2 + e_2^2 + 2ke_1e_2)$ . Efforts are complement if  $-\frac{1}{2} < k < 0$  and substitute if  $k > 0$

The principals observe performance outcome  $s_i = e_i + \epsilon_i$  where  $\epsilon_i \sim N(0, \sigma^2)$ . Principal  $i$  uses linear contracts and offers wage  $w_i = \beta_{i,0} + \beta_{i,1}s_1 + \beta_{i,2}s_2$ . Principal  $i$  maximizes expected profits minus compensation  $E(e_i - w_i)$

The agent has utility over effort and wage  $U = -\exp[-r(w_1 + w_2 - c(e_1, e_2))]$  and has reservation utility  $\underline{U}$

## Agent's effort choice

The agent maximizes  $E\{-\exp\{-r\{\sum_i(\beta_{i,0} + \beta_{i,1}s_1 + \beta_{i,2}s_2) + \frac{1}{2}(e_1^2 + e_2^2 + 2ke_1e_2)\}\}\}$ . The first order conditions give

$$\begin{cases} e_1 = \frac{\beta_{1,1} + \beta_{2,1} - k(\beta_{1,2} + \beta_{2,2})}{1 - k^2} \\ e_2 = \frac{\beta_{1,2} + \beta_{2,2} - k(\beta_{1,1} + \beta_{2,1})}{1 - k^2} \end{cases}$$

It immediately follows that  $de_1/d\beta_{1,1} = de_2/d\beta_{1,2}$  and  $de_2/d\beta_{1,2} = de_1/d\beta_{1,1}$

## Principal Problems

The agent's participation constraint can be rewritten as

$$w_1 = K - \beta_{2,1}e_1 - \beta_{2,2}e_2 + \frac{1}{2}(e_1^2 + e_2^2 + 2ke_1e_2) + \frac{r\sigma^2}{2} \left( (\beta_{1,1} + \beta_{2,1})^2 + (\beta_{1,2} + \beta_{2,2})^2 \right)$$

where  $K$  is a constant. The principal 1 takes principal 2's performance incentive as given and sets  $\beta_{1,1}$  and  $\beta_{1,2}$  to maximize  $e_1(\beta_{1,1}, \beta_{1,2}) - w_1(\beta_{1,1}, \beta_{1,2})$ . The first order conditions give

$$\begin{cases} \frac{1}{1-k^2} + \frac{\beta_{2,1}}{1-k^2} - k\frac{\beta_{2,2}}{1-k^2} - \left( \frac{e_1}{1-k^2} - k\frac{e_2}{1-k^2} + 2k\frac{e_2}{1-k^2} - 2kk\frac{e_1}{1-k^2} \right) = r\sigma^2 (\beta_{1,1} + \beta_{2,1}) \\ -\frac{k}{1-k^2} - k\frac{\beta_{2,1}}{1-k^2} + \frac{\beta_{2,2}}{1-k^2} - \left( -k\frac{e_1}{1-k^2} + \frac{e_2}{1-k^2} - 2k\frac{e_2}{1-k^2} + 2k\frac{e_1}{1-k^2} \right) = r\sigma^2 (\beta_{1,2} + \beta_{2,2}) \end{cases}$$

The optimal incentive weights are given by solving the system of 6 equations in 6 unknowns

**Case  $k = 0$**

We have

$$\begin{cases} e_1 = \beta_{1,1} + \beta_{2,1} \\ e_2 = \beta_{1,2} + \beta_{2,2} \end{cases}$$

Plugging this expression in the second FOC and using symmetry gives,

$$\beta_{1,2} = -\frac{r\sigma^2}{1 + 2r\sigma^2} < 0.$$

Principal 1 insures the agent against risk faced from variations in measure 2, that is, compensate the agent for bad shocks in measure 2

**General Case:** By symmetry we must have in equilibrium that the total incentive weight put on each measure is constant  $\beta_{1,2} + \beta_{2,2} = \beta_{1,1} + \beta_{2,1} \equiv \beta^*$ . This gives,  $e_1 = e_2 = \frac{\beta^*}{1+k}$ . Plugging these expressions in the FOC give

$$\beta^* = \frac{1}{1 + 2r\sigma^2(1 + k)}.$$

Clearly  $d\beta^*/dk < 0$ . The sum of the incentive weights on a given task decreases as tasks are more substitute. The reason is that the cost of providing effort increases as the agent tends to substitute away. The opposite applies for complement tasks. This implies that the principals should group together tasks that are complement and give these tasks to the same agent. They should separate substitute tasks or if this is not possible avoid using incentives

The principals are better off if they can commit to reward only the performance outcome that corresponds to the activity they care about (compartmentalization), because they will not insure the agent for performing poorly on the other principal's measure. Under compartmentalization, the multi-principal problem boils down to two independent principal agent problems

## Application: Identifying Distortions

A General Test for Distortions in Performance Measures (Courty and Marschke, 2006)

Is it possible to identify distortions (behavior that does not maximize the true objective of the organization) from simple statistics on performance outcomes?

This paper tries to capture the intuition that a performance measure 'degrades' after it is activated, that is, the measure becomes a worse indicator of value added

Consider the principal agent model with two measures. We are interested in the agent's responses to a change in the incentive contract and whether one can identify distortions from these responses (we are not interested in the design of the optimal contract)

The agent invests in multiple projects  $\alpha \in A$  ( $\alpha$  corresponds to the unit of observation for the econometrician). Each project has several dimensions of investment. We denote  $(e, g) = (e_0, e_1, e_2, g_1, g_2)$  the (unobservable) agent's productive efforts and gaming efforts. The performance outcome for project  $\alpha$  is

$$p_{i,\alpha}(e, g) = v_{0,\alpha}e_0 + (v_{i,\alpha} + \eta_{i,\alpha})e_i + w_{i,\alpha}g_i$$

The agent is rewarded on the basis of aggregate outcome  $\sum_{\alpha} p_{i,\alpha}(e, g)$ . The principal's objective is

$$V_{\alpha}(e, g) = v_{0,\alpha}e_0 + v_{1,\alpha}e_1 + v_{2,\alpha}e_2 - (\xi_{1,\alpha}g_1 + \xi_{2,\alpha}g_2)$$

We say that a performance measure is distorted if it either imperfectly captures the marginal productivity of effort, i.e.  $\eta_{i,\alpha} \neq 0$  for some  $\alpha$ , or if the agent can take gaming actions that increase the performance measure but not the objective, i.e.  $w_{i,\alpha} > 0$  for some  $\alpha$ . The agent maximizes

$$\sum_i \beta_i \left( \sum_{\alpha} p_{i,\alpha}(e_{\alpha}, g_{\alpha}) \right) - \frac{1}{2} \sum_{\alpha} \left( e_{0,\alpha}^2 + \sum_i (e_{i,\alpha}^2 + g_{i,\alpha}^2) \right)$$

The optimal investment is  $e_{0,\alpha}(\beta_1, \beta_2) = (\beta_1 + \beta_2)v_{0,\alpha}$ ,  $e_{i,\alpha}(\beta_1, \beta_2) = \beta_i(v_{i,\alpha} + \eta_{i,\alpha})$ ,  $g_{i,\alpha}(\beta_1, \beta_2) = \beta_i w_{i,\alpha}$ . One can compute simple statistics on the mean, variance, and covariance of performance outcome two

$$Ep_{2,\alpha}(\beta_1, \beta_2) = (\beta_1 + \beta_2)Ev_{0,\alpha}^2 + \beta_2 E(v_{2,\alpha} + \eta_{2,\alpha})^2 + \beta_2 Ew_{2,\alpha}^2$$

$$Varp_{2,\alpha}(\beta_1, \beta_2) = (\beta_1 + \beta_2)^2 Varv_{0,\alpha}^2 + \beta_2^2 Var(v_{2,\alpha} + \eta_{2,\alpha})^2 + \beta_2^2 Varw_{2,\alpha}^2$$

$$Cov(p_{2,\alpha}(\beta_1, \beta_2), V_\alpha(\beta_1, \beta_2)) = (\beta_1 + \beta_2)^2 Varv_{0,\alpha}^2 + \beta_2^2 Cor((v_{2,\alpha} + \eta_{2,\alpha})^2, v_{2,\alpha}(v_{2,\alpha} + \eta_{2,\alpha})) - \beta_2^2 Cov(\xi_{2,\alpha} w_{2,\alpha}, w_{2,\alpha}^2)$$

The mean and variance of measure 2 increase as  $\beta_2$  increases but this alone is not evidence of distortions. The covariance of measure 2 and  $V$  decreases only if there are distortions. A more general result can be derived

**Proposition:** Measure two is distorted if and only if  $\frac{cov(p_{2,\alpha}, V_\alpha)}{var(p_{2,\alpha})}$  decreases after an increase in  $\frac{\beta_2}{\beta_1}$

If there exists distortions, the regression coefficient of the measure on the true goal of the organization  $\left(\frac{cov(p_{2,\alpha}, V_\alpha)}{var(p_{2,\alpha})}\right)$  should decrease after the measure receives relatively more emphasis. In particular, the regression coefficient should decrease after the measure is activated

# Lecture 3: Matching and Contracts

## Part I: Two-Sided Matching Theory

Roth, Alvin E. and Marilda Sotomayor [1990], *Two-Sided Matching: A Study in Game-Theoretic Modeling and Analysis*, Cambridge, Cambridge University Press

Gale & Shapley, *American Mathematical Monthly* (1962)

These notes are based on the book by Roth and Sotomayor

## **Two sided matching**

Two sided: agents belong to disjoint sets

Matching: one-on-one due to bilateral nature of the exchange

Matching could be centralized (clearinghouse as in matching of medical interns to hospitals) or decentralized (as the Becker's marriage market)

In the medical intern market, interns and hospitals send preferences over matches to a clearing house that uses an algorithm to compute a proposed matching

## Issues

Do stable matchings exist?

Are there procedures such that parties will accept the match proposed by the procedure?

When will parties reveal their preferences truthfully in a centralized matching procedure?

When does unravelling (some parties move to decentralized matching) occur?

## One-to-one matching: The Marriage Market

A **marriage market** is a triplet  $R = (M, W, P)$  where  $M = \{m_1, \dots, m_p\}$  is a set of men,  $W = \{w_1, \dots, w_q\}$  a set of women, and  $P = (P_k)_{k \in M \cup W}$  a set of (strict) preference profiles

(To simplify, we assume that agents have strict preferences. Many of the results generalize when preferences are not strict.)

For any  $m$  and  $w$ ,  $P_m$  is a preference relation over  $W \cup \{m\}$  and  $P_w$  is a preference relation over  $M \cup \{w\}$

For example,  $m_2$ 's preferences are  $P_{m_2} = w_3, w_4, m_2, w_1, w_2$  and we write  $w_3 P_{m_2} w_4$

We assume that  $P$  is complete and transitive

We say that  $j$  is not **acceptable** to  $k$  if  $k$  prefers to remain single rather than be matched to  $j$ , i.e.  $kP_kj$

## Stable Matching

A **matching**  $\mu$  is a one-on-one function of  $M \cup W$  into itself such that  $\mu^2(m) = m$ ,  $\mu(w) \in M \cup \{w\}$  and  $\mu(m) \in W \cup \{m\}$ .

A matching  $\mu$  is blocked by  $k \in M \cup W$  if  $k$  prefers being single to being matched with  $\mu(k)$ , i.e.  $kP_k\mu(k)$ .

A matching  $\mu$  is blocked by pair  $(m, w) \in M \times W$  if they both prefer each other to their match under  $\mu$ , i.e.  $wP_m\mu(m)$  and  $mP_w\mu(w)$ .

A matching is **stable** if it isn't blocked by any agent or any pair of agents. Note that a stable matching is Pareto-efficient (Why?)

**Example:** Consider a market with 3 men and 3 women with the following preferences

$$P(m_1) = w_2, w_1, w_3 \qquad P(w_1) = m_1, m_3, m_2$$

$$P(m_2) = w_1, w_3, w_2 \qquad P(w_2) = m_3, m_1, m_2$$

$$P(m_3) = w_1, w_2, w_3 \qquad P(w_3) = m_1, m_3, m_2$$

All possible matchings are individually rational. The matching  $\begin{matrix} w_1 & w_2 & w_3 \\ m_1 & m_2 & m_3 \end{matrix}$  is unstable (blocked by  $(m_1, w_2)$ ) while matching  $\begin{matrix} w_1 & w_2 & w_3 \\ m_1 & m_3 & m_2 \end{matrix}$  is stable

Restricting attention to stable matching is reasonable if preferences are public information or if there exists a matchmaker who knows all agents' preferences

## Mechanisms

A **mechanism** is a systematic procedure which determines a matching for each marriage market. Examples include the medical intern mechanism or the typical 3 step application-admission-matriculation between students and universities

For a mechanism  $h$ , we have  $h[R]$  is the matching assigned for market  $R = (M, W, P)$  where  $h[R](k)$  denotes  $k$ 's match under mechanism  $h$  applied to market  $R$

A mechanism  $h$  is stable if  $h[R]$  is stable for any market  $R$

We start by asking whether there exists a  $h$  that is stable. Then, we will investigate whether parties have an incentive to reveal their preferences truthfully or if they can be better-off manipulating the information that is used to compute  $h[R]$

## Does a stable matching exist?

Could start with any matching, look for a blocking pair, match the blocking pair, and keep repeating until there exists blocking pairs. This procedure, however, may not converge to a stable matching if there exists cycles. In the above example, one starts

to match blocking pair  $(m_1, w_2)$  to form matching  $\begin{matrix} w_1 & w_2 & w_3 \\ m_2 & m_1 & m_3 \end{matrix}$ , then match

blocking pair  $(m_3, w_2)$  to form matching  $\begin{matrix} w_1 & w_2 & w_3 \\ m_2 & m_3 & m_1 \end{matrix}$ , then match blocking pair

$(m_3, w_1)$  to form matching  $\begin{matrix} w_1 & w_2 & w_3 \\ m_3 & m_2 & m_1 \end{matrix}$ , then match blocking pair  $(m_1, w_1)$  to

form matching  $\begin{matrix} w_1 & w_2 & w_3 \\ m_1 & m_2 & m_3 \end{matrix}$  which closes the cycle!

## Men Proposing Deferred Acceptance Algorithm (Gale-Shapley, 1962)

Step 1—Each man  $m$  proposes to his first choice (if he has any). Each woman rejects any unacceptable proposals and, if more than one acceptable proposal is received, holds the most preferred one

Step 2...

Step  $k$ —Any man who was rejected in previous round makes a proposal to his most preferred acceptable mate who hasn't yet rejected him. If no acceptable choice remains, he makes no proposal. Each woman holds her most preferred acceptable offer, and rejects the rest.

Algorithm terminates after a step where no offer or no rejection is made

The algorithm proposes to match each woman to the man (if any) whose proposal she is holding. Women who are not holding any proposal and men whose proposals have all been rejected remain single. We denote the resulting match  $\mu_M$

## Existence of Stable Matching

The following result holds for one-on-one two-sided markets

**Theorem 2-8** (Gale and Shapley 1962): A stable matching exists for every marriage market

Interestingly, this result holds only for two-sided and one-on-one markets. In particular, it does not hold when matching is one-sided (roommate problem where individuals have to be paired in rooms), when it is three sided (mother, father and child have to be matched into families), or when the market is not one-on-one (many-to-one allocation of pools of workers to firms)

The algorithm establishes existence of a stable matching and provides a method to construct stable matchings, assuming that preferences are known. Under the natural assumption that preferences are private information, the algorithm does not explain how one can get parties to reveal their preferences truthfully

## Preferences over Matchings

We write  $\mu >_M \mu'$  if all men prefer matching  $\mu$  over matching  $\mu'$ , that is,  $\mu(k) P_m \mu'(k)$

$\mu$  is **men-optimal** if every man likes  $\mu$  at least as much as any other stable matching

$(m, w)$  is **achievable** if there exists a stable matching that pairs  $m$  and  $w$

A men-optimal matching matches each man to his most preferred achievable woman

**Theorem 2-12** ( Gale and Shapley 1962) There always exists a men-optimal stable matching and it corresponds to the deferred acceptance algorithm with men proposing

$\mu_M$

All agents on one side of the market have common preferences regarding the choice of stable matching

## Strategic Behavior

The deferred acceptance algorithm produces stable matchings when preferences are publicly observed. When preferences are privately observed, can it be strategically manipulated by agents who would prefer to deliberately misrepresent their preferences? More generally, does there exist a mechanism that produce stable matchings and that cannot be manipulated?

Game theory approach—Players simultaneously reveal preferences over possible matches and these preferences are then plugged into a mechanism that proposes a match. The game is characterized by (a) a set of players:  $M \cup W$ , (b) a set of strategies: for men, preference order over women and oneself  $Q(m)$  and for women, preference order over men and oneself  $Q(w)$ , (c) the payoff for each player as a function of strategies:

matching  $h[R]$  computed using mechanism  $h$  and plugging preferences  $Q(m)$  and  $Q(w)$

We denote  $Q = (Q_{-i}, Q(i))$  to capture the strategy profile when player  $i$  deviates to  $Q(i)$

A mechanism  $h$  is **strategy-proof** if for any  $R$ ,  $h[R](k) P_k h[P_{-k}, P(k)](k)$  for any  $k \in M \cup W$ , for any deviation  $P(k)$ . It is a dominant strategy for all players to truthfully reveal their preferences

**Theorem 4-4** (Roth): There is no stable and strategy-proof mechanism.

This is an impossibility theorem. To prove it, it is sufficient to present an example of market for which there exists no stable matching mechanism such that truthful revelation of preferences is a dominant strategy. Consider the following 2x2 market

$$P(m_1) = w_1, w_2 \quad P(w_1) = m_2, m_1$$

$$P(m_2) = w_2, w_1 \quad P(w_2) = m_1, m_2$$

There exists only two stable matching: parallel pairs  $\begin{matrix} w_1 & w_2 \\ m_1 & m_2 \end{matrix}$  or crossing pairs

$$\begin{matrix} w_1 & w_2 \\ m_2 & m_1 \end{matrix} .$$

Under parallel pairs,  $w_2$  can deviate to  $Q(w_2) = m_1$ . Assuming that all other players report truthfully, the only stable matching to these preferences is crossing pairs and  $w_2$  is better off. Therefore, truthtelling is not a dominant strategy for  $w_2$ .

Under crossing pairs,  $m_2$  is better off reporting  $Q(m_2) = w_2$  given that everyone else reports truthfully and only stable matchings are selected.

## Part II: Assortative Matching

Instead of starting from exogenously given preferences over matches, we assume that individuals are endowed with bundles of characteristics, and that there is a production function that transforms endowments into output

Putting structure on the nature of the ultimate reason for why people have preferences over one-another and ultimately want to match (the joint production function) allows us to derive further implications on the outcome of matching

Sattinger, Michael, 1993. "Assignment Models of the Distribution of Earnings," *Journal of Economic Literature*, vol. 31(2), pages 831-80, June.

Li, Hao, 2007. "Assortative Matching." New Palgrave.

## One Dimensional Characteristic

Assume men and women can be ordered and ranked by a single characteristic or trait  $m \in \{m_1, \dots, m_M\}$  such that  $m_1 < \dots < m_M$  and  $w \in \{w_1, \dots, w_W\}$  such that  $w_1 < \dots < w_W$ . Individual characteristic or type is public information

Types  $m$  and  $w$  produce joint output  $h(m, w)$ . We assume throughout that  $h(m, w)$  is increasing in both traits. This imposes a great deal of commonality in agents' interest over types since all men produce more with  $w_W$  than with  $w_{W-1}$  and so on... and similarly all women produce more with  $m_M$  than with  $m_{M-1}$  and so on...

We distinguish the transferable and non-transferable utility cases. Under transferable utility, both parties can bargain over the joint output and agree on a sharing rule. Alternatively, we could assume that the surplus that each party obtains from a match

is given and nonnegotiable (non-transferable utility). This would be reasonable, for example, if the surplus corresponds to a public good, or if the sharing rule is exogenously given

We say that the matching is positive assortative, or PAM, if (among all married couples) the highest man marries the highest woman, the second highest types marry each other, and so on... Negative Assortative Matching (NAM) is defined as top marrying bottom and so on...

## Issues

Which agents marry and which prefer to remain single?

When should we expect positive assortative matching?

Under transferrable utility, how is the surplus shared between men and women?

How do changes in market characteristics (e.g. share of men relative to women, or shift in distribution of types) influence matching and surplus sharing?

Implications for how the distribution of utility across types depends on the distribution of types (explain changes in distribution of earning in a labor application)

## Non-Transferable Utilities (NTU)

The assumptions of NTU and  $h$  increasing in type imply that both men and women prefer to match with higher types

**Proposition:** There exists a unique stable matching and it is PAM

Because both men and women have commonality in preferences, both  $\mu_M$  and  $\mu_W$  are stable matchings

## Transferable Utilities (TU)

If there is no interaction in traits,  $h(m, w) = h^m(m) + h^f(w)$ , it does not matter who marries whom because utility is fully transferable. Any matching is stable. The interesting case occurs when there are interactions in traits

We say that  $h$  is super modular if for all  $m' > m$  and  $w' > w$

$$h(m', w') + h(m, w) \geq h(m, w') + h(m', w)$$

Supermodularity is equivalent to say that the contribution of  $f'$  relative to  $f$  increases with male characteristics since the above condition is equivalent to

$$h(m', w') - h(m', w) \geq h(m, w') - h(m, w)$$

In the continuous and twice differentiable case,  $h$  is supermodular iff  $h_{wm} > 0$

Assume that all men and women prefer to marry (with anyone) than to stay single

**Proposition:** If  $h$  is supermodular, the only stable matching is PAM

The intuition is that under supermodularity  $m'$  is willing to pay more than  $m$  to get  $w'$  instead of  $w$ . Respectively  $w'$  is willing to pay more than  $w$  to get  $m'$  instead  $m$ . Therefore, either  $m'$  (resp.  $w'$ ) will outbid  $m$  (resp.  $w$ ) for  $w'$  (resp.  $m'$ )

Supermodularity is a sufficient condition but not necessary. It is possible to have assortative matching for non-supermodular production functions

Under transferable utility, one needs a stronger assumption (supermodularity in addition to increasing surplus in type) to warranty PAM

## Continuous Type Case

Under discrete type, sharing rules cannot be uniquely pinned down. Discrete gaps between types imposes only bounds on sharing rules

To simplify, assume that types are continuous,  $m \in [0, 1]$  and  $w \in [0, 1]$ . We normalize the measure of men to 1 and we denote the female-male ratio as  $r$  such that  $r > 1$

We write the surplus function of couple  $m$  and  $w$  as  $h(m, w) + g$  where  $g$  is a common gain from marriage unrelated to characteristics. We assume that  $h$  is increasing in types, supermodular, and normalize  $h(0, 0) = 0$

Man  $m$  gets  $h(m, 0)$  if he remains single and similarly woman  $w$  gets  $h(0, w)$

The net surplus of  $m$  and  $w$  from getting married is

$$h(m, w) + g - h(0, w) - h(m, 0)$$

## Matching Function

In equilibrium, we have PAM. The top male marries the top female, and matching continues one-on-one till there is no man left,

$$1 - F(m) = r(1 - G(\psi(m)))$$

where  $\psi(m)$  is the female mate of man  $m$ . After simplification  $\psi(m) = G^{-1} \left[ 1 - \frac{1-F(m)}{r} \right]$  for all  $m \in [0, 1]$

Among married couples, man's trait increases as woman trait increases according to

$$\frac{dw}{dm} = \psi'(m) = \frac{1}{r} \frac{f(m)}{g(\psi(m))}$$

For example, if both  $F$  and  $G$  are uniform,  $\frac{dw}{dm} < 1$  and man's trait increases faster than woman's trait across married couples

## Sharing Rules

We denote  $v(m)$  the equilibrium utility of man  $m$  and  $u(w)$  the equilibrium utility of woman  $w$

In any stable matching we must have that no man  $m$  and woman  $w$  want to deviate. This implies that

$$v(m) + u(w) \geq h(m, w) + g$$

with equality only if  $w = \psi(m)$

Another way to state the problem is to look at the individual's choice problem. Any man  $m$  takes the women's demand  $u(\cdot)$  as given and maximizes his residual benefit from marriage. This implies

$$v(m) = g + \text{Max}_w \{h(m, w) - u(w)\}$$

The same applies for women. The first order conditions imply

$$h_w(m, w) = u'(w)$$

$$h_m(m, w) = v'(w)$$

**Remark:** Under supermodularity, the second order conditions to the men and women's optimization problems hold

The equilibrium surplus function must satisfy the differential equation

$$v'(m) = h_m(m, \psi(m))$$

The increase in men's utility depends only on men's contribution to marriage. The fact that better men marry better women cancels out and has no impact on the rate of increase of men's surplus. The rate of change, however, is higher if a man marries a higher woman

The last married women must be indifferent between marriage and singlehood

$$u(w_0) = h(0, w_0)$$

The lowest married women is of a higher type than the lowest married man,  $w_0 > 0$

Interestingly,  $w_0$  gets all the variable surplus from marriage. As a result, the worst man gets all the fixed rent from getting married

$$v(0) = g \geq h(0, 0) = 0$$

After integration, we can compute the equilibrium utilities as

$$u(w) = h(0, w_0) + \int_{w_0}^w h_w(\psi^{-1}(w'), w') dw'$$
$$v(m) = g + \int_0^m h_m(m', \psi(m')) dm'$$

**Remark:** If  $r = 1$ , then  $u(0)$  and  $v(0)$  is left undetermined up to  $u(0) + v(0) = g$  and  $u(0) \geq 0, v(0) \geq 0$ . Although the fixed gain from marrying is left undetermined (up to an upper bound), the marginal gain from marriage is the same as when  $r \neq 1$

## How do utility vary over types?

The partner who is worse off in the least productive marriage depends on  $g \lesseqgtr h(0, w_0)$ , that is, the relative difference between the rent from marriage scarcity and the outside option from marrying

Otherwise women's utility varies according to

$$\frac{du}{dm} = \frac{dv}{dm} \psi'$$

Under uniform distribution and  $h(m, w) = h(m + w)$ , man's utility increases faster than women's utility across marriages,  $\frac{du}{dm} = \frac{1}{r} \frac{dv}{dm} < \frac{dv}{dm}$

## Comparative Statics

Consider an increase in the female-male ratio  $r$

Denote  $\varphi(m|r)$  the type of the women married with man  $m$  when the sex ratio is  $r$ . When the sex ratio increases, a given type of man  $m$  marries a woman of higher type

$$\frac{d}{dr}\varphi(m|r) = \frac{1 - F(m)}{r^2 g(\varphi(m, r))} \geq 0$$

and receives a higher utility  $\frac{d}{dr}v(m|r) \geq 0$ . Women are less likely to marry and those who marry are worse off  $\frac{d}{dr}u(w|r) \leq 0$

Consider a shift in the distribution of type. For example, assume that the distribution of women shifts according to first order stochastic dominance

Men become better-off (they marry higher type women). A woman of given characteristic is worse-off (marry lower men) while a woman in a given percentile is better off

## Application to distribution of earnings (Sattinger III-b)

Assume the two traits are capital (e.g. machine, technology) and skill and both are positive

The production function is  $f(g, k) = g^\alpha k^\beta$  with  $\alpha, \beta > 0$ . Skill and capital are complement  $f_{gk}(g, k) = \alpha\beta g^{\alpha-1} k^{\beta-1} > 0$

Assume  $g$  and  $k$  are lognormal with variance  $\sigma_g^2$  and  $\sigma_k^2$

Under lognormality, the cumulative distribution of  $g$  is

$$G(g) = \int_0^g \frac{1}{\sqrt{2\pi}\sigma_g x} e^{-\frac{1}{2}\left(\frac{\ln x - \mu_g}{\sigma_g}\right)^2} dx$$

The cumulative distribution of  $k$  is similarly defined as  $K(k)$ . The matching function  $k(g)$  is given by

$$1 - G(g) = 1 - K(k(g))$$

$$\int_0^g \frac{1}{\sqrt{2\pi}\sigma_g x} e^{-\frac{1}{2}\left(\frac{\ln x - \mu_g}{\sigma_g}\right)^2} dx = \int_0^{k(g)} \frac{1}{\sqrt{2\pi}\sigma_k x} e^{-\frac{1}{2}\left(\frac{\ln x - \mu_k}{\sigma_k}\right)^2} dx$$

After a change of variable ( $y = \frac{\ln x - \mu}{\sigma}$ ) we get

$$\frac{\ln k(g) - \mu_k}{\sigma_k} = \frac{\ln g - \mu_g}{\sigma_g}$$

and rearranging terms gives

$$k(g) = A_1 g^{\frac{\sigma_k}{\sigma_g}}$$

where  $A_1$  is a constant

The wage function is given by

$$w'(g) = f_g(g, k(g)) = \alpha g^{\alpha-1} k(g)^\beta = A_2 g^{\alpha + \beta \frac{\sigma_k}{\sigma_g} - 1}$$

If all workers work with the same capital (machines are identical and  $\sigma_k = 0$ ), we have  $w'(g) = \alpha g^{\alpha-1}$ . Marginally better workers get paid their marginal product. Under matching, workers get paid more than their marginal product ( $\beta \frac{\sigma_k}{\sigma_g} > 0$ ) and the distribution of wage will be skewed relative to a world without matching

Integrating, and assuming  $w(0) = 0$ , we have

$$w(g) = A_3 g^{\frac{\alpha\sigma_g + \beta\sigma_k}{\sigma_g}}$$

Matching distorts the relation between skill and wage. Wage,  $w(g)$ , is convex in skill if  $\frac{\alpha\sigma_g + \beta\sigma_k}{\sigma_g} > 1$

For example, under constant return to scale,  $\alpha + \beta = 1$ ,  $w(g)$  is convex in skill if the variance of capital is greater than the variance of skill  $\sigma_k > \sigma_g$ . Better talents get matched with even greater capital and this increases their marginal productivity

## Referee report exercise

It is very difficult to assess other people's work (and even more difficult to assess your own work) but we have to do this all the time. This is even more true for new pieces of research that have not been legitimized and distilled by the profession. Although there is no unique formula to do so, the following points should be useful

State the main punchline of the paper and briefly summarize the argument. Keep your summary short (no more than 2 paragraphs or 15 lines) if you do not add anything to what the paper has already said. The goal of this summary is to convey the essence of the paper in very simple and understandable terms and in very few words

Your summary can add to the paper. Example of constructive comments include: (a) Restate some of the point of the paper in a language different from the one used by the authors clarifying the contribution, or suggesting new connections that were not evident in the paper. (b) Identify simplifications in the presentation of the argument that could further clarify the authors' contribution. (c) Discuss the robustness of the results. Point out that some conclusions may be more/less general than stated in the paper. (d) Add points that support or challenge the paper (facts, research piece not cited but (in)consistent with the argument, ...). (e) Point out assumptions that have not received enough emphasis (motivations missing) and explain why this is a problem. (f) Make connections with other economic fields or point out new applications where the paper has something relevant to say

Following your summary, provide an assessment of the importance of the article's contribution to the literature. How do you do that? (a) Do you think the paper

achieves its stated goal and makes the contribution it proposes to make. There is often a disconnect between what the paper states in the introduction and what it does in the core of the analysis. You may also have to discuss the issue of robustness and generality of the results presented. (b) How does the paper fit within the broader economic literature? Carefully put the paper within broader economic context and if necessary make connections with economic fields that the paper does not mention. (c) Note that it is sometimes difficult to identify and state the main contribution of the paper. Often, papers make many poorly connected points, putting in the same package parts that do not necessarily belong together. If so, you need to demonstrate that this is the case, and then chose the most important points that are worth focusing on

You should also point out potential flaws in the analysis. You should distinguish those major flaws that question the basic punchline of the paper (this was discussed

in the previous point above), and those flaws that can be fixed. For example, you could argue that the model needs to be extended in a more realistic and relevant set up (e.g. different assumptions on behavior, timing, ...). These are minor comments, however, at least as long as you think the analysis will still carry through. Similarly, you could ask for robustness check in the empirical analysis. If there are too many problems with the paper and too many issues to be addressed, then it is sometimes best to write a new paper from scratch

For an applied/empirical paper, you should explain how the article proposes to bring the theory to the data. Explain (a) What hypothesis the article tests? (b) What is the empirical strategy? (c) What are the strength and weaknesses of the data used to test the theory? Most importantly, you should investigate whether there are alternative models/hypothesis that could explain the evidence

## Course wrap-up

A Few thoughts on methodology... with some suggestions to carry on research in applied micro-economics

Pierre-André Chiappori and Steven D. Levitt. “An Examination of the Influence of Theory and Individual Theorists on Empirical Research in Microeconomics” *American Economic Review* 2003, 151-155

Hal Varian. “What Use is Economic Theory?” Mimeo, 1989. <http://www.sims.berkeley.edu/>

See also resources on the web e.g. <http://personal.lse.ac.uk/kudamats/tips4economists.htm>

Issues of methodology are rarely taught formally but are still essential. These issues are particularly relevant for our course since we are not following a well-defined research agenda (with a clear set of research questions and methodological tools) but rather we have been trying to go back and forth from theory to empirical evidence, covering various topics in information economics and contract theory. One can easily get lost in such a journey. I try to discuss some issues that are important to move on from this course

Discussion of methodology can quickly get boring with little value added. My objective is modest and I want to keep this short. First, I want to raise some important questions regarding how one can assess and improve the impact of one's own ideas and research. Second, I want to share a few practical tips that I have personally found useful

## **Influence of Theory on Empirical Research in Microeconomics**

Chiappori and Levitt ask the following questions: Does microeconomics theory influence empirical research? What fields influence empirical research the most? Where is the cross-fertilization the most productive? How long it takes for theory ideas to influence empirical work?

They break down the field of micro theory in different areas: price theory, application of price theory, information economics, game theory, behavioral economics, and general equilibrium. Then they sort theory and empirical contribution in these categories. They also associate for each empirical paper a referenced theory paper. Their main findings are:

Point 1: The ratio of theory and empirical work published is not constant across fields. More recent theories may not produce testable implication or may be more

difficult to test due to data constraints? The only recent field that has influenced empirical work is information economics

Point 2: Empirical work, however, disproportionately cite recent theory work over older work. Empiricists do refer to recent theory contributions

Point 3: A handful of economists have had a disproportionately important influence (winner take all)... although their influence has been through multiple ideas (the distribution of influence is less concentrated among papers than authors)

## **How to build and economic model?**

According to Varian, a model is an idealization of reality that has enough connection with reality to make it useful. We often forget this reality and waste much space trying to make our models too real, not making enough simplifications to focus on the essential message we want to convey

Where to start? One needs to start with an idea, insight, puzzle... and there are many ways to get and select ideas: literature, media, business insider story

Identify the central catch and build around it (example of ticket pricing). If the catch is good you should be able to express it with a simple story and illustrative example. Start with a simple model where the idea is transparent (the 2 principle—2 periods, 2 agents, 2 types, 2 states, 2 goods. . . )

“Simplify to get the result, complexify to see how general it is”

Search the literature and establish connections with your idea

Use techniques you know to make your idea more general and to embed it within the literature

Working on a paper is rarely an incremental and predictable process. We often end up spending much time on questions that are of little interest. To avoid that, some people follow a method of work: (a) Let your ideas compete with one another in a kind of elimination tournament (Stage A, B, and C), (b) Work in batch and leave time between iterations (ideas are like cheese)

Put simply, the idea is to try to spend time-consuming writing time only on thoughts that can be read and published

## **How to write an introduction?**

The goal is to raise interest and establish credibility: Get to the point quickly, use examples, keep the point as simple as possible and avoid details. Include in the introduction only statements that your reader can understand. Use an illustrative example but make sure it fits perfectly your story

Hook: Attract the reader's interest. Be careful in delineating the boundary of your problem. Show that your problem is economically relevant. (a) Plug into a policy debate (give figures showing economic importance). (b) Plug into an academic debate (argue that there is need for more knowledge on the issue)

Question: Define your problem and your contribution

Present your answer to the problem: explain the main idea and give also a sense of depth by discussing reasonable but non trivial implications, as well as logical consequence

Antecedent and value added: Acknowledge what was already known and make the case for your contribution. This section is central to convince your reader (editor/referee) that your paper presents something new that is worth circulating/citing/publishing.

## **The Value of Interaction**

As you work more on your model you tend to lose perspective. You tend to think that something is obvious when it really isn't. You tend to think something is complicated when it really isn't. To compensate for this, you should use all opportunities to interact, but beware of your audience's background and characteristics

For example, use classmates for in depth analysis of specific points, a working group as a captive audience from which it is more difficult to get targeted feedback, and seminar speakers and visitors to get feedback on the big picture (elevator talk)

## Literature Search

Research what others have said about your problem (Econlit, Repec, google, NBER, other working paper series)

Research the impact on the profession of past contributions on your topic (web of science, google scholar)

Use citation tools: Use both backward citations to go back to the roots and also forward citation to find out what has been done recently in your field. Is there any current research that cite the articles you cite?

More generally, citation research helps you to get an idea of what are the main ideas in your research area (citation impact) and how ideas have developed (forward citation). This will give you a better idea of how your ideas fit within the literature and whether they are likely to contribute to the profession

## Match Theory and Empirical Evidence

A large fraction of empirical work attempts to measure relations that rest on accepted theories (e.g. return on human capital)

To investigate the relevance of new theories, several approaches are available: (a) discuss descriptive statistics (typical at the early development stages), (b) consider formal tests either non-parametric or reduced form based on theoretical predictions (e.g. Chiappori Salanie Julien), (c) Fully structural model (Paulson-Townsend-Karaivanov)

The standard for testing a theory is much higher because one needs to rule out alternative explanations (Popperian approach). To be convincing, one needs to discuss in details how the data were generated (sources of heterogeneity, what is un/observed by the actors and the econometrician...). In addition, one may consider as many 'comparative static' dimensions as possible (e.g. how do the predictions change as a function of market parameters)

## Data Sources

Data can come from a wide variety of sources. For example, the different papers covered in class use data from:

Chiappori-Julien-Salanie-Salanie: Proprietary data (French car insurance company)

Finkelstein-McGarry: Publicly available survey data (Asset HEALth Dynamics)

Karlan-Zinman: Proprietary data + experiment (South African credit card)

Courty-Marschke: Publicly available and collected (Experimental study and DoL JTPA administrative data)

Bedard (NLSY)

Ackerman-Botticini (library archives—publicly available)

Paulson-Townsend-Karaivanov (collected data)

The data used come from a wide variety of sources. In almost two third of the cases the data were publicly available although some datasets were probably time consuming to put together