MICROECONOMICS III
Information Economics and Contract Theory
Final Exam—Pascal Courty
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ANSWER ALL QUESTIONS.
TOTAL POINTS: 120

Exercise 1 (36 pts)

1-(12 pts) A car insurance company offers two different contracts. The first contract fully reimburses claims while the other has a lower premium (costs less) but has a fixed deductible so that in the case of an accident the policy holder gets reimbursed the difference between the claim and the deductible if this difference is positive. What can be said about the existence of adverse selection and/or moral hazard in this market from the observation that those consumers who buy the contract with deductible are less likely to experience accidents?

This observation is consistent with both moral hazard and adverse selection. Under adverse selection, bad drivers are more likely to buy the no-deductible contract because they are more likely to benefit from full insurance. Under moral hazard, consumers are identical and they pick different contracts for exogenous reasons but those who end up with full insurance contracts have less incentives to drive carefully.

2-(12 pts) Is it true that in the two action moral hazard model with risk aversion and multiple states of the world, if there is a state of the world which never occurs under the high action and can occur under the low action, it is possible to achieve the first best outcome? What about if there is a state of the world which never occurs under the low action and can occur under the high action?

The first statement is true. The principal can put an infinite penalty on the outcome that occur only under low effort and give a constant payment for all other outcomes. The agent will take the high effort and will not face risk. In the other situation, the principal will put the largest payment on the outcome that never occurs under the low action but payment will still vary across outcomes and the first best cannot be achieved.

3-(12 pts) In Spence's signalling model, if firms can refuse to hire workers, a social planner cannot improve upon the Pareto efficient separating equilibrium of the signalling model. Discuss.

This is true. The social planner can improve welfare if it can impose the wage that has to be paid for any education level (see class notes), but if firms can cherry pick only a subset of contracts, no firm will offer the wage-education package that corresponds to the low type and the social planner's proposal will not be an equilibrium since the low type will try to mimic the high type.

Exercise 2: (24 pts) Moral Hazard with Limited Liability

Consider the two outcomes and two efforts moral hazard model with risk neutrality and assume it is efficient to supply high effort. The agent's utility is $w - e$ and $e \in \{e_l, e_h\}$ with $e_l < e_h$. The agent has outside option $\underline{u}$. Under high/low effort, the probability of success is p_h/ p_l. The pay-off to the principal when the outcome is $j = H, L$ is $x_j - w_j$. Assume $x_H > x_L$ and $p_h > p_l$. Assume that contracts must satisfy a limited liability constraint imposing that the agent's wage cannot be lower than a fixed constant $w_j \geq \underline{w}$ for $j = H, L$ with $\underline{w} < \underline{u}$.

1- (12 pts) Characterize the optimal contract. Distinguishing the case where the limited liability binds or not.

Define w_L^{RN} and w_H^{RN} the wages that bind the participation constraint and the incentive constraint that the agent supplies high effort. This contract is efficient in the absence of limited liability. If $w_L^{RN} = \underline{u} + \frac{p_h e_l - p_l e_h}{p_h - p_l} > \underline{w}$, then this is the optimal contract. Otherwise ($w_L^{RN} < \underline{w}$) the optimal contract that implements high effort sets $w_L^{RN} = \underline{w}$ and $w_H = \underline{w} + \frac{e_h - e_l}{p_h - p_l}$ to bind the incentive compatibility constraint. This is the optimal contract if the profits under this contract dominate the profits under low effort.

2- (12 pts) Does the optimal contract implement an efficient allocation? When?

The allocation is always efficient if the principal still implements the high action. (The agent gets positive rents $\underline{w} - \underline{u} + \frac{p_l e_h - p_h e_l}{p_h - p_l}$ if the limited liability constraint binds.) Inefficiency occurs if it is optimal to implement the low action which occurs if the limited liability constraint binds and when the rent given to the agent is too high relative to the benefit of high effort, $\underline{w} - \underline{u} + \frac{p_l e_h - p_h e_l}{p_h - p_l} + (e_h - e_l) > (p_h - p_l)(x_h - x_l)$.

Exercise 3: (60 pts) Collateral in Debt Contracts

Two lenders compete for a borrower who needs x to invest in a project that can succeed or fail. Both lenders and the borrower are risk-neutral, do not discount, and have outside option 0 in the event the project is not financed. The borrower can be of two types $k = L, H$. The project of a high type yields y (success) with probability θ^H and 0 (failure) with probability $1 - \theta^H$ and similarly for a low type with $\theta^H > \theta^L > \frac{x}{y}$. The probability that the borrower is type H is λ and we denote $\overline{\theta} = \lambda\theta^H + (1 - \lambda)\theta^L$.

Define a loan contract for the amount x as a pair (C, R) with $y \geq R \geq C \geq 0$. The lender gives x to finance the project and the borrower repays R in the event of success. C is the collateral pledged by the borrower which is fully returned if the project succeeds. If the project fails, the lender liquidates the collateral and the borrower looses C. There is a cost of liquidation so that the lender obtains only fraction $\delta \in [0,1]$ of the collateral.

The timing of events is as follows. Lenders first make contract offers. Each lender can offer multiple contracts. Then, the borrower selects a contract or gets her outside option. Finally, contracts are executed after nature has determined the project outcome. We focus on pure strategy subgame perfect equilibrium. Answer all questions *without* use of graphics.

1-(12) Write the expected utility $U^k(C,R)$ of a borrower of type k from contract (C,R) and the expected profit $\pi(C,R;k)$ from giving x to a borrower of type k to finance the project under contract (C,R). Denote by (C^k,R^k) the contract that is selected in equilibrium by type k. Write the constraint that type k is willing to participate $PC(k)$, the constraints that type k selects contract k over k', $IC(k,k')$ for $k \neq k'$, and the constraint that a lender who offers both equilibrium contracts earns non-negative profits ZP.

$U^k(C,R) = \theta^k(y-R) - (1-\theta^k)C$ and $\pi(C,R;k) = \theta^k R + (1-\theta^k)\delta C - x$
$U^k(C^k,R^k) \geq 0 \; PC(k)$
$U^k(C^k,R^k) \geq U^k(C^{k\prime},R^{k\prime}) \; IC(k,k\prime)$ for $k \neq k\prime$
$\lambda\pi(C^H,R^H;H) + (1-\lambda)\pi(C^L,R^L;L) \geq 0 \;\; ZP$

2-(12) Show that lenders earn zero expected profits in any equilibrium.

Proof by contradiction. Assume one lender earns positive profits. Consider the following deviation by the lender that earns the least expected profits: offer the same contracts as its competitor but lower the repayments R^k by a small amount. The deviator attracts the borrower for sure, gets the same contract selection as her competitor did, and therefore increases its profits. A contradiction.

3-(12) Show that there is no pooling equilibrium. (Hint: show that in any pooling equilibrium (C,R) one firm can earn positive profits by offering contract $(\widetilde{C},\widetilde{R}) = \left(C+\epsilon, R - \frac{1-\theta^H}{\theta^H}\epsilon - \epsilon'\right)$ for ϵ and ϵ' positive and small.) Conclude that in any separating equilibrium lenders must earn zero profits on all contracts offered.

Under pooling, the firms must earn positive profits from the high type since $\pi(C,R;H) > \pi(C,R;L)$ and $\lambda\pi(C,R;H) + (1-\lambda)\pi(C,R;L) = 0$. The new contract attracts only the high type since $U^H(\widetilde{C},\widetilde{R}) - U^H(C,R) = \theta^H\epsilon' > 0$ while $U^L(\widetilde{C},\widetilde{R}) - U^L(C,R) = \theta^L\left(\left(\frac{1-\theta^H}{\theta^H} - \frac{1-\theta^L}{\theta^L}\right)\epsilon + \epsilon'\right) < 0$ for ϵ' small enough. The firm earns positive profits with $(\widetilde{C},\widetilde{R})$. A contradiction.

In a separating equilibrium, the above argument implies that a lender must earn zero expected profits from the high type but since a lender earns zero profits on both types the lender also has to earn zero profits from the low type.

4-(12) Derive the only possible candidate equilibrium.

Competition for low types implies that $C^L = 0$ (proof by contradiction that if $C^L > 0$ there exist a profitable deviation). Zero profits imply that $(C^L, R^L) = \left(0, \frac{x}{\theta^L}\right)$. The high type contract is determined by $IC(L, H)$ binding and the zero profit condition $(C^H, R^H) = \left(x\frac{\theta^H-\theta^L}{\theta^H(1-\theta^L)-\delta\theta^L(1-\theta^H)}, x\frac{1-\theta^L-\delta(1-\theta^H)}{\theta^H(1-\theta^L)-\delta\theta^L(1-\theta^H)}\right)$. (Proof by contradiction showing that $C < C^H$ results in pooling and there exists a profitable deviation for $C > C^H$.) Both contracts satisfy the constraint $y \geq R \geq C \geq 0$.

5-(12) Under what condition there do not exist: (a) A pooling deviation attracting both borrower types. (b) A local separating deviation (hint: consider a slight decrease in C^H and R^L and increase in R^H).

The zero profit pooling contract with no collateral is $(C, R) = (0, \frac{x}{\bar{\theta}})$. A low type always prefers this contract to (C^L, R^L). The high type does not prefer this contract if $U^H(C^H, R^H) > U^H(0, \frac{x}{\bar{\theta}})$ which is equivalent to $\frac{\theta^H(1-\theta^L)-\delta\theta^L(1-\theta^H)}{\bar{\theta}} + \delta + (1-\delta)\theta^H + \frac{\theta^L}{\theta^H} \geq 2$.

Consider the alternative menu of contracts $\left(\widetilde{C^H}, \widetilde{R^H}\right) = \left(C^H - \varepsilon, R^H + \frac{1-\theta^H}{\theta^H}\varepsilon\right)$ and $\left(\widetilde{C^L}, \widetilde{R^L}\right) = \left(0, R^L - \frac{\theta^H-\theta^L}{\theta^H\theta^L}\varepsilon\right)$ for ε small and positive. By construction, the high type weakly prefers $\left(\widetilde{C^H}, \widetilde{R^H}\right)$ over (C^H, R^H) since $U^H(\widetilde{C^H}, \widetilde{R^H}) = U^H(C^H, R^H)$ while the low type weakly prefers $\left(\widetilde{C^L}, \widetilde{R^L}\right)$ over $\left(\widetilde{C^H}, \widetilde{R^H}\right)$ and (C^L, R^L) since $U^L(\widetilde{C^L}, \widetilde{R^L}) = U^L(\widetilde{C^H}, \widetilde{R^H}) > U^L(C^L, R^L)$. Therefore, both types select their intended new contracts and the profits change by $\varepsilon\left((1-\delta)\lambda(1-\theta^H) - (1-\lambda)\frac{\theta^H-\theta^L}{\theta^H}\right)$. Separating deviations are not profitable if the expression in brackets is negative.

Bonus: If these conditions hold, does an equilibrium exist? Discuss.

Due to the linearity of the problem, there do not exist any profitable separating deviations if the above condition excluding local separating deviations holds. Therefore the equilibrium exists if $\frac{\theta^H(1-\theta^L)-\delta\theta^L(1-\theta^H)}{\bar{\theta}} + \delta + (1-\delta)\theta^H + \frac{\theta^L}{\theta^H} \geq 2$ and $(1-\delta)\theta^H(1-\theta^H) \leq \frac{1-\lambda}{\lambda}(\theta^H - \theta^L)$. These conditions are more likely to hold if λ and θ^L are small and δ is high.