MICROECONOMICS III
Information Economics and Contract Theory
Mock Final Exam—Pascal Courty
EUI, Florence, 2006

ANSWER ALL QUESTIONS.
TOTAL POINTS: 120

Exercise 1 (36 pts)

1- (12 pts) Consider a standard principal agent model of incentive provision, and assume that the agent supplies positive effort under the optimal contract. Consider the possibility to renegotiate the contract after the agent has accepted the contract and chosen her effort level but before the performance outcome has been realized. At this point, it is efficient to fully insure the agent. This implies that when renegotiation is possible, the agent's payoff should not vary in equilibrium. Discuss.

2- (12 pts) Assume there are two kinds of travellers: business and leisure. Leisure travellers value travelling 6 for sure while business travellers value travelling 0 with probability half and 10 with probability half. A monopoly seller can offer a menu of refund contracts. For each refund contract, a consumer has to pay the up-front price and is eligible for a set-refund in the event of cancellation. Assume the cost of serving the consumer is 0 and consumers get 0 surplus in the event they do not travel. Derive the profit maximizing menu of contracts. (Hint: consider a menu of contracts that extracts all consumer surplus.)

3- (12 pts) Consider the auction framework with two bidders but assume that there are I valuations $v_1 < v_2 < .. < v_I$. As before, p_{ij} is the price paid by participant i and x_{ij} the probability that participant i gets the good under announcement v_i and v_j. Denote $IC_{i,i'}$ the incentive compatibility constraint that a bidder with valuation v_i does not report valuation $v_{i'}$ and PC_i the participation constraint of valuation v_i. Show that if $IC_{i,i-1}$ holds for any i and PC_1 holds, then PC_i holds for any $i > 1$.

Exercise 2: Moral Hazard and Monotonicity of Payment Rule (24 pts)

Consider the two outcomes and two efforts moral hazard model with risk aversion. The agent's utility is $u(w) - e$ where $u()$ is strictly concave and $e \in \{e_l, e_h\}$ with $e_l < e_h$. The agent has outside option $\underline{u}$. Under high/low effort, the probability of success is p_h/ p_l. The pay-off to the principal when the outcome is $j = H, L$ is $x_j - w_j$. Assume $x_H > x_L$ and $p_h > p_l$.

1- (8 pts) Assume it is optimal to implement the high action. Characterize the optimal contract.

2- (8 pts) Show that the incentive compatibility constraint implies that the payment has to increase with the level of outcome.

3- (8 pts) Assume there are 3 outcomes and let p_{ei} denote the probability of outcome $i \in \{L, M, H\}$ under effort $e \in \{l, h\}$. Assume the distibution under high effort first order stochastic dominante the distribution under low effort. Is it possible to use the same argument as in the case with 2 outcomes to show that the payment has to increase with the level of outcome?

Exercise 3: Regulation (60 pts)

A regulator wants to minimize the price charged by a natural monopolist who is privately informed about its cost. The monopolist produces a good at cost $c = \theta - e$ which is publicly observed but where $\theta \in \{\theta_H, \theta_L\}$ is a fixed type privately observed by the monopolist and $\Delta\theta = \theta_H - \theta_L > 0$ and $e \geq 0$ represents a level of effort chosen by the monopolist, and also not observed by the regulator, to reduce cost. The cost of exerting effort e is $\frac{1}{2}e^2$. The monopolist is low type with probability β. The regulator minimizes the payment $P = s + c$ that is given to the monopolist in exchange for the good where s is a fixed subsidy. The monopolist maximizes profits $P - c - \frac{1}{2}e^2$.

1- (10 pts) Assume the regulator observes θ. What is the optimal level of effort?

2- (10 pts) Assume the regulator observes only c and not θ nor e. Let (s_i, c_i) denote the contract taken in equilibrium by type $i = L, H$. Under contract i, the monopolist has to supply the good at cost c_i and then gets compensated s_i. Denote $e_{ii'}$ the level of effort that type i has to supply in equilibrium if she selects contract i' where $i, i' \in \{L, H\}^2$. We assume throughout that these levels of efforts are all positive. Show that $e_{LH} = e_{HH} - \Delta\theta$ and $e_{HL} = e_{LL} + \Delta\theta$.

3- (10 pts) Write down the participation constraints (PC_i) and incentive compatibility constraints $(IC_{ii'})$ that type i does not take contract i' using only the variables s_i and e_{ii}, $i = L, H$.

4- (10 pts) Show that PC_L is strict and that PC_H and IC_{LH} bind.

5- (10 pts) Solve for the optimal effort level e_{LL} and e_{HH}. assuming that IC_{HL} holds.

6- (10 pts) Derive the optimal contracts.

Answer key

1-1 The prediction is true only if the agent does not anticipate that renegotiation will take place. Under subgame perfection, the agent anticipates that he will be fully insured after the renegociation phase independently of effort supplied. Therefore, the agent supplies the minimum level of effort and the principal cannot provide incentives in the abscence of commitment. In the principal agent model, this possibility is ruled out because we implicitly assume that the principal can commit not to renegotiate.

1-2 Consider the menu of contracts: (a) business travel contract costs 10 and gives full refund (b) tourist contract costs 6 and gives no refund. Both types of travellers weakly prefer their contracts over (a) not travelling (PC) and (b) stictly prefer it over the other contract (IC). The allocation is first best and since consumers get zero surplus it has to be profit maximizing.

1-3 $IC_{i,i-1}$ can be written as $v_i E_j x_{ij} - E_j p_{ij} \geq v_i E_j x_{i-1,j} - E_j p_{i-1,j}$. But $v_i E_j x_{i-1,j} - E_j p_{i-1,j} > v_{i-1} E_j x_{i-1,j} - E_j p_{i-1,j}$. Using $IC_{i-1,i-2}$ $v_{i-1} E_j x_{i-1,j} - E_j p_{i-1,j} \geq v_{i-1} E_j x_{i-2,j} - E_j p_{i-2,j}$ and then again $v_{i-1} E_j x_{i-2,j} - E_j p_{i-2,j} > v_{i-2} E_j x_{i-2,j} - E_j p_{i-2,j}$ and doing so iteratively gives

$v_i E_j x_{ij} - E_j p_{ij} > v_1 E_j x_{1j} - E_j p_{1j}$ and since $v_1 E_j x_{1j} - E_j p_{1j} \geq 0$ by PC_1 we have $v_i E_j x_{ij} - E_j p_{ij} \geq 0$.

2-1 Using the first order condition to the principal problem, one can show as we did in class that the participation constraint and incentive compatibility constraint bind. The optimal contract (w_L, w_H) solves the system of two equations in two unknowns,

$p_h u(w_H) + (1 - p_h) u(w_L) - e_h = \underline{u}$

$p_h u(w_H) + (1 - p_h) u(w_L) - e_h = p_l u(w_H) + (1 - p_l) u(w_L) - e_l$

2-2 Rearranging terms in the incentive compatibility constraint imply

$(p_h - p_l)(u(w_H) - u(w_L)) = e_h - e_l > 0$ which implies $u(w_H) - u(w_L) > 0$ and since u is increasing we have $w_H > w_L$.

2-3 The proof does not follow with three outcomes. Incentive compatibility requires $(p_{hH} - p_{lH})(u(w_H) - u(w_M)) + (p_{lL} - p_{hL})(u(w_M) - u(w_L)) = e_h - e_l > 0$. First order stochastic dominance implies that $p_{hH} - p_{lH} \geq 0$ and $p_{lL} - p_{hL} \geq 0$ but this implies only that it is not possible that both $w_H < w_M$ and $w_M < w_L$. Stated differently, the above inequality can be rewritten as $\alpha(u(w_H) - u(w_M)) + (1 - \alpha)(u(w_M) - u(w_L)) > 0$ for $\alpha \in (0, 1)$ which only says that the payment has to increase in an average sense. This is not surprising since we already saw that the wage does not have to increase with outcome in the continuous outcome case.

3-1 The regulator minimizes $s + c$ subject to participation $s - \frac{1}{2} e^2 \geq 0$. Since the principal observes both c and θ she can deduct exactly $e = \theta - c$ so we can assume without loss of generality that e is contractible. Plugging s in the objective function gives $\theta - e + \frac{1}{2}e^2$ which is minimized for $e^* = 1$. The optimal subsidy is $s^* = 1/2$.

3-2 Use the condition that the monopolist has to deliver cost c_i if she selects contract i to derive $e_{LL} = \theta_L - c_L$, $e_{HH} = \theta_H - c_H$, $e_{LH} = \theta_L - c_H$, $e_{HL} = \theta_H - c_L$. After replacement, we get $e_{LH} = e_{HH} - \Delta\theta$ and $e_{HL} = e_{LL} + \Delta\theta$.

3-3 PC_i $s_i - \frac{1}{2}e_{ii}^2 \geq 0$
IC_{LH} $s_L - \frac{1}{2}e_{LL}^2 \geq s_H - \frac{1}{2}(e_{HH} - \Delta\theta)^2$
IC_{HL} $s_H - \frac{1}{2}e_{HH}^2 \geq s_L - \frac{1}{2}(e_{LL} + \Delta\theta)^2$

3-4 IC_{LH} and PC_H imply PC_L is strict. The other two claims are proved by contradiction. If PC_H does not bind then decrease both s_L and s_H by a small amount. If IC_{LH} does not bind decrease s_L by a small amount.

3-5 Solving for s_H from PC_H gives $s_H = \frac{1}{2}e_{HH}^2$ and for s_L from IC_{LH} gives $s_L = \frac{1}{2}(e_{HH}^2 + e_{LL}^2) - \frac{1}{2}(e_{HH} - \Delta\theta)^2$. Plug these values in the objective function $\beta(s_L + \theta_L - e_{LL}) + (1-\beta)(s_H + \theta_H - e_{HH})$ and take the first order conditions to obtain $e_{LL} = 1$ and $e_{HH} = 1 - \frac{\beta}{1-\beta}\Delta\theta$.

3-6 Check that IC_{HL} is satisfied. The optimal contract is $c_L = \theta_L - 1$, $s_L = \frac{1}{2}e_{LL}^2 + e_{HH}\Delta\theta - \frac{1}{2}\Delta\theta^2 = \frac{1}{2}\left(1 - \Delta\theta^2\right) + \Delta\theta\left(1 - \frac{\beta}{1-\beta}\Delta\theta\right)$, $c_H = \theta_H - 1 + \frac{\beta}{1-\beta}\Delta\theta$, $s_H = \frac{1}{2}\left(1 - \frac{\beta}{1-\beta}\Delta\theta\right)^2$. As expected, the high cost firm produces at a higher cost (both because it has higher initial cost and because it supplies less effort) and gets a lower subsidy.