Micro III: Information Economics and Contract Theory

Lecture 5: Signaling (MGW 13-C)

Lecture 6: Moral Hazard (MGW 14-B)

Lecture 7: Dynamic Adverse Selection (Bolton-Dewatripont 9.1)

Lecture 0: Course Introduction

- Introduction to information economics (Stiglitz, Joseph E. “The Contribution of the Economics of Information to Twentieth Century Economics”. Quarterly Journal of Economics, Nov. 2000, pp. 1441-1478.)

- Adverse selection model (MGW 13-B and Akerlof G (1970) “The Market for Lemon,” QJE 89: 488 – 500)

Introduction to Information Economics

- General equilibrium: Information plays a role only in "state of nature," with application to insurance and asset pricing

- Chicago school: information as an investment (i.e. investment in search, human capital; Shultz/Stigler/Beker)

- Economics of information: strategic role of information
  - Akerlof Lemon model (1970)
  - Mireless self-selection in taxation (1971)

- Arrow moral hazard and principal agent paradigm

- Main contributions of information economics and contract theory
  - Revisit main conclusions of general equilibrium theory (both positive and normative)
  - Explain behavior and outcomes difficult to understand otherwise (e.g. credit rationing)
  - Applications to finance (corporate governance, insurance), labour economics and personnel economics (compensation policies), industrial organization (pricing, procurement)

- Example of how informational issues may change standard theoretical predictions
  - Existence of equilibrium may not hold
  - Welfare conclusions may not hold (competitive equilibrium may not be Pareto optimal)
  - Competitive pricing (p=mc) may not hold
  - Supply may not equal demand in equilibrium (credit rationing, unemployment)
  - Law of one price may not hold (equilibrium distribution of price)

- Information Economics: Main Concepts
  - Imperfect information: Uncertainty due to nature
  - Private information: information observed privately by a party
  - Asymmetric information: Some agents know more than other
  - Constrained Pareto Optimality: Assume the social planner does not have directly access to the agent's private information
  - Self-selection: A party make a choice on the basis of her private information
  - Adverse selection: The choice made by the privately informed party goes against the interests of the uninformed party (as opposed to positive selection)

- Signalling: The privately informed party takes some action to signal her private information

- Screening: The uninformed party designs a menu of choice to screen the privately informed party

- Principal/Agent: A principal makes a take-it-or-leave-it offer to the agent (ignore bargaining and assume that the agent has an outside option taken as given)

- Moral hazard: A party (typically the agent) chooses an action that is not observed by the other party (typically the principal)

- Hidden action vs hidden information: Pure moral hazard vs agent taking (possibly unobservable) action after privately observing some information

- Methodological Issues
  - Partial equilibrium analysis
  - Often focus on interaction between a few parties (typically 2)
  - Parties write contracts subject to information constraints
  - Solution concept often based on non-cooperative game theory (Subgame perfect Nash, Bayesian updating, subgame perfect Bayesian equilibrium)

Lecture 1: Akerlof Lemon Model (MGW 13-B)

Adverse selection model (MGW 13-B)

Akerlof G (1970) “The Market for Lemon,” QJE 89: 488 – 500): Model of market collapse under asymmetric information with application to second hand car market, labor market, credit market, and development economics

The market collapse prediction is a rather dramatic conclusion but the concept of adverse selection is very general and had been applied in personnel economics (early retirement, compensation policies), insurance, corporate finance, to name just a few fields

Labor market application: Identical firms hire workers who are privately observe their productivity

Firms: Any number greater than one. Transform labor into output using CRS technologies. Firms are risk neutral. Assume that output price is one (partial equilibrium analysis)

Workers: Measure N of heterogeneous workers. Worker of type θ produces θ in a firm and $r(\theta)$ in home production. $\theta \in [\underline{\theta}, \overline{\theta}]$ with $0 \leq \underline{\theta} < \overline{\theta} < \infty$. The distribution of types is $F(\theta)$ with density $f(\theta)$ such that $f(\theta) > 0$ for $\theta \in [\underline{\theta}, \overline{\theta}]$

Market outcomes to be determined:

Equilibrium wage(s?)

Sorting of workers between firms and home production

Equilibrium concept:

Competitive equilibrium

Game theoretic approach

Possible scenarios:

Full information on workers' type or productivity (standard approach)

Workers are privately informed about their productivity (asymmetric information)

Economic questions of interest:

1. Does the introduction of asymmetric information matter?

2. Does it change market outcomes?

3. Does it change the efficiency properties of the equilibrium?

Case 1: Symmetric Information and Competitive Equilibrium (Benchmark)

The wage can be function of the type $w^*(\theta)$

$w^*(\theta) = \theta$

$\{\theta \;\; s.t. \; r(\theta) \leq \theta\}$ are employed in a firm

Firms earn zero profits

Sorting according to principle of comparative advantage

Equilibrium is Pareto efficient

Standard conclusions from competitive equilibrium analysis

Case 2: Asymmetric Information and Competitive Equilibrium

Focus on case where (a) $r(\theta) \leq \theta$ and (b) $r'(\theta) \geq 0$

Firms and workers are price takers. We assume that those workers who are indifferent between working at a firm and home employment chose the former

Asymmetric information implies that the wage rate must be independent of the workers' type. There is a single wage rate in equilibrium

A competitive equilibrium is characterized by a wage w^* and a sorting rule $\Theta \subset [\underline{\theta}, \overline{\theta}]$ such that worker $\theta \in \Theta$ is employed in a firm and worker $\theta \notin \Theta$ is self-employed

Supply of labor: Worker occupational choice (binary optimization problem) imply that $\Theta(w) = \{\theta \;\; s.t. \; r(\theta) \leq w\}$

Demand for labor: Firms demand for labor depend on their expectations regarding the type of workers who apply to work in firms. Let μ represent the expected productivity of a worker who is not self employed. Each firm demands zero unit of labor if $\mu < w$, any non-negative amount if $\mu = w$, and an infinite amount if $\mu > w$

Firm rational expectation: $\mu = E[\theta|\theta \in \Theta(w)]$ if positive employment ($\Theta \neq \emptyset$) and otherwise we will assume $\mu = E\theta$ ($\Theta \neq \emptyset$)

Remark: Any equilibrium has to have positive employment, that is, $\Theta \neq \emptyset$. (Proof by contradiction)

Equilibrium in labor market: In any equilibrium with non-zero level of employment, demand for labor equals supply of labor implies that $w = E[\theta|\theta \in \Theta(w)]$

Equilibrium characterization: Any competitive equilibrium is a pair (w^*, Θ^*) such that (a) $\Theta^*(w^*) = \{\theta \;\; s.t. \; r(\theta) \leq w^*\}$ and (b) $w^* = E[\theta|\theta \in \Theta^*(w^*)]$

Existence of equilibrium

Define the function $H(w) = E[\theta|\theta \in \Theta(w)]$. Any w such that $w = H(w)$ is an equilibrium wage

Properties of the function $H(.)$:

1. $H()$ is continuous and increasing
2. $H(r(\underline{\theta})) = \underline{\theta} \geq r(\underline{\theta})$
3. $H(w) = E\theta < \overline{\theta}$ for any $w \geq r(\overline{\theta})$

An equilibrium always exists since the function $H()$ crosses the 45 degree line at least once

Possibility of multiple equilibria (H may cross the 45 degree line multiple times)

No-trade (with positive measure) if $H(w) < w$ for $w > r(\underline{\theta})$ (only type $\underline{\theta}$ are employed and earn $w = r(\underline{\theta}) = \underline{\theta}$)

Inefficiency as long as $r(\overline{\theta}) > E\theta$ (the highest type cannot be employed)

Case 3: Game theoretic approach

Are all equilibria equally reasonable?

Consider for example and equilibrium w^* and assume there exist a $w > w^*$ such that $H(w) > w$

Firms want to deviate from w^*: If a firm offers wage w, workers earn more and the deviating firm earns positive profits

This type of deviation, however, is ruled out under the price taking assumption

2-stage game

Assume there are only 2 firms (w.l.o.g.)

Stage 1: Firms announce wages (w_1, w_2)

Stage 2: Workers chose between self-employment or work with firm 1 or 2

Equilibrium concept: Subgame perfect Nash equilibrium

Issue: Does the set of equilibrium change?

Let $w^{**} = Max\{w \text{ s.t. } H(w) = w\}$

Assume that $w^{**} > r(\underline{\theta})$ and that $H(w)$ crosses the 45 degree line from above at w^{**}

Stage 2: workers pick the maximum of w_1, w_2, and $r(\theta)$ (we assume that if they are indifferent they pick employment and randomize between firms)

Stage 1: Firm wage offers

Lemma 1: Firms earn zero profits

Standard Bertrand competition argument

Lemma 2: One firm must offer at least w^{**} in equilibrium

If no firm offer at least w^{**}, one of the two firms can earn positive profits by offering a wage higher than its competitor and slightly lower than w^{**}. A contradiction with Lemma 1

Note the role played by the assumption that $H(w)$ crosses the 45 degree line from above at w^{**}. The proof of Lemma 2 does not hold without this assumption

Proposition: There exists a unique SPNE and $w_1 = w_2 = w^{**}$ and $\Theta^* = \{\theta \;\; s.t.\, r(\theta) \leq w^{**}\}$

Wage w^{**} is the only wage from which there does not exist profitable deviations

Constrained Pareto optimum

Equilibrium typically fails to be Pareto optimum

A social planner who knows the workers' types could improve efficiency

What about a social planner who does not know the workers' types?

Although the general treatment of this problem requires more advance tools, it is possible to show that the planner cannot implement an allocation that dominates the Pareto dominant equilibrium

Lecture 2: Competitive Screening (MGW 13-D)

No-trade (with positive measure) conclusion is unrealistic if gains-from-trade are large

Market players should come up with schemes to capture these gains from trade (evolutionary/Darwinist view that only the most efficient institutions survive)

Screening and signalling models: Intuition is that high types could reveal their types by engaging in an activity that is less costly for them than for low types

Example: Low risk drivers may find it less costly to accept low deductible than high risk drivers

Rothchild and Stiglitz, "Equilibrium in Competitive Insurance Markets: An Essay on the Economics of Imperfect information," QJE, (November 1976):629-650

Screening Model

Labor market application: Two firms hire two types of workers who are privately informed about their productivity

Firms: Two risk neutral firms transform labor into output using CRS technologies. Assume that output price is one (partial equilibrium analysis). Firms can assign a worker to tasks of different level of difficulty $t \geq 0$. Firm profits from a worker of productivity θ who is assigned to task t is $\theta - w$ where w is the wage

Workers: Two types of workers, θ_L and θ_H. A worker of type $\theta \in \{\theta_L, \theta_H\}$ produces θ output. The fraction of workers of type θ_H is λ. Workers have zero outside option (no type dependent home production) The utility of worker of type

θ who receives wage w in task t is $u(w, t|\theta) = w - c(t, \theta)$ where $c(0, \theta) = 0$, $c_t(t, \theta) > 0$, $c_{tt}(t, \theta) > 0$, $c_\theta(t, \theta) < 0$, and $c_{t\theta}(t, \theta) < 0$

Remark: We have assumed that more difficult tasks do not increase a worker's output. This is to focus on the possibility that difficult tasks may be used only for informational reasons

Two-stage game:

Stage1: Firms announce wage-task contracts (w, t)

Stage 2: Workers chose a contract or get the outside option of zero

As tie-break rule, we assume that indifferent workers always prefer low task contracts over high ones, and employment over outside option. If two firms offer the same contract workers randomize with equal probability

Focus on pure strategy subgame perfect Nash equilibria

Market outcomes to be determined:

Equilibrium wage-task contracts

Sorting of workers between contracts

Possible scenarios:

Full information on workers' type (productivity)

Workers are privately informed about their productivity

Economic questions of interest:

1. Does the introduction of asymmetric information matter?
2. Does it change market outcomes?
3. Does it change the efficiency properties of the equilibrium?
4. Do workers work under the same contract (pooling equilibrium) or under different contracts (separating equilibrium)?

Workers' indifference curves in $(\mathbf{t}, \mathbf{w})$

Proofs make extensive use of graphics in the (w, t) quadrant

Differentiating $u(w, t|\theta) = K$, where K is a constant,, $dw - c_t dt = 0$

$\frac{dw}{dt} = c_t > 0$ increasing

$\frac{d^2w}{dt^2} = c_{tt} > 0$ convex

$\frac{d^2w}{dtdw} = c_{t\theta} < 0$ type L has higher slope than type H

Since $c_\theta > 0$ type H indifference curves lie above type L

Single crossing condition: Indifference curves of the two types cross at most once since high types have lower marginal disutility for task difficulty. In addition the high type indifference curves always cross the low type indifference curves from above

Remark: the single crossing condition is central concept to sort privately informed agents and it will also play a role in the model of monopoly screening and auctions

Case 1: Symmetric Information (Benchmark)

- The wage can be function of the worker's type $w^*(\theta_i, t_i^*)$
- $(w_i^*, t_i^*) = (\theta_i, 0)$ for $i = L, H$
- Firms earn zero profits
- Worker work in easiest task since no productivity gain from working in more difficult tasks (no efficiency role for task difficulty)
- Equilibrium is Pareto efficient

- Standard conclusions from competitive equilibrium analysis

Case 2: Asymmetric Information

Note that the symmetric information equilibrium cannot be implemented because firms do not observe the worker's type. If a firm offers the two equilibrium contracts offered under symmetric information, it will earn loss $E\theta - \theta_H$ since both workers will accept the high wage contract

Separating equilibrium: each type of worker accepts a different contract

Pooling equilibrium: both types of worker accept the same contract

Lemma 1: In any equilibrium, firms earn zero overall profits

Competition implies that firms will bid profits down to zero under a Bertrand type of argument

Lemma 2: No pooling equilibrium exists

Under pooling and zero profits, high types cross subsidize low types. Therefore, a firm can benefit by creaming-off the high types and this can be achieved by increasing the task difficultly and paying a premium that is not attractive to low types which is always possible under single crossing

Lemma 3: In any separating equilibrium, firms earn zero profits on all contracts

In a separating equilibrium, firms can give targeted contracts to each type of workers. This drives the profits on each type to zero

Lemma 4: In any separating equilibrium, low types receive the competitive contract $(\theta_L, 0)$

Firms can pay the low types their productivity and assign them to tasks with zero difficulty because they break even if only low types accept and they can only benefit if high types accept as well. Not possible to do the same with high types, however, since low types prefer to work at the high type competitive contract $(\theta_H, 0)$ and firm would earn negative profits $(E\theta - \theta_H)$

Lemma 5: In any separating equilibrium, the high type contract must be such that $\theta_H - c(t_H, \theta_L) = \theta_L - c(0, \theta_L)$

Firms pay high types more but also increase the task difficulty to make sure that low types do not want to pretend to be high types. This is known as the separating or indifference condition. Low types have to be indifferent between both contracts

Proposition: Any SPNE is a separating equilibrium such that low types accept $(\theta_L, 0)$ and high types accept (θ_H, t_H)

Remark: Type θ_L wants to mimic type θ_H while the reverse does not hold. As a result the contract of type θ_H is distorted while the contract of θ_L isn't!

Conclusions:

Only separating equilibria can exist and even a separating equilibrium does not always exists

- For example, assume there are very few low types and the separating condition requires high types to endure difficult tasks (under separation). Since there are very few low types $\theta_H - E\theta$ is small. A firm could offer a wage of $E\theta - \epsilon$ and task difficulty $t = 0$. All workers are better off under this contract (high types take a slight pay break $(\theta_H - E\theta + \epsilon)$ but get to work on much easier task) and the firm earns positive profits. Therefore, there may exist pooling deviations

- Similarly, there may exist separating deviations where firms reduce task difficulty as well as wages for high types and use some of the surplus to increase the wage of the low types making sure that low types still do not want to pretend they are high types

Distortions only for type θ_H

Existence of equilibrium is sensitive to small changes in preferences!

Lecture 3: Monopoly Screening (Salanie 2.2)

Private information changes the analysis of market interactions under competition

What about a monopolist who faces privately informed buyers?

Can the monopolist increase profits by explicitly taking into account the fact that consumers are privately informed?

Mussa, M., and S. Rosen (1978) “Monopoly and Product Quality,” Journal of Economic Theory 18: 301-317

Applications: product line (vertical differentiation or quality), non-linear pricing (quantity price discrimination or Ramsey pricing), regulation (cost privately observed), taxation (private information on willingness to work)...

Monopoly screening

Second degree price discrimination: A monopolist faces two types of privately informed buyers

Demand: Two types of consumers $\theta \in \{\theta_1, \theta_2\}$ such that $\theta_1 < \theta_2$ consuming at most one unit of good. The proportion of consumers of type θ_1 is π. Consumers have heterogeneous preferences for quality. Utility of a type $\theta \in \{\theta_1, \theta_2\}$ for quality $q > 0$ who has to pay $t \geq 0$ is $u(q, t|\theta) = \theta q - t$. Consumers get zero utility if they do not consume

Monopolist: The monopolist produces quality q at cost $c(q)$ where c is an increasing and convex function such that $c\prime(0) = 0$ and $c\prime(\infty) = \infty$. The profits from selling one unit of quality q at price t is $t - c(q)$. We assume throughout that it is optimal to sell to both types of consumers

Take-it-or-leave-it offers

Monopolist announces price quality menu of contracts (q_i, t_i) and these are take-it-or-leave-it offers

Buyers select a contract or get the outside option of zero

As tie-break rule, we assume that indifferent buyers always prefer higher quality contracts over lower ones, and buying over outside option

Remark: The problem is sometimes presented as a principal agent problem where the monopolist is the principal who faces a representative buyer with unknown type who is the agent. One could also write the problem as a two-stage game but this would not add further insights

Market outcomes to be determined:

Contracts offered by the monopolist

Consumer choice of product

Possible scenarios:

Competition

Monopoly and full information on consumers' type (as in first, or third in our case, degree price discrimination)

Consumers are privately informed about their marginal valuation for quality and monopolist offers a single product

Private information and multiple products

Economic questions of interest:

1. Does the introduction of asymmetric information changes the monopolist product offers and pricing?

2. Does the monopolist offer a single or multiple qualities?

3. Do consumers end up to consume inefficient quality products?

4. Do consumers end up with surplus? Which consumers?

Consumers' indifference curves and monopolist isoprofit curves in $(\mathbf{t}, \mathbf{q})$

Differentiating $u(q, t|\theta) = K$, where K is a constant, $\theta dq - dt = 0$

$\frac{dt}{dq} = \theta > 0$ increasing, $\frac{d^2t}{dq^2} = 0$ linear, $\frac{d^2t}{dqd\theta} = 1 > 0$ type 1 has lower slope than type 2

Differentiating $\pi(q, t) = K$, $dt - c_q dq = 0$

$\frac{dt}{dq} = c_q > 0$ increasing, $\frac{d^2t}{dq^2} = c_{qq}$ convex

Single crossing condition: Indifference curves of the two types cross at most once since high types have higher marginal valuations for quality. In addition the high type indifference curves always cross the low type indifference curves from below

Case 1: Competition (Benchmark 1)

Competitive firms offer products of quality q_i^* such that $\theta_i = c'(q_i^*)$ at price $p_i = c'(q_i^*)$ for $i = 1, 2$

$q_1^* < q_2^*$ (since $c\prime$ is increasing)

Higher type consume higher quality products

Type i consumer gets surplus $\theta_i q_i^* - c(q_i^*)$

Higher types get more surplus

Case 2: Symmetric Information (Benchmark 2)

$\underset{t_i,q_i}{Max}\,(t_i - c(q_i))$

Subject to participation $\theta_i q_i - t_i \geq 0$

Participation constraint binds $\theta_i q_i = t_i$ (full rent extraction)

$c\prime(q_i^*) = \theta_i$ efficient provision of quality

Same product offering as under competition but the monopolist extracts all the consumer surplus

Case 3: Asymmetric information and single product (Benchmark 3)

Assume the monopolist offers a single product

If $\theta_1 q_1^* - c(q_1^*) > (1-\pi)(\theta_2 q_2^* - c(q_2^*))$ then the monopolist sells to all consumers a product of quality q_1^*

Otherwise the monopolist sells only to high types a product of quality q_2^*

Either high types under consume quality or low types over consume quality

Case 4: Asymmetric information and menu of contracts

Assume the monopolist offers the efficient pair of contracts. Will the profits be equal to the symmetric information profits?

Yes, if both types of consumers chose the contracts they are supposed to chose

Low types get zero utility under contract 1 and negative utility $(\theta_1 - \theta_2)q_2^* < 0$ under contract 2. Low types do not deviate

High types get zero utility under contract 2 and positive utility $(\theta_2 - \theta_1)q_1^* > 0$ under contract 1. High types will deviate!

All types pick contract 1

Monopolist earns less than under symmetric information!

Monopolist could do better by changing contracts (e.g. give a better deal to high types).

What is the optimal combination of contracts?

Monopoly screening problem

Monopolist offers two contracts (t_1, q_1) and (t_2, q_2) to maximize profits

$$\pi(t_1 - c(q_1)) + (1 - \pi)(t_2 - c(q_2))$$

subject to the constraints that all consumers participate $\theta_i q_i - t_i \geq 0$ (PC_i)

and that consumers select the contract that is designed for them

$\theta_1 q_1 - t_1 \geq \theta_1 q_2 - t_2$ (IC_1)

$\theta_2 q_2 - t_2 \geq \theta_2 q_1 - t_1$ (IC_2)

(IC_1) says that low types (weakly) prefer contract (t_1, q_1) over contract (t_2, q_2) while (IC_2) says that the opposite holds for high types

These two new constraints are known as incentive compatibility constraints or truth-telling constraints and nests within the monopoly optimization problem the consumer decision problem

The participation constraint is also known as the individual rationality constraint

Remark 1: We could consider more complex offers or 'mechanisms' where the monopolist asks the agent to send more complex messages and makes offers conditional on messages sent by the agent. One can show, however, that we can restrict to the above problem without loss of generality. This result is known as the revelation principle

Remark 2: The optimization problem has 4 constraints. We know that both PC_i cannot bind (otherwise, $0 = \theta_2 q_2 - t_2 \geq \theta_2 q_1 - t_1 > \theta_1 q_1 - t_1 = 0$ where the first inequality holds by IC_2. A contradiction). One could assume that some constraints bind and others don't (make a guess), solve for the optimal solution (t_1, q_1) and (t_2, q_2), and then check if the guess on the status of the constraints was actually correct. Could work but long process with 4 constraints (two types) and the number of constraints is of the order of the square of the number of types

Trick: The structure of the problem may imply that some constraints necessarily bind and others don't

Analysis of incentive constraints

We focus for the moment on the incentive constraints IC and PC

Lemma 1: $PC_1 + IC_2$ imply PC_2

The high type necessarily gets some surplus so s/he is always willing to participate

Lemma 2: $q_2 \geq q_1$

High types have to consume (weakly) higher quality than low types

Lemma 3: $q_2 \geq q_1$ and IC_2 binding imply IC_1

Low types do not want to deviate as long as high types are indifferent between the two contracts and quality is increasing

To conclude, IC_1 and IC_2 binding is equivalent to $q_2 \geq q_1$ and IC_2 binding

Remark: The three lemmas follow from the incentive constraints alone. These lemmas would also hold for a social planner trying to maximize consumer surplus subject to a self-financing constraint as in Ramsey Pricing

Monopoly maximization

Lemma 4: PC_1 binds

The monopolist does not have to give any surplus to the low type

Lemma 5: IC_2 binds

If IC_2 does not bind, it means that the monopolist is leaving too much surplus to the high type

Since IC_2 binds, Lemma 3 implies that we can simplify the IC constraints as $\theta_2 q_2 - t_2 = \theta_2 q_1 - t_1$, and $q_2 \geq q_1$

Plugging the value of t_1 from PC_1 and t_2 from IC_2 in the objective function, we can rewrite the monopoly profits as $\pi(\theta_1 q_1 - c(q_1)) - (1-\pi)(\theta_2 - \theta_1)q_1 + (1 - \pi)(\theta_2 q_2 - c(q_2))$

The only constraint left is $q_2 \geq q_1$ which we ignore for the moment keeping in mind that we will have to check in the end that it holds

Lemma 6: $q_2 = q_2^*$ and q_1 is such that $c\prime(q_1) = \theta_1 - \frac{1-\pi}{\pi}(\theta_2 - \theta_1)$

We have $q_1 < q_2^*$ since $c\prime(q_1) < \theta_1 < \theta_2 = c\prime(q_2^*)$

The monopolist can always gain by maximizing gains from trade with the high type so $q_2 = q_2^*$

On the other hand, there is a new cost from increasing the low product's quality. The profits from the low type can be decomposed as

$$\underbrace{\pi(\theta_1 q_1 - c(q_1))}_{\substack{\text{Social surplus from} \\ \text{increasing type 1 quality}}} \quad - \underbrace{(1-\pi)(\theta_2 - \theta_1) q_1}_{\substack{\text{Informational rent} \\ \text{given to type 2}}}$$

Since $c\prime(q_1) < \theta_1$, $q_1 < q_1^*$ and the low type quality is distorted downward (under supply quality)

Summary

High types get the efficient quality, q_2^*, and rent $(\theta_2 - \theta_1)q_1$

Low types get inefficiently low quality and no rent

High types are indifferent between both contracts and low types strictly prefer their contracts

More generally with multiple types $i = 1..I$, there is no distortion only at the top or highest type ($\theta_I = c'(q_I^*)$), zero rent only at the bottom or lowest type (PC_1 binds), rent increases with types, and all types are indifferent between their contract and the contract of the next type below ($IC_{i,i-1}$ specifying that type i is indifferent between contract i and contract $i-1$ binds)

Lecture 4: Auctions (Part I) (Bolt-Dewat 7.3)

Simple discrete model (two buyers and two valuations) to investigate how screening tools carry from price discrimination to auction and derive many, if not most, insights

Generalization to multiple buyers and continuous valuations to prove the revenue equivalence theorem

Vickrey, W., (1961) "Counterspeculation, Auctions, and Competitive Sealed Tenders," Journal of Finance 16, 8-37

BD 7 and also MGW 23

Introduction

In auctions, one party (the seller) designs a game involving the strategic behavior of several bidders

Auction theory belongs to the class of multilateral contracting problem also known as game, or mechanism, design

Need to model complex strategic interactions: how do agents play the game given that they do not know what other agents know?

Important applications:

- Auctions: one seller and several privately informed participants

- Bilateral trade: one seller and one buyer both privately informed

- Public project implementation: a social planner must decide whether to implement a project that benefits many taxpayers and how to finance it

Auction Model

One seller with a single unit of good and two participants or bidders

Bidders privately observe their willingness to pay for the good, v, where $v \in \{v_L, v_H\}$

The probability that a bidder values the good v_H is β and valuations are independent

Both the seller and the bidders have reservation value 0 under no trade

Outcomes to be determined:

Which bidder, if any, gets the good?

How much do participants pay the seller?

Possible scenarios:

Monopoly price

More complex mechanism: real world auction, general mechanism

Economic questions of interest:

1. What mechanism maximizes revenues?

2. Is the resulting allocation efficient?

3. How do standard auction schemes perform? Both in term of revenue and efficiency?

Monopoly Price

Single participant: take-it-or-leave-it offer

- (a) $p = v_L$ generates revenue v_L, (b) $p = v_H$ generates expected revenue βv_H
- Optimal price is $p = v_H$ if $\beta v_H > v_L$

Two participants: take-it-or-leave-it offer

- (a) $p = v_L$ generates revenue v_L, (b) $p = v_H$ generates expected revenue $(1 - (1 - \beta)^2)v_H$

- Optimal price is $p = v_H$ if $(1-(1-\beta)^2)v_H > v_L$

Inefficiencies decrease and revenues increase with 2 participants due to reduction in events of no-trade

Can seller use competition to further increase revenue? Will this change efficiency?

Auction design

Focus on schemes such that participants first announce their types to the seller. Then the seller implements an allocation and payment that are conditional on announcements

p_{ij} is the price paid by participant i under announcement v_i and v_j

x_{ij} is the probability that participant i gets the good under announcement v_i and v_j

Remark: we implicitly assume that the seller treats both participants symmetrically

An auction scheme (p_{ij}, x_{ij}) must satisfy three types of constraints

(a) Feasibility constraints: $2x_{HH} \leq 1$, $2x_{LL} \leq 1$, $x_{HL} + x_{LH} \leq 1 \quad FC$

(b) Participation (or incentive rationality) constraints

$$\beta(x_{HH}v_H - p_{HH}) + (1-\beta)(x_{HL}v_H - p_{HL}) \geq 0 \quad IRH$$

$$\beta(x_{LH}v_L - p_{LH}) + (1-\beta)(x_{LL}v_L - p_{LL}) \geq 0 \quad IRL$$

(c) Incentive compatibility constraints

$$\beta(x_{HH}v_H - p_{HH}) + (1-\beta)(x_{HL}v_H - p_{HL}) \geq \beta(x_{LH}v_H - p_{LH}) + (1-\beta)(x_{LL}v_H - p_{LL}) \quad ICH$$

$\beta(x_{LH}v_L - p_{LH}) + (1-\beta)(x_{LL}v_L - p_{LL}) \geq \beta(x_{HH}v_L - p_{HH}) + (1-\beta)(x_{HL}v_L - p_{HL}) \quad ICL$

Remark: One can focus on schemes p_{ij} and x_{ij} that satisfy the above 3 sets of constraints without loss of generality (revelation principle)

Define $p^e_L = \beta p_{LH} + (1-\beta)p_{LL}$ and $p^e_H = \beta p_{HH} + (1-\beta)p_{HL}$ as the expected payment of a low and high type

Seller revenues under scheme (p_{ij}, x_{ij}) that satisfies FC, IR and IC is $2(\beta p^e_H + (1-\beta)p^e_L)$

Efficient allocation: $x_{HL} = 1$, $x_{LH} = 0$, $x_{LL} = x_{HH} = 1/2$

Plan of analysis

Step 1: Treatment of the incentive constraints (IR, IC)

Step 2: Optimal revenue under efficient auction

Step 3: Optimal revenue under general auction

Step 4: Equilibrium and revenue under standard auctions

Step 1: Treatment of the incentive constraints (IR, IC)

Lemma 1: IRL and ICH imply IRH

Same argument as before that the high type is willing to participate as long as the low type is willing as well and the high type does not want to deviate

Lemma 2: ICH and ICL imply $\beta(x_{HH}-x_{LH})+(1-\beta)(x_{HL}-x_{LL}) \geq 0$ MC

We refer to the last inequality as the monotonicity condition (MC). Note that MC can be expressed as $E_j x_{ij}$ increases with i, since

$$E_H x_{Hj} = \beta x_{HH} + (1-\beta)x_{HL} \geq \beta x_{LH} + (1-\beta)x_{LL} = E_L x_{Lj},$$

that is, the probability of consumption increases with the level of valuation announced. In expectation, a participant is more likely to consume if she announces a higher valuation. This result is again very similar to what we had under monopoly price discrimination (quality increases with type, that is, with marginal willingness to pay for quality). The only difference is that the level of consumption is now a random variable so we have to take expectations

Lemma 3: ICH binding and MC imply ICL

Conclusion: If we can show that ICH binds, we will be able to replace ICH and ICL by ICH and MC (again this is similar as before)

Note that the proofs of Lemmas 1-3 use only the incentive constraints. Using the fact that the seller maximizes revenue gives more implications

Lemma 4: IRL and ICH bind

Again, this is proved by contradiction: it is possible to increase revenues if these constraints do not bind

Step 2: Optimal revenue under efficient auction

Consider the efficient allocation $x_{HL} = 1$, $x_{LH} = 0$, $x_{LL} = x_{HH} = 1/2$

MC holds under the efficient auction

Efficient allocation and IRL binding imply $p^e_L = \frac{1-\beta}{2} v_L$

Efficient allocation and IRL as well as ICH binding imply $p^e_H = \frac{v_H}{2} + \frac{1-\beta}{2} v_L$

If $\beta v_H > v_L$, expected revenue increases to $\beta v_H + (1 - \beta) v_L$

Step 3: Optimal revenue under general auction

The seller maximizes $2(\beta p_H^e + (1-\beta)p_L^e)$ subject to IRL, ICH, FC and MC

IRL implies that $p_L^e = (\beta x_{LH} + (1-\beta)x_{LL})v_L$

ICH implies that $(\beta x_{HH} + (1-\beta)x_{HL})v_H - p_H^e = (\beta x_{LH} + (1-\beta)x_{LL})v_H - p_L^e$

Plugging these values back in the expression for the revenue implies that the monopolist sets $x_{HL} = 1$, $x_{LH} = 0$, $x_{HH} = 1/2$, and $x_{LL} = 1/2$ if $\beta v_H < v_L$ and $x_{LL} = 0$ otherwise

The revenue under a reserve price $(x_{LL} = 0)$ is $v_H\beta(2-\beta)$ while the revenue under no reserve price $(x_{LL} = 1/2)$ is $\beta v_H + (1-\beta)v_L$

Relative to single price with two participants, revenue increases under the optimal auction and distortions increase as well since the seller is now more likely to withhold the good. The intuition for this last result is that although revenue increases both with and without a reserve price, it does so by a larger amount with a reserve price

Step 4: Equilibrium and revenue under standard auctions

English auction

Second price sealed bid auction (SPSB) or Vickrey auction

Dutch auction

First price sealed bid auction (FPSB)

SPSB/English

1. Both bidders make sealed bid

2. The highest bid wins the good and pays an amount corresponding to the second highest bid

Lemma 1: $b(v) = v$ is a weakly dominant strategy equilibrium

Remark: the English auction is strategically equivalent to a SPSB auction

FPSB/Dutch

1. Both bidders make sealed bid
2. The highest bid wins the good and pays an amount corresponding to her bid

Lemma 2: There does not exist a pure strategy equilibrium in the FPSB auction

Under pure strategies, a bidder can always overbid her opponent, driving $b(v_H)$ to v_H which cannot be an equilibrium

Define $\bar{b}$ such that $v_H - \bar{b} = (1 - \beta)(v_H - v_L)$, and cummulative distribution $F()$ such that $(v_H - x)(\beta F(x) + (1 - \beta)) = (1 - \beta)(v_H - v_L)$ with associated density f

Lemma 3: $b(v_L) = v_L$ and $b(v_H)$ randomizes according to density f over $(v_L, \bar{b})$ is a symmetric Bayesian Nash equilibrium

Remark: the Dutch auction is strategically equivalent to a FPSB auction

Conclusions

The equilibrium winning probabilities are independent of the auction chosen:

- A high valuation received the good with probability $E_H x_{Hj} = \frac{\beta}{2} + 1 - \beta$

- For a low valuation $E_L x_{Lj} = \frac{1-\beta}{2}$

The equilibrium surpluses are independent of the auction chosen:

- A high valuation gets $(1 - \beta)(v_H - v_L)$ surplus

- A low valuation gets 0 surplus

Revenue equivalence between the four auctions, $\beta^2 v_H + (1-\beta^2)v_L$

Remark: Revenue is higher under the optimal auction $(\beta v_H + (1-\beta)v_L)$ because the high type's incentive compatibility constraint binds in the optimal auction while it doesn't under the standard auctions

Lecture 4': Auctions (Part II) Continuous valuations

Revisit the analysis of optimal auction under a more general model with continuous valuations and arbitrary number of types

n bidders with identically and independently distributed willingness to pay

Willingness to pay of bidder i is $\theta_i \in [\underline{\theta}, \overline{\theta}]$ with $0 \leq \underline{\theta} < \overline{\theta} < \infty$

The distribution of θ_i is $F(\theta_i)$ with density $f(\theta_i)$ (could easily extend analysis to non-identical distributions (F_i, f_i))

Utility of bidder i when s/he receives the good with probability x and pays t is $\theta_i x - t$

Denote $\theta = (\theta_i, \theta_{-i})$ the vector of realizations where θ_{-i} captures the $n-1$ valuations of the bidders other than i

Assume bidders are asked to make announcements on their types. Announcement $\widetilde{\theta} = (\widetilde{\theta}_i, \widetilde{\theta}_{-i})$ says that bidder i announces to have valuation $\widetilde{\theta}_i$ or to be of type $\widetilde{\theta}_i$, while other bidders announce type vector $\widetilde{\theta}_{-i}$. Bidders could lie, that is, announce a different valuation than their actual realization

Let $x(\theta_i, \theta_{-i})$ and $t(\theta_i, \theta_{-i})$ denote the allocation rule and transfers under announcement θ_i, θ_{-i}. $x(\theta_i, \theta_{-i})$ is the probability that type θ_i gets the good and $t(\theta_i, \theta_{-i})$ is the payment θ_i has to make to the seller

Feasibility constraint: $\sum_{i=1..n} x(\theta_i, \theta_{-i}) \leq 1$ for any (θ_i, θ_{-i}) FC

Let $\pi(\widetilde{\theta}_i, \theta_i)$ denote θ_i's expected utility from announcing $\widetilde{\theta}_i$ when her true valuation is θ_i

$$\pi(\widetilde{\theta}_i, \theta_i) = E_{\theta_{-i}} \left(\theta_i x(\widetilde{\theta}_i, \theta_{-i}) - t(\widetilde{\theta}_i, \theta_{-i}) \right)$$

where the conditional expectation is taken over all possible realizations of the non-i bidders assuming that they tell the truth

Incentive Rationality Constraint: $\pi(\theta_i, \theta_i) \geq 0$ for all θ_i $IRC(\theta_i)$

Incentive Compatibility Constraint: $\pi(\theta_i, \theta_i) = Max_{\widetilde{\theta}_i} \pi(\widetilde{\theta}_i, \theta_i)$ ICC

Seller revenue maximization: $Max_{x(),t()}\ nEt(\theta_i, \theta_{-i})$ subject to FC, IRC, and ICC

Again, we start by simplifying the set of constraints IRC and ICC

Local Incentive Compatibility Constraint: $\frac{\partial}{\partial \widetilde{\theta}_i}\pi(\theta_i, \theta_i) = 0$, $LICC$

$LICC$ says that announcement $\widetilde{\theta}_i = \theta_i$ is a local optimum

Monotonicity Condition: $E_{\theta_{-i}} x(\theta_i, \theta_{-i})$ increasing in θ_i, MC

Under MC, bidder i is more likely to get the good if s/he announces a higher valuation

Lemma 1: ICC is equivalent to MC and $LICC$

Same conclusion as before. Note that ICC now corresponds to optimality conditions (local, or FOC, plus global, or SOC). Lemma 1 says that local plus global conditions is equivalent to local plus a monotonicity condition on the delivery rule

Lemma 2: $IRC(\underline{\theta})$ and ICC imply $IRC(\theta_i)$ for $\theta_i \geq \underline{\theta}$

The next step is to simplify the expression for expected per bidder transfer $T^e = Et(\theta_i, \theta_{-i})$. Taking full derivative w.r.t. θ_i in $\pi(\theta_i, \theta_i)$ gives $\frac{d}{d\theta_i}\pi(\theta_i, \theta_i) = \frac{\partial}{\partial\widetilde{\theta}_i}\pi(\theta_i, \theta_i) + E_{\theta_{-i}}(x(\theta_i, \theta_{-i}))$ but $LICC$ implies that the former term cancels giving

$$\frac{d}{d\theta_i}\pi(\theta_i, \theta_i) = E_{\theta_{-i}}(x(\theta_i, \theta_{-i}))$$

Integrating this expression between $\underline{\theta}$ and θ_i simplifies the expression of type θ_i's expected utility to

$$\pi(\theta_i, \theta_i) = \pi(\underline{\theta}, \underline{\theta}) + E_{\theta_{-i}}\left(\int_{\underline{\theta}}^{\theta_i} x(y, \theta_{-i})dy\right)$$

The expected surplus of a bidder who has valuation θ_i is equal to the outside option of the bidder with lowest valuation plus an informational rent that is a function of the allocation rule to all types lower than θ_i. This is similar to the expression we had in the monopoly pricing problem and the implication there was that increasing quality to the low type (here delivery to the low valuations) increases the informational rent that has to be granted to higher types to maintain incentive compatibility (here it corresponds to second term in above expression)

But $\pi(\theta_i, \theta_i) = E_{\theta_{-i}}(\theta_i x(\theta_i, \theta_{-i})) - E_{\theta_{-i}}(t(\theta_i, \theta_{-i}))$ so

$$E_{\theta_{-i}}(t(\theta_i, \theta_{-i})) = E_{\theta_{-i}}(\theta_i x(\theta_i, \theta_{-i})) - E_{\theta_{-i}}\left(\int_{\underline{\theta}}^{\theta_i} x(y, \theta_{-i}) dy\right) - \pi(\underline{\theta}, \underline{\theta})$$

Taking expectations

$$T^e = \int_{\underline{\theta}}^{\overline{\theta}} \left(E_{\theta_{-i}} \theta_i x(\theta_i, \theta_{-i}) - E_{\theta_{-i}} \left(\int_{\underline{\theta}}^{\theta_i} x(y, \theta_{-i}) dy \right) \right) f(\theta_i) d\theta_i - \pi(\underline{\theta}, \underline{\theta})$$

Expressing the second term in the integral as

$$E_{\theta_{-i}} \left(\int_{\underline{\theta}}^{\overline{\theta}} \left(\left(\int_{\underline{\theta}}^{\theta_i} x(y, \theta_{-i}) dy \right) \left(\frac{d}{d\theta_i} (1 - F(\theta_i)) \right) \right) d\theta_i \right)$$

and integrating this term by part gives $E_{\theta_{-i}} \int_{\underline{\theta}}^{\overline{\theta}} x(\theta_i, \theta_{-i})(1 - F(\theta_i)) d\theta_i$. After replacement,

$$T^e = \int_{\underline{\theta}}^{\overline{\theta}} E_{\theta_{-i}} \left(\theta_i - \frac{1 - F(\theta_i)}{f(\theta_i)}) \right) x(\theta_i, \theta_{-i}) f(\theta_i) d\theta_i - \pi(\underline{\theta}, \underline{\theta})$$

or

$$T^e = E_\theta \left(\theta_i - \frac{1 - F(\theta_i)}{f(\theta_i)}) \right) x(\theta_i, \theta_{-i}) - \pi(\underline{\theta}, \underline{\theta})$$

Revenue Equivalence

The seller's revenue maximization problem simplifies to

$$Max_{x(\theta_i,\theta_{-i})} nE_\theta \left(\theta_i - \frac{1-F(\theta_i)}{f(\theta_i)})\right) x(\theta_i, \theta_{-i}) - \pi(\underline{\theta}, \underline{\theta}) \text{ s.t. } FC,\ MC,\ \pi(\underline{\theta}, \underline{\theta}) \geq 0$$

Revenue depends only on the allocation rule $x(\theta_i, \theta_{-i})$ and the surplus given to the lowest type $\pi(\underline{\theta}, \underline{\theta})$

Any 2 auction schemes that share the same allocation rule $x(\theta_i, \theta_{-i})$ and give the surplus rent to the lowest type $\pi(\underline{\theta}, \underline{\theta})$ will raise the same revenue (revenue equivalence theorem)

Remark: The revenue is the same only on average. It may not be the same for all vectors of realizations (θ_i, θ_{-i})

Optimal Auction

Assume $J(\theta) = \theta - \frac{1-F(\theta)}{f(\theta)}$ is increasing in θ

Define $\widetilde{\theta} = Inf(\theta$ such that $J(\theta) \geq 0)$

Optimal allocation sets $x(\theta_i, \theta_{-i}) = 1$ if and only if $\theta_i \geq \widetilde{\theta}$ and $\pi(\underline{\theta}, \underline{\theta}) = 0$

This allocation rule satisfies both FC and MC

If one sets the reserve price to $\widetilde{\theta}$ in the standard auctions (e.g. FPSB or SPSB), the revenue will be maximized since the lowest type gets zero expected surplus and the allocation is identical to the allocation in the optimal auction

Lecture 5: Signaling (MGW 13-C)

In the screening model (Lecture 2), firms move first and attach eligibility restrictions to more attractive contracts.

Specifically, in the labor market application, firms require that those workers who want higher wages have to endure more difficult tasks

Another possibility is that workers move first and invest in costly activities hoping to consequently receive better offers

Different class of games since we need to consider how firms interpret the signals sent by workers' investments in costly activities

In particular, we need to define rules for how firms form beliefs about productivity based on the signals they receive

Cover the original application of Spence to education in "Job Market Signaling" QJE (1973) 87: 355 - 74

The signalling idea has been applied to advertising, financial contracts...

Signaling Model

Labor market application: Two firms compete for a single worker who is privately informed about his/her productivity

Worker: A single worker privately observes her productivity $\theta \in \{\theta_L, \theta_H\}$ such that $\theta_L < \theta_H$. The probability that the worker is of type θ_H is λ. The worker either works and produce θ output or gets zero outside option (no type dependent home production). The worker can invest in education e. A worker of type θ, who is paid w, and invests e in education gets utility $u(w, e|\theta) = w - c(e, \theta)$ where $c(0, \theta) = 0$, $c_e(e, \theta) > 0$, $c_{ee}(e, \theta) > 0$, $c_\theta(e, \theta) < 0$, and $c_{e\theta}(e, \theta) < 0$

Firms: Two risk neutral firms transform labor into output using CRS technologies. Assume that output price is one (partial equilibrium analysis). Firm profits from paying w a worker of type θ who has education e is $\theta - w$

Remark: We assume that education does not increase firms' profits. This is to focus on the possibility that education may be used only for informational reasons

Timing of the game:

1. Nature sets the worker's type θ

2. The worker invests in education e

3. Firms simultaneously make wage offers $w(e)$ conditional on e

4. The worker decides which offer to accept if any

As tie-break rule, we assume that if the worker is indifferent, s/he chooses employment over outside option. If the two firms offer the same contract, workers randomize with equal probability

Remark: One can think of the model as a single worker of unknown type or of a continuum of workers who can be of two types. The exposition is easier under the single worker interpretation

This is a game of imperfect information because the firms do not observe the worker's type

Firms observe only the level of education and they must make an offer based on this observation. Firms' belief on the worker's type will depend in equilibrium on the education level observed. But the worker's choice of investment in education depends on wage offers. This is a chicken and egg problem in the sense that the rational for investment in education has to be self-fullfiling

Read about the initial treatment of the issue in the original work by Spence. Since then, these concepts have been formalized under different equilibrium concepts

Each equilibrium concept needs to specify how the firms form beliefs

The restrictions imposed on how firms form beliefs will greatly constrain the set of wage offers that can be sustained as part of an equilibrium

Equilibrium concept: Perfect Bayesian Equilibrium (PBE) in pure strategy

Let $\mu(e)$ represent the firms' common belief that the worker is of high type after observing education level e. An equilibrium is a profile of strategies $e(\theta)$, $w(e)$ and a system of belief $\mu(e)$. We define a perfect Bayesian equilibrium as a strategy profile and beliefs such that

1. The worker's investment strategy $e(\theta)$ is optimal given the firm's strategy

2. The firm's belief that the worker is of high type, $\mu(e)$, is computed according to Bayes' rule whenever possible

3. Wage offers constitute a Nash equilibrium of the simultaneous game starting after stage (3) given beliefs $\mu(e)$

Stronger concept that weak PBE since impose (a) NE in stage (3) and (b) firms share a common belief off the equilibrium path

Formally, the worker's work decision in stage (4) should be specified as part of the equilibrium definition but we omit it to keep the exposition simple

Market outcomes to be determined:

Equilibrium wage and education

Sorting of workers between work and outside option

Possible scenarios:

Full information on workers' type (productivity)

Workers cannot invest in education (signalling ban)

Workers are privately informed about their productivity and can invest in education

Economic questions of interest:

1. Existence of equilibrium?
2. Set of separating/pooling equilibria?
3. Investment in education $e(\theta)$ and wage schedule $w(e)$?
4. Efficiency properties of different equilibria?
5. Role of belief in equilibrium refinement?

Workers' indifference curves in (e, w)

Proofs make extensive use of graphics in the (e, w) quadrant

Differentiating $u(w, e|\theta) = K$, where K is a constant, $dw - c_e de = 0$

$\frac{dw}{de} = c_e > 0$ increasing

$\frac{d^2w}{de^2} = c_{ee} > 0$ convex

$\frac{d^2w}{ded\theta} = c_{t\theta} < 0$ type L has higher slope than type H (single crossing condition)

Since $c_\theta > 0$ type H indifference curves lie above type L

Case 1: Symmetric Information (Benchmark 1)

- The wage can be function of the worker's type $w^*(\theta_i, e_i^*)$

- $(w^*(\theta_i, e_i^*), e_i^*) = (\theta_i, 0)$ for $i = L, H$

- Firms earn zero profits

- All types work and no type invest in education since no productivity gain from doing so

- Equilibrium is Pareto efficient: Standard conclusion from competitive equilibrium analysis

Case 2: Equilibrium without signaling (Benchmark 2)

- Workers cannot invest in signaling (signalling ban)
- $w^* = E\theta$ and all types work is the only equilibrium
- Firms earn zero profits
- Equilibrium is Pareto efficient

Remark: No adverse selection since no outside option

Case 3: Asymmetric Information

Note that the symmetric information equilibrium cannot be implemented because firms do not observe the worker's type

If a firm offers the two equilibrium contracts from benchmark 1 (symmetric information), it will earn loss $E\theta - \theta_H$ since both types will accept the high wage contract. Therefore, these contracts cannot be part of an equilibrium

Solve the game by looking at decisions in later stages first similarly as you would do under backward induction

Stage 4: Worker accepts highest offer if non-negative

Stage 3: Expected productivity of worker who has education e is $\mu(e)\theta_H + (1 - \mu(e))\theta_L$

Lemma 1: $w(e) = \mu(e)\theta_H + (1 - \mu(e))\theta_L$ is the only Nash equilibrium in the stage 3 simultaneous game given $\mu(e)$

Simple Bertrand argument taking into account firms' beliefs

Since $\mu(e)$ has to be computed according to Bayes rule whenever possible, this implies that firms earn zero profits

Stage 2: Distinguish two types of equilibria, separating and pooling

Separating equilibrium

Lemma 2: In any PBE, $w^*(e^*(\theta_H)) = \theta_H$ and $w^*(e^*(\theta_L)) = \theta_L$

Conditional on education level, firms know the worker's type. Lemma 1 says that firms pay the worker his/her productivity

Lemma 3: In any PBE, $e^*(\theta_L) = 0$

There is no productivity gain from education and all types produce at least θ_L so a firm cannot earn only non-negative profits by paying wage θ_L to a worker who has no education

Define $\widetilde{e}$ and e_1 such that $\theta_L = \theta_H - c(\widetilde{e}, \theta_L)$ and $\theta_L = \theta_H - c(e_1, \theta_H)$. $\widetilde{e}$ corresponds to the level of education that leaves θ_L indifferent between contracts

$(\theta_H, \widetilde{e})$ and $(\theta_L, 0)$. e_1 is corresponds to the level of education that leaves θ_H indifferent between contracts (θ_H, e_1) and $(\theta_L, 0)$

Lemma 4: Any education level $e^*(\theta_H)) \in [\widetilde{e}, e_1]$ can be supported as part of a PBE

Welfare implication: Recall that when education is banned, all workers are paid $E\theta$ (benchmark 2). With the introduction of education: θ_L is strictly worse off and θ_H is better off if and only if $\theta_H - c(e^*(\theta_H), \theta_H) > E\theta$. It is possible that both types are worse off with education. This is more likely to be the case if λ is close to 1

Remark: When both types are worse off with education, θ_H still does not deviate because $\theta_H - c(e^*(\theta_H), \theta_H) > \theta_L)$

Pooling equilibrium

In a pooling equilibrium firms expect the worker to produce $E\theta$

Lemma 5: In any PBE, $w^*(e^*) = E\theta$

This is a direct implication of Lemma 1 since Bayes rule imply that $\mu(e^*) = \lambda$

Define e' such that $E\theta - c(e', \theta_L) \geq \theta_L - c(0, \theta_L)$. e' corresponds to the level of education that leaves θ_L indifferent between contracts $(E\theta, e')$ and $(\theta_L, 0)$

Lemma 6: Any education level $e^* \leq e'$ can be sustained as part of a PBE

Pooling equilibria are dominated by the equilibrium without signaling

Welfare Conclusions

Signaling is inefficient under pooling

θ_H may prefer a signaling ban (pooling with no education as in benchmark 2) under separating

Even if θ_H prefers the separating outcome over pooling with no education, a market intervention mandating a fixed wage schedule $w(e)$ may Pareto dominate the separating equilibrium. This intervention reduces the level of education required from the high type, reduces as well the high type wage, and subsidizes the low type wage to make sure that the low type does not want to deviate

$w(e)$ is not part of an equilibrium under competition because firms break even only on average; they do not do so on each contract as required by Lemma 1

Equilibrium refinement

Bayes rule constrains only the beliefs on the equilibrium path

Could add some restrictions on the beliefs that are allowed off the equilibrium path

Refinement example: A firm cannot believe that a type, who cannot benefit from a given deviation under any possible firm belief, could have taken that deviation

Can show that a proper formalization of this restriction on belief formation can narrow down the set of equilibrium to the Pareto dominant separating equilibrium $\widetilde{e}$. In particular, it eliminates all other separating equilibria as well as all pooling equilibria

Lecture 6: Moral Hazard (MGW 14-B)

Introduction to the principal-agent paradigm

Two effort levels and two outcomes model

Two efforts and continuous outcome model

Holmstrom B. (1979), "Moral Hazard and Observability," Bell Journal of Economics, 10: 74-91

Principal Agent Paradigm

Under an agency relationship, one party, the principal, hires another party, the agent, to perform some task

Agency problems occur when the agent does not have the same preferences as the principal

The agent faces a moral hazard problem because she is confronted with the dilemma of doing what's best for her or what's best for the principal. Economists put moral issues aside and assume that the agent does what's best for her

What can the principal do to address the problem? (a) Find a perfect agent: Selection or screening. (b) Get the right behavior: Incentive provision

Short-Term Incentives: Piece Rates and Bonuses

Piece rates pay workers based on the amount of output they produce regardless of the amount of time actually worked

Bonus are lump-sum payments (made in addition to other forms of compensation) usually conditional on some kind of performance evaluation

The common point to piece rate and bonus is that they provide short term performance incentives

Short-Term Incentives: How does it work?

The principal (e.g. firm) cannot observe the agent's (e.g. worker) effort or true contribution

The firm observes only an imperfect measure of effort (e.g. profits, sales...)

The performance measure is a function of the worker's effort and also some random noise

For example, a CEO's performance depends on the level of industry competition, a sales person performance depends on the product sold, a farmer's output depends on the weather...

Two effort levels and two outcomes model

The worker can either supply high (e_H) or low effort (e_L) and this decision is not observed by the principal. The cost of effort e is $g(e)$ such that $g(e_H) > g(e_L)$ and we denote the incremental cost of effort $c = g(e_H) - g(e_L)$. The worker has reservation utility $\underline{u}$

The principal observes a performance outcome that can be either high or low. p_H and p_L are the probability of high performance under high and low efforts respectively. High effort is more likely to generate high performance $p_H > p_L$. Perfect performance measure has $p_H = 1$ and $p_L = 0$

The principal makes an offer which consists in a fixed salary s plus a bonus b if performance is high

Timing of events:

1. Firm sets the compensation policy
2. Worker chooses effort level
3. Nature draws performance according to probability conditional on effort
4. Worker gets compensated

Analysis

Assume there is no performance bonus ($b = 0$)

Since $s - c < s$ the worker prefers to supply low effort

The worker will not supply effort unless $s + p_H b - c > s + p_L b$. The lowest bonus such that e_H is incentive compatible is,

$$b = \frac{c}{p_H - p_L}$$

The incremental benefit of supplying effort has to be greater than (or equal to) the incremental cost of doing so

Under perfect performance measure ($p_H = 1$, $p_L = 0$) the bonus is equal to the cost of effort $b = c$

The more noisy the performance measure ($p_H - p_L$ low) the greater the bonus $\frac{\partial b}{\partial(p_H - p_L)} < 0$

The worker earns $s + b$ with probability p_H and s with probability $1 - p_H$. The principal sets the fixed salary s such that the worker is indifferent between working under contract (s, b) and the outside option $\underline{u} = p_H(s + b) + (1 - p_H)s$ or

$$s = \underline{u} - p_H b$$

A risk averse worker gets disutility from incentive compatible compensation since pay is variable

Two efforts and continuous outcome model

$e \in \{e_L, e_H\}$

$\pi \in [\underline{\pi}, \overline{\pi}]$, and $f(\pi|e)$ represent the conditional density function of profits given effort

$F(\pi|e_L) \geq F(\pi|e_H)$ for $\pi \in [\underline{\pi}, \overline{\pi}]$ with strict inequality for some interval Π

Agent: $u(w,e) = v(w) - g(e)$ with $v' > 0$, $v'' \leq 0$, and $g(e_H) > g(e_l) \geq 0$. Reservation utility $\underline{u}$

Principal: $E(\pi - w)$ (risk neutral)

Game: Principal makes a take-it-or-leave-it contract offer to the agent. The contract can be conditional on effort level under symmetric information, $w(\pi, e)$ but not under asymmetric information $w(\pi)$

Let $e^* = e_L$ if $\int \pi f(\pi|e_L)d\pi - v^{-1}(\underline{u} + g(e_L)) \geq \int \pi f(\pi|e_H)d\pi - v^{-1}(\underline{u} + g(e_H))$ and $e^* = e_H$ if the opposite (strict) inequality holds

Efficient outcome: set $e = e^*$ and pay the agent a fix wage

Discussion

Stylized model of an employment relationship: the assumption that the agent has a reservation utility gives all bargaining power to principal

In a market with many agents and many principals the surplus may be shared differently but this is beyond the point since our focus here is on the nature of the contract that maximizes constrained efficiency and not on the division of surplus

The principal agent framework is a starting point to model incentive problems in organizations

Market outcomes to be determined:

Wage schedule

Worker's effort

Possible scenarios:

Observable effort $w(e, \pi)$

Unobservable effort $w(\pi)$ and risk neutral agent

Unobservable effort $w(\pi)$ and risk averse agent

Economic questions of interest:

1. What is the optimal compensation contract?

2. Is the contracting outcome efficient? Efficient level of effort? Efficient risk sharing?

3. Is the equilibrium contract consistent with observed compensation policies (i.e. fixed salary, piece rate)?

Case 1: Symmetric information (Benchmark 1)

Under a take-it-or-leave-it offer the principal must satisfy the agent's participation constraint:

$$\int v(w(\pi))f(\pi|e)d\pi - g(e) \geq \underline{u} \quad PC$$

The principal sets the level of effort e and wage schedule $w(e, \pi)$ to maximize $\int(\pi - w(\pi))f(\pi|e)d\pi$ subject to PC

Solve the problem in two steps: (a) set the optimal compensation conditional on effort, (b) set the optimal level of effort

Lemma 1: Wage schedule $w(e, \pi) = v^{-1}(\underline{u} + g(e))$ and $w(e', \pi) < v^{-1}(\underline{u})$ for $e' \neq e$ implements effort level e

The principal binds the participation constraint and pays the agent only if the requested level of effort is observed

Lemma 2: The optimal level of effort is e^*

The principal selects the efficient level of effort because she captures all the surplus form effort ending up with a surplus of $E\pi - v^{-1}(\underline{u}+g(e^*))$

Conclusions: (a) Full insurance if the agent is risk averse. (b) The optimal contract implements the efficient level of effort

Case 2: Asymmetric information and risk neutral worker (Benchmark 2)

Assume the agent's utility is $u(w, e) = w - g(e)$

Consider the contract $w(\pi) = \pi - E(\pi|e^*) + \underline{u} + g(e^*)$

This contract implements the first best level of effort since (a) The agent chooses e^* because she receives all the surplus from effort. (b) The agent accepts the contract since she is indifferent with the outside option

The principal earns $E(\pi|e^*) - \underline{u} - g(e^*)$ which is identical to the symmetric information profits (under the assumption that $v(w) = w$). Since $w(\pi)$ maximizes total surplus and gives the agent exactly her outside option, this contract has to be an optimal contract

The agent faces risk (level of utility varies) while the principal faces no risk (receives a fixed payment)

In a context where the principal owns a productive asset (e.g. firm, land), this contract can be interpreted as the principal selling the project to the agent at price $E\pi - \underline{u} - g(e^*)$. The agent supplies the efficient level of effort because she fully appropriates the marginal benefit of effort

This can be interpreted as a transfer of ownership. The agent is the residual claimant to firm 's return

Remark: The contract $w(\pi)$ is not the only contract that implements the optimal profits for the principal. Any contract that varies wage enough to satisfy the incentive constraint that the agent chooses the high action when it is optimal to do so and leaves no surplus to the agent is also optimal

Case 3: Asymmetric information and risk aversion

The wage schedule under asymmetric information and risk neutrality (benchmark 2) $w(\pi)$ is no more first best because the agent faces some risk

The wage schedule under symmetric information (benchmark 1) $w(e, \pi)$ is not implementable because the wage cannot depend directly on the level of effort

If $e^* = e_H$ and the principal pays the agent $w(e_H, \pi)$ then the agent will accept the contract and select e_L since $\int v(w(e_H, \pi))f(\pi|e_H)d\pi - g(e_H) = v(w(e_H, \pi)) - g(e_H) = \underline{u} < \underline{u} + g(e_H) - g(e_L) = \int v(w(e_H, \pi))f(\pi|e_L)d\pi - g(e_L)$

The full-insurance contract is not incentive compatible!

Contract $w(\pi)$ is incentive compatible for effort e_H if

$$\int v(w(\pi))f(\pi|e_H)d\pi - g(e_H) \geq \int v(w(\pi))f(\pi|e_L)d\pi - g(e_L) \quad IC(e_H)$$

Contract $w(\pi)$ is incentive compatible for effort e_L if

$$\int v(w(\pi))f(\pi|e_L)d\pi - g(e_L) \geq \int v(w(\pi))f(\pi|e_H)d\pi - g(e_H) \quad IC(e_L)$$

The principal chooses $e \in \{e_L, e_H\}$ and $w(\pi)$ to maximize $E(\pi - w(\pi))$ subject to PC and $IC(e)$

Case 1: $e^* = e_L$

Assume for now that it is efficient for the agent to supply low effort

Lemma 3: $IC(e_L)$ always hold when the wage is constant

In fact, for any non increasing compensation rule, the agent always prefers to supply low effort

If $e^* = e_L$, the optimal contract sets $w(\pi) = v^{-1}(\underline{u} + g(e_L))$ and implements the first best outcome

Case 2: $e^* = e_L$

Assume now that $e^* = e_H$. Let λ and μ the Lagrange multipliers on PC and $IC(e_H)$

Assume the principal chooses the function $w()$ point by point. The first order condition for $w(\pi)$ is

$$\frac{1}{v'(w(\pi))} = \lambda + \mu \left(1 - \frac{f(\pi|e_L)}{f(\pi|e_H)}\right) \quad FOC(\pi)$$

Lemma 4: $e^* = e_H$ implies $\lambda > 0$ and $\mu > 0$

The optimal contract is defined by a set of $\lambda > 0$, $\mu > 0$, and $w(\pi)$ such that PC and $IC(e_H)$ bind and $FOC(\pi)$

Discussion

There is a statistical interpretation to the optimal compensation rule: Let $\widehat{w}$ such that $\frac{1}{v'(\widehat{w})} = \lambda$. For any π such that $w(\pi) > \widehat{w}$ we have $\frac{f(\pi|e_L)}{f(\pi|e_H)} < 1$ and the opposite inequality holds for any π such that $w(\pi) < \widehat{w}$. The fraction $\frac{f(\pi|e_L)}{f(\pi|e_H)}$ is the likelihood ratio. For a given outcome π the likelihood ratio is high if the chance that this outcome could have occurred is high under effort e_L and/or low under effort e_H. Stated differently, conditional on π, e_L is more likely than e_H if the likelihood ratio is high

The principal pays the agent more in those states of the world where, an outsider who would observe only the outcome realization π and would know nothing about which action e the agent has taken, would conclude that it is more likely that the agent has taken the high action. This is just an interpretation since there is no inference to be made here: given the incentives in place, the principal knows that the agent takes the high action

$w(\pi)$ is not necessarily linear as in a piece rate. More problematically, $w(\pi)$ is not necessarily increasing in π. In fact, taking full derivative of $FOC(\pi)$ with respect to π

$$\frac{d}{d\pi}w(\pi) = \mu v''(w(\pi))\frac{d}{d\pi}\left[\frac{f(\pi|e_L)}{f(\pi|e_H)}\right]$$

The wage schedule increases if the likelihood ratio is decreasing in π and this is known as the monotone likelihood ratio condition. More surprisingly, the wage can decrease with profits and this happens over intervals where the likelihood ratio decreases with π!

More costly to implement the high effort under asymmetric information since $Ew(\pi) > v^{-1}(\underline{u} + g(e_H))$. The increase in utility cost is called risk premium

Inefficiency necessarily occur when e_H is the first best level of effort because either the principal settle for e_L (when the risk premium is too high) or the agent faces some residual risk (which is an inefficient risk allocation given that the principal is risk neutral)

Assume the principal observes an additional measure y in addition to π. In the landlord/farmer application, the landlord may observe the crop and also the weather. Should the principal use this new measure in the compensation contract. To answer this question define $f(\pi, y|e)$ as the joint distribution of y and π conditional on e. One can write

$$f(\pi, y|e) = f_1(\pi|e) f_2(y|\pi, e)$$

If $f_2(y|\pi, e)$ does not depend on e then $\frac{f_2(y|\pi, e_L)}{f_2(y|\pi, e_H)}$ cancels out in the FOC and y does not enter the FOC. Therefore, the optimal wage does not depend on y. In statistical terms, one says that π is a sufficient statistic for y with respect to e when $f_2(y|\pi, e)$ does not depend on e

Lecture 7: Dynamic Adverse Selection (Bolton-Dewatripont 9.1)

Coase conjecture (1972): A monopolist selling a perfectly durable good in a world with no discounting will be forced to sell at marginal cost. The intuition is that early buyers would not accept to pay any price above cost since they anticipate that prices will decline once the monopolist starts to sell to lower valuation buyers

Coasian dynamics says that monopoly power may be reduced due to inter-temporal competition between current and future incarnation of the monopolist. This is also known as the time inconsistency problem because sequential optimization is dominated by overall ex-ante optimization

The Coase inter-temporal monopoly problem belongs to the class of dynamic (in the sense of repeated interaction) adverse selection problems where the type of the privately informed party is fixed. This class of problems explicitly models the concepts of commitment, renegotiation, and spot versus long term contracts

Applications to firm financing (soft-budget constraint and decentralization of credit), incentives in regulation (ratchet effect)

Coase, Ronald H, 1972. "Durability and Monopoly," Journal of Law & Economics, vol. 15(1), pages 143-49

Hart, Oliver and Jean Tirole (1988), "Contract Renegotiation and Coasian Dynamics" Review of Economic Studies 55, pages 509-540

Model

Monopoly model with one buyer and one seller who interact over two periods $t \in \{1, 2\}$. Both parties discount payoffs in the second period by $\delta \in (0.1]$ with $\Delta = 1 + \delta$. Denote $x_{i,t}$ and $t_{i,t}$ the delivery rule for and transfer paid by type $i \in \{L, H\}$ in period $t \in \{1, 2\}$

Seller: The seller sells a fully durable good that is produced at zero production cost, and makes take it or leave it offers. The seller maximizes the net present value of revenues $\beta(t_{H,1} + \delta t_{H,2}) + (1 - \beta)(t_{L,1} + \delta t_{L,2})$

Buyer: Single buyer who values the good v_H with probability β and v_L with probability $1 - \beta$ such that $v_H > v_L > 0$. The buyer gets zero utility in any period where she does not consume. The buyer maximizes the net present value from consumption $v_i(x_{i,1} + \delta x_{i,2}) - (t_{i,1} + \delta t_{i,2})$

Game of incomplete information since the seller does not know the buyer's type. Focus on pure strategy perfect Bayesian equilibrium

Review: In the one period version of the model, the monopolist sells at v_H if $\beta v_H > v_L$ and v_L otherwise. In the former case, the monopolist does not sell with probability $1-\beta$. Inefficiency occurs because the monopolist has to trade-off allocative efficiency (sell more often) with informational rent (lower price)

Scenarios

One can distinguish different scenarios along three dimensions: (a) spot and long term contracts, (b) situations where the monopolist can and cannot commit to what she will do in the second period, and (c) situations where the monopolist may sell the durable good or rent it. Overall, we consider four scenarios of particular interest:

1. Full commitment to contract $x_{i,j}$ and $t_{i,j}$ (no renegotiation)

2. Selling under no commitment to future offers

3. Renting under no commitment to future offers

4. Long term contract with renegotiation

Economic questions of interest:

How does the trade-off between allocative efficiency and informational rent change when the interaction is repeated?

Understand the role of commitment and renegotiation in determining monopoly pricing strategies

Explain commonly observed practices (Xerox policy to lease only)

Case 1: Full commitment

The monopolist maximizes $\beta(t_{H,1} + \delta t_{H,2}) + (1 - \beta)(t_{L,1} + \delta t_{L,2})$ subject to participation $v_i(x_{i,1}+\delta x_{i,2})-(t_{i,1}+\delta t_{i,2}) \geq 0$ $(PC(i))$ for $i \in \{L, H\}$, incentive compatibility $v_i(x_{i,1} + \delta x_{i,2}) - (t_{i,1} + \delta t_{i,2}) \geq v_i(x_{i,'1} + \delta x_{i,'2}) - (t_{i,'1} + \delta t_{i,'2})$ $(IC(i, i'))$ for $i \neq i' \in \{L, H\}$, and feasibility $x_{i,j} \in [0, 1]$

One can focus without loss of generality on total discounted consumption $X_i = x_{i,1} + \delta x_{i,2}$ and total discounted payment $T_i = t_{i,1} + \delta t_{i,2}$

A standard treatment of the information constraints shows that $PC(L)$ and $IC(H, L)$ bind. Define $\beta' = \frac{vL}{v_H}$. The optimal contract is such that $X_H = \Delta$ and

$$\begin{cases} \beta < \beta' \implies X_L = \Delta,\ T_H = T_L = \Delta v_L \\ \beta > \beta' \implies X_L = 0,\ T_H = \Delta v_H,\ T_L = 0 \end{cases}$$

The monopolist cannot do better than replicating the one period pricing strategy. The profits are just the repeated profits and the trade off between allocative efficiency and rent extraction remains unchanged

If $\beta < \beta'$, the seller does not learn anything after period one so there is no incentive to change the contract. If $\beta > \beta'$, the seller finds out the buyer's type at the end of period one. Under no sale, the seller knows that the buyer is low type and it is (ex-post) optimal to lower the price. This, however, will be anticipated by the high type in the first period!

From now on, we focus on the interesting case, that is $\beta > \beta'$, and consider situations where the monopolist cannot fully commit to what she will do in the second period

Remark: The monopolist can implement the optimal profits with contract (X_H, T_H) alone. This contract can be interpreted as selling the good in period one at price

Δv_H. Alternatively, the monopolist could implement the optimal profits by committing to rental price $R_H = v_H$ in both periods. Therefore, one can restrict to rental or sales and still implement the optimal profits under full commitment

Case 2: Selling without commitment

The seller makes take it or leave it offers to purchase the durable good in both periods. Timing of the game:

1. Seller offers the good at P_1 in period one

2. If the buyer buys, the game ends. Otherwise, let $\mu(P_1)$ represent the updated belief that the buyer is of type H

3. Seller offers the good at P_2 in period two

4. The buyer accepts or rejects and the game ends

Stage 4: Any buyer of type i accepts a price no greater than v_i

Stage 3: $P_2 = v_H$ if $\mu(P_1) \geq \beta'$ and $P_2 = v_L$ if $\mu(P_1) < \beta'$. The low type gets zero surplus in period 2

State 2: The low type accepts any $P_1 \leq \Delta v_L$ in period one. The high type's decision depends on P_2. If the high type expect $P_2 = v_H$ then she accepts any $P_1 \leq \Delta v_H$ in period one. If the high type expect $P_2 = v_L$ then she accepts any $P_1 \leq v_H + \delta v_L$ in period one since the indifference condition between buying and waiting is $\Delta v_H - P_1 = \delta(v_H - P_2)$

Stage 1: (a) The seller may pool and sell to both types in the first period at price Δv_L with profits Δv_L. (b) Alternatively, the seller may separate by selling at $P_1 = v_H + \delta v_L$ and $P_2 = v_L$ with expected profits $\beta v_H + \delta v_L$. The separating profits dominate the pooling profits when $\beta v_H > v_L$ (as assumed)

Conclusion: The monopolist cuts price when she becomes more pessimistic. Because the high type anticipates this, the monopolist has to settle for a lower period one price and total profit decrease relative to full commitment $\beta v_H + \delta v_L < \beta \Delta v_H$

Remark: One can show that the with continuous types (v) and continuous time offers, the Coase conjecture holds and the initial price converges to cost as the time between any two offers converge to zero

Case 3a: Renting without commitment and with buyer anonymity

Under buyer anonymity, the seller cannot keep track of the buyer's type from one period to the other. This is realistic in a setting where there is a continuum of buyers who all have the same distribution of valuation as before and the valuations are independent

Note that the Coase time inconsistency problem presented above in the no commitment case still holds with a continuum of anonymous buyers. The interpretation is slightly different, however. The high valuation types anticipate that $P_2 = v_L$ because the monopolist will serve its residual demand. They are willing to pay at most $P_1 = v_H + \delta v_L$ in the first period. In equilibrium, the monopolist sells to β high types at price $P_1 = v_H + \delta v_L$ in period one and to $1 - \beta$ low types at price $P_2 = v_L$ in period two

Consider next the case of rental with buyer anonymity. Since the seller does not know the type of a buyer who shows up in the second period, the seller has no incentive to lower the price. The rental problem is identical to the one period revenue maximization problem and the (sequentially) optimal rent is $R_1 = R_2 = v_H$. The time inconsistency problem disappear. The intuition is that under anonymity the monopolist does not lean anything new at the end of period one

Conclusion: Rental is a solution to the lack of commitment problem under buyer anonymity

Case 3b: Renting without commitment and with buyer non-anonymity

If a buyer accepts a price above v_L the seller learns that she must be a high type. The seller cannot commit not to increase the price after finding out that information. This is known as the ratchet effect

Stage 4: Any buyer of type i accepts a rent no greater than v_i

Stage 3: $R_2 = v_H$ if $\mu(R_1) \geq \beta'$ and $R_2 = v_L$ if $\mu(R_1) < \beta'$

Stage 2: The low type accepts any $R_1 \leq v_L$ and rejects any rent above. The high type anticipates that if she accepts a rent above v_L she will get zero informational rent in period two since $R_2 = v_H$. If $R_1 > v_L$ the high type knows that only the high type would rent in period one. Therefore the seller will update her belief to

$\mu(R_1) = 1$ if the buyer rents. If the high type expect $\mu(R_1) = 0$ if the buyer does not rent, then she will rent in period one only if $R_1 \leq (1-\delta)v_H + \delta v_L$ since she is indifferent between renting in period one and waiting when $v_H - R_1 = \delta(v_H - R_2)$ for $R_2 = v_L$

Stage 1: (a) The seller may pool at $R_1 = v_L$ and $R_2 = v_H$ with profits $v_L + \delta\beta v_H$. Alternatively, the monopolist may separate with $R_1 = (1-\delta)v_H + \delta v_L$ and $R_2 = v_L$ if the buyer does rent in period one and $R_2 = v_H$ if she does. The separating profits are $\delta v_L + \beta v_H$. Again, the separating profits dominate the pooling profits

Conclusion: Under non-anonimity, renting does not solve the commitment problem and profits are lower than under full commitment. In the absence of commitment and with two periods, the profits are the same under rental and sale. One can show that with more than two periods, however, sales dominate rental and the intuition is that it is more difficult to achieve separation under rental due to the ratchet effect

Case 4: Long term contract and renegotiation

Under long term contract and renegotiation, the seller can write a long term contract that can be enforced by the buyer, but the seller can renegotiate the contract with the buyer at the end of period one. The initial contract protect the buyer against opportunistic behavior but it does not have to be enforced as was the case under full commitment. Renegotiation allows only for bilateral voluntary changes in the contract

The seller anticipates that ex-post renegotiation will take place in the second period if new gains from trade, that cannot be captured under the terms of the initial contract, appear at the end of the first period. How does the possibility of renegotiation influence the initial contract?

Long term sales contract: Sequential optimization implies that under separation consumption always take place in the second period since the seller finds out the buyer's type at the end of period one. This imposes the same constraint as in the no commitment case. Ex-post renegotiation is identical to no commitment The seller can reproduce the separation outcome with the renegotiation proof menu of contracts: (a) consume in both periods at total price $v_H + \delta v_L$ (b) consume only in period 2 at price δv_L. This menu of contract gives profits $\beta(v_H + \delta v_L) + (1 - \beta)\delta v_L = \beta v_H + \delta v_L$

Long term rental contract: Can get rid of the ratchet effect since it is now possible to insure the agent against opportunistic behavior. Offer the menu of contracts: (a) rent in both periods at rent $R_1 = v_H$ and $R_2 = v_L$ (b) rent in period two only at $R_2 = v_L$. This menu of contracts again gives profits $\beta(v_H + \delta v_L) + (1 - \beta)\delta v_L = \beta v_H + \delta v_L$. In the case where there are more than two periods, a similar menu of contracts increases revenues relative to spot market rental under no commitment

Conclusion: With two periods, the monopolist the monopolist achieves the same profits with long term contracts and renegotiation as under no commitment. (Long term contracting increases profits only under rental and with more than two periods.) Technically, one needs to introduce a renegotiation proof constraint to take into account the fact that the new information that is revealed over time may suggest new gains from trade