MICROECONOMICS III: Information and Contract Theory

PROBLEM SET 4

Karel Mertens
European University Institute

Problem 1: Moral Hazard

An Agent (A) needs to complete a task for a Principal (P). A can choose between working hard, $a = 1$ or shirking $a = 0$. If A accepts the task, he is paid a wage w is his utility is $u(w) - a$, where $u(\cdot)$ is strictly concave. A can always reject the task and obtain an outside option $\underline{u}$. The only thing P can observe is whether A succeeds or fails his task. He can make the wage w contingent on that observation. If $a = 1$, the probability of success is p_H and the payoff to P is x_H. If he $a = 0$, the probability of success is p_L and the payoff to P is x_L. P's total pay-off, conditional on his observation, is $x_j - w_j$, $j = H, L$. Assume $x_H > x_L$ and $p_H > p_L$.

1) Suppose P wants A to work hard, write down the incentive compatibility constraint (ICC). Also write down $A's$ individual rationality constraint (IRR).

2) Argue that the optimal (for P) wage schedule (w_H, w_L) implies both constraints holding with equality.

3) Let W_1 denote $P's$ expected payoff if he makes A work hard. Off course it might also be optimal for P to have $a = 0$, in which case the wage is w and his expected pay-off is W_0. When is $W_1 > W_0$?

4) Show that $p_L w_H + (1 - p_L) w_L > w$.

5) Show that if $x_H - w_H \leq x_L - w_L$, then $W_1 < W_0$ and explain.

Problem 2: Moral Hazard and Credit Market Rationing

Assume an investor can choose between two projects, A and B. Both projects require a fixed investment of I. The pay-off $\tilde{X}_j$ for $j = A, B$ is

$$\tilde{X}_i = \begin{cases} X_j & \text{with probability} \quad p_j \\ 0 & \text{with probability} \quad 1 - p_j \end{cases}$$

Assume $p_A X_A > p_B X_B > I$, $1 > p_A > p_B > 0$ and $X_A > X_B$. The investor must borrow the amount I from a bank. Assume the gross interest payment R will only be paid if the project is successful. I.e. the bank cannot get payments from a bankrupt investor. The expected payoff to the investor is $U_j(R) = p_j(X_j - R)$. The bank's expected profit is $\pi_j = p_j R - I$.

1) Assume that R cannot be made contingent on the project choice. Define $\hat{R}$ such that the investor chooses project A if $R \leq \hat{R}$. Find $\hat{R}$.

2) Suppose the bank is a monopolist and there is a single investor. What is the bank's optimal interest policy?

3) Suppose now there are N identical investors. Also suppose the bank has a fixed amount L of loanable funds, with $I < L < NI$. Discuss why and when *credit rationing* might arise, i.e demand for loanable funds necessarily exceeds the supply of loanable funds.