MICROECONOMICS III
Information Economics and Contract Theory
Final Exam—Pascal Courty
EUI, Florence, 2007

ANSWER ALL QUESTIONS.
TOTAL POINTS: 120

Exercise 1 (36 pts)

1-(12 pts) Consider the Rothchild and Stiglitz model of insurance with risk averse consumers and two types of probability of accident (risk) $\pi_B > \pi_G$. Under the separating equilibrium (assuming existence), which incentive compatibility constraint binds under competitive supply and monopoly supply? Discuss.

Under competition, firms make zero profit on each contract and we have shown that the $IC(B, G)$ binds. Monopoly is more tricky. If consumers have no outside options, then one of consumer will get zero surplus. The good type can always get some surplus from taking the contract that ensures that the bad type participates. Therefore, $IC(G, B)$ will bind. The constraint that binds depends on the market structure because this determines how the rents are shared and the incentive to deviate. If consumers use their initial endowments as outside option, then the monopolist faces a population of consumers with varying outside options, a case not covered in class.

2-(12 pts) Consider the adverse selection labor market model (Lecture 1) where the only equilibrium is such that no worker works and such that $\theta - r(\theta)$ is increasing. Assume firms can observe a worker's type at cost c. Firms can propose contracts that make pay conditional on type. Can this possibility increase welfare?

Consider the lowest solution to $\theta^* - r(\theta^*) = \frac{c}{\int_{\theta^*}^{\overline{\theta}} f(\theta)d\theta}$. Denote it θ^* if it exists and $\theta^* = \overline{\theta}$ otherwise. Firms can commit to pay a type θ worker $\theta - \frac{c}{\int_{\theta^*}^{\overline{\theta}} f(\theta)d\theta}$ after observing her type. There exists an equilibrium where all workers apply, those $\theta \geq \theta^*$ accept wage offer $\theta - \frac{c}{\int_{\theta^*}^{\overline{\theta}} f(\theta)d\theta}$, and firms break even. Firms do not want to deviate. Welfare increases if $\theta^* < \overline{\theta}$. As c goes to zero, θ^* converges to $\underline{\theta}$ and the equilibrium approaches the first best outcome.

3-(12 pts) After using a final exam for many years, a professor of a compulsory course decides to offer students the option to choose between the same type of final or write an essay. Do you expect average grades on the final and on the course to change?

Students who have a comparative advantage at the essay will self-select and not take the final. Overall grades will increase as long as the professor uses the same grading rule as assumed. Grades on the final may increase or decrease depending on which pool of students have a comparative advantage at writing an essay. Grades increase if the worst students do, decrease if the best students do, and stay the same if a random pool of students opt for the essay.

Exercise 2: (24 pts) Cheap Talk with Aligned Preferences

Consider the model of strategic information transmission with uniform signal and identical quadratic preferences, that is, F is uniform on $[0,1]$, $U^R(y,m) = U^S(y,m) = -(y-m)^2$. Define a N partition equilibrium as $0 = a_0 < a_1 < ... < a_N = 1$ such that the sender sends message $n = 1$ if $m \in [a_0, a_1)$, $n = 2$ if $m \in [a_1, a_2)$, ..., $n = N$ if $m \in [a_{N-1}, a_N]$.

1-(12 pts) (i) Show that exactly N distinct actions can be taken in a N partition equilibrium. (ii) Derive these actions as a function of the signal. (iii) Write the arbitrage condition that the sender is indifferent between signal n and $n-1$ when $m = a_{n-1}$.

(i) The sender can have only N different posterior corresponding to the N intervals $[a_n, a_{n+1})$. For each posterior, there is a unique optimum that maximizes $ArgMax_y \int_{a_n}^{a_{n+1}} U^R(y,m) \frac{f(m)}{\int_{a_n}^{a_{n+1}} f(t)dt} dm$ because U^R is striclty concave. Therefore, only N actions will be taken in equilibrium. (ii) Solving for the above maximization problem gives $y(n) = \frac{a_{n-1}+a_n}{2}$. (iii) The sender is indifferent if $U^S(\frac{a_{n-2-1}+a_{n-1}}{2}, a_{n-1}) = U^S(\frac{a_{n-1}+a_n}{2}, a_{n-1})$ or $a_n = 2a_{n-1} - a_{n-2}$.

2-(12 pts) Derive all N partition equilibria. What happens to the equilibrium when N increases to infinity? Discuss.

Rearrangin (iii), all steps have equal length $a_{i+1} - a_i = a_i - a_{i-1}$ implying that $a_i = \frac{i}{N}$. This is the only N equilibrium since there are no other N partitions of N equal segments. As N goes to infinity, we have $y(n(m))$ converges to the first best action m.

Exercise 3: (60 pts) Moral Hazard with Heterogeneous Types

Consider a moral hazard model with a risk neutral principal and a risk averse agent. The agent can exert effort e at cost $\frac{c}{2}e^2$. Effort e generates a noisy signal of performance $e + \varepsilon$ where ε is a random variable with mean 0 and variance σ^2. The principal pays the agent a variable rate based on the signal realization $\beta(e+\varepsilon)$ in addition to a fixed salary t. Both the principal and the agent have 0 reservation utilities. The principal's profits are

$$e - t - E\beta(e+\varepsilon)$$

The agent has mean-variance preferences of the form

$$U(e, \widetilde{m}) = E(\widetilde{m}) - \frac{c}{2}e^2 - \frac{1}{2} rVar(\widetilde{m})$$

where $\widetilde{m}$ is any random variable with mean $E\widetilde{m}$ and variance $Var\widetilde{m}$. In contrast with the standard model, we assume that the agent initially makes a take it or leave it offer (t, β) to the principal. The principal can accept or reject the offer. If the principal accepts, then the agent selects an effort level, nature draws ε, and finally payments are made.

1-(10 pts) Write the agent's optimization problem after the principal has accepted the contract and show that the agent's expected utility from contract (t, β) is $\frac{\beta^2}{2c} + t - \frac{1}{2}r(\sigma\beta)^2$.

The agent's expected utility is $\beta e + t - \frac{c}{2}e^2 - \frac{1}{2}r\left(\beta\sigma\right)^2$. The optimal effort is $e = \frac{\beta}{c}$. Plugging that value gives the above expression.

2-(10 pts) Write the agent's optimization problem at the contact offer stage. What contract does the agent offer? What are the principal's profits, and the agent's expected utility?

The participation constraint binds $t = (1-\beta)e$. The agent maximizes $\frac{\beta^2}{2c} + (1-\beta)\frac{\beta}{c} - \frac{1}{2}r\left(\beta\sigma\right)^2$. We have

$$\beta = \frac{1}{1 + rc\sigma^2}, \quad t = \frac{r\sigma^2}{\left(1 + rc\sigma^2\right)^2}, \; U = \frac{1}{2c\left(1 + rc\sigma^2\right)}$$

3-(10 pts) Assume there are two types of agents with cost of effort $\frac{c_L}{2}e^2$ and $\frac{c_h}{2}e^2$ such that $c_L < c_H$ and the cost type is privately observed. Assume 2 principals compete with contract offers of the type (t, β). Write the utility $U(i, j)$ that type i agent gets from contract j.

Type i agent working under contract j exerts effort $\frac{\beta_j}{c_i}$. Plugging this value in the previous answer gives $U(i, j) = t_j + \frac{\beta_j}{2c_i}(1 - rc_i\sigma^2)$.

4-(10 pts) The fraction of H type is α. Write all conditions that a separating equilibrium must satisfy.

Any equilibrium must satisfy truth telling $U(H, H) \geq U(H, L)$ and $U(L, L) \geq U(L, H)$, participation $U(H, H) \geq 0$ and $U(L, L) \geq 0$, and no profitable deviation stating that there do no exist $(\widetilde{t}_l, \widetilde{\beta}_l)$ and $(\widetilde{t}_h, \widetilde{\beta}_h)$ such that $U(H, \widetilde{H}) \geq U(H, H)$, $U(L, \widetilde{L}) \geq U(L, L)$, and $\alpha\left(\frac{\widetilde{\beta}_h}{c_h} - \widetilde{t}_h - \frac{\widetilde{\beta}_h^2}{c_h}\right) + (1-\alpha)\left(\frac{\widetilde{\beta}_l}{c_l} - \widetilde{t}_l - \frac{\widetilde{\beta}_l^2}{c_l}\right) \geq 0$.

5-(10 pts) Show that in any separating equilibrium $\beta_L > \beta_H$. (Hint: use the incentive compatibility constraints from (3)).

Adding $U(H, H) \geq U(H, L)$ and $U(L, L) \geq U(L, H)$ and rearranging gives $\frac{1}{c_l c_h}\left(\beta_L + \beta_H\right)\left(\beta_L - \beta_H\right)\left(c_L - c_H\right) \leq 0$. Since $c_L < c_H$, we have $\beta_L > \beta_H$.

6-(10 pts) Show that the take it or leave it contracts you derived in (2), applied to c_L and c_H, cannot be part of an equilibrium.

The high type want to deviate since $U(H,H) = \frac{1}{2c_h(1+rc_h\sigma^2)} < \frac{1+rc_h\sigma^2}{2c_h(1+rc_l\sigma^2)^2} = U(H,L)$.

7-(Extra Credit) Characterize (using a set of equations) the only candidate separating equilibrium. (Sketch of proof is sufficient.)

t_H and β_H correspond to the expression derived (2) replacing with c_h. t_L and β_L are such that firms earn zero profit $\frac{\beta_L}{c_l} - t_L - \frac{\beta_{Ll}^2}{c_l} = 0$ and $U(H,H) = U(H,L)$ or $\frac{1}{2c_h(1+rc_h\sigma^2)} = t_L + \frac{\beta_l^2}{2c_h} - \frac{1}{2}r(\sigma\beta_l)^2$.