MICROECONOMICS III

Information Economics and Contract Theory

PROBLEM SETS

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MICROECONOMICS III - Information Economics and Contract Theory
PROBLEM SET 1
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1 Adverse Selection and Monopoly Screening

A monopolist can produce goods of different qualities. The cost of production of a unit of quality q is

$$c(q) = \frac{1}{2}q^2$$

Consumers buy at most one unit. A consumer of type θ has utility

$$u(q|\theta) = \theta q - p$$

if she consumes a unit of quality q and 0 otherwise. There are N consumers and consumer n has preference θ_n such that $\theta_1 > \theta_2 > .. > \theta_N$.

1. What is the first best allocation?

2. The monopolist cannot observe the consumers' types but can offer different qualities at different prices. Assume in equilibrium consumer n choses quality q_n at price p_n for $n = 1..N$.

   - Write the incentive compatibility constraints $IC_{n,n'}$ that type n does not deviate to n' and the incentive rationality constraints IR_n.
   - How many constraints are there?
   - Write the monopoly profit maximizing problem.

3. Show that q_n is decreasing in n.

4. Show that if $IC_{n,n+1}$ bind for $n = 1..N-1$ and IR_N binds, and q_n is decreasing in n, then all the other constraints hold.

5. Show that $IC_{n,n+1}$ and IR_N have to bind in equilibrium.

6. Solve for the profit maximizing allocation.

2 Adverse Selection

Consider the model outlined in MWG Chapter 13 - B. Assume $r(\theta)$ to be continuous and non decreasing in θ. Notice that $r(\theta) \leq \theta$.

1. Show that any competitive equilibrium allocation is pareto inefficient, iff $r(\bar{\theta}) > E\theta$.

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PROBLEM SET 2
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3 Competitive Screening in the Insurance Market

A risk neutral company offers insurance against the risk of a car accident. There are two types of agents, characterized by their probability to have an accident. Careful drivers (G) face a low probability of accident π_G while risky drivers (B) are more likely to have an accident with probabilty $\pi_B > \pi_G$. Denote the proportion of G type agents with q. If no accident occurs, the value of the car is $\omega^{NA} = w$, while if an accident occurs the damaged car values $\omega^A = w - d$. Agents are risk averse with Von Neumann Morgenstern utility $u(.)$. Agents pay a premium $\alpha_1 = pz$ and receive z in case of an accident We denote the net payoff in case of an accident $\alpha_2 = z - pz$.

1. What is the expected utility of agent $i = G, B$ when offered insurance (α_1, α_2)? What are the expected profits of the firm?
2. Solve for the first best insurance coverage for agent $i = G, B$.
3. Describe the situation graphically. (Hint: use the ω_A, ω_{NA} space.)
4. Competition: Assume there are ≥ 2 firms in the market competing for agents. Show that firms earn 0-profits.
5. Can a pooling contract (α_1, α_2) exist? Why (not)?
6. Consider a separating equilibrium candidate $\left\{(\alpha_1^G, \alpha_2^G), (\alpha_1^B, \alpha_2^B)\right\}$.
   - Show that in any equilibrium $(\alpha_1^B, \alpha_2^B) = (\alpha_1^{B*}, \alpha_2^{B*})$.
   - Show that in any equilibrium $U(\alpha_1^{B*}, \alpha_2^{B*}) = (1-\pi_B)U(w-\alpha_1^G)+\pi_B U(w-d+\alpha_2^G)$ has to hold.
   - Characterize and describe graphically the set of restrictions (4) that any separating equilibrium has to satisfy.
7. Show graphically under which conditions a separating equilibrium does exist. What role does q play?

4 Monopoly Price Discrimination

A monopolist can sell two types of products l and h to two types of consumers L and H. .There are n_H and n_L consumers of type H and L respectively. A consumer of type $A \in \{L, H\}$ gets utility $v(A, a)$ if she consumes product $a \in \{l, h\}$ and 0 if she does not consume any product. The cost of producing product type $a \in \{l, h\}$ is c_a. We have $v(H, h) \geq v(L, h) \geq c_h$, $v(H, l) \geq v(L, l) \geq c_l$, $v(H, h) - v(H, l) \geq v(L, h) - v(L, l)$, and $v(L, h) - c_h \geq v(L, l) - c_l$. Let $\pi(a, b)$ denote the profits from selling product a to H and b to L.

1. Write down $\pi(h, l)$, $\pi(h, h)$, and $\pi(h, \varnothing)$.

2. Show that no other product allocation dominate the 3 allocations considered in the previous question (e.g. $\pi(l, l)$, $\pi(\varnothing, h)$, ... are dominated by one of the above profits).

3. Show that $\pi(h, l)$ dominates $\pi(h, h)$, and $\pi(h, \varnothing)$ only if

$$\frac{v(H, h) - c_h}{v(H, l) - c_l} \geq \frac{v(L, h) - c_h}{v(L, l) - c_l}$$

4. Discuss the implication for when one should expect to observe price discrimination.

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PROBLEM SET 3

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5 Signaling and Corporate Finance:

A risk neutral firm wants to finance a project. The initial cost to start the project is 1 euro. The project can be good (G) with probability $0 \leq \alpha \leq 1$ or bad (B) with probability $1 - \alpha$. The probability of success for project $i = G, B$ is p_i leading to profits $\Pi > 0$. In case of failure profits are 0. Assume $p_G > p_B$. Assume that the firm does not have any initial funds, so it needs to borrow from competitive and risk neutral banks. The firm knows the type of project it wants to finance while banks do not. The amount R , which is specified in a loan contract, should be repaid to the bank per unit of the initial loan. In the case of project failure the firm defaults and the bank receives 0. Let

$$p_G \Pi > 1 + r > p_B \Pi \tag{1}$$

where r is the risk free rate of interest. Initially assume the following timing: The bank first offers R and then the firm either accepts R or does not finance the project at all. (We consider SPNE.)

1. - Show that in equilibrium both projects are financed or neither is.
   - Show that the bank earns $0 - profits$.
   - Derive the equilibrium.
   - When are projects financed and how does this depend on p_i, α, r and Π?

2. Suppose now that the firm can finance fraction $z \in [0, 1]$ out of its own budget. The effective cost of doing so is $(1 + l)z$ where $l > r$. l can be interpreted as the cost of giving up liquidity.

   - Write the firm's pay-off as a function of R, z and project type.
   - Assume the following sequence of events: First the firm offers to self finance z. Second, banks respond by offering a contract R. Last, the firm decides to do the planned project or to drop it. Discuss existence and characterize the pareto dominant separating equilibrium.

- Compare the equilibria in 1) and 2). Discuss when good and bad projects are financed and whether efficiency is achieved.

6 Equity Financing

A firm has a project that requires an investment $I = 20$ at t=0 for a sure return of 30 at t=1. There is no discounting. I has to be raised from the financial market via a new equity issue. Potential new investors are uncertain about the value of the firm's assets in place: $A \in \{A_B = 50, A_G = 100\}$ with $\Pr[A = 100] = 0.1$.

1. Assume that investors believe that both types of firms invest. What fraction of the firm's equity has to be issued to new investors? What are the payoffs to existing shareholders (i.e the firm before issueing new equity) if they undertake the project? Can these beliefs be part of a Perfect Bayesian Equilibrium?

2. Asume that investors believe that only firms of type A_B issue new equity. Adress the same questions as in 1.

3. Now shareholders commit at t=0 to a wasteful advertising campaign at t=1, i.e. after the project return is realized. The advertising expenditure is irreversible- and results in a drop in profits of K. The size of the expenditure is a choice variable. Can a good firm signal its type via such an expenditure? Discuss.

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PROBLEM SET 4
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7 Moral Hazard and Motivation

A risk neutral principal (P) hires a risk neutral agent (A) to carry out a project. The project's outcome can be high or low such that $Y_H > Y_L \geq 0$ and $\Pr(Y_H) = e \in [0, 1]$, where e is agent's effort. The cost of exerting effort is $c(e) = \frac{1}{2}e^2$. Effort cannot be observed. The minimum wage paid to the agent has to be positive ($\underline{w} \geq 0$), because the agent benefits from limited liability. A's reservation utility is $\overline{u} \geq 0$ and P's reservation payoff is 0. A contract between P and A has two components: the fixed wage w (independent of outcome) and the bonus payment $b \geq 0$ paid if Y_H occurs. If the project outcome is high, P gets profit $\pi > 0$. (Assume $\pi + \theta < 1$) Agents care about something more than just money and receive a non-pecuniary benefit θ if the project outcome is Y_H.

1. Assume effort is contractible. Solve for the first best effort level.
2. Write P's optimal contracting problem when effort is not observable. (Hint: The incentive compatability constraint says that the equilibrium effort level maximizes the agent's utility given (b, w).)
3. Show that under any optimal incentive contract the participation and/or the limited liability constraint bind.
4. Show that the limited liability constraint binds. Hint: Suppose the LLC does not bind, then show that
   - e is at its first best level and
   - the expected payoff of P is strictly negative.

   Let

$$\overline{v} = \frac{1}{2}\left(\frac{\theta + \pi + \sqrt{(\theta + \pi)^2 - 4\underline{w}}}{2}\right)^2 + \underline{w} \tag{2}$$

be the reservation payoff of A such that P makes 0−Expected Profits under an optimal contract. Assume that $\pi^2 - 4\underline{w} > 0$. Denote $\underline{v}$ as the reservation payoff of A such that for $\overline{u} \geq \underline{v}$ A's participation constraint is binding. They are both positive real numbers and $\overline{v} > \underline{v}$ holds. It follows that there are three cases to consider when characterizing the optimal contract:

5. Case 1: LLC and PC are binding. Characterize the optimal contract and show that it is feasible iff $\overline{u} \in [\underline{v}, \overline{v}]$.

6. Case 2: PC is not binding: We will distinguish two subcases:

   - Characterize the optimal contract when $\pi > \theta$ and show that it is feasible iff $\overline{u} \in [0, \underline{v}]$.
   - Characterize the optimal contract when $\pi < \theta$ and show that it is feasible iff $\overline{u} \in [0, \underline{v}]$.

7. Discuss how the optimal contract depends on the role of θ.