MICROECONOMICS III
Information Economics and Contract Theory
Final Exam—Pascal Courty
EUI, Florence, 2008

ANSWER ALL QUESTIONS.
TOTAL POINTS: 120

Exercise 1 (36 pts)

1-(12 pts) An insurance company offers a unique policy against home accidental damage (e.g. fire, flooding...). The company increases the deductible and observes a decrease in claims. Explain.

Under moral hazard, insurees have an incentive to self-insure after the introduction of a deductible, that is, to make investments that reduce the chance of a claim. Under adverse selection, the deductible makes the contract less attractive to high risks (people who are more likely to incur a claim). Both moral hazard and adverse selection can explain the decrease in claims.

2-(12 pts) Consider the moral hazard model with 2 efforts and 2 states of the world (assume $p_h > p_l$ where p_e denote the probability of high outcome under action e), risk averse agent and risk neutral principal, and assume it is efficient to implement the high action. Give a necessary and sufficient condition on the p_e's such that the first best applies.

The first best will take place if the high action is implemented and compensation does not vary. The first statement is equivalent to $(p_h - p_l)(w_h - w_l) \geq c_h - c_l$ which will hold only if $w_h > w_l$. The second statement will not be violated only if $p_h = 1$. Therefore, the first best can occur only if $p_h = 1$. When this is the case, the first best can be implemented with $w_h = U + c_h$ and w_l arbitrarily low. A necessary and sufficient condition is $1 = p_h > p_l$.

3-(12 pts) Consider the basic adverse selection model under competition with privately informed workers but assume that $r'(\theta) < 0$. Can inefficiencies exist?

Since $r'(\theta) < 0$, the higher the wage the lower the average quality of workers. Too many workers may work. To illustrate, if $E\theta > r(\underline{\theta}) > \underline{\theta}$ firms offer $E\theta$ and all workers work although it would be efficient for $\underline{\theta}$ to remain self-employed.

Exercise 2: (42 pts) Procurement

A government agency can outsource a contract to a firm who is privately informed about its cost. The agency believes that the firm's cost is c_H with probability α and $c_L < c_H$ with probability $1 - \alpha$. The agency receives V if the project is implemented

1-(10 pts) Assume the agency makes a take it or leave it offer to the firm. Solve for the optimal offer and agency surplus.

The monopolist can either outsource to both types or only to the low cost type. In the first case PC_H implies $c_H \leq t$ which also implies PC_L. The agency surplus is $V - c_H$ under $t = c_H$. In the second case, the surplus is $(1-\alpha)(V - c_L)$ under $t = c_L$. The agency outsource to both types if $V - c_H > (1-\alpha)(V - c_L)$.

2-(10 pts) Assume now that the agency can offer a menu of contract of the type (q, t) where t is the up-front payment to the firm and q is the probability that the project is implemented. Stated differently, a firm that selects contract (q, t) receives t for sure and implements the project with probability q. Write down the participation constraints, the incentive compatibility constraints, and the agency screening problem.

$PC_i : t_i - q_i c_i \geq 0$

$IC_{i->j} : t_i - q_i c_i \geq t_j - q_j c_i$.

The agency maximizes $\alpha(q_H V - t_H) + (1-\alpha)(q_L V - t_L)$

s.t $.PC_L, PC_H, IC_L, IC_H$.

3-(10 pts) Which constraints bind in any optimal contract?

PC_H binds. Otherwise, the agency could reduce both transfers and all constraints would still hold while surplus would increase. A contradiction. IC_L binds. Otherwise the agency could increase t_L and all constraints would hold while surplus would increase. A contradiction.

4-(12 pts) Solve for the optimal contract. Can the agency benefit from screening? Explain.

We have $t_h = q_H c_H$ and $t_L = c_L(q_L - q_H) + q_H c_H$. The FOC implies $q_L = 1$. Two cases may occur. Either $q_H = 1$ and $t_h = t_L = c_H$ or $q_H = t_H = 0$ and $t_L = c_L$. The monopolist's surplus does not increase relative to take it or leave it offer. We have a corner solution. The agency does not trade-off at the margin rent and allocation as would be the case if the V or C would a non-linear functions of q.

Exercise 3: (42 pts) Cheap Talk: Toward a General Result

Consider the model of strategic information transmission with uniformly distributed signal and quadratic preferences. The sender's type is distributed uniformly on $[0, 1]$, $U^R(y, m) = -(y - m)^2$ and $U^S(y, m, b) = -(y - (m + b))^2$. Assume two actions y_0 and y_1 such that $y_0 < y_1$ are taken in equilibrium and denote by m_0 and m_1 (one of) the sender types who initiate these actions.

1-(14 pts) Show that there must exist a sender of type $\widetilde{m} \in [m_0, m_1]$ such that $\widetilde{m}$ is indifferent between actions y_0 and y_1, any $m > \widetilde{m}$ strictly prefers y_1, and any $m < \widetilde{m}$ strictly prefers y_0.

The $\widetilde{m}$ who is indifferent between actions y_0 and y_1 is $\widetilde{m} = \frac{y_0 + y_1}{2} - b$. To show that $\widetilde{m} \in [m_0, m_1]$ note that, $\widetilde{m}$ has to be to the right of m_0 since those types to the left of m_0 strictly prefer y_0. Similarly, $\widetilde{m}$ has to be to the left of m_1. Since higher types strictly prefer higher actions, all types to the left of $\widetilde{m}$ strictly prefer y_0 and all types to the right of $\widetilde{m}$ strictly prefer y_0.

2-(14 pts) Show that R's optimal action in state $\widetilde{m}$ has be belong to $[y_0, y_1]$.

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Since any $m > \widetilde{m}$ prefers action y_1 over action y_0, R must believe that $m > \widetilde{m}$ when she implements y_1. Since R's optimal action is increasing in m, R's optimal action under $\widetilde{m}$ has to be lower than y_1. A symmetric argument shows that R's optimal action under $\widetilde{m}$ has to be greater than y_0.

3-(14 pts) Conclude that $y_1 - y_0 \geq b$ and that at most a finite number of actions can be taken in any equilibrium.

In state $\widetilde{m}$, the optimal action of S is in the middle of y_0 and y_1 and we showed in the previous question that R's optimal action belongs to $[y_0, y_1]$. Therefore, in state $\widetilde{m}$ the optimal actions of both R and S belong to $[y_0, y_1]$. But in any state, the distance between the optimal action of S and R is b. Therefore, the distance between actions y_0 and y_1 has to be at least b. The maximum number of actions such that no two actions are less than b apart is at most $1/b$.

4-(Extra Credit) How would the conclusion that at most a finite number of actions can be taken in any equilibrium generalize to non-quadratic utility functions that are single-peaked and such that $U_{ym}^{R,S} > 0$?

The proof assumes that two actions y_0 and y_1 are taken in equilibrium and identifies a state of the world such that the optimal actions of both S and R in that state belongs to the interval $[y_0,y_1]$. If we assume that in any state S and R disagree by at least ϵ (their optimal actions are at least ϵ apart) then a similar proof as above will carry through. We can demonstrate the existence of a $\widetilde{m}$ such that both S and R's optimal actions in that state belong to $[y_0,y_1]$. We cannot derive $\widetilde{m}$ or S's optimal action in $\widetilde{m}$, but we can show that $\widetilde{m} \in [m_0, m_1]$ and that S's optimal action belongs to $[y_0, y_1]$. We use the monotonicity of the optimal actions and also the fact that $U^S(y_0, ., b)$ and $U^S(y_1, ., b)$ can cross at most once.