MICROECONOMICS III – Information Economics and Contract Theory
Problem Sets
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Problem Set 1

1 Adverse Selection and Monopoly screening in the Financial Market

There is a continuum of possible investors. They can invest one to start a project. If they do, their project will succeed with probability p_i and they will receive revenue π. Investors are risk-neutral and $p_i \sim uniform[0, 1]$.

There is perfect competition in the banking sector and symmetric information. Banks can offer individual loan contracts to investors and make interests rates r_i conditional on success probabilities p_i.

a) What is the Pareto-Optimal allocation of loans?
b) What is the equilibrium allocation of loans?
c) How do the results change if there is a single monopolistic bank?

Now assume that investors privately observe their success probability p_i.

d) Derive the loan equilibria under asymmetric information for the cases of competitive and monopolistic banking sector respectively.

In the following we consider the case of a single monopolistic bank. Assume there are only two types of investors H and L, with $p_H > p_L$. Each investor owns illiquid property worth C, which can be used as collateral for the bank loan, but not directly to pay for the investment. The bank can offer a menu of controls (r_i,c_i), where:
r_i: interest rate
c_i: collateral, with $c_i \in [0, 1]$

e) Write the participation constraint for an investor to sign a loan contract.
f) Write the incentive compatibility constraint for an investor to choose contract (r_i,c_i) over any alternative contract (r_j,c_j) .
g) Write the monopoly maximization problem.
h) Solve for profit maximization.

Problem Set 2

1 Entrepreneurial incentives and taxation

There are two types of entrepreneurs. A share of α belongs to type 1 and a share of $(1-\alpha)$ belongs to type 2. They have different abilities γ with $\gamma_{2>}\gamma_1$. They can exert an effort e in their enterprise which causes them disutility of $\frac{1}{2}e^2$ measured in monetary units. Both effort and ability are only observable by the entrepreneur. The effort e and the ability γ result in the generation of revenue $R_i = \gamma_i e_i$.

Their utility function has the form:
$U_i = (\gamma_i e_i - \delta_i - \frac{1}{2}e_i^2)^\beta$ with $\beta \in (0, 1)$.

There is an entrepreneurial organization (EO). In the EO membership is compusory (assume outside option has value $-\infty$). It finances its activities total cost C via membership fees δ_i. The EO maximizes total welfare of all members subject to its budget constraint, giving each member the same weight. It can observe individual revenues and impose fees conditional on revenues: (δ_i, R_i)

a) write the budget constraint of the EO and the incentive compatibility constraints of the entrepreneurs.
b) show which constraint(s) bind(s) in equilibrium
c) find the constrained pareto-optimal scheme for the membership fees and derive the equilibrium values for δ_i, U_i and e_i,

2. Licensing to a monopolist

There is a university researcher, who has developed a new technology and owns the patent. She wants to license the patent to a monopolist. The average production costs of the monopolist are constant and equal to $c_.$. The new technology would reduce these average costs from c to $c_1 < c$. Demand for the monopolist is $Q(p)$. The licensing contract consists of two controls. A fixed payment $L \geq 0$ and a variable per unit payment $\alpha \geq 0$. There is the following time structure: First, the researcher makes a take it or leave it offer to the monopolist: either one contract or a set of contracts of the type (L, α). Then, the monopolist accepts one of the contracts or rejects.

a) write the profit maximization problem of the monopolist and compare profits with and without licensing
b) write the profit maximization problem of the researcher and the constraint(s) she has to take into account
c) find the optimal licensing contract under symmetric information

Now the new technology can have two different outcomes: good or bad, with $c_G < c_B < c$

Assume now, that the quality of the technology can only be observed by the monopolist (because he/she has a better understanding of his/her own production technology), but not.by the researcher. Analyze the optimal contract designed by the researcher and answer the following questions:

d) write up all relevant constraints for this problem (hint: this includes constraint(s) relevant in part b and any constraint(s) arising from information asymmetry).

e) show which constraint(s) bind(s) and which constraint(s) hold(s) in equilibrium.

f) show that the optimal contract has the following characteristics:

$\alpha_G = 0$ and $\alpha_B > 0$ and $L_B < L_G < L_{G^*}$,where L_{G^*} is the optimal licensing fee under symmetric information.

Problem Set 3

1 Moral hazard in farming

There is a landowner and a farmer. On the land corn can be grown. The yield depends on the effort of the farmer. Effort is continous with $e \in [0, 1]$. Yields can be high or low. With probability e yields are good (y_G) with probability $(1 - e)$ yields ar bad (y_B).

This setting can be summarized in the following expected yield function:

$E(\Pi \mid e) = p(y_G)y_G + p(y_B)y_B = (e)y_G + (1 - e)y_B$

The farmer has outside option $U(notfarm) = 0$

The utility function of the farmer has the following form:

$U(I, e) = I^{\frac{1}{2}} - \frac{1}{2}e^2$, where I is total income.

Landowners are risk-neutral. There utility is equal to their total net income.

Symmetric Information

The landowner can make a take-it or leave-it offer to the farmer specifying a wage w to farm the land.

a) What are the equilibriuim contract, equilibrium level of effort, and equilibrium utilities if effort is perfectly observable?

Asymmetric Information

Now assume that effort is no longer observable. In the following four different contract settings will be considered:

(i) take-it or leave-it offer (w) wage employment contract.

–>The landowner can make a take-it or leave-it offer to the farmer specifying a wage w to farm the land.

(ii) take-it or leave-it offer (R) rental contract.

–> The landowner offers the farmer a take-it or leave-it annual fixed rent cost. That means the farmer pays R to the landlord and keeps all the revenue Π from the farming. Here, you do not have to solve explicitely for the fixed rent cost R, but you should characterize its solution and explain its role briefly.

(iii) take-it or leave-it offer (β) sharecropping contract.

–> There is a sharecropping agreement, i.e. the farmer has to give a share of β of the yield Π to the landowner.

(iv) take-it or leave-it offer (w, b) wage with extra payment employment contract.

–> The landowner can offer a contract with a fixed basic wage $w \geq 0$ plus a share γ of yields if y_G is achieved. In this case assume that farmers are risk neutral, i.e. their utility function now becomes:

$U(I, e) = I - \frac{1}{2}e^2$

b) what are the equilibriuim contract, equilibrium level of effort, and equilibrium utilities in cases (i)-(iv)?

c) discuss whether there is a welfare ordering of the four contracting strategies possible (with risk-averse agents).

Problem Set 4

1 Advertisment and signaling

There is one firm and one price-taking consumer. First the firm developes a new product. The quality of the new product is drawn randomly and can be high or low $q_H > q_L$. The quality is private information of the firm. A consumer can consume up to one unit of the good per time period. The consumer values a unit of the high quality product with θ_H and a unit of the low quality product with θ_L, with $\theta_H > \theta_L$. The consumer valuation is common knowledge. The firm has per unit production costs depending on the quality of the product, with $\theta_L < c_L < c_H < \theta_H$. The firm has a time discount rate of δ. There are two time periods. The product is an experience good, i.e. if the consumer buys the product she learns the quality of the product.

a) Define a Perfect Bayesian Equilibrium for this setting.
b) Derive all Perfect Bayesian Equilibria.

Now suppose the firm can spend amount A on advertising in period one. There is no direct effect of this action, i.e. the firm is basically 'burning ' money. Yet, the high quality firm can use advertising as a signal to improve its profits.

c) Derive the Perfect Bayesian Equilibrium with signaling.

2 Cheap Talk in Business

A manager runs a team of two juniors $i = 1, 2$. Junior i has ability $a_i \geq 0$. Abilities are independently drawn according to a uniform distribution with support $[0, 1]$. Due to her experience, the manager privately observes the juniors' abilities. Junior i gets utility $U(e_i, a_i) = a_i e_i - \frac{1}{2} e_i^2$ from exerting effort e_i if she believes that her ability is a_i. The manager earns profits $W(e_1, e_2, a_1, a_2) = a_1 e_1 + a_2 e_2$. The game is as follows. Nature draws the juniors' abilities. The manager observes the abilities and can make any public announcement. The agents select a level of efforts.

a) Characterize a babbling equilibrium where no information is transmitted.
b) Show that no information can be transmitted if there is a single junior.
c) Show that full information cannot be an equilibrium with two juniors.

Consider the following set of announcements. The manager can say that junior 1 is more able than 2, junior 2 is more able than 1, or both juniors are equally equal.

d) Assume the juniors believe the manager. What effort do they take after the announcement?

e) Show that there exist an equilibrium where the manager truthfully adopts the above announcement.

f) bonus question (Difficult) Show that the equilibrium you characterizes in 4 holds when abilities are independently drawn from any distribution f with support $[0, 1]$. (Hint: the expectation of the maximum of two iid random variables drawn from distribution f is $\int_0^1 a2F(a)f(a)da$.)