MICROECONOMICS III
Information Economics and Contract Theory
Final Exam—Pascal Courty
EUI, Florence, 2009

ANSWER ALL QUESTIONS.
TOTAL POINTS: 120

Exercise 1 (25 pts) **(**Say whether the statement is true/false/uncertain and explain.**)**

1-(12.5 pts) Under competitive insurance and perfect information, consumers are fully insured.

Uncertain/True. Full insurance means that income does not vary across states of the world. Under fair insurance risk averse consumer are fully insured. The statement is true if consumers are risk averse (typically assumed) and insurers are risk neutral. The later will be the case if risks are small and idiosyncratic.

2-(12.5 pts) In the (Crawford/Sobel) cheap talk model, is it possible that more than two actions could be taken in a given state of the world? Does it happend that two actions are taken in a given state of the world in equilibrium?

False. In Crawford/Sobel the players have single peaked utilities. The receiver will take a single action given her belief about the state of the world. The sender could send at most two messages (that trigger different actions) for a given state of the world. The second statement is true. Two actions can be taken when the state of the world is such that the sender is indifferent between to adjancent actions. In the quadractic case, this will happen only when $a_i + b = (a_i + a_{i+1})/2$, a set of events of measure zero.

Exercise 2: (45 pts)

A risk neutral borrower can invest in a project requiring one unit of investment in period 1 and yielding a return $\widetilde{\omega}$ in period 2. The borrower has no financial resource. A risk neutral lender has one unit of investment and could get r in the second period from investing in a competitive capital market. The random variable $\widetilde{\omega}$ has full support on $[a, b]$, cumulative distribution function G, and probability density function g, with $E(\tilde{\omega}) = \int_a^b \omega g(\omega) d\omega > r$, and $a < r$. The realization of the project return ω is costlessly observable by the borrower while the lender has to pay a monitoring cost γ to (perfectly) observe it. The borrower proposes the following financial contract to the lender who can only accept or reject. The lender invests in the project in period 1. In period 2, the borrower reports the project return $s \in [a, b]$. If $s \in [a, \overline{R}]$ then the lender pays the monitoring cost, finds out ω, pays the borrower $f(\omega) \geq 0$ and keeps $\omega - f(\omega)$. If $s \in [\overline{R}, b]$ then the borrower gives $\overline{R}$ to the lender. The function $f()$ and the constant $\overline{R}$ are to be determined.

1-(10 pts) Write down the incentive compatibility constraint that the borrower reports truthfully the return, the participation constraint of the lender, and the maximization problem of the borrower.

IC: for $\omega \in [a, \overline{R}]$, $f(\omega) \geq \omega - \overline{R}$ and for $\omega \in [\overline{R}, b]$, $\omega - \overline{R} \geq f(\omega)$

PC $\int_a^{\overline{R}} (\omega - f(\omega) - \gamma) g(\omega) d\omega + (1 - G(\overline{R}))\overline{R} \geq r$

Borrower maximizes $\int_a^{\overline{R}} f(\omega) g(\omega) d\omega + \int_{\overline{R}}^b (\omega - \overline{R}) g(\omega) d\omega$ subject to IC and PC

2-(15 pts) Show that $f(\omega) = 0$ $\forall \omega$ satisfies the IC and characterize the corresponding optimal .$\overline{R}$. Can the borrower be better off with a different $f(\omega)$? Argue that the optimal contract ressembles a debt contract.

IC clearly holds, and $\overline{R}$ is the lowest value that solves $\int_a^{\overline{R}} (\omega - \gamma) g(\omega) d\omega + (1 - G(\overline{R}))\overline{R} = r$.

Setting $f(\omega) > 0$ increases $\overline{R}$ from the PC ($\int_a^{\overline{R}} (\omega - f(\omega) - \gamma) g(\omega) d\omega + (1 - G(\overline{R}))\overline{R}$ decreases with $f(\omega)$). The borrower is worse off because her payoff ($\int_a^{\overline{R}} (\omega - \gamma) g(\omega) d\omega + (1 - G(\overline{R}))\overline{R} - r) + \int_{\overline{R}}^b (\omega - \overline{R}) g(\omega) d\omega = E(\tilde{\omega}) - \gamma G(\overline{R}) - r$) decreases with $\overline{R}$. The intuition is that the monitoring cost has to be paid more often.

3-(10 pts) Compute the inefficiency cost relative to the first best outcome. When will the project be undertaken?

Efficiency cost is $\gamma G(\overline{R})$. The project is undertaken is there exists a $\overline{R}$ such that.$\int_a^{\overline{R}} (\omega - \gamma) g(\omega) d\omega + (1 - G(\overline{R}))\overline{R} = r$.

Exercise 3: (50 pts)

A monopolist sells a product line composed of packages of different *sizes* to a unit mass of consumers. There are two types of consumers $\theta \in \{\theta^L, \theta^H\}$ and fraction $\alpha \in [0, 1]$ is θ^L. Type θ has utility $u(q, \theta)$ from consuming quantity q such that $u_q > 0$, $u_{qq} < 0$, $u(q, \theta^H) > u(q, \theta^L)$, and $u_q(0, \theta^H) > u_q(0, \theta^L)$. Both types receive u^0 in the event they don't consume. The cost of producing a package of size q is $C(q) = cq$. The monopolist offers product line $\{(p_L, q_L), (p_H, q_H)\}$ where one has to pay p to obtain package q.

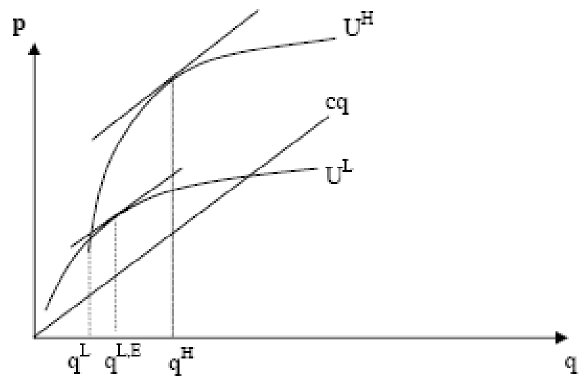
1-(10 pts) Write down the monopoly profit maximization problem.

See class notes. The only difference is that the utility is concave (rather than linear) while the cost is linear (rather than convex).

2-(20 pts) Solve analytically the monopoly problem.

Proceed as in class notes with the treatment of IC and PC. We are left with the constraints that PC_L and $IC_{H,L}$ bind.and quality has to be increasing. Plugging these 2 constraints in the objective function and ignoring for now the condition that quantity has to be increasing, we solve for the optimal quantities: $\alpha(u_q(q_L, \theta^L) - c) = (1 - \alpha)(u_q(q_H, \theta^H) - u_q(q_L, \theta^H))$ and $u_q(q_H, \theta^H) = c$. We check that quantities indeed increases and compute the optimal prices from the two binding constraints.

3-(10 pts) Provide a graphical interpretation for the solution you derived above in the (q, p) quandrant.



4-(10 pts) Assume now that every once in a while the monopolist would like to sell to a third type of consumer, the bargain hunters. There is a unit mass of bargain hunters and they buy Q units only if the *price per unit* is lower than u. Rewrite the monopoly problem with the additional constraint that one of the product offered is affordable to the bargain hunters. How many products will the monopolist sell? Which incentive compatibility constraint and participation constraint bind?

Call the new product offered (p_B, q_B). The problem becomes $Max\ \alpha(p_L - cq_L) + (1-\alpha)(p_H - cq_H) + q_B(p_B - c)$ subject to the usual PC, the 2 IC for L and H, and in addition $p_B \leq u$, $q_B \leq Q$, and the IC constraint for the bargain hunters $uq_B - p_B \geq uq_i - p_i$ for $i = L, H$.

If q_B is positive then $q_B = Q$. If Q is large, then the monopolist can separate the bargain hunters by offering them a large package that is not attractive to either the low or high types (assuming that marginal utility goes to zero when q increases). Three products will be offered and the monopolist will extract all the surpulus of the bargain hunters without any loss from the other consumers. If Q is not so large, then some new constraints will bind. One, two, or three products may be offered depending on how the monopolist wants to sort consumers. The treatment of the IC constraint for L and H shows that PC_H is slack and q is increasing with type but the proof that PC_L and $IC_{H,L}$ bind do not carry through.