Micro III: Information Economics and Contract Theory

Website: http://www.iue.it/Personal/Courty/courses.html

Lecture 0: Course introduction

Lecture 1: Akerlof Lemon Model (MGW 13-B)

Lecture 2: Choice under Uncertainty

Lecture 3: Moral Hazard (MGW 14-B)

Lecture 4: Monopoly Screening (Salanie 2-2, 2-3)

Lecture 5: Competitive Screening (MGW 13-D)

Lecture 6: Signaling (MGW 13-C)

Lecture 7: Strategic Transmission of Information (Gibbons 4.3A)

Lecture 0: Course Introduction

- Introduction to information economics (Stiglitz, Joseph E. "The Contribution of the Economics of Information to Twentieth Century Economics". Quarterly Journal of Economics, Nov. 2000, pp. 1441-1478.)

- Adverse selection model (MGW 13-B and Akerlof G (1970) "The Market for Lemon," QJE 89: 488 – 500)

Introduction to Information Economics

- General equilibrium: Information plays a role only in "state of nature," with application to insurance and asset pricing

- Chicago school: information as an investment (i.e. investment in search, human capital; Shultz/Stigler/Beker)

- Economics of information: strategic role of information
  - Akerlof Lemon model (1970)
  - Mireless self-selection in taxation (1971)

  - Arrow moral hazard and principal agent paradigm

- Main contributions of information economics and contract theory
  - Revisit main conclusions of general equilibrium theory (both positive and normative)
  - Explain behavior and outcomes difficult to understand otherwise (e.g. credit rationing)
  - Applications to finance (corporate governance, insurance), labour economics and personnel economics (compensation policies), industrial organization (pricing, procurement)

- Example of how informational issues may change standard theoretical predictions
  - Existence of equilibrium may not hold
  - Welfare conclusions may not hold (competitive equilibrium may not be Pareto optimal)
  - Competitive pricing (p=mc) may not hold
  - Supply may not equal demand in equilibrium (credit rationing, unemployment)
  - Law of one price may not hold (equilibrium distribution of price)

- Information Economics: Main Concepts
  - Imperfect information: Uncertainty due to nature
  - Private information: information observed privately by a party
  - Asymmetric information: Some agents know more than other
  - Constrained Pareto Optimality: Assume the social planner does not have directly access to the agent's private information
  - Self-selection: A party make a choice on the basis of her private information
  - Adverse selection: The choice made by the privately informed party goes against the interests of the uninformed party (as opposed to positive selection)

- Signalling: The privately informed party takes some action to signal her private information

- Screening: The uninformed party designs a menu of choice to screen the privately informed party

- Principal/Agent: A principal makes a take-it-or-leave-it offer to the agent (ignore bargaining and assume that the agent has an exogeneously determined outside option)

- Moral hazard: A party (typically the agent) chooses an action that is not observed by the other party (typically the principal)

- Hidden action vs hidden information: Pure moral hazard vs agent taking (possibly unobservable) action after privately observing some information

- Methodological Issues
  - Partial equilibrium analysis
  - Often focus on interaction between a few parties (typically 2)
  - Parties write contracts subject to information constraints
  - Solution concept often based on non-cooperative game theory (Subgame perfect Nash, Bayesian updating, Perfect Bayesian Equilibrium)

Lecture 1: Akerlof Lemon Model (MGW 13-B)

Adverse selection model (MGW 13-B)

Akerlof G (1970) “The Market for Lemon,” QJE 89: 488 – 500): Model of market collapse under asymmetric information with application to second hand car market, labor market, credit market, and development economics

The market collapse prediction is a rather dramatic conclusion but the concept of adverse selection is very general and had been applied in personnel economics (early retirement, compensation policies), insurance, corporate finance, to name just a few fields

Labor market application: Identical firms hire workers who privately observe their productivity

Firms: Any number greater than one. Transform labor into output using CRS technologies. Firms are risk neutral. Assume that output price is one (partial equilibrium analysis)

Workers: Measure N of heterogeneous workers. Worker of type θ produces θ in a firm and $r(\theta)$ in home production. $\theta \in [\underline{\theta}, \overline{\theta}]$ with $0 \leq \underline{\theta} < \overline{\theta} < \infty$. The distribution of types is $F(\theta)$ with density $f(\theta)$ such that $f(\theta) > 0$ for $\theta \in [\underline{\theta}, \overline{\theta}]$

Market outcomes to be determined:

Equilibrium wage(wage function?)

Sorting of workers between firms and home production

Equilibrium concept:

Competitive equilibrium

Game theoretic approach

Possible scenarios:

Full information on workers' type or productivity (standard approach)

Workers are privately informed about their productivity (asymmetric information)

Economic questions of interest:

1. Does the introduction of asymmetric information matter?
2. Does it change market outcomes?
3. Does it change the efficiency properties of the equilibrium?

Case 1: Symmetric Information and Competitive Equilibrium (Benchmark)

The wage can be function of the type $w^*(\theta)$

$w^*(\theta) = \theta$

$\{\theta \;\; s.t. \; r(\theta) \leq \theta\}$ are employed in a firm

Firms earn zero profits

Sorting according to principle of comparative advantage

Equilibrium is Pareto efficient

Standard conclusions from competitive equilibrium analysis

Case 2: Asymmetric Information and Competitive Equilibrium

Focus on case where (a) $r(\theta) \leq \theta$ and (b) $r'(\theta) \geq 0$

Firms and workers are price takers. We assume that those workers who are indifferent between working at a firm and home employment chose the former

Asymmetric information implies that the wage rate must be independent of the workers' type. There is a single wage rate in equilibrium

A competitive equilibrium is characterized by a wage w^* and a sorting rule $\Theta \subset [\underline{\theta}, \overline{\theta}]$ such that worker $\theta \in \Theta$ is employed in a firm and worker $\theta \notin \Theta$ is self-employed

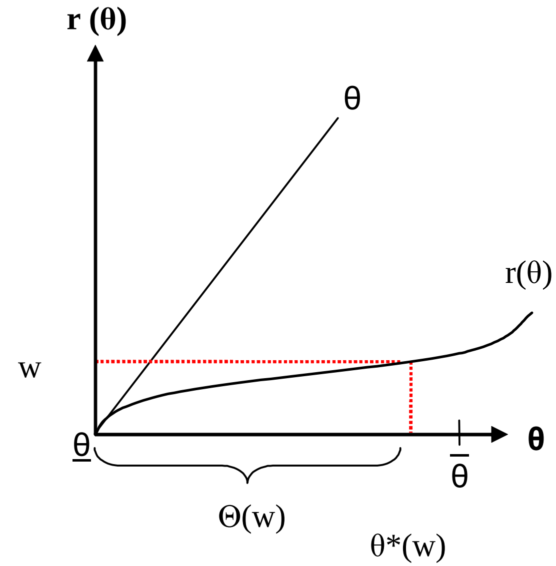
Supply of labor: Worker occupational choice (binary optimization problem) imply that $\Theta(w) = \{\theta \ \ s.t. \ r(\theta) \leq w\}$

Demand for labor: Firms demand for labor depend on their expectations regarding the type of workers who apply to work in firms. Let μ represent the expected productivity of a worker who is not self employed (applies to a job and accept an equilibrium offer). Each firm demands zero unit of labor if $\mu < w$, any non-negative amount if $\mu = w$, and an infinite amount if $\mu > w$

Firm rational expectation: $\mu = E[\theta|\theta \in \Theta(w)]$ if positive employment ($\Theta \neq \varnothing$) and otherwise ($\Theta = \varnothing$) we will assume $\mu = E\theta$

Remark: Any equilibrium has to have positive employment, that is, $\Theta \neq \varnothing$. (Proof by contradiction)

Equilibrium in labor market: In any equilibrium with non-zero level of employment, demand for labor equals supply of labor implies that $w = E[\theta|\theta \in \Theta(w)]$



Equilibrium characterization: Any competitive equilibrium is a pair (w^*, Θ^*) such that (a) $\Theta^*(w^*) = \{\theta \ \ s.t. \ r(\theta) \leq w^*\}$ and (b) $w^* = E[\theta|\theta \in \Theta^*(w^*)]$

Existence of equilibrium

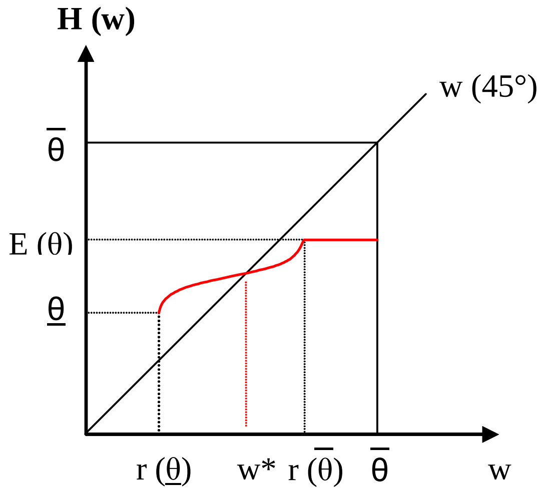
Define the function $H(w) = E[\theta|\theta \in \Theta(w)]$. Any w such that $w = H(w)$ is an equilibrium wage

Properties of the function $H(.)$:

1. $H()$ is continuous and increasing

2. $H(r(\underline{\theta})) = \underline{\theta} \geq r(\underline{\theta})$

3. $H(w) = E\theta < \overline{\theta}$ for any $w \geq r(\overline{\theta})$

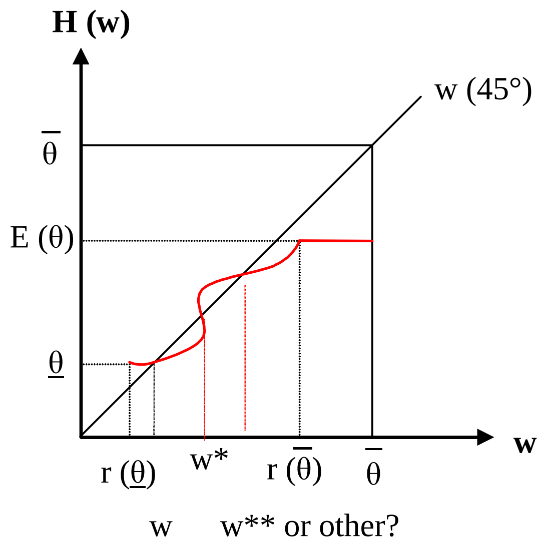


Equilibrium: w*=r (θ*(w))

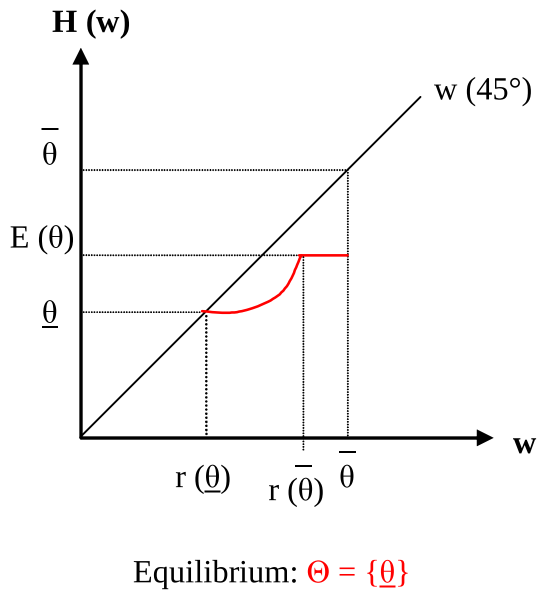
An equilibrium always exists since the function $H()$ crosses the 45 degree line at least once

Inefficiency in any equilibrium if and only if $r(\overline{\theta}) > E\theta$ (the highest type cannot be employed)

Possibility of multiple equilibria (H may cross the 45 degree line multiple times)



No-trade (with positive measure) if $H(w) < w$ for $w > r(\underline{\theta})$ (only type $\underline{\theta}$ are employed and earn $w = r(\underline{\theta}) = \underline{\theta}$)



Case 3: Game theoretic approach

Are all equilibria equally reasonable?

Consider for example an equilibrium wage w^* and assume there exist a $w > w^*$ such that $H(w) > w$

Firms want to deviate from w^*: If a firm offers wage w, workers earn more and the deviating firm earns positive profits

This type of deviation, however, is ruled out under the price taking assumption

2-stage game

Assume there are only 2 firms (w.l.o.g.)

Stage 1: Firms announce wages (w_1, w_2)

Stage 2: Workers chose between self-employment or work with firm 1 or 2

Equilibrium concept: Subgame perfect Nash equilibrium

Issue: Does the set of equilibrium change?

Let $w^{**} = Max\{w \text{ s.t. } H(w) = w\}$

Assume that $w^{**} > r(\underline{\theta})$ and that $H(w)$ crosses the 45 degree line from above at w^{**}

Stage 2: workers pick the maximum of w_1, w_2, and $r(\theta)$ (we assume that if they are indifferent they pick employment and randomize between firms)

Stage 1: Firm wage offers

Lemma 1: Firms earn zero profits

Standard Bertrand competition argument

Lemma 2: One firm must offer at least w^{**} in equilibrium

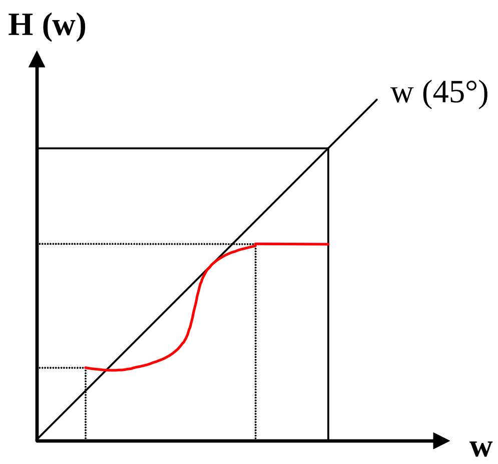
If no firm offers at least w^{**}, one of the two firms can earn positive profits by offering a wage above its competitor but lower than w^{**}. A contradiction with Lemma 1

Proposition: There exists a unique SPNE and $w_1 = w_2 = w^{**}$ and $\Theta^* = \{\theta \;\; s.t.\; r(\theta) \leq w^{**}\}$

Wage w^{**} is the only wage from which there does not exist profitable deviations

Note the role played by the assumption that $H(w)$ crosses the 45 degree line from above at w^{**}.

The proof of Lemma 2 does not hold without this assumption



Constrained Pareto optimum

Equilibrium typically fails to be Pareto optimum

A social planner who knows the workers' types could improve efficiency

What about a social planner who does not know the workers' types?

Although the general treatment of this problem requires more advance tools, it is possible to show that the planner cannot implement an allocation that dominates the Pareto dominant equilibrium

Lecture 2: Choice under Uncertainty (MGW 6)

Expected utility theorem: utility function representation of choice under uncertainty

Measures of risk aversion: how does risk taking depends on attitude toward risk?

Measures of riskiness: comparison of payoff distributions

Expected Utility Theorem

How to model choice between risky alternatives?

$C = \{1, .., N\}$ set of possible outcomes

Definition: (a) A simple lottery is $L = \{p_1, .., p_N\}$ s.t. $p_n \geqslant 0$ and $\sum_n p_n = 1$

(b) A compound lottery is $(L_1, , .., L_K; \alpha_1, , .., \alpha_K)$ where $L_k = \{p_1^k, .., p_N^k\}$ are simple lotteries, $\alpha_k \geqslant 0$, and $\sum_k \alpha_k = 1$

(c) The reduced lottery $\{p_1, .., p_N\}$ corresponding to compound lottery $(L_1, , .., L_K; \alpha_1, , .., \alpha$
is such that $p_n = \sum_k \alpha_k p_n^k$

Let $\mathcal{L}$ denote the set of simple lotteries over C.

Simplex representation: linear structure of the space of lotteries

Consequentialism: only care about final outcomes

Preferences over lotteries

$\succsim$ denotes a complete and transitive preference relation over $\mathcal{L}$

Definition: $\succsim$ is continuous if $\forall\ L, L', L'' \in \mathcal{L}$, the sets $\{\alpha \in [0,1]$ s.t. $\alpha L + (1-\alpha)L' \succeq L''\}$ and $\{\alpha \in [0,1]$ s.t. $L'' \succeq \alpha L + (1-\alpha)L'\}$ are closed

Remark: continuity rules out lexicographic preferences, for example, and implies the existence of a utility function representing $\succeq$. The intuition is the following. Let L_0 and L_1 denote respectively the worst and best lotteries. For any lottery L denote $S_u = \{\alpha \in [0,1]$ s.t. $\alpha L_0 + (1-\alpha)L_1 \succeq L\}$ and $S_L = \{\alpha \in [0,1]$ s.t. $\alpha L_0 + (1-\alpha)L_1 \preceq L\}$. The sets S_u and S_l are closed, non-empty, and $S_u \cup S_l = [0,1]$. Therefore there exists an $\alpha_L \in S_u \cup S_l$, that is $L \sim \alpha_L L_0 + (1-\alpha_L)L_1$. Can easily show uniqueness of α_L and this delivers representation of $\succeq$ with a real valued function

Definition: $\succsim$ satisfies the independence axiom if $\forall\ L, L', L'' \in \mathcal{L}$ and $\alpha \in [0,1]$, $L \succeq L' \Longleftrightarrow \alpha L + (1-\alpha)L'' \succeq \alpha L' + (1-\alpha)L''$

The independence axiom is unlike anything encountered in consumer theory and the reason is that when we consider choice under uncertainty, the consumer does not consume L or L' together with L'' but instead of L''. Independence implies that the indifference curves in the simplex representation are linear and parallel

Definition: U :$\mathcal{L} \longrightarrow \mathbb{R}$ has expected utility form if $\exists(u_1, .., u_N)$ s.t. $\forall L = \{p_1, .., p_N\} \in \mathcal{L}$, $U(L) = \sum_n u_n p_n$

$U(.)$ is the von Neumann Morgenstern (VNM) utility function while u_n is the Bernoulli utility function

Expected utility is linear over lotteries

Proposition: $\succsim$ over $\mathcal{L}$ is complete and transitive and satisfies the continuous and independence axioms. Then $\succsim$ can be represented with an expected utility form. $\exists(u_1, .., u_N)$ s.t. $\forall L, L' \in \mathcal{L}$, $L \succeq L'$ iff $\sum_n u_n p_n \geq \sum_n u_n p'_n$

The intuition is that the continuity axiom delivers representation (existence of a real valued function) and the independence axiom delivers linearity (new feature of choice under uncertainty)

Proposition: Assume the VNM function U represents $\succsim$. Then U' represents $\succsim$ if and only iff there exists for γ and $\beta > 0$ such that $U'(L) = \beta U(L) + \gamma$, $\forall L \in \mathcal{L}$

Example

Suppose $C = \{10, 4, -2\}$

For any lottery $L = \{p_1, p_2, p_3\}$ the proof of the representation proposition shows that there exists a number x such that $L \sim \{x, 0, 1-x\}$

Define $u(n)$ to be equal to x such that the individual is indifferent with a lottery that gives outcome n for sure and lottery $\{x, 0, 1-x\}$

We have $u(10) = 1$ and $u(-2) = 0$. Assume that $\{0, 1, 0\} \sim \{0.6, 0, 0.4\}$. Therefore, $u(4) = 0.6$

The individual prefers to avoid risk since $(0.6)(10) + (0.4)(-2) = 5.2 > 4$

With this representation, we can now rank any pair of lotteries. For example $U((0.8, 0.2, 0)) = 0.92 < 0.918 = U((0.9, 0.03, 0.07))$

Discussion:

Analytical convenience: simple way to capture choice under uncertainty

Normative implications: how to act in the presence of complex risky choice

The axioms are violated by some choice (Allais Paradox, Machina Paradox)

Allais Paradox

$C = (2.5\ million, 0.5\ million, 0)$.

$L_1 = (0, 1, 0) \succ L_1' = (0.1, 0.89, 0.01)$ if certainty of receiving half million is preferred (insurance)

$L_2 = (0, 0.11, 0.89) \prec L_2' = (0.1, 0, 0.9)$ if prefers to get two millions with a slightly smaller probability than half million (prefer gambling in the presence of risk)

This pair of choices violates the expected utility representation since the first choice says that $u_2 > 0.1u_1 + 0.89u_2 + 0.01u_3$ and adding $0.89u_3 - 0.89u_2$ on both sides gives $0.11u_2 + 0.89u_3 > 0.1u_1 + 0.9u_3$ which says $L_2 \succ L_2'$

Risk aversion

Assume now that lotteries can take continuous values on the real line ($C = \mathbb{R}$) and are represented by a distribution function $F()$

Generalization of the expected utility theorem gives $U(L) = \int u(x)dF(x)$

Definition: An individual is risk neutral if for any L, $U(L) = U\left(\int xdF(x)\right)$, risk averse if $U(L) < U\left(\int xdF(x)\right)$, and risk lover if $U(L) > U\left(\int xdF(x)\right)$

Definition: The certainty equivalent $c(F,u)$ of lottery L is such that $u(c(F,u)) = \int u(x)dF(x)$

Proposition: The following 3 properties are equivalent (i) Risk aversion, (ii) $u()$ is concave, (iii) $c(F,u) \leq \int x dF(x)$ for any F

Definition: Arrow-Pratt Coefficient of Absolute Risk Aversion (CARA) at x is $r_a(x) = -\frac{u''(x)}{u'(x)}$

Definition: u exhibits decreasing CARA (DARA) if $r_a(x)$ is decreasing in x

Definition: u_2 is a concave transformation of u_1 if there exist an increasing concave function ψ such that $u_2(x) = \psi(u_1(x))$

Proposition: The following properties are equivalent: (i) DARA, (ii) $x - c_x$ deceasing in x for any lottery ($u(c_x) = \int u(x+z)dF(z)$), (iii) for any $x_2 < x_1$, $u_2(z) = u(x_2 + z)$ is a concave transformation of $u_1(z) = u(x_1 + z)$

Application: Investment in a risky asset

An individual can invest wealth w in a riskless asset with zero return or a risky asset that gives return r_i with probability p_i such that $\sum_i p_i r_i > 0$

If the individual invest β in the risky asset her wealth is $(w-\beta)+\beta(1+r_i) = w+\beta r_i$ in state i

The individual maximizes $\sum_i p_i u(w + \beta r_i)$ with $\beta \in [0, w]$

A-Investment in risky asset

We have $\left(\frac{d}{d\beta} EU\right)_{\beta=0} = u'(w) \sum_i p_i r_i > 0$

The individual always invests a positive amount in the risky asset

B-How does risk-taking depend on wealth?

DARA implies $\frac{d}{dw}\beta > 0$

Under DARA, an individual invests more in the risky asset when she gets wealthier

B-How does risk-taking depend on risk-attitudes?

Assume individual u is more risk averse than individual v

Then u invests less in the risky asset $\beta_u < \beta_v$

Comparison of payoff distributions

Level versus dispersion

Restrict to distributions such that $F(0) = 0$, $F(x) = 1$ for $x > 0$

Definition: F first order stochastically dominates G (FOSD) if for any non-decreasing function $u()$, $\int u(x)dF(x) \geq \int u(x)dG(x)$

Proposition: F FOSD G is equivalent to $F(x) \leq G(x)$

Definition: F second order stochastically dominates G (SOSD) if for any non-decreasing concave function $u()$ with non negative support, $\int u(x)dF(x) \geq \int u(x)dG(x)$

Proposition: F FOSD G is equivalent to $\int_0^t F(x)dx \geq \int_0^t G(x)dx$ for any t

Lecture 3: Moral Hazard (MGW 14-B)

Introduction to the principal-agent paradigm

Two effort levels and two outcomes model

Two efforts and continuous outcome model

Linear-Normal-Exponential Moral Hazard Model

Holmstrom B. (1979), "Moral Hazard and Observability," Bell Journal of Economics, 10: 74-91

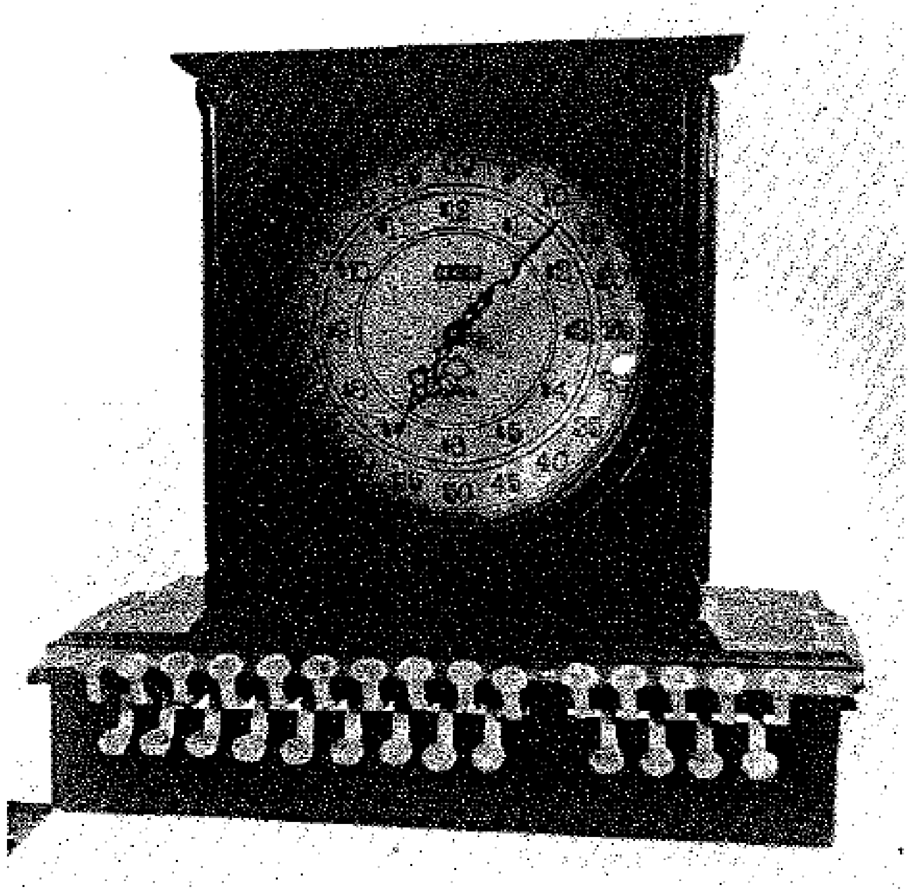


Fig. 2-2: NCR replica of original Ritty patent dial register.

Principal Agent Paradigm

Under an agency relationship, one party, the principal, hires another party, the agent, to perform some task

Agency problems occur when the agent does not have the same preferences as the principal

The agent faces a moral hazard problem because she is confronted with the dilemma of doing what's best for her or what's best for the principal. Economists put moral issues aside and assume that the agent does what's best for her

What can the principal do to address the problem? (a) Find a perfect agent: Selection or screening. (b) Get the right behavior: Incentive provision

Short-Term Incentives: Piece Rates and Bonuses

Piece rates pay workers based on the amount of output they produce regardless of the amount of time actually worked

Bonus are lump-sum payments (made in addition to other forms of compensation) usually conditional on some kind of performance evaluation

The common point to piece rate and bonus is that they provide short term performance incentives

Short-Term Incentives: How does it work?

The principal (e.g. firm) cannot observe the agent's (e.g. worker) effort or true contribution

The firm observes only an imperfect measure of effort (e.g. profits, sales...)

The performance measure is a function of the worker's effort and also some random noise

For example, a CEO's performance depends on the level of industry competition, a sales person performance depends on the product sold, a farmer's output depends on the weather...

Two effort levels and two outcomes model

The worker can either supply high (e_H) or low effort (e_L) and this decision is not observed by the principal. The cost of effort e is $g(e)$ such that $g(e_H) > g(e_L)$ and we denote the incremental cost of effort $c = g(e_H) - g(e_L)$. The worker has reservation utility $\underline{u}$

The principal observes a performance outcome that can be either high or low. p_H and p_L are the probability of high performance under high and low efforts respectively. High effort is more likely to generate high performance $p_H > p_L$. Perfect performance measure has $p_H = 1$ and $p_L = 0$

The principal makes an offer which consists in a fixed salary s plus a bonus b if performance is high

Timing of events:

1. Firm sets the compensation policy
2. Worker chooses effort level
3. Nature draws performance according to probability conditional on effort
4. Worker gets compensated

Analysis

Assume there is no performance bonus ($b = 0$)

Since $s - c < s$ the worker prefers to supply low effort

The worker will not supply effort unless $s + p_H b - c > s + p_L b$. The lowest bonus such that e_H is incentive compatible is,

$$b = \frac{c}{p_H - p_L}$$

The incremental benefit of supplying effort has to be greater than (or equal to) the incremental cost of doing so

Under perfect performance measure ($p_H = 1$, $p_L = 0$) the bonus is equal to the cost of effort $b = c$

The more noisy the performance measure ($p_H - p_L$ low) the greater the bonus $\frac{\partial b}{\partial(p_H - p_L)} < 0$

The worker earns $s + b$ with probability p_H and s with probability $1 - p_H$. The principal sets the fixed salary s such that the worker is indifferent between working under contract (s, b) and the outside option $\underline{u} = p_H(s+b) + (1 - p_H)s - g(e_H)$or

$$s = \underline{u} + g(e_H) - p_H b$$

A risk averse worker gets disutility from incentive compatible compensation since pay is variable

Two efforts and continuous outcome model

$e \in \{e_L, e_H\}$

$\pi \in [\underline{\pi}, \overline{\pi}]$, and $f(\pi|e)$ represent the conditional density function of profits given effort

$F(\pi|e_L) \geq F(\pi|e_H)$ for $\pi \in [\underline{\pi}, \overline{\pi}]$ with strict inequality for some interval Π

Agent: $u(w, e) = v(w) - g(e)$ with $v' > 0$, $v'' \leq 0$, and $g(e_H) > g(e_l) \geq 0$. Reservation utility $\underline{u}$

Principal: $E(\pi - w)$ (risk neutral)

Game: Principal makes a take-it-or-leave-it contract offer to the agent. The contract can be conditional on effort level under symmetric information, $w(\pi, e)$ but not under asymmetric information $w(\pi)$

Let $e^* = e_L$ if $\int \pi f(\pi|e_L)d\pi - v^{-1}(\underline{u} + g(e_L)) \geq \int \pi f(\pi|e_H)d\pi - v^{-1}(\underline{u} + g(e_H))$ and $e^* = e_H$ if the opposite (strict) inequality holds

Efficient outcome: set $e = e^*$ and pay the agent a fix wage

Discussion

Stylized model of an employment relationship: the assumption that the agent has a reservation utility gives all bargaining power to principal

In a market with many agents and many principals the surplus may be shared differently but this is beyond the point since our focus here is on the nature of the contract that maximizes constrained efficiency and not on the division of surplus

The principal agent framework is a starting point to model incentive problems in organizations

Market outcomes to be determined:

Wage schedule

Worker's effort

Possible scenarios:

Observable effort $w(e, \pi)$

Unobservable effort $w(\pi)$ and risk neutral agent

Unobservable effort $w(\pi)$ and risk averse agent

Economic questions of interest:

1. What is the optimal compensation contract?

2. Is the contracting outcome efficient? Efficient level of effort? Efficient risk sharing?

3. Is the equilibrium contract consistent with observed compensation policies (i.e. fixed salary, piece rate)?

Case 1: Symmetric information (Benchmark 1)

Under a take-it-or-leave-it offer the principal must satisfy the agent's participation constraint:

$$\int v(w(e,\pi))f(\pi|e)d\pi - g(e) \geq \underline{u} \quad PC$$

The principal sets the level of effort e and wage schedule $w(e,\pi)$ to maximize $\int(\pi - w(e,\pi))f(\pi|e)d\pi$ subject to PC

Solve the problem in two steps: (a) set the optimal compensation conditional on effort, (b) set the optimal level of effort

Lemma 1: Wage schedule $w(e,\pi) = v^{-1}(\underline{u} + g(e))$ and $w(e',\pi) < v^{-1}(\underline{u})$ for $e' \neq e$ implements effort level e

The principal binds the participation constraint and pays the agent only if the requested level of effort is observed

Lemma 2: The optimal level of effort is e^*

The principal selects the efficient level of effort because she captures all the surplus form effort ending up with a surplus of $E\pi - v^{-1}(\underline{u}+g(e^*))$

Conclusions: (a) Full insurance if the agent is risk averse. (b) The optimal contract implements the efficient level of effort

Case 2: Asymmetric information and risk neutral worker (Benchmark 2)

Assume the agent's utility is $u(w, e) = w - g(e)$

Consider the contract $w(\pi) = \pi - E(\pi|e^*) + \underline{u} + g(e^*)$

This contract implements the first best level of effort since (a) The agent chooses e^* because she receives all the surplus from effort. (b) The agent accepts the contract since she is indifferent with the outside option

The principal earns $E(\pi|e^*) - \underline{u} - g(e^*)$ which is identical to the symmetric information profits (under the assumption that $v(w) = w$). Since $w(\pi)$ maximizes total surplus and gives the agent exactly her outside option, this contract has to be an optimal contract

The agent faces risk (level of utility varies) while the principal faces no risk (receives a fixed payment)

In a context where the principal owns a productive asset (e.g. firm, land), this contract can be interpreted as the principal selling the project to the agent at price $E\pi - \underline{u} - g(e^*)$. The agent supplies the efficient level of effort because she fully appropriates the marginal benefit of effort

This can be interpreted as a transfer of ownership. The agent is the residual claimant to firm 's return

Remark: The contract $w(\pi)$ is not the only contract that implements the optimal profits for the principal. Any contract that varies wage enough to satisfy the incentive constraint that the agent chooses the high action when it is optimal to do so and leaves no surplus to the agent is also optimal

Case 3: Asymmetric information and risk aversion

The wage schedule under asymmetric information and risk neutrality (benchmark 2) $w(\pi)$ is no more first best because the agent faces some risk

The wage schedule under symmetric information (benchmark 1) $w(e, \pi)$ is not implementable because the wage cannot depend directly on the level of effort

If $e^* = e_H$ and the principal pays the agent $w(e_H, \pi)$ then the agent will accept the contract and select e_L since $\int v(w(e_H, \pi))f(\pi|e_H)d\pi - g(e_H) = v(w(e_H, \pi)) - g(e_H) = \underline{u} < \underline{u} + g(e_H) - g(e_L) = \int v(w(e_H, \pi))f(\pi|e_L)d\pi - g(e_L)$

The full-insurance contract is not incentive compatible!

Contract $w(\pi)$ is incentive compatible for effort e_H if

$$\int v(w(\pi))f(\pi|e_H)d\pi - g(e_H) \geq \int v(w(\pi))f(\pi|e_L)d\pi - g(e_L) \quad IC(e_H)$$

Contract $w(\pi)$ is incentive compatible for effort e_L if

$$\int v(w(\pi))f(\pi|e_L)d\pi - g(e_L) \geq \int v(w(\pi))f(\pi|e_H)d\pi - g(e_H) \quad IC(e_L)$$

The principal chooses $e \in \{e_L, e_H\}$ and $w(\pi)$ to maximize $E(\pi - w(\pi))$ subject to PC and $IC(e)$

Case 1: $e^* = e_L$

Assume for now that it is efficient for the agent to supply low effort

Lemma 3: $IC(e_L)$ always hold when the wage is constant

In fact, for any non increasing compensation rule, the agent always prefers to supply low effort

If $e^* = e_L$, the optimal contract sets $w(\pi) = v^{-1}(\underline{u} + g(e_L))$ and implements the first best outcome

Case 2: $e^* = e_H$

Assume now that $e^* = e_H$. Let λ and μ the Lagrange multipliers on PC and $IC(e_H)$

Assume the principal chooses the function $w()$ point by point. The first order condition for $w(\pi)$ is

$$\frac{1}{v'(w(\pi))} = \lambda + \mu \left(1 - \frac{f(\pi|e_L)}{f(\pi|e_H)}\right) \quad FOC(\pi)$$

Lemma 4: $e^* = e_H$ implies $\lambda > 0$ and $\mu > 0$

The optimal contract is defined by a set of $\lambda > 0$, $\mu > 0$, and $w(\pi)$ such that PC and $IC(e_H)$ bind and $FOC(\pi)$

Discussion

There is a statistical interpretation to the optimal compensation rule: Let $\widehat{w}$ such that $\frac{1}{v'(\widehat{w})} = \lambda$. For any π such that $w(\pi) > \widehat{w}$ we have $\frac{f(\pi|e_L)}{f(\pi|e_H)} < 1$ and the opposite inequality holds for any π such that $w(\pi) < \widehat{w}$. The fraction $\frac{f(\pi|e_L)}{f(\pi|e_H)}$ is the likelihood ratio. For a given outcome π the likelihood ratio is high if the chance that this outcome could have occurred is high under effort e_L and/or low under effort e_H. Stated differently, conditional on π, e_L is more likely than e_H if the likelihood ratio is high

The principal pays the agent more in those states of the world where, an outsider who would observe only the outcome realization π and would know nothing about which action e the agent has taken, would conclude that it is more likely that the agent has taken the high action. This is just an interpretation since there is no inference to be made here: given the incentives in place, the principal knows that the agent takes the high action

$w(\pi)$ is not necessarily linear as in a piece rate. More problematically, $w(\pi)$ is not necessarily increasing in π. In fact, taking full derivative of $FOC(\pi)$ with respect to π

$$\frac{d}{d\pi}w(\pi) = \mu\frac{\left(v'(w(\pi))\right)^2}{v''(w(\pi))}\frac{d}{d\pi}\left[\frac{f(\pi|e_L)}{f(\pi|e_H)}\right]$$

The wage schedule increases if and only if the likelihood ratio is decreasing in π and this is known as the monotone likelihood ratio condition. More surprisingly, the wage can decrease with profits and this happens over intervals where the likelihood ratio increases with π!

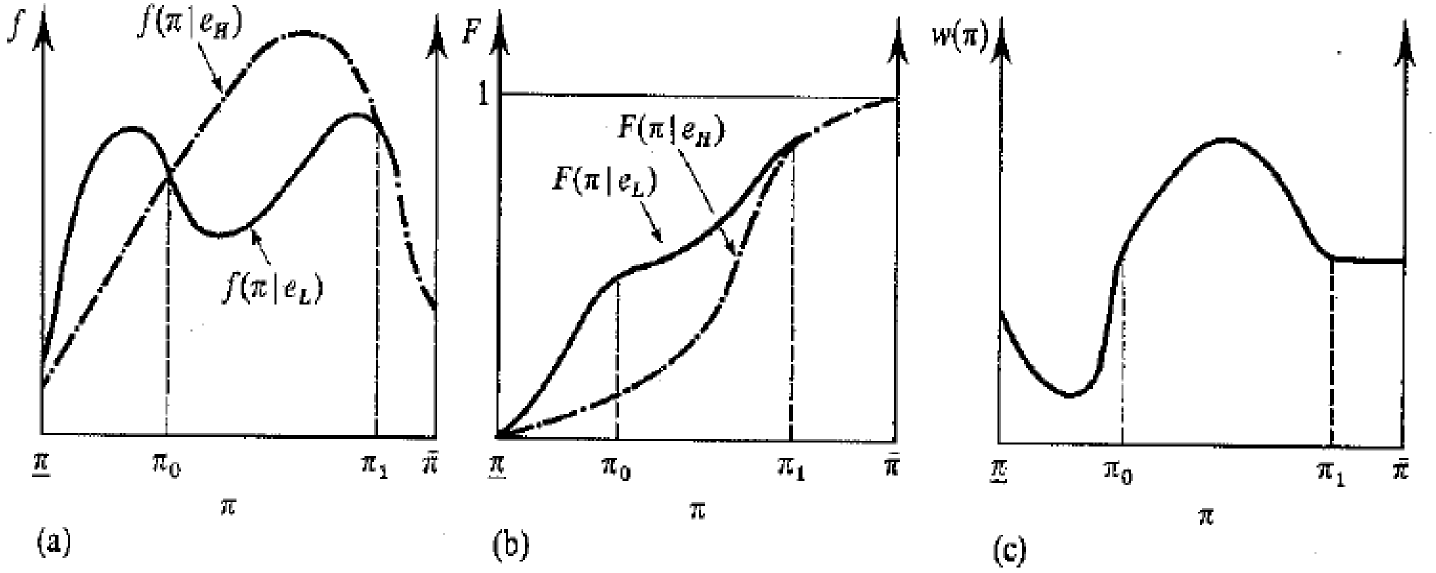


Figure 14.B.1
A violation of the monotone likelihood ratio property. (a) Densities. (b) Distribution functions. (c) Optimal wage scheme.

More costly to implement the high effort under asymmetric information since $Ew(\pi) > v^{-1}(\underline{u} + g(e_H))$. The increase in utility cost is called risk premium

Inefficiency necessarily occur when e_H is the first best level of effort because either the principal settle for e_L (when the risk premium is too high) or the agent faces some residual risk (which is an inefficient risk allocation given that the principal is risk neutral)

Assume the principal observes an additional measure y in addition to π. In the landlord/farmer application, the landlord may observe the crop and also the weather. Should the principal use this new measure in the compensation contract. To answer this question define $f(\pi, y|e)$ as the joint distribution of y and π conditional on e. One can write

$$f(\pi, y|e) = f_1(\pi|e)f_2(y|\pi, e)$$

If $f_2(y|\pi, e)$ does not depend on e then $\frac{f_2(y|\pi,e_L)}{f_2(y|\pi,e_H)}$ cancels out in the FOC and y does not enter the FOC. Therefore, the optimal wage does not depend on y. In statistical terms, one says that π is a sufficient statistic for y with respect to e when $f_2(y|\pi, e)$ does not depend on e

Linear-Normal-Exponential Moral Hazard Model

Basic Setup (Holmstrom and Milgrom 1991, BD 4.2)

Agent chooses effort $e \geq 0$ at cost $c(e)$ such that $c(e) = \frac{ce^2}{2}$. The agent has utility over effort e and wage w, $U(e, w) = -\exp[-r(w - c(e))]$ and reservation utility $\underline{U}$. (The coefficient of absolute risk aversion, CARA, $r = -\frac{u''}{u'}$, is constant)

The principal observes performance outcome $s = e + \epsilon_s$, where $\epsilon_s \sim N(0, \sigma_s^2)$

The principal uses linear compensation contracts (Holmström and Milgrom 1987), i.e. $w = \beta_0 + \beta s$, where β_0 is the agent's fixed salary and β is the "piece rate" on signal s

The Principal is risk neutral and maximizes expected profits, assumed to be equal to the effort expensed by the agent, minus compensation, $E(e - w)$

Lemma: If variable x is normal, $x \sim N(\mu, \sigma^2)$, then $E[\exp(x)] = \exp\left(\mu + \frac{\sigma^2}{2}\right)$

Issues:

1. Compute the agent's effort choice given incentive scheme (β_0, β)
2. Compute the optimal incentive scheme (β_0, β)
3. Comparative static: how does the piece rate β vary with r, c, and σ_s^2?

The Agent's Problem

$$\underset{e}{Max} E \left\{ -\exp\left[-r \left(\beta_0 + \beta(e + \epsilon_s) - \frac{ce^2}{2} \right) \right] \right\}$$

The above Lemma simplifies this expression to,

$$\underset{e}{Max} - \exp\left[-r \left(\beta_0 + \beta e - \frac{ce^2}{2} \right) + \frac{(r\beta\sigma_s)^2}{2} \right]$$

The expression in brackets (minus) is called the certainty equivalent. This corresponds to the level of riskless compensation that gives the agent the same level utility as under the risky compensation contract (β_0, β). $\frac{(r\beta\sigma_s)^2}{2}$ is the risk premium the principal has to give the agent to face risk. The agent maximizes her certainty equivalent

$$\underset{e}{Max} \left\{ r \left(\beta_0 + \beta e - \frac{ce^2}{2} \right) - \frac{(r\beta\sigma_s)^2}{2} \right\}$$

taking first order condition and solving for e gives

$$e = \frac{\beta}{c}$$

The Principal's Problem

$$\begin{aligned} & \underset{\beta_0,\beta}{Max} E\left[e - (\beta_0 + \beta s)\right] \\ s.t. E(U) &\geq \underline{U} \quad PC \\ e &= \frac{\beta}{c} \quad IC \end{aligned}$$

The principal's objective simplifies to $e-(\beta_0+\beta e)$. Assuming that the participation constraint binds, this constraint can be written as

$$-\exp\left[-r\left(\beta_0 + \beta e - \frac{ce^2}{2}\right) + \frac{(r\beta\sigma_s)^2}{2}\right] = \underline{\mathsf{U}}$$

$$-r\left(\beta_0+\beta e-\frac{ce^2}{2}\right)+\frac{(r\beta\sigma_s)^2}{2}=\log -\underline{U}$$

$$\beta_0+\beta e=\frac{ce^2}{2}+\frac{r(\beta\sigma_s)^2}{2}+k$$

where k is a constant. Plugging both constraints into the principal's maximization problem gives

$$\underset{\beta_0,\beta}{MaxE}\left[\frac{\beta}{c}-\left(\frac{c\left(\frac{\beta}{c}\right)^2}{2}+\frac{r(\beta\sigma_s)^2}{2}+k\right)\right]=\frac{\beta}{c}-\frac{\beta^2}{2c}-\frac{r(\beta\sigma_s)^2}{2}-k$$

The first order condition with respect to β gives

$$\beta=\frac{1}{1+rc\sigma_s^2}$$

Interpretation

- $0 < \beta \leq 1$

- $\beta = 1$ if $r = 0$ or $\sigma_s^2 = 0$

- $\beta < 1$ if $r > 0$ and $\sigma_s^2 > 0$

- $\frac{d\beta}{dr} < 0$, more risk averse agents face less powerful incentives

- $\frac{d\beta}{d\sigma} < 0$, less powerful incentive in more risky environments

- $\frac{d\beta}{dc} < 0$, less powerful incentive when greater disutility of effort

Lecture 4a: Monopoly Screening (Salanie 2.2)

Assume consumers are privately informer about their preferences. Can the monopolist increase profits by offering a menu of options?

Price discrimination with two types

Mussa, M., and S. Rosen (1978) "Monopoly and Product Quality," Journal of Economic Theory 18: 301-317

Applications: product line (vertical differentiation or quality), non-linear pricing (quantity price discrimination or Ramsey pricing), regulation (cost privately observed), taxation (private information on willingness to work)...

Monopoly screening

Second degree price discrimination: A monopolist faces two types of privately informed buyers

Demand: Two types of consumers $\theta \in \{\theta_1, \theta_2\}$ such that $\theta_1 < \theta_2$ consuming at most one unit of good. The proportion of consumers of type θ_1 is π. Consumers have heterogeneous preferences for quality. Utility of a type $\theta \in \{\theta_1, \theta_2\}$ for quality $q > 0$ who has to pay $t \geq 0$ is $u(q, t|\theta) = \theta q - t$. Consumers get zero utility if they do not consume

Monopolist: The monopolist produces quality q at cost $c(q)$ where c is an increasing and convex function such that $c\prime(0) = 0$ and $c\prime(\infty) = \infty$. The profits from selling one unit of quality q at price t is $t - c(q)$. We assume throughout that it is optimal to sell to both types of consumers

Take-it-or-leave-it offers

Monopolist announces price quality menu of contracts (q_i, t_i) and these are take-it-or-leave-it offers

Buyers select a contract or get the outside option of zero

As tie-break rule, we assume that indifferent buyers always prefer higher quality contracts over lower ones, and buying over outside option

Remark: The problem is sometimes presented as a principal agent problem where the monopolist is the principal who faces a representative buyer with unknown type who is the agent. One could also write the problem as a two-stage game but this would not add further insights

Market outcomes to be determined:

Contracts offered by the monopolist

Consumer choice of product

Possible scenarios:

Competition

Monopoly and full information on consumers' type (as in first, or third in our case, degree price discrimination)

Consumers are privately informed about their marginal valuation for quality and monopolist offers a single product

Private information and multiple products

Economic questions of interest:

1. Does the introduction of asymmetric information change the monopolist product offers and pricing?
2. Does the monopolist offer a single or multiple qualities?
3. Do consumers end up to consume inefficient quality products?
4. Do consumers end up with surplus? Which consumers?

Consumers' indifference curves and monopolist isoprofit curves in (t, q)

Differentiating $u(q, t|\theta) = K$, where K is a constant, $\theta dq - dt = 0$

$\frac{dt}{dq} = \theta > 0$ increasing, $\frac{d^2t}{dq^2} = 0$ linear, $\frac{d^2t}{dqd\theta} = 1 > 0$ type 1 has lower slope than type 2

Differentiating $\pi(q, t) = K$, $dt - c_q dq = 0$

$\frac{dt}{dq} = c_q > 0$ increasing, $\frac{d^2t}{dq^2} = c_{qq}$ convex

Single crossing condition: Indifference curves of the two types cross at most once since high types have higher marginal valuations for quality. In addition the high type indifference curves always cross the low type indifference curves from below

Case 1: Competition (Benchmark 1)

Competitive firms offer products of quality q_i^* such that $\theta_i = c'(q_i^*)$ at price $p_i = c(q_i^*)$ for $i = 1, 2$

$q_1^* < q_2^*$ (since $c\prime$ is increasing)

Higher type consume higher quality products

Type i consumer gets surplus $\theta_i q_i^* - c(q_i^*)$

Higher types get more surplus

Case 2: Symmetric Information (Benchmark 2)

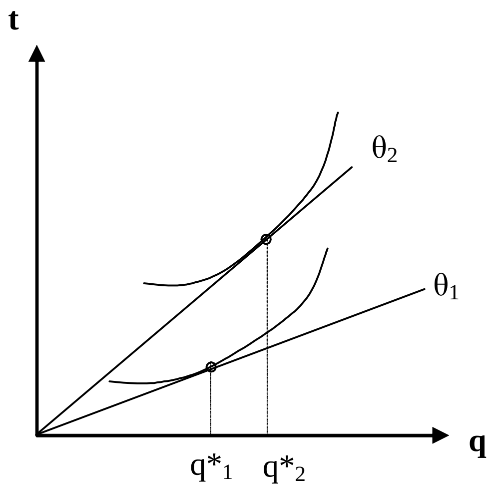
$\underset{t_i, q_i}{Max}\,(t_i - c(q_i))$

Subject to participation $\theta_i q_i - t_i \geq 0$

Participation constraint binds $\theta_i q_i = t_i$ (full rent extraction)

$c\prime(q_i^*) = \theta_i$ efficient provision of quality

Same product offering as under competition but the monopolist extracts all the consumer surplus


t
θ2
θ1
q
q*1
q*2

Case 3: Asymmetric information and single product (Benchmark 3)

Assume the monopolist offers a single product

If $\theta_1 q_1^* - c(q_1^*) > (1-\pi)(\theta_2 q_2^* - c(q_2^*))$ then the monopolist sells to all consumers a product of quality q_1^*

Otherwise the monopolist sells only to high types a product of quality q_2^*

Either high types under consume quality or low types over consume quality

Case 4: Asymmetric information and menu of contracts

Assume the monopolist offers the efficient pair of contracts. Will the profits be equal to the symmetric information profits?

Yes, if both types of consumers chose the contracts they are supposed to chose

Low types get zero utility under contract 1 and negative utility $(\theta_1 - \theta_2)q_2^* < 0$ under contract 2. Low types do not deviate

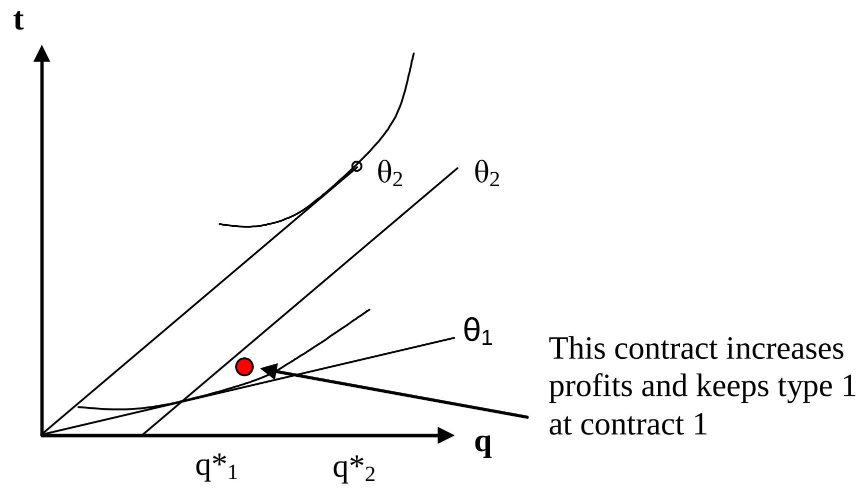
High types get zero utility under contract 2 and positive utility $(\theta_2 - \theta_1)q_1^* > 0$ under contract 1. High types will deviate!

All types pick contract 1

Monopolist earns less than under symmetric information!

Monopolist could do better by changing contracts (e.g. give a better deal to high types).

What is the optimal combination of contracts?



t
θ2
θ2
θ1
q
q*1
q*2
This contract increases profits and keeps type 1 at contract 1

Monopoly screening problem

Monopolist offers two contracts (t_1, q_1) and (t_2, q_2) to maximize profits

$$\pi(t_1 - c(q_1)) + (1 - \pi)(t_2 - c(q_2))$$

subject to the constraints that all consumers participate $\theta_i q_i - t_i \geq 0$ (PC_i)

and that consumers select the contract that is designed for them

$\theta_1 q_1 - t_1 \geq \theta_1 q_2 - t_2$ (IC_1)

$\theta_2 q_2 - t_2 \geq \theta_2 q_1 - t_1$ (IC_2)

(IC_1) says that low types (weakly) prefer contract (t_1, q_1) over contract (t_2, q_2) while (IC_2) says that the opposite holds for high types

These two new constraints are known as incentive compatibility constraints or truth-telling constraints and nests within the monopoly optimization problem the consumer decision problem

The participation constraint is also known as the individual rationality constraint

Remark 1: We could consider more complex offers or 'mechanisms' where the monopolist asks the agent to send more complex messages and makes offers conditional on messages sent by the agent. One can show, however, that we can restrict to the above problem without loss of generality. This result is known as the revelation principle

Remark 2: The optimization problem has 4 constraints. We know that both PC_i cannot bind (otherwise, $0 = \theta_2 q_2 - t_2 \geq \theta_2 q_1 - t_1 > \theta_1 q_1 - t_1 = 0$ where the first inequality holds by IC_2. A contradiction). One could assume that some constraints bind and others don't (make a guess), solve for the optimal solution (t_1, q_1) and (t_2, q_2), and then check if the guess on the status of the constraints was actually correct. Could work but long process with 4 constraints (two types) and the number of constraints is of the order of the square of the number of types

Trick: The structure of the problem may imply that some constraints necessarily bind and others don't

Analysis of incentive constraints

We focus for the moment on the incentive constraints IC and PC

Lemma 1: $PC_1 + IC_2$ imply PC_2

The high type necessarily gets some surplus so s/he is always willing to participate

Lemma 2: $q_2 \geq q_1$

High types have to consume (weakly) higher quality than low types

Lemma 3: $q_2 \geq q_1$ and IC_2 binding imply IC_1

Low types do not want to deviate as long as high types are indifferent between the two contracts and quality is increasing

To conclude, IC_1 and IC_2 binding is equivalent to $q_2 \geq q_1$ and IC_2 binding

Remark: The three lemmas follow from the incentive constraints alone. These lemmas would also hold for a social planner trying to maximize consumer surplus subject to a self-financing constraint as in Ramsey Pricing

Monopoly maximization

Lemma 4: PC_1 binds

The monopolist does not have to give any surplus to the low type

Lemma 5: IC_2 binds

If IC_2 does not bind, it means that the monopolist is leaving too much surplus to the high type

Since IC_2 binds, Lemma 3 implies that we can simplify the IC constraints as $\theta_2 q_2 - t_2 = \theta_2 q_1 - t_1$, and $q_2 \geq q_1$

Plugging the value of t_1 from PC_1 and t_2 from IC_2 in the objective function, we can rewrite the monopoly profits as $\pi(\theta_1 q_1 - c(q_1)) - (1-\pi)(\theta_2 - \theta_1)q_1 + (1-\pi)(\theta_2 q_2 - c(q_2))$

The only constraint left is $q_2 \geq q_1$ which we ignore for the moment keeping in mind that we will have to check in the end that it holds

Lemma 6: $q_2 = q_2^*$ and q_1 is such that $c\prime(q_1) = \theta_1 - \frac{1-\pi}{\pi}(\theta_2 - \theta_1)$

We have $q_1 < q_2^*$ since $c\prime(q_1) < \theta_1 < \theta_2 = c\prime(q_2^*)$

The monopolist can always gain by maximizing gains from trade with the high type so $q_2 = q_2^*$

On the other hand, there is a new cost from increasing the low product's quality. The profits from the low type can be decomposed as

$$\underbrace{\pi(\theta_1 q_1 - c(q_1))}_{\substack{\text{Social surplus from} \\ \text{increasing type 1 quality}}} \quad - \underbrace{(1-\pi)(\theta_2 - \theta_1)q_1}_{\substack{\text{Informational rent} \\ \text{given to type 2}}}$$

Since $c\prime(q_1) < \theta_1$, $q_1 < q_1^*$ and the low type quality is distorted downward (under supply quality)

Summary

High types get the efficient quality, q_2^*, and rent $(\theta_2 - \theta_1)q_1$

Low types get inefficiently low quality and no rent

High types are indifferent between both contracts and low types strictly prefer their contracts

More generally with multiple types $i = 1..I$, there is no distortion only at the top or highest type ($\theta_I = c'(q_I^*)$), zero rent only at the bottom or lowest type (PC_1 binds), rent increases with types, and all types are indifferent between their contract and the contract of the next type below ($IC_{i,i-1}$ specifying that type i is indifferent between contract i and contract $i-1$ binds)

Lecture 4b: Monopoly Screening: Continuous type case

Maskin, E., and J. Riley (1984), "Monopoly with Incomplete Information," Rand Journal of Economics 15(2): 171-196

Mussa, M., and S. Rosen (1978) "Monopoly and Product Quality," Journal of Economic Theory 18: 301-317

Salanie 2-3

Non-Linear Pricing: Parametric demand approach (Tirole/Salanie)

A monopolist faces a population of consumers indexed by $\theta \in [\underline{\theta}, \overline{\theta}]$ with utility $u(q, \theta) - p$. The distribution of θ is $F(\theta)$ with density $f(\theta)$. Consumers have reservation utility $\underline{u} = 0$

The variable q could be interpreted as quality or quantity

The cost of producing q is $c(q)$

We assume single crossing $u_{q\theta}(q, \theta) \geq 0$ in addition to the standard assumptions $u_q > 0$, $u_\theta > 0$, and $u_{qq} < 0$

The monopolist offers the menu of contracts $((p(\theta), q(\theta))$, where $p(\theta)$ is the price paid by type θ for quantity (or quality) $q(\theta)$ in equilibrium

The monopoly problem is

$$\underset{p(\theta),q(\theta)}{Max} \int_{\underline{\theta}}^{\overline{\theta}} [p(\theta) - c(q(\theta))]\, dF(\theta)$$

$$s.t. \;\; u(q(\theta), \theta) - p(\theta) \geq 0 \;\; \forall\theta \text{ PC}$$

$$q(\theta) \in ArgMax_{\widetilde{\theta}}\{u(q(\widetilde{\theta}), \theta) - p(\widetilde{\theta})\} \;\; \forall\theta \text{ ICC}$$

Define $v(\theta)$ as the utility of type θ under her best contract

$$v(\theta) = Max_{\widetilde{\theta}}\{u(q(\widetilde{\theta}), \theta) - p(\widetilde{\theta})\} = u(q(\theta), \theta) - p(\theta)$$

Remark: We loosely use the variables $p(\theta)$ and $q(\theta)$ to refer to both the choice variables in the objective function and to the optimal pricing policy in $v(\theta)$

Lemma 1: ICC is equivalent to $v_\theta(\theta) = u_\theta(q(\theta), \theta)$ and $q_\theta(\theta) \geq 0$ for all θ

The FOC and SOC to the agent's optimization problem can be simplified by the FOC and a monotonicity condition. The rent given to the agent increases according to

$$v_\theta(\theta) = u_\theta(q(\theta), \theta) \geq 0$$

This implies that we can rewrite $v(\theta)$ as $v(\theta) = \int_{\underline{\theta}}^{\theta} u_\theta(q(\widetilde{\theta}), \widetilde{\theta}) d\widetilde{\theta} + v(\underline{\theta})$

Lemma 2: Assume ICC hold. PC is equivalent to $v(\underline{\theta}) \geq 0$

We can now restate the monopoly problem as

$$\underset{q(\theta), v(\theta)}{Max} \int_{\underline{\theta}}^{\overline{\theta}} [u(q(\theta), \theta) - v(\theta) - c(q(\theta))] \, dF(\theta)$$

$$s.t. \;\; v(\underline{\theta}) \geq 0 \;\; \text{PC}$$

$$v_\theta(\theta) = u_\theta(q(\theta), \theta) \text{ and } q_\theta(\theta) \geq 0 \;\; \text{ICC}$$

Lemma 3: $v(\underline{\theta}) = 0$

The lowest type gets no rent since $u(q(\underline{\theta}), \underline{\theta}) = p(\underline{\theta})$

The second term in the objective function is $-\int_{\underline{\theta}}^{\overline{\theta}} v(\theta) dF(\theta) = \int_{\underline{\theta}}^{\overline{\theta}} v(\theta) d(1 - F(\theta))$ and after integration by part

$$\begin{aligned} -\int_{\underline{\theta}}^{\overline{\theta}} v(\theta) dF(\theta) &= -\int_{\underline{\theta}}^{\overline{\theta}} v_\theta(\theta)(1 - F(\theta)) d\theta + \underbrace{\left[\int_{\underline{\theta}}^{\theta} v_\theta(\theta)(1 - F(\theta))\right]_{\underline{\theta}}^{\overline{\theta}}}_{=0} \\ &= -\int_{\underline{\theta}}^{\overline{\theta}} u_\theta(q(\theta), \theta) \frac{1 - F(\theta)}{f(\theta)} dF(\theta) \end{aligned}$$

The monopolist sets $q(\theta)$ to maximize

$$\underset{q(\theta)}{Max} \int_{\underline{\theta}}^{\overline{\theta}} \left(u(q(\theta),\theta) - u_\theta(q(\theta),\theta)\frac{1-F(\theta)}{f(\theta)} - c(q(\theta)) \right) dF(\theta)$$
$$s.t. \;\; q_\theta(\theta) \geq 0$$

Define the argument in the integral as $\Lambda(q(\theta),\theta)$ which is also known as the virtual profit

Assuming the constraint holds, the optimal $q(\theta)$ is such that $\Lambda_q(q(\theta),\theta) = 0$,

$$u_q(q(\theta),\theta) - \frac{1-F(\theta)}{f(\theta)} u_{q\theta}(q(\theta),\theta) - c_q(q(\theta)) = 0$$

The constraint $q_\theta(\theta) \geq 0$ holds as long as $\Lambda_{q\theta} \geq 0$ since $q_\theta = -\frac{\Lambda_{q\theta}}{\Lambda_{qq}}$

Interpretation: Equalize marginal profit from selling one more unit to θ with the marginal cost of informational rent that has to be granted to all higher types

$$f(u_q - c_q) = (1 - F)u_{q\theta}$$

Application: Non linear pricing

Consumer $\theta \in [\underline{\theta}, \overline{\theta}]$ has utility $U(q,\theta) = q - \frac{1}{2\theta}q^2$ if she consumes q units. The distribution of θ is uniform over $[\underline{\theta}, \overline{\theta}]$. The cost of producing q is $c(q) = cq$ and $c \leq 1$. Consumers have reservation utility $\underline{u} = 0$. The first best consumption quantity is

$$q^{FB}(\theta) = (1-c)\theta$$

The monopoly first order condition, $u_q(q(\theta),\theta) = u_{\theta q}(q(\theta),\theta)\frac{1-F(\theta)}{f(\theta)} + c_q(q(\theta))$ implies

$$q^M(\theta) = \frac{\theta^2(1-c)}{\overline{\theta}}$$

This is the solution to the monopoly problem because $q(\theta)$ is increasing in θ. The monopolist under supplies quantity for all types but $\overline{\theta}$ since, $q^{FB}(\theta) - q^M(\theta) =$

$\theta(1-c)\left(1-\frac{\theta}{\overline{\theta}}\right) \geq 0$. The price of the q^{th} unit under the optimal non-linear price is

$$p(\theta) = u(q(\theta), \theta) - v(\theta)$$

But $v(\theta) = \int_0^\theta u_\theta(q(\widetilde{\theta}), \widetilde{\theta})d\widetilde{\theta} = \frac{1}{6}\left(\frac{1-c}{\overline{\theta}}\right)^{1/2} q(\theta)^{3/2}$. So $p(\theta) = q(\theta) - \frac{2}{3}\left(\frac{1-c}{\overline{\theta}}\right)^{1/2} q(\theta)^{3/2}$ and the price of q units is

$$\widetilde{p}(q) = q - \frac{2}{3}\left(\frac{1-c}{\overline{\theta}}\right)^{1/2} q^{3/2}$$

The price of the qth unit is $\frac{d\widetilde{p}}{dq} = 1 - \left(\frac{1-c}{\overline{\theta}}q\right)^{1/2}$

Application: Quality price discrimination

Consumer $\theta \in [0,1]$ has utility $u(q,\theta) = (1+\theta)q$ with θ uniformly distributed on $[0,1]$, $\underline{u} = 0$, and $c(q) = \frac{1}{2}q^2$

$$q^{FB}(\theta) = 1 + \theta$$

Monopoly $u_q(q(\theta),\theta) = u_{\theta q}(q(\theta),\theta)\frac{1-F(\theta)}{f(\theta)} + c_q(q(\theta))$ implies

$$q^M(\theta) = 2\theta$$

Rent $v(\theta) = \int_0^\theta u_\theta(q(\widetilde{\theta}),\widetilde{\theta})d\widetilde{\theta} = \theta^2$, $p(\theta) = u(q(\theta),\theta) - v(\theta) = \theta(\theta+2)$, and the price of good of quality q is

$$P(q) = p(\theta(q)) = p\left(\frac{q}{2}\right) = q\left(\frac{q}{4} + 1\right)$$

No distortion at the top ($q^M(1) = q^{FB}(1) = 2$), zero surplus at the bottom, and rent increases while distortion decrease with θ

Lecture 5: Competitive Screening (MGW 13-D)

No-trade (with positive measure) conclusion is unrealistic if gains-from-trade are large

Market players should come up with schemes to capture these gains from trade (evolutionary/Darwinist view that only the most efficient institutions survive)

Screening and signalling models: Intuition is that high types could reveal their types by engaging in an activity that is less costly for them than for low types

Example: Low risk drivers may find it less costly to accept deductible than high risk drivers

Rothchild and Stiglitz, "Equilibrium in Competitive Insurance Markets: An Essay on the Economics of Imperfect information," QJE, (November 1976):629-650

Competitive Screening: Application to a labor market (MGW 13-D)

Labor market application: Two firms hire two types of workers who are privately informed about their productivity

Firms: Two risk neutral firms transform labor into output using CRS technologies. Assume that output price is one (partial equilibrium analysis). Firms can assign a worker to tasks of different level of difficulty $t \geq 0$. Firm profits from a worker of productivity θ who is assigned to task t is $\theta - w$ where w is the wage

Workers: There is a unit continuum of workers who can be of two types, θ_L and θ_H. A worker of type $\theta \in \{\theta_L, \theta_H\}$ produces θ output. The fraction of workers of type θ_H is λ. Workers have zero outside option (no type dependent home production)

The utility of worker of type θ who receives wage w in task t is $u(w, t|\theta) = w - c(t, \theta)$ where $c(0, \theta) = 0$, $c_t(t, \theta) > 0$, $c_{tt}(t, \theta) > 0$, $c_\theta(t, \theta) < 0$, and $c_{t\theta}(t, \theta) < 0$

Remark: We have assumed that more difficult tasks do not increase a worker's output. This is to focus on the possibility that difficult tasks may be used only for informational reasons

Two-stage game:

Stage1: Firms announce wage-task contracts (w, t)

Stage 2: Workers chose a contract or get the outside option of zero

As tie-break rule, we assume that indifferent workers always prefer low task contracts over high ones, and employment over outside option. If two firms offer the same contract workers randomize with equal probability

Focus on pure strategy subgame perfect Nash equilibria

Market outcomes to be determined:

Equilibrium wage-task contracts

Sorting of workers between contracts

Possible scenarios:

Full information on workers' type (productivity)

Workers are privately informed about their productivity

Economic questions of interest:

1. Does the introduction of asymmetric information matter for the allocation of task difficulty?
2. Does it change market outcomes?
3. Does it change the efficiency properties of the equilibrium?
4. Do workers work under the same contract (pooling equilibrium) or under different contracts (separating equilibrium)?

Workers' indifference curves in $(\mathbf{t}, \mathbf{w})$

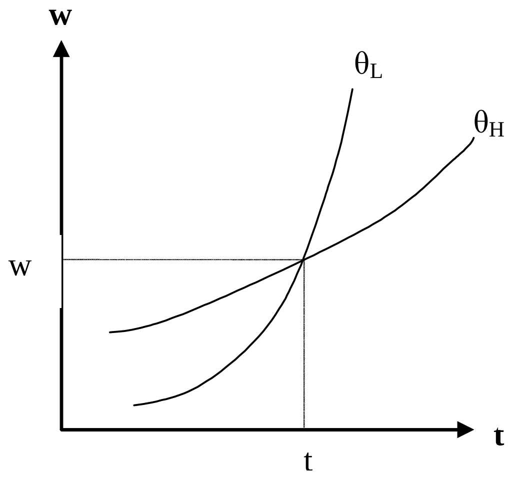
Proofs make extensive use of graphics in the (w, t) quadrant

Differentiating $u(w, t|\theta) = K$, where K is a constant, $dw - c_t dt = 0$

$\frac{dw}{dt} = c_t > 0$ increasing

$\frac{d^2w}{dt^2} = c_{tt} > 0$ convex

$\frac{d^2w}{dtd\theta} = c_{t\theta} < 0$ type L has higher slope than type H



SINGLE CROSSING PROPERTY: Θ_H crosses θ_L only once from above

Single crossing condition: Indifference curves of the two types cross at most once since high types have lower marginal disutility for task difficulty. In addition the high type indifference curves always cross the low type indifference curves from above

Remark: the single crossing condition is central concept to sort privately informed agents (it also plays a role in the model of monopoly screening and auctions)

Case 1: Symmetric Information (Benchmark)

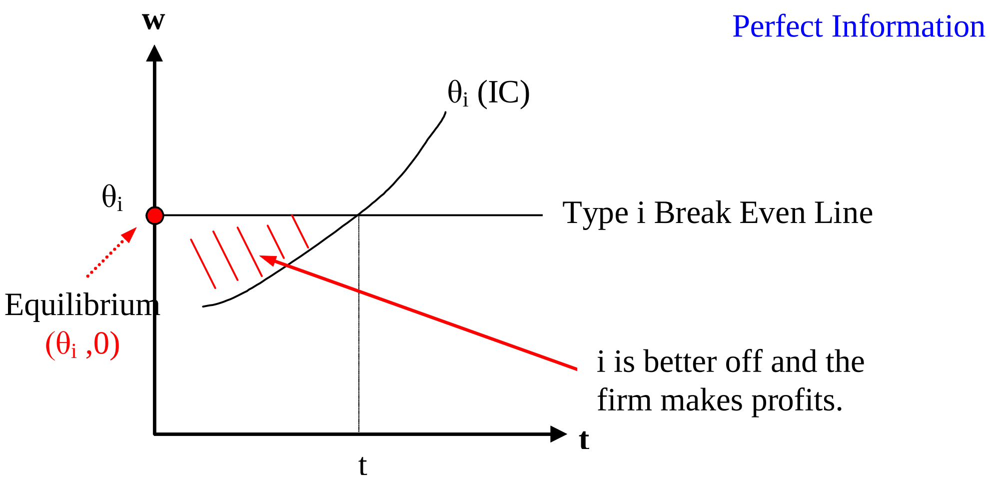
- The wage can be function of the worker's type $w^*(\theta_i, t_i^*)$

- $(w_i^*, t_i^*) = (\theta_i, 0)$ for $i = L, H$

- Firms earn zero profits

- Worker work in easiest task since no productivity gain from working in more difficult tasks (no efficiency role for task difficulty)

- Equilibrium is Pareto efficient



- Standard conclusions from competitive equilibrium analysis

Case 2: Asymmetric Information

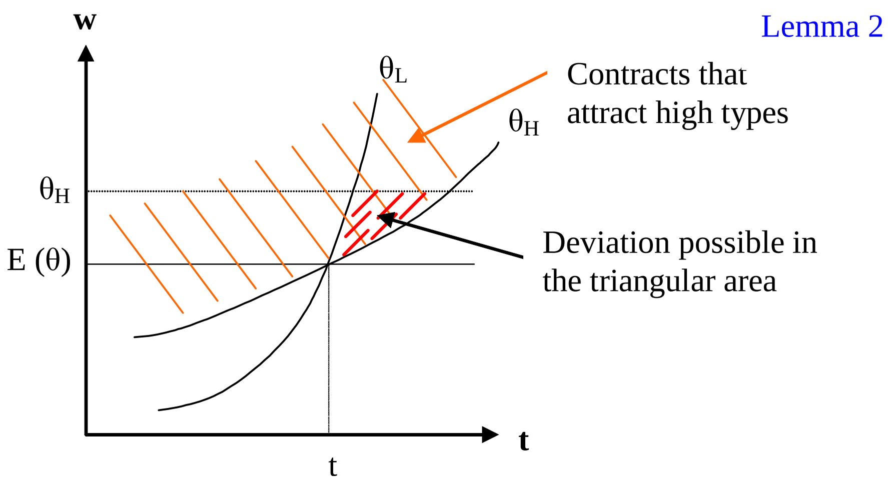
Note that the symmetric information equilibrium cannot be implemented because firms do not observe the worker's type. If a firm offers the two equilibrium contracts offered under symmetric information, it will earn loss $E\theta - \theta_H$ since both workers will accept the high wage contract

Separating equilibrium: each type of worker accepts a different contract

Pooling equilibrium: both types of worker accept the same contract

Lemma 1: In any equilibrium, firms earn zero overall profits

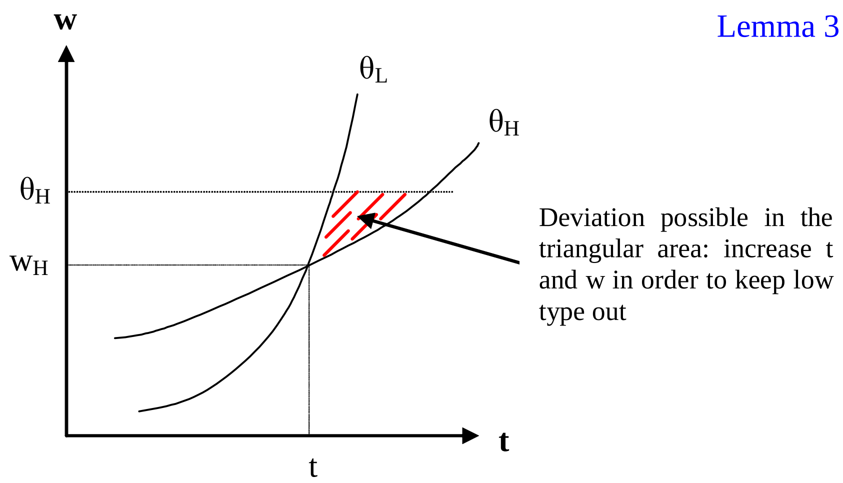
Competition implies that firms will bid profits down to zero under a Bertrand type of argument



Notice that any contract below the θ_L line will make the low types reject it, which we want to make sure.

Lemma 2: No pooling equilibrium exists

Under pooling and zero profits, high types cross subsidize low types. Therefore, a firm can benefit by creaming-off the high types and this can be achieved by increasing

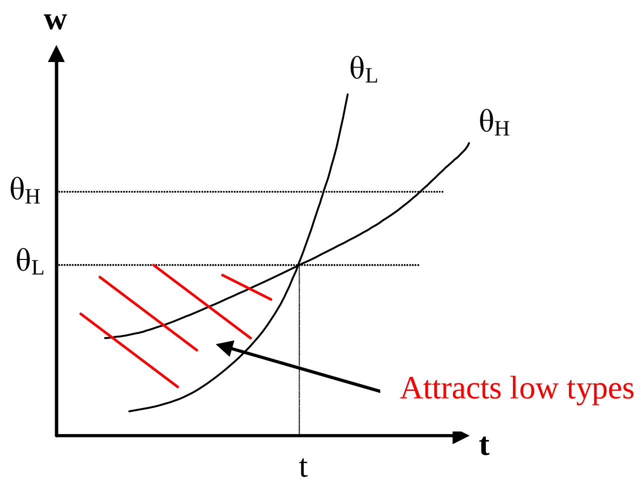


Lemma 3

the task difficultly and paying a premium that is not attractive to low types which is always possible under single crossing

Lemma 3: In any separating equilibrium, firms earn zero profits on all contracts

Lemma 4



In a separating equilibrium, firms can give targeted contracts to each type of workers. This drives the profits on each type to zero

Lemma 4: In any separating equilibrium, low types receive the competitive contract $(\theta_L, 0)$

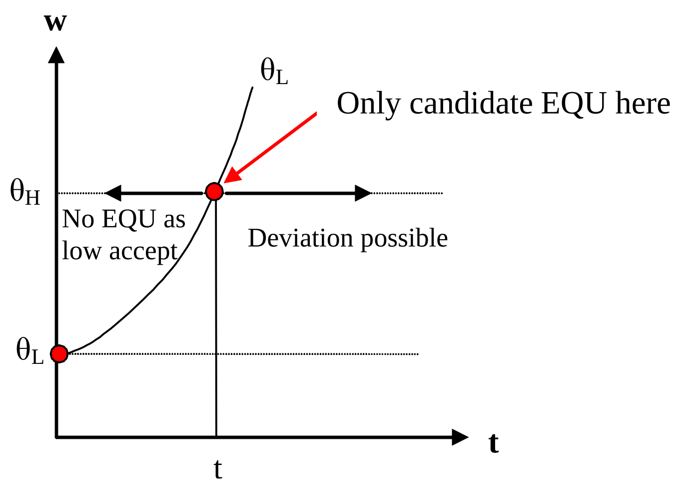
Firms can pay the low types their productivity and assign them to tasks with zero difficulty because they break even if only low types accept and they can only benefit if high types accept as well. Not possible to do the same with high types, however, since low types prefer to work at the high type competitive contract $(\theta_H, 0)$ and firm would earn negative profits $(E\theta - \theta_H)$

Lemma 5: In any separating equilibrium, the high type contract must be such that $\theta_H - c(t_H, \theta_L) = \theta_L - c(0, \theta_L)$

Firms pay high types more but also increase the task difficulty to make sure that low types do not want to pretend to be high types. This is known as the separating or indifference condition. Low types have to be indifferent between both contracts

Proposition: Any SPNE is a separating equilibrium such that low types accept $(\theta_L, 0)$ and high types accept (θ_H, t_H)

Lemma 5


w
θL
Only candidate EQU here
θH
No EQU as
low accept
Deviation possible
θL
t
t

Remark: Type θ_L wants to mimic type θ_H while the reverse does not hold. As a result the contract of type θ_H is distorted while the contract of θ_L isn't!

Conclusions:

We considered a richer setup than the simplest adverse selection model by introducing the possibility to make payoff-relevant decisions (task allocation)

Only separating equilibria can exist and even a separating equilibrium does not always exists

- For example, assume there are very few low types and the separating condition requires high types to endure difficult tasks (under separation). Since there are very few low types $\theta_H - E\theta$ is small. A firm could offer a wage of $E\theta - \epsilon$ and task difficulty $t = 0$. All workers are better off under this contract (high types take a slight pay break $(\theta_H - E\theta + \epsilon)$ but get to work on much easier task) and the firm earns positive profits. Therefore, there may exist pooling deviations

- Similarly, there may exist separating deviations where firms reduce task difficulty as well as wages for high types and use some of the surplus to increase the wage of the low types making sure that low types still do not want to pretend they are high types

Distortions only for type θ_H, upward constraint bind, and all consumers get some surplus

Existence of equilibrium is sensitive to small changes in preferences!

Rothchild Stiglitz (BD 13.1.2)

Similar to MGW 13-D but application to insurance. Policy holders are privately informed about their risk. There is competition in insurance provision. Contracts are exclusive

Consumers may have an accident in which case their wealth w is decreased by L. Type is risk or probability to have an accident. p_i is the probability that type i has an accident with $p_1 < p_2$ and the fraction of type i in the population is α_i. There are two competing insurers

RS consider the following game. (1) In the first stage, insurers offer contracts (I_i, D_i) where I_i is the insurance premium and D_i is the deductible. Individual i utility from contract i' is $p_i u(w - D_{i'} - I_{i'}) + (1 - p_i) u(w - I_{i'})$. (2) In stage 2, individuals choose the best possible contract or no insurance (outside option). (3) Finally, nature determine accident outcomes and contracts are executed

Remark: Single crossing condition (the indifference curves of two different types do not cross more than once) holds in insurance markets. High risk types are willing to pay more for a marginal reduction in risk (i.e. higher coverage or lower deductible). It should be possible to separate high and low risks: offer high risks greater coverage at a higher premium so that low risks do not want to imitate high risks

Conclusion from RS (The analysis follows MGW13-D)

No pooling equilibrium

Complete insurance to high risk, $D_2 = 0$, and zero profit implies $I_2 = p_2 L$

Deductible to low risk that maximizes low risk 's utility $p_1 u(w - D_1 - I_1) + (1 - p_1)u(w-I_1)$ subject to separation $u(w-I_2) \geq p_2 u(w-D_1-I_1)+(1-p_2)u(w-I_1)$ and non-negative profits $I_1 \geq p_1(L - D_1)$

IC(2,1) and non-negatif profit constraints bind. The low risk contract is given by $u(w-I_2) = p_2 u(w-(1-p_1)D_1-p_1 L)+(1-p_2)u(w-I_1)$ and $I_1 = p_1(L-D_1)$

No equilibrium with few high risks (pooling deviation)

Lecture 6: Signaling (MGW 13-C)

In the screening model (Lecture 2), firms move first and attach eligibility restrictions to more attractive contracts.

Specifically, in the labor market application, firms require that those workers who want higher wages have to endure more difficult tasks

Another possibility is that workers move first and invest in costly activities hoping to consequently receive better offers

Different class of games since we need to consider how firms interpret the signals sent by workers' investments in costly activities

In particular, we need to define rules for how firms form beliefs about productivity based on the signals they receive

Cover the original application of Spence to education in "Job Market Signaling" QJE (1973) 87: 355 - 74

The signalling idea has been applied to advertising, financial contracts...

Signaling Model

Labor market application: Two firms compete for a single worker who is privately informed about his/her productivity

Worker: A single worker privately observes her productivity $\theta \in \{\theta_L, \theta_H\}$ such that $\theta_L < \theta_H$. The probability that the worker is of type θ_H is λ. The worker either works and produce θ output or gets zero outside option (no type dependent home production). The worker can invest in education e. A worker of type θ, who is paid w, and invests e in education gets utility $u(w, e|\theta) = w - c(e, \theta)$ where $c(0, \theta) = 0$, $c_e(e, \theta) > 0$, $c_{ee}(e, \theta) > 0$, $c_\theta(e, \theta) < 0$, and $c_{e\theta}(e, \theta) < 0$

Firms: Two risk neutral firms transform labor into output using CRS technologies. Assume that output price is one (partial equilibrium analysis). Firm profits from paying w a worker of type θ who has education e is $\theta - w$

Remark: We assume that education does not increase firms' profits. This is to focus on the possibility that education may be used only for informational reasons

Timing of the game:

1. Nature sets the worker's type θ

2. The worker invests in education e

3. Firms simultaneously make wage offers $w(e)$ conditional on e

4. The worker decides which offer to accept if any

As tie-break rule, we assume that if the worker is indifferent, s/he chooses employment over outside option. If the two firms offer the same contract, workers randomize with equal probability

Remark: One can think of the model as a single worker of unknown type or of a continuum of workers who can be of two types. The exposition is easier under the single worker interpretation

This is a game of imperfect information because the firms do not observe the worker's type

Firms observe only the level of education and they must make an offer based on this observation. Firms' belief on the worker's type will depend in equilibrium on the education level observed. But the worker's choice of investment in education depends on wage offers. This is a chicken and egg problem in the sense that the rational for investment in education has to be self-fullfiling

Read about the initial treatment of the issue in the original work by Spence. Since then, these concepts have been formalized under different equilibrium concepts

Each equilibrium concept needs to specify how the firms form beliefs

The restrictions imposed on how firms form beliefs will greatly constrain the set of wage offers that can be sustained as part of an equilibrium

Equilibrium concept: Perfect Bayesian Equilibrium (PBE) in pure strategy

Let $\mu(e)$ represent the firms' common belief that the worker is of high type after observing education level e. An equilibrium is a profile of strategies $e(\theta)$, $w(e)$ and a system of belief $\mu(e)$. We define a perfect Bayesian equilibrium as a strategy profile and beliefs such that

1. The worker's investment strategy $e(\theta)$ is optimal given the firm's strategy

2. The firm's belief that the worker is of high type, $\mu(e)$, is computed according to Bayes' rule whenever possible

3. Wage offers constitute a Nash equilibrium of the simultaneous game starting after stage (3) given beliefs $\mu(e)$

Stronger concept that weak PBE since impose (a) NE in stage (3) and (b) firms share a common belief off the equilibrium path

Formally, the worker's work decision in stage (4) should be specified as part of the equilibrium definition but we omit it to keep the exposition simple

Market outcomes to be determined:

Equilibrium wage and education

Sorting of workers between work and outside option

Possible scenarios:

Full information on workers' type (productivity)

Workers cannot invest in education (signalling ban)

Workers are privately informed about their productivity and can invest in education

Economic questions of interest:

1. Existence of equilibrium?
2. Set of separating/pooling equilibria?
3. Investment in education $e(\theta)$ and wage schedule $w(e)$?
4. Efficiency properties of different equilibria?
5. Role of belief in equilibrium refinement?

Workers' indifference curves in (e, w)

Proofs make extensive use of graphics in the (e, w) quadrant

Differentiating $u(w, e|\theta) = K$, where K is a constant, $dw - c_e de = 0$

$\frac{dw}{de} = c_e > 0$ increasing

$\frac{d^2w}{de^2} = c_{ee} > 0$ convex

$\frac{d^2w}{ded\theta} = c_{t\theta} < 0$ type L has higher slope than type H (single crossing condition)

Since $c_\theta > 0$ type H indifference curves lie above type L

Case 1: Symmetric Information (Benchmark 1)

- The wage can be function of the worker's type $w^*(\theta_i, e_i^*)$
- $(w^*(\theta_i, e_i^*), e_i^*) = (\theta_i, 0)$ for $i = L, H$
- Firms earn zero profits
- All types work and no type invest in education since no productivity gain from doing so
- Equilibrium is Pareto efficient: Standard conclusion from competitive equilibrium analysis

Case 2: Equilibrium without signaling (Benchmark 2)

- Workers cannot invest in signaling (signalling ban)

- $w^* = E\theta$ and all types work is the only equilibrium

- Firms earn zero profits

- Equilibrium is Pareto efficient

Remark: No adverse selection since no outside option

Case 3: Asymmetric Information

Note that the symmetric information equilibrium cannot be implemented because firms do not observe the worker's type

If a firm offers the two equilibrium contracts from benchmark 1 (symmetric information), it will earn loss $E\theta - \theta_H$ since both types will accept the high wage contract. Therefore, these contracts cannot be part of an equilibrium

Solve the game by looking at decisions in later stages first similarly as you would do under backward induction

Stage 4: Worker accepts highest offer if non-negative

Stage 3: Expected productivity of worker who has education e is $\mu(e)\theta_H + (1 - \mu(e))\theta_L$

Lemma 1: $w(e) = \mu(e)\theta_H + (1 - \mu(e))\theta_L$ is the only Nash equilibrium in the stage 3 simultaneous game given $\mu(e)$

Simple Bertrand argument taking into account firms' beliefs

Since $\mu(e)$ has to be computed according to Bayes rule whenever possible, this implies that firms earn zero profits

Stage 2: Distinguish two types of equilibria, separating and pooling

Separating equilibrium

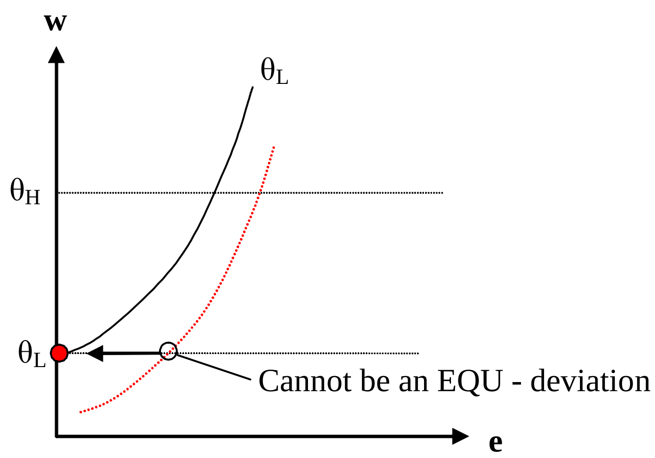
Lemma 2: In any PBE, $w^*(e^*(\theta_H)) = \theta_H$ and $w^*(e^*(\theta_L)) = \theta_L$

Conditional on education level, firms know the worker's type. Lemma 1 says that firms pay the worker his/her productivity

Lemma 3: In any PBE, $e^*(\theta_L) = 0$

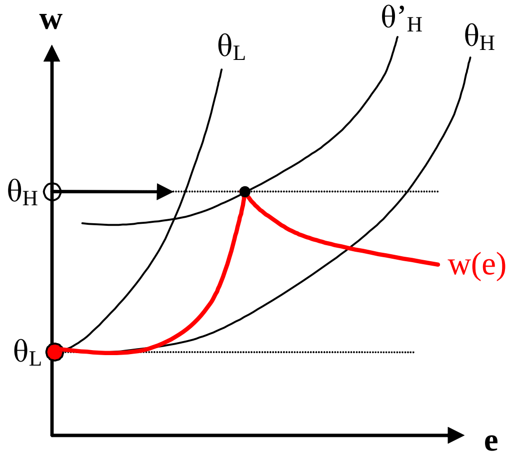
There is no productivity gain from education and all types produce at least θ_L so a firm cannot earn only non-negative profits by paying wage θ_L to a worker who has no education

Define $\widetilde{e}$ and e_1 such that $\theta_L = \theta_H - c(\widetilde{e}, \theta_L)$ and $\theta_L = \theta_H - c(e_1, \theta_H)$. $\widetilde{e}$ corresponds to the level of education that leaves θ_L indifferent between contracts



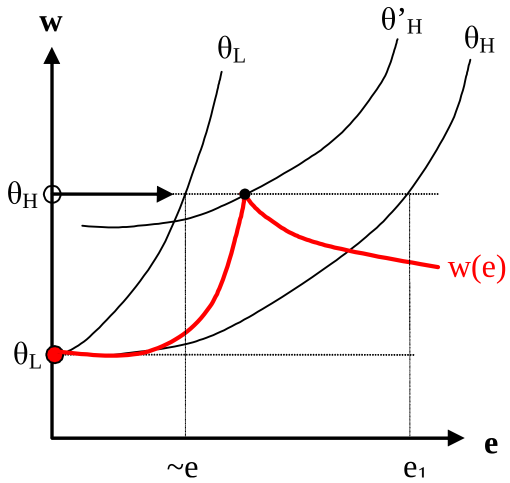
Lemma 3

Separating Equilibrium



$(\theta_H, \widetilde{e})$ and $(\theta_L, 0)$. e_1 is corresponds to the level of education that leaves θ_H indifferent between contracts (θ_H, e_1) and $(\theta_L, 0)$

Lemma 4: Any education level $e^*(\theta_H)) \in [\widetilde{e}, e_1]$ can be supported as part of a PBE



Lemma 4

Welfare implication: Recall that when education is banned, all workers are paid $E\theta$ (benchmark 2). With the introduction of education: θ_L is strictly worse off and θ_H is better off if and only if $\theta_H - c(e^*(\theta_H), \theta_H) > E\theta$. It is possible that both types are worse off with education. This is more likely to be the case if λ is close to 1

Remark: When both types are worse off with education, θ_H still does not deviate because $\theta_H - c(e^*(\theta_H), \theta_H) > \theta_L$)

Pooling equilibrium

In a pooling equilibrium firms expect the worker to produce $E\theta$

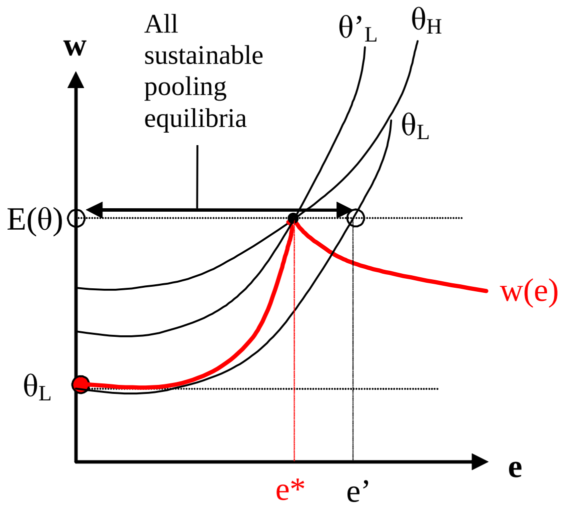
Lemma 5: In any PBE, $w^*(e^*) = E\theta$

This is a direct implication of Lemma 1 since Bayes rule imply that $\mu(e^*) = \lambda$

Define e' such that $E\theta - c(e', \theta_L) \geq \theta_L - c(0, \theta_L)$. e' corresponds to the level of education that leaves θ_L indifferent between contracts $(E\theta, e')$ and $(\theta_L, 0)$

Lemma 6: Any education level $e^* \leq e'$ can be sustained as part of a PBE

Pooling equilibria are dominated by the equilibrium without signaling



Lemma 6

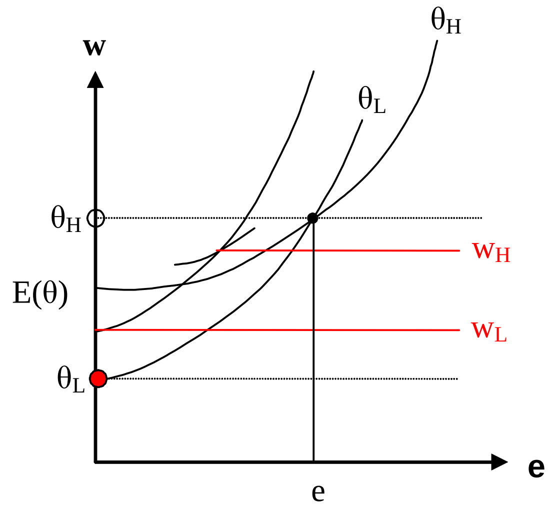
Welfare Conclusions

Signaling is inefficient under pooling

θ_H may prefer a signaling ban (pooling with no education as in benchmark 2) under separating

Even if θ_H prefers the separating outcome over pooling with no education, a market intervention mandating a fixed wage schedule $w(e)$ may Pareto dominate the separating equilibrium. This intervention reduces the level of education required from the high type, reduces as well the high type wage, and subsidizes the low type wage to make sure that the low type does not want to deviate

$w(e)$ is not part of an equilibrium under competition because firms break even only on average; they do not do so on each contract as required by Lemma 1



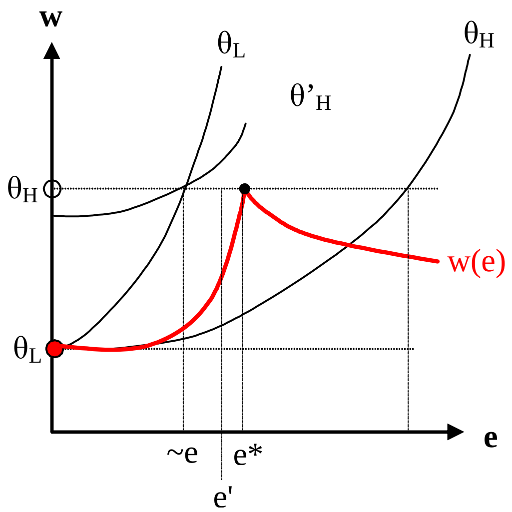
Deviation from separting equilibrium

Equilibrium refinement

Bayes rule constrains only the beliefs on the equilibrium path

Could add some restrictions on the beliefs that are allowed off the equilibrium path

Refinement example: A firm cannot believe that a type, who cannot benefit from a given deviation under any possible firm belief, could have taken that deviation. A proper formalization of this restriction on belief formation can narrow down the set of equilibrium to the Pareto dominant separating equilibrium $\widetilde{e}$



Equilibrium
Refinement

Intuitive criterion equilibrium refinement

Signal e is **equilibrium dominated** for type θ if the equilibirum payoff of θ is greater than θ's highest possible payoff from choosing e (in the simultaneous offer game that takes place between firms after the signal e is sent)

Intuitive critirion: If the information set following e is off the equilibrium path and e is equilibrium dominated for type θ but not for some type θ' then $\Pr(\theta \mid e) = 0$

The intuitive criterion implies that $\mu(e) = 1$ for $e > \widetilde{e}$. This eliminates all separating equilibria where the high type take an effort greater than $\widetilde{e}$. The intuitive criterion also rules out all pooling equilibria

Lecture 7: Strategic Information Transmission (Gibbons 4.3A)

Introduction to 'cheap talk game' literature and concept of partition equilibrium

Costless signalling in contrast to Spense's costly signalling model

Applications to literature on expert and committee decision making

Crawford and Sobel. 'Strategic Information Transmission.' Econometrica 50 (6) 1982, 1431-51

Robert Gibbons, 'A Primer in Game Theory' Chapter 4.3A

Cheap Talk Games

Exchange of information: experts are used in many situations to help making informed decisions

Applications include committee decision, tribunal case, parlementary committee, government design of policy...

Several key features characterize such situations: (a) Experts are privately informed, (b) The expert's information can improve decision making, (c) Experts may have different preferences over decision making than decision makers, (d) Difficult to use competition to get experts to reveal their information (because of monopoly reasons or impossibility to verify the information), (e) Monetary transfers are often ruled out (cannot buy the information or difficult to provide incentives conditional on outcomes)

In practice, experts provide an opinion that influences decision making. Relevant issues include:

(a) Does the expert transmit her information truthfully?

(b) How is the expert's information incorporated into the decision making process?

Model

Two players: sender (S) and receiver (R)

S privately observes the realization of a random variable, or ‘state of the world’ $m \in [0, 1]$ distributed according to F with density f

S's utility is $U^S(y, m, b)$ where $y \in \mathbb{R}$ corresponds to the action to be taken by R, m to the state of the world, and b to a preference parameter which captures how aligned the preferences of S and R are

R receives S's message $n \in N \subset \mathbb{R}$ and choose y to maximize $U^R(y, m)$ conditional on her belief about the state of nature

We leave for now the message space N undetermined. Note that n does not enter the utility functions anyway. This is why this is called costless signalling or cheap talk

Both U^S and U^R are twice continuously differentiable with a single optimum, that is, for any m and b, we have, $\forall y$, $U^i_{11} < 0$ and $\exists! y$ s.t. $U^i_1 = 0$

We assume in addition that $U^i_{12} > 0$ which implies that the best value of y increases in m for both players (sorting condition)

Uniform Quadratic Example

Assume F is uniform on $[0, 1]$

$U^R(y, m) = -(y - m)^2$

$U^S(y, m, b) = -(y - (m + b))^2$ for $b > 0$

$y^S(m, b) = m + b$ and $y^R(m) = m$

Parameter b measures the similarity of the players' preferences

Timing of events:

1. S privately observes the state of the world m
2. S sends signal $n \in N$ to R
3. R takes action y after receiving the signal
4. Payoffs $U^R(y, m)$ and $U^S(y, m, b)$

Remark: Although m necessarily precedes n in time, we assume that S 's choice of signalling rule and R's choice of action rule are chosen simultaneously. (Players cannot commit to a strategy)

For each state of the world m, we denote the preferred action of S, $y^S(m, b) = ArgMax_y U^S(y, m, b)$ and of R, $y^R(m) = ArgMax_y U^R(y, m)$. These functions are well-defined continuous and increasing in m

Equilibrium Concept: Perfect Bayesian Equilibrium (PBE)

An PBE is a triplet composed of a signalling rule, an action rule, and a belief system $\{q(n|m), y(n), p(m|n)\}$ such that

1. $\forall m \in [0,1]$, $\int_N q(n|m)dn = 1$ and $q(n'|m) > 0$ if $n' \in ArgMax_{n \in N} U^S(y(n), m, b)$
2. $\forall n$, $y(n) \in ArgMax_y \int_0^1 U^R(y,m)p(m|n)dn$
3. $p(m|n) = \frac{q(n|m)f(m)}{\int_0^1 q(n|t)f(t)dt}$

Remark 1: S may randomize but R would not randomize ($y(n)$ deterministic in (1) and (2)) because $U_{11}^R < 0$

Remark 2: For any realization of m, at most two actions can be taken in equilibrium

Analysis: Uniform Quadratic Case

There always exists a babbling PBE, $p(m|n) = f(m)$, $\forall n$

Full information is an equilibrium when $b = 0$ (preferences perfectly aligned)

Full information cannot be an equilibrum when $b > 0$

Concept of partition equilibrium: Define a N-step partition of $[0, 1]$ as $0 = a_0 < a_1 < ... < a_N = 1$ such that sender type in $[a_{i-1}, a_i]$ sends message i and type a_i is indifferent between i and $i + 1$ (arbitrage condition)

Two-Step Partition Equilibrium

Assume that $S \in [0, a_1]$ says $[0, a_1]$ while $S \in [a_1, 1]$ says $[a_1, 1]$

If the receiver believes that the sender belongs to interval (a, b), she takes action $\overline{y}(a, b) = \frac{a+b}{2}$

Therefore, R takes action $\frac{a_1}{2}$ if she receives signal $[0, a_1]$ and action $\frac{a_1+1}{2}$ if she receives signal $[a_1, 1]$

If type a_1 is indifferent between $\frac{a_1}{2}$ and $\frac{a_1+1}{2}$ then lower types strictly prefer to truthfully reveal that they belong to $[0, a_1]$ and similarly for higher types

Type a_1 is indifferent between revealing $[0, a_1]$ and $[a_1, 1]$, with the consequence that actions $\frac{a_1}{2}$ and $\frac{a_1+1}{2}$ are taken respectively, if her optimal action, $a_1 + b$, is in between these two actions

$$a_1 + b = \frac{1}{2}\left(\frac{a_1}{2} + \frac{a_1 + 1}{2}\right)$$

or

$$a_1 = \frac{1}{2} - 2b$$

and the equilibrium exists if $b < 1/4$ so that $a_1 > 0$. As expected, the equilibrium exists as long as players' preferences are not too different

N-Step Partition Equilibrium

We repeat the same argument as in the 2 step partition.

R's optimal action for step $[a_{i-1}, a_i]$ is $\frac{a_{i-1}+a_i}{2}$. Sender of type a_i is indifferent between $[a_{i-1}, a_i]$ and $[a_i, a_{i+1}]$ if her optimal action $(a_i + b)$ is in the middle of the receiver's responses to $[a_{i-1}, a_i]$ and $[a_i, a_{i+1}]$ which are respectively $\frac{a_{i-1}+a_i}{2}$ and $\frac{a_{i+1}+a_i}{2}$, that is, if

$$\frac{1}{2}\left(\frac{a_{i+1}+a_i}{2}+\frac{a_{i-1}+a_i}{2}\right)=a_i+b$$

Thus steps increase at the rate of $4b$

$$a_{i+1}-a_i=c+4b=a_i-a_{i-1}+4b$$

This second order difference equation fully characterizes all the steps given initial values $a_0 = 0$ and a_1. If the first step is of length $d = a_1 - a_0 = a_1$, then the Nth step must end at 1 which implies that

$$d + (d + 4b) + .. + (d + (N - 1)4b) = 1$$

$$Nd + N(N - 1)2b = 1$$

As long as $Nd + N(N - 1)2b < 1$, there exists a d that solves the above equation and the equilibrium exists

Define $N(b)$ as the largest integer such that an equilibrium exists $2N(b)(N(b) - 1)b \leq 1$. The highest solution to this equation is

$$1/2\left(1 + \sqrt{1 + 2/b}\right)$$

An N-step partition equilibrium exists as long as $N(b)$ is less than $1/2\left(1+\sqrt{1+2/b}\right)$. As b converges to zero, $N(b)$ increases to infinity. There is a sense in which the amount of information communicated depends on how similar the players' preferences are

Example: $b = 1/20$ implies $N(b) = 3$ and there is a 2 step equilibrium at $0, 2/5, 1$ and a 3 step equilibrium at $0, 2/15, 7/15, 1$

Although both S and R prefer the most informative equilibrium $N(b)$ from an ex-ante point of view, it is not true ex-post, after S has observed her signal. For some realizations of the signal, S may prefer not to play the 'most informative' equilibrium $N(b)$!

Analysis: General Case

Are partition equilibrium the only type of equilibrium?

Lemma 1: No information revelation (babbling, $p(m|n) = f(m)$, $\forall n$) is a PBE

Lemma 2: Assume preferences are perfectly aligned ($\forall m \quad y^S(m,b) = y^R(m)$), then full information is a PBE

Lemma 3: Assume preferences are not perfectly aligned ($\exists m$ s.t. $y^S(m,b) \neq y^R(m)$), then full information is not a PBE

There are multiple equilibria even when preferences are perfectly aligned.

An implication of Lemma 3 is that full separation, as in Spence, is not feasible when preferences are misaligned

Lemma 4: Assume $\forall m,\ \ y^S(m,b) \neq y^R(m)$. (a) $\exists\epsilon$ such that for any actions u and v taken in equilibrium $|u-v| > \varepsilon$. (b) The set of actions taken in equilibrium is finite

Any equilibrium has to be a partition equilibrium. Define a N partition of $[0,1]$ as $0 = a_0 < a_1 < ... < a_N = 1$ and R's best response function to the belief that $m \in [a,b]$

$$\overline{y}(a,b) = ArgMax_y \int_a^b U^R(y,m)\frac{f(m)}{\int_a^b f(t)dt}dm$$

Define the arbitrage condition stating that when $m = a_i$, S is indifferent between action $\overline{y}(a_i, a_{i+1})$ and action $\overline{y}(a_{i-1}, a_i)$

$$U^S(\overline{y}(a_i, a_{i+1}), a_i, b) = U^S(\overline{y}(a_{i-1}, a_i), a_i, b) \qquad (A)$$

for $i = 1..N-1$. This implies that when $m < a_i$, S strictly prefers $\overline{y}(a_{i-1}, a_i)$ over $\overline{y}(a_i, a_{i+1})$, and the opposite holds when $m > a_i$

Proposition: Assume $\forall m,\ \ y^S(m,b) \neq y^R(m)$. (a) $\exists N(b)$ such that for $N \leq N(b)$, $\exists$ an equilibrium $\{q(n|m), y(n), p(m|n)\}$ such that (i) $y(n) = \overline{y}(a_i, a_{i+1})$ for $n \in [a_i, a_{i+1}]$, (ii) $q(n|m)$ is uniform over each partition interval taking different values on different intervals, (iii) $p(m|n) = \frac{q(n|m)f(m)}{\int_{a_i}^{a_{i+1}} q(n|t)f(t)dt}$ if $n \in [a_i, a_{i+1}]$

Any $N \leq N(b)$ partition equilibrium can be implemented using any signal set of N distinct elements

When $N > 1$, S communicates some information but this information is imprecise since R knows that $m \in [a_i, a_{i+1}]$ but does not know which value m has taken in this interval

The arbitrage condition (A) imposes an endogenous gap between actions and this is what enforces truth-telling by the sender