Buying Frenzies

Pascal Courty [1]

August 18, 2005

Abstract: I extend DeGraba's model of buying frenzies. I identify conditions under which buying frenzies are the only possible equilibrium and under which rationing occurs in equilibrium.

Keywords: Buying Frenzy, Rationing.
J.E.L: D21, D42, D45.

[1]Department of Economics, European University Institute, Via della Piazzuola 43, 50133, Firenze, Italy. Comments welcome at pcourty@iue.it. I would like to thank Marco Galbiati, Andrea Gallice, Gregor Langus and Jochen Schanz.

Buying frenzies occur when some consumers rush to buy a product while those who wait are rationed out. Such phenomena happen, for example, in ticket concerts (Courty, 2004) and for the launch of some new products (DeGraba,1995). DeGraba argues that a candidate explanation is that consumers discover new information about their willingness to pay over time. A monopolist may create a buying frenzy to make sure that consumers buy before they become informed. DeGraba's main contribution is to identify conditions under which buying frenzies are the only possible equilibrium outcome. I derive two new results. First, I show that DeGraba's analysis does not hold when consumer valuations take continuous values. Second, one can restore DeGraba's results if one assumes that the monopolist may receive, in the event a frenzy does not take place, an outside option for the unsold production. A new implication is that rationing occurs only if the outside option is low or uncertain.

1 Model

The model is pictured in Figure 1. There are two periods. On the demand side, there is a continuum of N consumers with unit demands. In period 1, consumers have the same expected valuations μ. Consumers privately learn their valuations in period 2. Consumer valuations are identically and independently distributed with cumulative distribution $F(.)$ and density $f(.)$. F and f are continuous functions with support $[v_L, v_H]$ and $\int_{v_L}^{v_H} vf(v)dv = \mu$. This demand specification is an approximation of a large market where consumers have idiosyncratic preferences that are discovered over time (Lewis and Sappington, 1994). On the supply side, a monopolist chooses output Q in period 1 and cannot produce more units in period 2. The marginal cost of production is $M \leq v_L$. The monopolist then sets the period 1 price p_1 and consumers can either buy or wait. If all output is sold at the end of period 1, the game ends. If no sale takes place in period 1, the producer receives with probability α outside offer w for the unsold production. If the monopolist receives and accepts the offer, the game ends. Otherwise, the monopolist sets the period 2 price p_2. Those consumers who have not yet bought may buy. For simplicity, the monopolist and consumers are risk-neutral and do not discount. In the event of excess demand, consumers are served according to a rationing rule that is left unspecified. I assume that $v > \frac{1-F(v)}{f(v)}$ for $v \in [v_L, v_H]$. I could replace this assumption with the assumption that the monopolist faces a Coase commitment problem in period 2.

Either assumption imply that the monopolist sells all leftover units in period 2. Finally, I make the technical assumption that (at most) mass ϵ of consumer may coordinate their decisions where ϵ can be arbitrarily small. The role of this assumption, which would hold in a discrete version of the model with N consumers, will become clear in the proof of proposition 2.

The case $\alpha = 0$ is a straightforward extension of DeGraba's model to continuous distributions of valuations. The novel part of my setup is the feature that the monopolist may receive an outside offer in period 2 (case $\alpha > 0$). Illustrations can easily be found. If the monopolist is a concert promoter, for example, it may be possible to cancel the event and to recoup part of the fixed cost. In the application to new product development, the monopolist may be able to sell the project to a third party or to direct unsold output to a different geographic market.

I focus on subgame perfect equilibrium. An equilibrium is a sextuplet $(Q, p_1, r_1, k, p_2, r_2)$ where r_1 is a function that maps Q and p_1 for each consumer into a reservation price in period one, $k \in \{0, 1\}$ and p_2 map all possible outcomes up to the beginning of period 2 into a decision to cancel the project ($k = 1$) and a period 2 price, and r_2 maps for each consumer all possible outcomes, including the realization of the consumer's valuation, into a reservation price in period 2. Following DeGraba, I say that a buying frenzy equilibrium occurs if sales take place only in period 1, a 'safe' buying frenzy equilibrium occurs if in addition there is a unique subgame perfect continuation equilibrium after announcement (Q, p_1), a market clearing equilibrium if sales take place only in period 2, and an asymmetric equilibrium if some sales take place in both periods.

In period 2, there is no aggregate uncertainty. $N(1 - F(p))$ consumers are willing to pay at least p. A lower bound on the monopoly profits is the market clearing profits $\Pi^{MC} = \max_p N(1 - F(p))(p - M)$. The monopolist could also earn more in a buying frenzy equilibrium. Questions of interest are: (a) Is it possible that only buying frenzies are equilibrium outcomes? That is, can the monopolist earn more than Π^{MC} in any equilibrium? (b) Are there safe buying frenzy equilibrium? (c) Does rationing always take place in a safe buying frenzy equilibrium? The answer to these questions can be positive in DeGraba's setup ($\alpha = 0$ and discrete valuations). Proposition 1 ($\alpha = 0$) shows that these results do not hold with continuous valuations. Propositions 2 and 3 show that the answer can again be positive when $\alpha > 0$.

2 Analysis

Consider the case $\alpha = 0$. $S_2(q, n)$ is the subgame that starts at the beginning of period 2 when n consumers have not bought and there is q unsold units. Since $v > \frac{1-F(v)}{f(v)}$, it is always optimal for the monopolist to sell all units. $n(1 - F(p))$ consumers are willing to pay at least p. The equilibrium period 2 price in $S_2(q, n)$ is

$$p_2(q, n) = F^{-1}\left(1 - \frac{q}{n}\right).$$

The expected utility of a consumer who has not bought in $S_2(q, n)$ is

$$U_2(q, n) = \int_{p_2(q,n)}^{v_H} (v - p_2(q, n))\, f(v)dv.$$

$S_1(Q, p_1)$ denotes the subgame that takes place after the monopolist has set Q and p_1.

Proposition 1: In any $S_1(Q, p_1)$ such that $Q \leq N$ and $Q(p_1 - M) \geq \Pi^{MC}$, there exists a market clearing continuation equilibrium if $\alpha = 0$.

Proof: Consider the following continuation equilibrium: $r_1 < p_1$, $p_2 = p_2(Q, N)$, and $r_2 = p_2$. I only need to show that r_1 is a best reply. Since $Q(p_1 - M) \geq \Pi^{MC} = \max_q q(p_2(q, N) - M) \geq Q(p_2(Q, N) - M)$, I have $p_1 \geq p_2(Q, N)$. A best reply for consumers to set $r_1 < p_1$ since $\mu - p_1 \leq \mu - p_2(Q, N) \leq U_2(Q, N)$.□

Proposition 1 implies that the answer to questions (a) and (b) is negative: There is no safe buying frenzy equilibrium and the market clearing equilibrium ($p_1 = p_2 = v_L$, $r_1 < p_1$, $k = 0$, $r_2 = p_2$) with payoff Π^{MC} always exists. The monopolist cannot rule out this equilibrium because this would imply lowering prices too much. In fact, the monopolist has to offer consumers a utility from buying early that is greater than the highest possible utility they can get by waiting. For a given Q, the highest price the monopolist can charge to rule out the market clearing equilibrium is $p_1 = \mu - U_2(Q, N)$. But the monopolist can do better under market clearing since $p_2(Q, N) \geq \mu - U_2(Q, N)$.

Proposition 1 reaches an opposite conclusion from DeGraba's analysis that shows that it is possible that 'every subgame perfect equilibrium is a buying-frenzy equilibrium' (Proposition 3, page 337). DeGraba's conclusion rests on the fact that, in contrast with Proposition 1, the market clearing equilibrium can be ruled out in his setup, and this is driven by the combination of two specific features of his model. First, period 2's valuations can take only a finite number of values (2 in his model). This makes the period 2 inverse demand a step function. At the equilibrium period 2's price, a positive mass of consumers is indifferent between purchasing and not doing so. Second, DeGraba assumes that all indifferent consumers actually demand the good and he considers a rationing rule that results in inefficient allocations. Under continuous distribution, however, the goods are efficiently allocated in period 2 and it is never optimal to eliminate the market clearing equilibrium.

Consider next the case $\alpha > 0$. (CT) is the ex-post cancellation threat that the monopolist weakly prefers to cancel the project in $S_2(q, N)$ when she receives an outside offer,

$$w \geq qp_2(q, N) \qquad (CT)$$

(IGC) is the incentive constraint that consumers prefer to buy than to wait as a group in $S_1(Q, p_1)$

$$\mu - p_1 \geq (1 - \alpha)U_2(Q, N) \qquad (IGC)$$

(IC) is the incentive constraint that mass ε of consumers prefer to buy than to wait given that mass $N - \varepsilon$ buy in $S_1(Q, p_1)$

$$\mu - p_1 \geq U_2(\max(0, Q - N + \varepsilon), N - Q + \varepsilon) \qquad (IC)$$

(IC) is not really constraining. It holds for any $p_1 \leq \mu$ as long as $Q \leq N - \varepsilon$. Finally, let $\widetilde{\Pi}^{MC} = \max\left(\Pi^{MC}, \alpha w + (1 - \alpha)\Pi^{MC}\right)$ and $(q^*, p_1^*) = \arg\max_{q,p_1}\{q(p_1 - M);\ s.t.\ (CT),$ (IGC), $(IC)\}$ where for presentation's sake I assume that the maximum is unique.

Proposition 2: Every equilibrium is a buying frenzy equilibrium if and only if $q^*(p_1^* - M) > \widetilde{\Pi}^{MC}$ $(C1)$, holds.

Proof: The 'if' part of the proposition proceeds in three steps.
Claim 1: There is no asymmetric continuation equilibrium in any $S_1(Q, p_1)$ where $0 < n_1 < Q \leq N$ consumers buy early. In any asymmetric continuation equilibrium,

$$U_2(Q - n_1, N - n_1) \geq \mu - p_1 \geq U_2(Q - n_1 + \widetilde{\varepsilon}, N - n_1 + \widetilde{\varepsilon}).$$

since mass $\widetilde{\varepsilon} \in [0, \varepsilon]$ of consumer may jointly deviate. But U_2 increases with ε

$$\frac{dU_2}{d\varepsilon}(Q - n_1 + \varepsilon, N - n_1 + \varepsilon) = \frac{N - Q}{(N - n_1 + \varepsilon)^2} \frac{1 - F(p_2(Q - n_1 + \varepsilon, N - n_1 + \varepsilon))}{f\left(F^{-1}\left(1 - \frac{Q - n_1 + \varepsilon}{N - n_1 + \varepsilon}\right)\right)} > 0.$$

A contradiction.

Claim 2: For any $\eta > 0$ there exists a unique continuation equilibrium in $S_1(q^*, p_1^* - \eta)$. Consider the continuation buying frenzy equilibrium where $r_1 = p_1^*$, $p_2 = p_2(q^* - n_1, N - n_1)$ if n_1 consumers wait, and $r_2 = p_2$. (IC) implies that r_1 is a best reply for any mass $\widetilde{\varepsilon} \in [0, \varepsilon]$ of consumers. The only other possible continuation equilibrium is that all consumers wait which is ruled out by (IGC).

Claim 3: There is no market clearing equilibrium. A lower bound on profits is $q^*(p_1^* - M)$. $(C1)$ implies that the highest possible profits under any market clearing equilibrium are dominated by $q^*(p_1^* - M)$. Therefore market clearing cannot be part of an equilibrium.

The 'only if' part is proved by contradiction. Assume $(C1)$ does not hold and consider the market clearing equilibrium with equilibrium strategy profile $Q = N$, $p_1 = p_2 = v_L$, $k = 1$ iff $w \geq Qp_2(Q, N)$, $r_1 > r_2 = v_L$ and for any initial announcement $(Q, p_1) \neq (N, v_L)$

$$r_1(Q, p_1) = \begin{cases} \mu - (1 - \alpha)U_2(Q, N) \text{ if } w \geq Qp_2(Q, N) \\ \mu - U_2(Q, N) \text{ if } w < Qp_2(Q, N) \end{cases},$$

$k = 1$ iff $w \geq Qp_2(Q, N)$, and $r_2 = p_2(Q, N)$. Consumer decisions are optimal. If $w \geq Qp_2(Q, N)$ they rationally anticipate that the project will be cancelled with probability α and are better off waiting if $p_1 > \mu - (1 - \alpha)U_2(Q, N)$. If $w < Qp_2(Q, N)$ the project is never cancelled and consumers are better off waiting if $p_1 > \mu - U_2(Q, N)$. Given consumer responses, the highest possible profits under any deviation is $q^*(p_1^* - M)$ which is dominated by $\widetilde{\Pi}^{MC}$ if $(C1)$ is violated.□

The set of equilibrium under $(C1)$ is any (Q, p_1) such that $Q(p_1 - M) \geq q^*(p_1^* - M)$, (IC) holds, and the continuation equilibrium is a buying frenzy. Propositions $1 - 2$ imply that the monopolist must receive an outside option with non-negative probability to eliminate the market clearing equilibrium. The outside option reduces consumers' utility from waiting and increases the price the monopolist can charge early. (The price p_1 that satisfies (IGC) increases as α increases.) If $\alpha = 1$ and $w \geq Nv_L$, the monopolist extracts almost all consumer surplus (profits are no less than $(N - \epsilon)(\mu - M)$).

If $(C1)$ does not hold, then there is no safe buying frenzy equilibrium. If $(C1)$ holds, there is a unique safe buying frenzy equilibrium (q^*, p_1^*). Consider the possibility of rationing in the safe buying frenzy equilibrium. To rule out the uninteresting case where rationing occurs because (IC) binds, I say that rationing occurs if $q^* < N - \epsilon$. Let $(q^{**}, p_1^{**}) = \arg\max_{q,p_1}\{q(p_1 - M) \text{ s.t. } (IGC), (IC)\}$.

Proposition 3: Assume ϵ is small. Rationing occurs in the safe buying frenzy equilibrium if $(C1)$ holds and either $\alpha\mu + (1-\alpha)[v_L - \frac{1}{f(v_L)}] - M < 0 \quad (C2)$, or $w < q^{**}p_2(q^{**}, N)$ $(C3)$, hold.

Proof: I show that profits decrease at $Q = N$ under $(C2)$. Let $\pi(q) = q(\mu - (1 - \alpha)U_2(q, N) - M)$.

$$\frac{d}{dq}\pi(q) = \mu - (1-\alpha)U_2(q,N) - M - (1-\alpha)\left(\frac{q}{N}\right)^2 \frac{1}{f\left(F^{-1}\left(1 - \frac{q}{N}\right)\right)}$$

$\frac{d}{dq}\pi(N) = \alpha\mu + (1-\alpha)[v_L - \frac{1}{f(v_L)}] - M < 0$ and by continuity $\frac{d}{dq}\pi(N-\epsilon) < 0$ for ϵ small enough. If $(C3)$ holds then $q^* < q^{**} \leq N$. For ϵ small enough $q^* < N - \epsilon$.$\square$

If $\alpha = 1$, $(C2)$ cannot hold. If w is large $(C3)$ cannot hold. Therefore, rationing can occur only if the outside option is uncertain or low.

References

1. Courty, Pascal. "Some Economics of Ticket Resale." *Journal of Economic Perspectives.* Spring 2003, 17 (2) p.85-97.
2. DeGraba, Patrick. "Buying Frenzies and Seller-Induced Excess Demand." *RAND Journal of Economics.* Vol. 26, No 2, (Summer 1995):331-42.
3. Lewis, Tracy and David Sappington. "Supplying Information to Facilitate Price Discrimination". *International Economic Review.* 35 (1994):309-327.

Figure 1: Timing of Events