SECOND MID-TERM EXAM
Microeconomics II
Pascal Courty
EUI, Florence, December 15, 2004

ANSWER ALL QUESTIONS. TIME: TWO HOURS.
TOTAL POINTS: 100

Exercise 1 (35 pts)

Indicate whether each of the following is TRUE, FALSE, or UNCERTAIN. Justify your answer. No credit given without justification.

(a) (7 pts) It is optimal for the UK premier league to sell television rights collectively and exclusively to a single television operator.

(b) (7 pts) Consider an economy with preferences $u_i(l_i, x_i) = l_i + \phi_i(x_i)$ where l_i is consumption of leisure and x_i is a consumption good. The budget constraint is $x_i \leq w(T - l_i) + I$ where w is an exogenously given wage, T is total time endowment, $T - l_i$ is labor, and I is non-labor income. The equilibrium consumption of good x_i is independent of I since there is no wealth effect for quasi-linear preferences.

(c) (7 pts) In 1995, the European Court of Justice issued the Bosman judgement stating that when a contract with a footballer expires, the club may not prevent the player from signing a new contract with another club in another Member State. This judgement should not change the allocation of players or the quality of games.

(d) (7 pts) Under the assumption that insurance markets are competitive, insurance prices should be fair $\left(\frac{p_1}{p_2} = \frac{\pi_1}{\pi_2}\right)$ for idiosyncratic risk (i.e. personal accident) but not necessarily for aggregate risk (i.e. regional hurricane).

(e) (7 pts) Truthtelling is an equilibrium of the modified Clark-Groves mechanism where the music lover can play but pays $\widehat{c} + \varepsilon$ and the neighbor receives $\widehat{b} - \varepsilon$ (with $\varepsilon > 0$) in the event $\widehat{b} > \widehat{c}$, and no music is allowed and no transfer takes place otherwise.

Exercise 2 (25 pts)

Consider a 2x2 exchange economy where consumer 1's utility is $u_1(x_{1,1}, x_{2,1}) = Min(x_{1,1}, x_{2,1})$ and consumer 2's utility is $u_2(x_{1,2}, x_{2,2}) = \alpha \ln(x_{1,2}) + (1 - \alpha)\ln(x_{2,2})$. Consumers' endowments are $w_1 = (1, 0)$ and $w_2 = (0, 1)$. Throughout the exercise we normalize the price of good 1 to 1 and denote the price of good 2, p.

(a) (6.25 pts) Let the function $x_{j,i}(p)$ denote consumer i's demands for good j. Derive $x_{j,i}(p)$ for $i = 1, 2$ and $j = 1, 2$.

(b) (6.25 pts) Show that $\sum_{i=1,2}(x_{1,i}(p)+px_{2,i}(p)-w_{1,i}-pw_{2,i})=0$. Argue that this implies Walras law stating that if market for good j clears then market for good j' with $j \neq j'$ also clears.

(c) (6.25 pts) Find the competitive equilibrium of this exchange economy.

(d) (6.25 pts) Assume a social planner allocates total endowment $w=(1,1)$ to maximize $u_1(x_{1,1},x_{2,1})+u_2(x_{1,2},x_{2,2})$. Derive the set of socially optimal allocations. Is the allocation you derived in (c) socially optimum?

(e) (Bonus) How does the analysis change if $w_1=(w_{1,1},0)$ and $w_2=(0,w_{2,2})$ with $w_{1,1} \neq w_{2,2}$.

Exercise 3 (40 pts)

Two lovers $i=1,2$ have to share a cake of size y over lunch and dinner. Each lover can request any amount r_i for lunch and receives that amount if $r_1+r_2 \leq y$ and $\frac{y}{2}$ otherwise. (The lovers eat at lunch whatever amount they receive.) When dinner comes, the lovers equally share whatever is left of the cake, if anything. Lover i's utility from consuming c_1 over lunch and c_2 over dinner is $u_i(c_1,c_2)=\ln(c_1)+\ln(c_2)$.

(a) (10 pts) Assume lover 1 requests r_1 for lunch. What is is lover 2's best response request? What is the (symmetric Nash) equilibrium of the game?

(b) (10 pts) Assume a family counselor can set lover i's consumption in meal t to $c_{i,t}$ for $t=l,d$ (lunch/dinner). Find the optimal consumption allocation $c_{i,t}$ under the assumption that the counselor equally likes both lovers. Does it differ from the equilibrium consumption you derived in (a). Interpret.

(c) (10 pts) Consider the corresponding I-lover game where lover i requests any amount r_i for lunch and receives that amount if $\sum_{i=1..I} r_i \leq y$ and $\frac{y}{I}$ otherwise. As before, the lovers share the cake's leftovers for dinner. Find the equilibrium.

(d) (10 pts) What is the efficient allocation $c_{i,t}$? How does total lunch and dinner consumption (summed over all lovers) differ in the Nash equilibrium and in the efficient allocation as I increases to infinity? Interpret.

(e) (Bonus) Return to the two lovers game and assume they have to share the cake over breakfast, lunch, and dinner. To start, assume that the lovers have eaten $\sum_{i=1,2} c_{i,b}$ over breakfast, and consider what happens after breakfast, given that the cake is now of size $y-\sum_{i=1,2} c_{i,b}$. As before, lovers make requests for lunch and share the leftover at dinner. What is the equilibrium of the lunch-dinner game? Now consider the breakfast request decisions. Assume lovers make requests for breakfast under the assumption that they will subsequently play the lunch-dinner game. What is the Nash equilibrium in the breakfast-lunch-dinner game?

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Exercise 1 (35 pts)

(a) (7 pts) TRUE. Assuming that the UK football league maximizes profits, it is optimal to sell TV rights exclusively to a single TV operator. Selling to multiple TV operators would increase competition in the TV market and reduce the overall surplus the TV operators can extract from consumers. Similarly, breaking-up TV rights (non-collective sale) would also introduce competition and reduce the total amount that can be extracted from consumers. Essentially, the UK football league can achieve the first-degree price discrimination outcome by selling TV rights to a single operator, who will itself resell the bundled TV rights to consumers for a fixed access fee.

(b) (7 pts) FALSE. The optimal consumption of good x is $x^* = Min(wT + I, \widehat{x})$ where $\phi'(\widehat{x}) = 1/w$. Clearly x^* is a function of I. The intuition is simply that the consumer cannot borrow labor and this imposes the constraint $l_i \geq 0$. As a result, the consumer cannot always consume the unconstrained level of good consumption $\widehat{x}$.

(c) (7 pts) TRUE. The Bosman ruling has reallocated the property right over international transfer from clubs to players. The Coase theorem says that the allocation of players is independent of the property right's allocation under no transaction cost, symmetric information, no wealth effect, and no bargaining inefficiency. The allocation of players and quality of games may change with the reallocation of property rights if one, or more, of these assumption fail. (For example, if players do not have access to financial markets, they may not be able to invest in soccer-human-capital and the quality of player may decrease. But this remains an empirical issue.)

(d) (7 pts) UNCERTAIN. Competitive insurance markets implies zero profit. In addition, idiosyncratic risk implies by the law of large numbers that an insurance company does not face residual risk on its capital. Insurance prices should be fair, $\frac{p_1}{p_2} = \frac{\pi_1}{\pi_2}$. (If consumers have subjective probabilities they may not fully insure at market prices.) Under aggregate shocks, an insurance company cannot apply the law of large numbers and there will be a risk premium in those states of the world where total endowment is lower. Therefore, market prices will typically not be fair ($\frac{p_1}{p_2} \neq \frac{\pi_1}{\pi_2}$).

(e) (7 pts) FALSE. A contradiction can be reached by considering ε large. ML gets negative surplus even if $b > c$. It is optimal to reveal the lowest possible b so that music is never allowed. More formally, ML maximizes $\int_0^{\widehat{b}}(b -$

$(c+\varepsilon))g(c)dc$ where g is the density of c and the lowest possible value of c is 0. The first order condition imply that it is optimal to reveal $\widehat{b} = b - \varepsilon$ which is not truthtelling. The intuition is simply that ML prefers not to trade when $c \in [b-\varepsilon, b)$.

Exercise 2 (25 pts)

(a) (6.25 pts)
Consumer A: The budget constraint is $x_{11} + px_{21} = 1$. Because of Leontief preferences $x_{11} = x_{21} = \frac{1}{1+p}$.
Consumer B: The budget constraint is $x_{12} + px_{22} = p$. Because of Cobb-Douglas preferences $\frac{1-\alpha}{\alpha}\frac{x_{12}}{x_{22}} = p$ must hold, such that the Marshallian demands are $x_{12} = \alpha p$, $x_{22} = 1 - \alpha$.

(b) (6.25 pts) We need to show that $\sum_{i=1,2} x_{1i}(p) - w_{1i} + p(x_{2i}(p) - w_{2i}) = 0$ for any p. Substituting in the optimal demands we get

$$\sum_{i=1,2} x_{1i}(p) - w_{1i} + p(x_{2i}(p) - w_{2i}) = \frac{1}{1+p} - 1 + p\left(\frac{1}{1+p}\right) + \alpha p + p(1-\alpha-1)$$

which indeed sums to zero. If the market for good 1 clears, the market for good 2 must clear as well since the sum of all excess demands in the economy equals zero for *any* p.

(c) (6.25 pts) Taking the market clearing condition for good 1, $x_{11} + x_{12} = 1 \Leftrightarrow \alpha p + \frac{1}{1+p} = 1$ we find the equilibrium price $p = \frac{1-\alpha}{\alpha}$. The equilibrium quantities are (α, α) for consumer 1 and $(1-\alpha, 1-\alpha)$ for consumer 2.

(d) (6.25 pts) For this welfare function, the social planner will allocate $(1,1)$ (i.e. everything) to consumer 2. The Pareto set must be such that $x_{11} = x_{21}$ and $x_{21} = x_{22}$ such that the set of Pareto optimal allocations is just the 45 degree line in the Edgeworth box. Hence, the allocation for this welfare function is a single element of the Pareto set. Also note that by the first welfare theorem, the competitive equilibrium calculated in (c) is also an element of the Pareto set.

(e) (Bonus) In this case, the Edgeworth box is no longer square. It is not true that $x_{11} = x_{21}$ and $x_{21} = x_{22}$ holds for every element of the Pareto Set. (There is a subset of Pareto optimal allocations for which consumer 2 does not consume one of the goods, see Problem Set 1 Exercise 2.3 for a very similar case). It is also no longer true that for the welfare function specified in (d) the social planner always allocates everything to consumer 2. In some cases, consumer 1 also receives an (equal) nonnegative amount of both goods.

Exercise 3 (40 pts)

(a) (10 pts) First note that because of Cobb-Douglas utility, we can rule out equilibria where the whole pie is eaten in the first period. So restricting attention to cases in which $r_1 < y/2$, lover 2 chooses r_2 in order to $\max ln(r_2) + ln(y - r_1 - r_2)$. The best response (conditional on $r_1 < y/2$) is found to be

$r_2(r_1) = \frac{y-r_1}{2}$. In a symmetric Nash equilibrium, the requests for lunch then are $(y/3, y/3)$. Consequently, each lover eats $y/6$ for dinner.

(b) (10 pts) Clearly, such an allocation requires $c_{11} = c_{12} = c_{21} = c_{22}$. This is because consumption in each period is equally weighted in the Cobb-Douglas specification and because the counselor weights both lovers equally. Hence, the first best eating pattern is given by $(y/4, y/4)$ for each lover. In the Nash outcome both lovers eat too much in the first period because of strategic behavior.

(c) (10 pts) Similarly, the best response function is given by $r_i(r_{-i}) = \frac{y-\sum_{-i} r_i}{2}$ and the Nash outcome is one in which every person requests $\frac{y}{I+1}$ of the cake for lunch. For dinner, everybody eats $\frac{y}{I(I+1)}$.

(d) (10 pts) Similarly, the efficient allocation is one in which each lover has an eating pattern $\left(\frac{y}{2I}, \frac{y}{2I}\right)$. As I grows bigger, a larger part of the pie is eaten for lunch, whereas in the social optimum only half is eaten for any I. For $I \rightarrow \infty$, the whole pie is eaten for dinner if there is strategic behavior.

(e) (Bonus) Define $c_b < y$ as the total amount eaten for breakfast. Repeating the analysis of a), the best response function at lunchtime is $r_{i,l}(r_{-i,l}) = \frac{y-c_b-r_{-1,i}}{2}$ such that in a symmetric Nash equilibrium both lovers eat $\frac{y-c_b}{3}$ for lunch each. The first period problem for lover 2 is to choose $r_{i,b}$ to max $ln(r_{i,b}) + ln\left(\frac{y-r_{i,b}-r_{-i,b}}{3})\right) + ln\left(\frac{y-r_{i,b}-r_{-i,b}}{6}\right)$. The best response function is $r_{i,b}(r_{-i,b}) = \frac{y-r_{-i,b}}{3}$. The Nash equilibrium eating pattern is for each lover $(y/4, y/6, y/12)$. Note that the socially optimal individual meal would be $y/6$.