3. Labor and consumption taxation

Consider an economy in which the preferences of individual i are quasi-linear, namely:

$$w^i = c^i + V(x^i),$$

where c^i represents the individual's consumption and x^i his leisure. Moreover, $V(\cdot)$ is increasing and concave in x^i. The private budget constraint of each agent is given by

$$(1 + q_C)c^i \leq (1 - q_L)l^i + f,$$

where q_L is the income tax rate, q_C the consumption tax rate, and f a fixed subsidy from the government. The real wage of each agent is exogenous and normalized to unity. Furthermore, each agent has a private productivity parameter α^i, so that agents have different amounts of effective time available. More precisely, they face the following time constraint:

$$1 + \alpha^i \geq x^i + l^i.$$

Assume that α^i is drawn from a distribution with mean α and median α^m.

a. Compute each individual's optimal labor supply. What effects does an increase in q_L (respectively, q_C) have on the individual labor supply? Discuss the result.

b. Write the government budget constraint and derive the level of the subsidy as a function of $\mathbf{q} = (q_L, q_C)$. Compute the policy preferences $W(\mathbf{q}; \alpha^i)$ of individual i.

c. Does a Condorcet winner exist in that case? If yes, who is the Condorcet winner?

d. Compute the utilitarian welfare and determine the socially optimal policy. What is the winning policy $\mathbf{q}$ when $\alpha^i = \alpha$ for all i? What happens if agents are heterogeneous?

4. Multidimensional public consumption in the presence of a Condorcet winner

Suppose that all individuals in the economy have the same exogenous income $y > 4$ and are subject to the same income tax τ. Government revenue per capita τy is spent on two types of publicly provided goods 1 and 2 in per capita amounts of q_1 and q_2. Individuals also consume a privately provided good, denoted by c. Agents have heterogeneous preferences for public goods and their utility function is summarized by

$$w^i = U(c) + \alpha^i G(q_1) + (1 - \alpha^i)F(q_2),$$

where α^i is an intrinsic parameter of agent i. The functions $U(\cdot)$, $G(\cdot)$, and $F(\cdot)$ are continuous, twice continuously differentiable, strictly increasing, and strictly concave.

a. Write each individual's budget constraint as well as that of the government. Derive the policy preferences of agent i and verify that they satisfy the intermediate-preference property. What does this imply? Determine the optimal quantity $q_1(q_2, \alpha^i)$ (respectively, $q_2(q_1, \alpha^i)$) from agent i's perspective for a given level q_2 (respectively, q_1). When q_2 (respectively, q_1) increases, what is the effect on the optimal provision of q_1 (respectively, q_2) for agent i? How does α^i affect these quantities?

b. To simplify the analysis, suppose that $U(x) = G(x) = F(x) = \ln(x)$. Compute agent i's bliss point $(q_1(\alpha^i), q_2(\alpha^i))$. Suppose the economy consists of three agents (or three groups of agents) $i = \{1, 2, 3\}$ with different intrinsic parameters. More precisely, $\alpha^1 = 0$, $\alpha^2 = \frac{1}{2}$, and $\alpha^3 = 1$. Determine the optimal provision of public goods for each agent. Which policy is implemented under majority rule?

c. Suppose now that the preferences of agent i are summarized by

$$w^i = U(c) + \alpha^i G(q_1) + (1 - \alpha^i) F(q_2) + h(\alpha^i) H(q_1, q_2),$$

where $U(x) = G(x) = F(x) = \ln(x)$ and $H(q_1, q_2) = \ln(q_1 q_2)$. Derive each individual's policy preferences. Discuss. Suppose that $h(\alpha^i) = (\alpha^i)^2$. Determine the optimal quantity $q_1(q_2, \alpha^i)$ (respectively, $q_2(q_1, \alpha^i)$) from agent i's perspective for a given level q_2 (respectively, q_1). When q_2 (respectively, q_1) increases, what is the effect on the optimal provision of q_1 (respectively, q_2) for agent i? Compute the equilibrium public consumptions. How does α^i affect these quantities?

d. Suppose that the economy consists of three agents $i = \{1, 2, 3\}$ for which $\alpha^1 = 0, \alpha^2 = \frac{1}{2}$, and $\alpha^3 = 1$. Determine the optimal provision of public goods for each agent. Is there a Condorcet winner?

e. Suppose now that voters' preferences are given by

$$w^i = U(c) + K(\alpha^i)[G(q_1) + F(q_2)],$$

with $U(x) = G(x) = F(x) = \ln(x)$. Compute public consumption levels in equilibrium. Suppose that $K(\alpha^i) = (\alpha^i - \frac{1}{2})^2$ and that the economy consists of agents with type $\alpha^1 = \frac{1}{4}$, $\alpha^2 = \frac{1}{3}$, and $\alpha^3 = 1$. Is there a Condorcet winner in this economy?

5. Structure-induced equilibrium and multidimensional public consumption

Consider the same model as in problem 4 but suppose now that the economy consists of three types of agents. The preferences w^i of type i are

$$w^1 = \ln(c) + \ln(q_1 + 1)$$

$$w^2 = \ln(c) + a \ln(q_2 + 1)$$

$$w^3 = \ln(c) + \ln(q_1 + 1) + b \ln(q_2 + 1).$$

We assume that the intrinsic parameters of voters α^i are drawn from a common knowledge distribution $F(\cdot)$. At the date the platforms are offered, voters have beliefs about the politicians' preference parameters, α^P. They are represented by the probability distributions $F^P(\cdot)$.

a. Determine the policy q_2^P that the winner selects. Characterize the expected utility of voter i when politician P announces q_1^P. Show that the voters' preferences over politician P's policy depend on their beliefs.

b. Characterize the voter who is indifferent between voting for politician A and voting for politician B when q_1^A and q_1^B are announced. Characterize the vote share of politician A for all (q_1^A, q_1^B).

c. Suppose that agents have the same beliefs about politicians, that is, $F^A = F^B$. Which platforms guarantee half of the electorate for each politician? What happens if beliefs differ?

2. Downsian competition in a simple public-good model

Consider the economy described in problem 2 of chapter 2. More precisely, agent i's preferences over a publicly provided good y and a privately provided good c^i is expressed by

$$w^i = c^i + \alpha^i V(y),$$

where $V(\cdot)$ is a concave, well-behaved function and α^i is the intrinsic parameter of agent i that is drawn from distribution $F(\cdot)$ with mean α. Again, all individuals have initial resources only in the private good, $e^i = 1$ for all i, and one unit of private good is required to produce one unit of public good. To finance the public-good production, the government raises a tax q on each individual so that agent i's budget constraint is $c^i \leq 1 - q$.

a. Derive the policy preferences of each agent $W(q; \alpha^i)$ as well as the social optimum in this economy.

Suppose that two politicians $P = A, B$ select platforms q^A and q^B. Assume that each maximizes the expected value of some exogenous rent R. Call π_P the vote share for politician P; then P's probability of winning the election is $p_P = \text{Prob}\,(\pi_P \geq \frac{1}{2})$ and his expected utility is then $p_P R$. First, the two candidates announce their platforms simultaneously and noncooperatively. Then, elections are held. Last, the elected politician implements his announced policy.

b. Assume that $\alpha^i = \alpha$. Determine the candidates' probability of winning. What are the announced platforms and which one is implemented? Discuss.

c. Determine each candidate's probability of winning when agents are heterogeneous. What are the selected platforms in that case? Which one is implemented?

d. What are the model's economic predictions? Discuss.

3. A simple model of probabilistic voting

Consider the same model as in problem 2, but assume that three factors affect voter i's voting strategy: (1) the economic policy implemented q, (2) his individual ideological bias σ^i toward candidate B, and (3) the popularity δ of politician B. We assume that σ^i is uniformly distributed on $[-\frac{1}{2\phi}, \frac{1}{2\phi}]$. Moreover, δ is the same for all voters and is drawn from the uniform distribution on

$$\left[-\frac{1}{2\psi}, \frac{1}{2\psi}\right].$$

The distributions are common knowledge, but only agent i observes his own parameter σ^i. Then, i's preferences over the policy implemented by A are summarized by $W(q^A; \alpha^i)$, whereas the preferences over the policy implemented by politician B take the final form

$$W(q^B, \alpha^i) + \sigma^i + \delta.$$

The timing is as follows: first, each voter observes σ^i, and politicians simultaneously and noncooperatively announce platforms q^A and q^B. Second, δ is realized. Third, elections take place, and last, the announced policy is implemented.

a. Give an interpretation of σ^i. Characterize the agent who is indifferent between voting for politician A and voting for politician B for given policies q^A and q^B. Suppose that $\alpha^i = \alpha$. Deduce candidate A's vote share as well as his probability of winning.

b. Which platforms do the politicians select? Which one is implemented? Discuss.

c. Suppose that agents are heterogeneous. What does this imply for the equilibrium?

d. Discuss your results and compare them with the results obtained in problem 2.

4. Probabilistic voting in the presence of groups of voters

Consider a modified version of the previous model. More precisely, we assume that the population consists of three kinds of voters $J = \{R, M, P\}$ with intrinsic parameters α^J. The proportion of agents in group J is denoted by λ^J, and $\sum_{J=1}^{3} \lambda^J = 1$. Besides, $\sum_{J=1}^{3} \alpha^J \lambda^J = \alpha$. Once more, the voting strategy of voter i in group J is affected by (1) the economic policy that is implemented q, (2) his individual ideological bias σ^{iJ} toward candidate B, and (3) the popularity δ of politician B. We assume that σ^{iJ} is uniformly