d. Finally, consider the case in which the candidates do not know the value of δ, and δ is uniformly distributed on

$$\left[\alpha - \frac{1}{2\varphi}, \alpha + \frac{1}{2\varphi}\right],$$

that is, one candidate may have a competitive advantage. Solve for the equilibrium levels of taxes and rents as well as the equilibrium probabilities of winning the election. Are expected rents higher in this equilibrium than in the equilibrium without advantages for either candidate? It is not necessary to specify the conditions for existence of equilibrium.

2. Rents with endogenous value of being in office

Consider the model in section 4.4. An incumbent politician proposes a level of spending on public goods, g, and a level of private rents for himself, r. The public good g is financed through proportional income taxes. A continuum of citizens of measure one, indexed by i, all have the same income y. The government's budget constraint is $\tau y = \theta g + r$, where θ is a parameter measuring the cost of providing public goods. Citizen i's preferences over private consumption c^i and a public good g are described by

$$u^i = c^i + H(g),$$

where $c^i = y(1 - \tau)$. The incumbent's utility consists only of consumption of the rents, $u = \gamma r$. The following game is repeated an infinite number of periods. (1) θ_t is realized and observed by everybody. (2) Voters set a reservation utility for reelecting the incumbent. (3) The incumbent sets the policy variables, r_t and g_t. (4) Elections are held in which the voters choose between the incumbent and an opponent with the same characteristics as the incumbent. A politician maximizes

$$\sum_{t=0}^{\infty} \beta^t p_t \gamma r_t,$$

where β^t is the subjective discount factor and p_t is the probability of the incumbent's being in office at period t. Assume that a politician who is voted out of office cannot be reelected.

The incumbent at period 0 maximizes

$$\gamma r_0 + \beta p_1 R_{I,1},$$

where $R_{I,1}$ is the value of being an incumbent in period 1. The voters coordinate on the same retrospective voting strategy, voting for the incumbent if their utility is higher than or equal to $\varpi_t(\theta_t)$.

a. Solve for the optimal voting strategy $\varpi_t(\theta_t)$.

b. Suppose that term limits are imposed that do not allow the incumbent to stay in office more than three periods. How will this affect the voters' ability to discipline the incumbent?

c. Now suppose that there are two parties to which the candidates may belong. As before, the politician may stay in office for a maximum of three terms. Assume that the voters use the rule to vote for a candidate belonging to the same party as the incumbent if and only if rents are below some specific level. Suppose further that a new party candidate may bribe the incumbent not to keep rents too high in his third term. What is the new equilibrium level of rents?

3. Electoral cycles with seignorage

Assume the following model of electoral cycles with seignorage. Let us write the government budget constraint as

$$g_t = \eta_t(\bar{\tau} y + s_t),$$

where $\bar{\tau}$ denotes fixed taxes, η_t denotes the incumbent's competency level, there are no endogenous rents r, and the variable s_t denotes "seignorage" or, more generally, a hidden and distorting tax observed and paid by the voters only after the elections. Therefore, voters' welfare is

$$w_t = y - \bar{\tau} - s_t - V(s_t) + \alpha g_t,$$

where $V(\cdot)$ is a convex function capturing the distortions of seignorage. As is common in these models, the politician's competency, η_t, is determined by

$$\eta_t = \mu_t + \mu_{t-1},$$

where μ_t is uniformly distributed with mean 1 and density ξ and is serially uncorrelated. Politicians maximize voters' welfare and reelection rents according to

$$E(w_t \mid \mu_{t-1}) + p_t R,$$

where p_t is the probability of reelection.

The stage game at time t is given by: The politician chooses s_t, given μ_{t-1} and without observing μ_t. Nature determines μ_t. Voters observe g_t only. If t is an on-election period voters reelect the incumbent politician or elect a new contender, drawn from the same distribution. If t is an off-election period, we move to the election period. The stage game is infinitely repeated.

a. Show that in off-election periods, the incumbent sets s_t optimally.

b. Find the equilibrium seignorage in on-election periods. Show that they are larger than off-election seignorage (the social optimum level).

More references to its applications are given in later chapters. The aforementioned survey by Osborne (1995) carefully discusses many of the theoretical issues in sections 5.1 through 5.3.

As mentioned in chapter 2, the agenda-setting model in section 5.4 is due to Romer and Rosenthal (1978, 1979). This model was developed to deal with a number of issues, including the effects of different proposal, amendment, veto, and gatekeeping powers in political systems. Rosenthal (1990) surveys its further developments and applications. Baron and Ferejohn (1989) developed the so-called legislative bargaining model, which can be seen as a generalization of the Stahl-Rubinstein model of noncooperative two-person bargaining and decisions by unanimity to multiagent noncooperative bargaining and decision by majority under different rules for proposals and amendments. The model has since become one of the workhorse models in the rational choice approach to U.S. congressional politics. We give references to its many applications in subsequent chapters.

5.7 Problems

1. Probabilistic voting with outcome-seeking politicians

Assume that the indirect utility function of tax policy is described by

$$w^i = -(\tau - \tau^i)^2.$$

There are two political parties, one with the preferred tax rate zero, the other with preferred tax rate one. The parties can commit to a party platform that will be implemented should the party win the election. The political parties are uncertain about the most preferred tax rate τ^m of the median voter and assign a uniform probability distribution between $(\frac{1}{2} - a)$ and $(\frac{1}{2} + a)$ to τ^m. The parameter a lies in the interval (0, 1). The parties are exclusively policy motivated. Let τ_0 and τ_1 be the policies proposed by parties 0 and 1, respectively.

a. Show that the parties will never choose their bliss points and will never converge completely.

b. Solve for the equilibrium policy, given that it is symmetric, that is, $\tau_0 = 1 - \tau_1$. Discuss how the equilibrium policies depend on the level of uncertainty, as described by a.

c. Show that the equilibrium must be of the form $\tau_0 = 1 - \tau_1$.

2. The citizen-candidate model

Consider a society inhabited by a continuum of citizens with incomes uniformly distributed between zero and two. Each citizen i has preferences over private consumption c and a publicly provided private good g that are given by

$$w^i = \sqrt{c^i} + \sqrt{g}.$$

The public good g is financed through a proportional income tax τ, and the government budget constraint is $\tau y = g$, where y is the average income. Private consumption is $c^i = (1 - \tau)y^i$.

Consider the following timing: (1) Any citizen may enter as a political candidate at a cost ε. (2) An election is held among the candidates; the candidate who gets plurality wins the election, and a tie is resolved by the toss of a coin. (3) The winning candidate selects a tax rate; if there are no candidates, then a default tax rate $\overline{\tau}$ is implemented.

a. What policy would be implemented by a winning candidate with income y_i?

b. Suppose that $\varepsilon = \sqrt{2} - \sqrt{\frac{3}{4}} - \sqrt{\frac{1}{4}}$. In what region must the status quo policy, $\overline{\tau}$, lie in order for the equilibrium to exist where only the citizen with median income is a candidate? Are there other one-candidate equilibria?

c. Characterize the two-candidate equilibria.

d. Describe what would happen in a two-candidate equilibrium if the median candidate were to enter. How would the set of possible equilibria change if voters voted sincerely, that is, voted for the candidate giving them the highest utility.

3. Lobbying in a representative democracy

This problem is based on Besley and Coate 1997, 1999. Consider a model with N citizens, $i \in \{1, \ldots, N\}$, who derive a utility from consuming a private and a public good. Each citizen is endowed with $y^i = 1$ units of private good. The public good can be produced at any level g. One unit of private good is required to produce one unit of public good. Let c^i be the quantity of private good consumed by agent i, his utility being

$$w^i = c^i + \theta^i \ln(g),$$

where θ^i represents the preference of agent i for the public good. Assume that $\theta^i \in \{\theta^1, \ldots, \theta^T\}$, where $\theta^{k+1} = \theta^k + 1$ for all $k \in \{1, \ldots, (T-1)\}$. Let η^k be the number of citizens with type θ^k. Naturally, $\sum_{k=1}^{T} \eta^k = N$. There exists a median type denoted by θ^m. The timing of the game is as follows. First, each citizen decides whether to become a candidate and pays a cost δ if he does. Second, the voting stage takes place, and at most one candidate is elected. Denote the elected candidate by e. Assume that citizens vote sincerely and abstain when indifferent. Moreover, when n candidates receive the same vote share, each is elected with probability $\frac{1}{n}$. If nobody is elected, the game ends. Last, there exists a