Politics & Economics: Theory and Applications

Solutions to Problem Set 2

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Exercise 4.2 Rents with endogenous value of being in office, PT p.92
g level of spending on public good proposed by incumbent politician
r private rent incumbent politician extracts
$y_i = y \; \forall i$, income of a continuum of citizens i with measure 1
θ cost of providing public goods
$\tau y = \theta g + r$ government's budget constraint
$u = \gamma r$ incumbent's utility function
$u^i = c^i + H(g)$ utility function describing citizen i's preferences
$c^i = y(1-\tau)$ citizen i's budget constraint

Voter-politician game repeated infinite number of periods t:
1) θ_t is realised and observed
2) voters set a reservation utility for the incumbent
3) the incumbent sets the policy variables r_t and g_t
4) elections are held and citizens vote for either the incumbent or an opponent with identical features
$\omega_t(\theta_t)$ voter's state-contingent reservation utility determining her retrospective voting strategy for the incumbent under the assumption of coordination by voters;
$\gamma r_0 + \beta p_1 R_{I,1}$ incumbent's problem in $t = 1$ with $R_{I,1}$ value of being incumbent in period 1;
infinite horizon problem of politician with subjective discount factor β^t, probability p_t that the incumbent is in office in t and constrained by budget constraints and and consumer's reservation utility

$$\max_{\{r_t(\theta_t), g_t(\theta_t)\}} \sum_{t=0}^{\infty} \beta^t p_t \gamma r_t \text{ s.t. } \begin{cases} \omega_t(\theta_t) = y(1-\tau) + H(g(\theta_t)) \\ \tau y = \theta g(\theta_t) + r(\theta_t) \end{cases}$$

4.2.a. optimal voting strategy $\omega_t(\theta_t)$
Let $g^*(\theta_t)$ be the optimal state-contingent level of public good.
Then solve from the budget constraint in the politician's problem for the state-contingent rent $r(\theta_t)$ which the voter allows the incumbent to set so as to be re-elected

Then, by

$$\begin{aligned} r(\theta_t) &= y - \omega_t(\theta_t) + H(g^*(\theta_t)) - \theta_t g^*(\theta_t) \text{ if incumbent aims at re-election} \\ r_t &= y \text{ otherwise} \end{aligned}$$

In period t the incumbent prefers re-election in $t+1$ if

$$\gamma r_t + \delta V_{I,t+1} > \gamma y \tag{1}$$

Given the incumbent's strategy, voters' optimal one is to allow rent extraction r_t^* by making the incumbent indifferent between extracting r and being re-elected or y and no re-election, $\gamma r_t^* + \delta V_{I,t+1} = \gamma y$

$$r_t^* = max\left[0, y - \frac{\delta}{\gamma}V_{I,t+1}\right] \tag{2}$$

r_{t+1}^* is set with the same reasoning as r_t^*

$$\begin{cases} \gamma r_t^* + \delta V_{I,t+1} = \gamma y \\ V_{I,t+1} = \gamma r_{t+1}^* + \delta V_{I,t+2} = \gamma y \end{cases}$$

Then by solving for r one gets one gets $r_t^* = r_t = (1-\delta)y$

4.2.b. voters' ability to discipline incumbent given term limits in office

$$\begin{aligned} r_3^* &= y \text{ in } t=3 \\ r_2^*\delta\gamma y \geq y \Rightarrow r_2^* &= (1-\delta)y \text{ in } t=2 \\ r_1^*\delta(1-\delta)\gamma y \geq y \Rightarrow r_1^* &= (1-\delta+\delta^2)y \text{ in } t=1 \end{aligned}$$

In period $t = 3$ the incumbent extracts all income as rent. Then re-electing the incumbent is not a subgame-perfect strategy for voters who would be better off by electing in the third period a new incumbent to whom can extract $r_1^* = (1-\delta+\delta^2)y$. However, note that the incumbent would then extract $r_2 = y$ in period $t = 2$ and this is not subgame-perfect for voters either.
Then the only subgame-perfect equilibrium in the presence of term limits is full rent extraction by incumbents.

4.2.c. two parties and term limits in office
Define as b the bribe level. Then, the incumbent sets

$$\begin{aligned} \gamma r_3 b \geq \gamma y \Rightarrow r_3 &= y - \frac{1}{\gamma}b \text{ in } t=3 \\ \gamma r_2 + \delta\gamma y \geq \gamma y \Rightarrow r_2 = r_1 &= (1-\delta)y \text{ in } t=2 \text{ and } t=1 \end{aligned}$$

Recall incumbent's utility $u = \gamma r$. Then one obtains the value $\delta\gamma y$ of being a first period incumbent that a new candidate is happy to pay to the old candidate.

$$\underbrace{\gamma(1-\delta)y}_{u_1} + \underbrace{\underbrace{\delta\gamma(1-\delta)y}_{\delta u_2} + \underbrace{\delta^2\gamma y}_{\delta^2 u_3}}_{\delta\gamma y} = \gamma y \tag{3}$$

Voters must prefer to re-elect the incumbent for the third time to electing an incumbent for the equilibrium to be subgame perfect and this occurs if

$$\begin{aligned} r_3 \leq r_1 &= r_2 = (1-\delta)y \\ \text{equivalently } \gamma\delta y &\leq b \end{aligned}$$

Then in equilibrium $r_1 = r_2 = r_3 = (1-\delta)y$ and $b = \gamma\delta y$. In words, an incumbent is elected for 3 periods and the value of being a politican is greater than zero only for a new candidate.

Exercise 5.2 The citizen-candidate model, PT p. 110-111
let a continuum of citizens i have income $y^i \sim U[0,2]\ \forall i$
citizen i's preferences $\omega^i = \sqrt{c^i} + \sqrt{g}$
$\tau y = g$ government's budget constraint
$c^i = (1-\tau)y^i$ citizen's budget constraint
Timing of the game:
1) any citizen can stand for elections at cost ϵ
2) elections are held and the candidate who gets majority/plurality wins. A coin is tossed in case of a tie
3) the elected politician sets a tax rate τ and this equals $\bar{\tau}$ in the absence of candidates

5.2.a. Policy of winning citizen-candidate i
Replace citizen i's and the government's budget constraints in citizen i's policy preferences to obtain the preferred tax rate τ^i by citizen i as a solution to the following problem

$$\max_{\tau} \sqrt{(1-\tau)y^i} + \sqrt{\tau y} \iff \frac{y^i}{\sqrt{(1-\tau)y^i}} = \frac{y}{\sqrt{\tau y}} \text{ setting foc to 0}$$
$$\tau^i = \frac{y}{y^i + y}$$

5.2.b. Status quo policy $\bar{\tau}$ and existence of equilibrium with median income citizen
Let $\epsilon = \sqrt{2} - \sqrt{\frac{3}{4}} - \sqrt{\frac{1}{4}}$ be the cost of standing for elections.
From the uniform distribution of income, average and median income coincide and equal 1, $y = y^m = 1$. Then $\tau^m = \frac{1}{2}$ is the most preferred tax rate for the median voter and her decision to stand for elections is determined by

$$\underbrace{\sqrt{\frac{1}{2}} + \sqrt{\frac{1}{2}}}_{\omega^m(\tau^m)} - \underbrace{\left(\sqrt{(1-\bar{\tau})} + \sqrt{(\bar{\tau})}\right)}_{\omega^m(\bar{\tau})} \geq \underbrace{\sqrt{2} - \sqrt{\frac{3}{4}} - \sqrt{\frac{1}{4}}}_{\epsilon}$$
$$-\left(\sqrt{(1-\bar{\tau})} + \sqrt{(\bar{\tau})}\right) + \frac{3}{4} + \sqrt{\frac{1}{4}} \geq 0$$

The inequality holds if $\bar{\tau} \leq \frac{1}{4}$ or $\bar{\tau} \geq \frac{3}{4}$ (you can check by opening poleco_ps_1_2s_swpgraph.tex with Scientific Workplace).
Consider other 1-candidate equilibria by letting another candidate standing for elections. If her preferred tax rate is close enough to the one of the median voter, the other candidates that would defeat her in elections have so close preferences to her that they don't find it profitable to stand.

5.2.c. Two-candidate equilibria
Consider 2 citizens 1 and 2 running for office:

$$\tau^1 = (1-\tau^2) \text{ , they have some chance to win} \Rightarrow \text{median voter indifferent}$$
$$\omega^1(\tau^1) - \omega^1(\tau^2) \geq \epsilon \text{ , 1 prefers to run rather than dropping out}$$
$$\omega^2(\tau^2) - \omega^2(\tau^1) \geq \epsilon \text{ , 2 prefers to run rather than dropping out}$$

By rearranging the solution to 5.2.a. and using $y = y^m = 1$ one obtains

$$y^1 = \frac{1-\tau^1}{\tau^1}, \quad y^2 = \frac{\tau^1}{1-\tau^1} = \frac{1}{y^1}$$
$$\tau^1 = \frac{1}{1+y^1}, \quad \tau^2 = \frac{y^1}{1+y^1}$$

By plugging them into citizen 1's utility function,

$$\underbrace{\sqrt{\left(1-\frac{1}{1+y^i}\right)y^i}+\sqrt{\frac{1}{1+y^i}}}_{\omega^1(\tau^1)}-\underbrace{\sqrt{\left(1-\frac{y^1}{1+y^1}\right)y^i}-\sqrt{\frac{y^1}{1+y^1}}}_{\omega^1(\tau^2)}=\epsilon$$

$y^1 = 1.64$ and $y^2 = 0.61$ are obtained as numerical solutions (you can check by opening poleco_ps_1_2s_swpgraph.tex with Scientific Workplace) if $\epsilon = \sqrt{2} - \sqrt{\frac{3}{4}} - \sqrt{\frac{1}{4}}$ as in 5.2.b. Then $\{(y^1, y^2) : (y^1, y^2) \in (1.64, 2) \times (0.5, 0.61) \wedge y^2 = \frac{1}{y^1}\}$ is the set of equilibria.

5.2.d. Median candidate entry in 2-candidates equilibrium and sincere voting

The entry of a median candidate does not alter a citizen's voting choice in the 2-candidates equilibrium as voting for the median candidate would lead to a defeat of their preferred candidate.

With sincere voting the platforms of the candidates with extreme preferences would no longer qualify as equilibria as a third candidate with median preferences and entering the race would win elections.