

# Politics & Economics: Theory and Applications

## Problem Set 4

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### Separation and unification of regions

Consider a country with 2 regions  $A$  and  $B$  and exogenously and immutable boundaries and a continuum of agents who differ in their initial endowments  $K_{ui}$  of capital and  $L_{ui}$  of labour. Also assume competitive product, labour and capital markets;

$L_i$  and  $K_i$  population and capital in each region  $i$ ;

$L = L_A + L_B$  inelastic labour supply;

$K = K_A + K_B$  total capital stock in the country;

$Y_i = K_i^\beta L_i^{1-\beta}$  output in region  $i$  with  $0 < \beta < 1$ ;

$y_i = \frac{Y_i}{L_i}$  and  $k_i = \frac{K_i}{L_i}$  per capita regional output and capital;

**i)** compute the equilibrium wage rate  $s_i$  and return on capital  $r_i$  when there is factor mobility inside but not across regions. What happens when factors are also mobile across regions? **ii)** write down the expression for the individual agent's income and interpret.

Let  $h(w_u) = h_A(w_u) + h_B(w_u)$  be the income distribution in the country on the support  $[0, \bar{w}]$  and with  $h_i(w_u)$  being income distribution in region  $i$ ; then  $Y = \int_0^{\bar{w}} w_u h(w_u) dw_u$  is the expression for total income and output.

Consider now the event that the 2 regions separate to form independent countries and assume separation leads to efficiency losses such that after it an individual only gets  $\alpha w_u$ ,  $\alpha \leq 1$ . You can see the loss as arising from the impossibility to replicate in the separate regions allocations that were available and chosen in the unified country under decentralisation.

Also assume perfect substitutability between private consumption  $c_u$  and public good consumption  $g$  and a linear per capita tax  $\tau y$  finances the consumption of the public good and  $\frac{\tau^2}{2} y$  is the cost of its provision.

$$U(c_u, g) = c_u + g$$

**iii)** write down the individual's and the government's budget constraints.

**iv)** compute the preferred income tax rate  $\tau^*$  for an individual in the unified country.

**v)** by looking at individuals' preferences over taxes and redistribution, what is the equilibrium rate unde majority rule?

Now focus on the circumstances determining separation of the regions by assuming no factor mobility across regions and that separation is triggered when the majority of voters in at least one region

favour it.

**vi)** write down the individual's utility function  $U_i(w_u)$  in the separated region  $i$ .

**vii)** verify that  $u$ 's  $U_i(w_u) - U(w_u)$  is either always increasing or decreasing in  $w_u$  by using the relevant results from Persson & Tabellini.

**viii)** compute the equilibrium tax rate  $\tau_i^*$  of region  $i$  under separation and write down the individual  $u$ 's indirect utility function.

**ix)** compute the transfer/subsidy  $S$  that makes an individual indifferent between living in a unified country or a separated region; interpret the effects triggering the separation choice in the expression you get.