Politics & Economics: Theory and Applications
Solutions to Problem Set 4

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Separation and unification of regions

Consider a country with 2 regions A and B, exogenously immutable boundaries and a continuum of agents u who differ in their initial endowments K_{ui} of capital and L_{ui} of labour in region i. Also assume competitive product, labour and capital markets;
L_i and K_i population and capital in each region i;
$L = L_A + L_B$ inelastic labour supply;
$K = K_A + K_B$ total capital stock in the country;
$Y_i = K_i^{\beta} L_i^{1-\beta}$ output in region i with $0 < \beta < 1$;
$y_i = \frac{Y_i}{L_i}$ and $k_i = \frac{K_i}{L_i}$ per capita regional output and capital;

i) compute the equilibrium wage rate s_i and return on capital r_i when there is factor mobility inside but not across regions. What happens when factors are also mobile across regions?

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\begin{aligned}
\max_{L_i,K_i} \Pi_i &= K^{\beta} L^{1-\beta} - sL - rK \quad \text{region } i\text{'s problem} \\
\frac{\partial \Pi_i}{\partial L_i} &= (1-\beta) \underbrace{K^{\beta} L^{-\beta}}_{\frac{K^{\beta}L^{1-\beta}}{L} = y} - s_i = 0 \iff \\
s_i &= (1-\beta) y_i \\
\frac{\partial \Pi_i}{\partial K_i} &= \beta \underbrace{K^{\beta-1} L^{1-\beta}}_{\frac{K^{\beta}L^{1-\beta}}{L}\frac{L}{K} = \frac{y_i}{k_i}} - r_i = 0 \iff \\
r_i &= \beta \frac{y_i}{k_i}
\end{aligned}
$$

with factor mobility across regions, factor prices s_i and r_i, the capital-labour ratio and $k_i = \frac{K_i}{L_i}$ and per capita output $y_i = \frac{Y_i}{L_i}$ are equalised.

ii) write down the expression for the individual's income and interpret.

agents have heterogeneous initial endowments L_{ui} and K_{ui}. then agent u's income in region i is $w_{ui} = s_i L_{ui} + r_i K_{ui}$; this is given by the sum of her initial endowments each weighted by equilibrium factor prices.

Let $h(w_u) = h_A(w_u) + h_B(w_u)$ be the income distribution in the country on the support $[0, \bar{w}]$ and with $h_i(w_u)$ being income distribution in region i; then $Y = \int_0^{\hat{w}} w_u h(w_u) dw_u$ is the expression for total income and output.

Consider now the event that the 2 regions separate to form independent countries and assume separation leads to efficiency losses such that after it an individual only gets αw_u, $\alpha \leq 1$. You can see the loss as arising from the impossibility to replicate in the separate regions allocations that were available and chosen in the unified country under decentralisation.
Also assume perfect substitutability between private consumption c_u and public good consumption g and a linear per capita tax τy finances the consumption of the public good and $\frac{\tau^2}{2}y$ is the cost of its provision.

$$U(c_u, g) = c_u + g$$

iii) write down the individual's and the government's budget constraints.

$$\begin{aligned} c &= (1-\tau)w_u \text{ individual's budget constraint} \\ g &= \left(\tau - \frac{\tau^2}{2}\right)y \text{ government's budget constraint} \end{aligned}$$

note that the individual's budget constraint contains her income w_u on which the tax is levied while the governments' one contains the per capita or average output y using which the per capita transfer g is computed.

iv) compute the preferred income tax rate τ^* for an individual in the unified country.

$$\begin{aligned} \max_{c_u} U(c_u, g) &= c_u + g \text{ s.t.} \begin{cases} c^i = (1-\tau)w_u \\ g = \left(\tau - \frac{\tau^2}{2}\right)y \end{cases} \\ \equiv \max_{\tau} U(\tau) &= (1-\tau)w_u + \left(t - \frac{\tau^2}{2}\right)y \\ \frac{\partial U(\tau)}{\partial \tau} &= -w + (1-\tau)y = 0 \iff \\ \tau^*(w_u) &= \frac{y - w_u}{y} \end{aligned}$$

the differences in preferences over taxes among individuals depend on the difference between per capita or average output y and individual u's one whose distribution is $h(w_u)$.

v) by looking at individuals' preferences over taxes and redistribution, what is the equilibrium tax rate under majority rule?
preferences of individuals over income tax rate in iv) are single-peaked. then by the median voter equilibrium, the equilibrium tax rate is the one the median voter prefers. the utility $U(w_m)$ associated to it and $U(w_u)$ to any other voter u under the median voter equilibrium are

$$\begin{aligned} U_m &= c_m + g \\ &= \left(1 - \frac{y - w_m}{y}\right)w_m + \left[\frac{y - w_m}{y} - \frac{1}{2}\left(\frac{y - w_m}{y}\right)^2\right]y \\ &= \ldots \\ &= w_m + \frac{1}{2}\frac{(y - w_m)^2}{y} \\ U(w_u) &= c_u + g \\ &= w_m + \frac{1}{2}\frac{(y - w_m)}{y}[(y - w_u) + (w_m - w_u)] \end{aligned}$$

Now focus on the circumstances determining separation of the regions by assuming no factor mobility across regions and that separation is triggered when the majority of voters in at least one region favour it.

vi) write down the individual u's utility function $U_i(w_u)$ in the separated region i.
individual u gets income αw_u. then total output is $\alpha Y = \alpha \int_0^{\hat{w}} w_u h(w_u) dw_u$ and the per capita one αy.
then given equilibrium tax rate t_i

$$\begin{aligned} U_i(w_u) &= \alpha(c_u + g) \ , \ i \in \{A, B\} \\ &= \alpha \left[w_m + \frac{1}{2} t_i [(y - w_u) + (w_m - w_u)] \right] \end{aligned}$$

vii) verify that individual u's $U_i(w_u) - U(w_u)$ is either always increasing or decreasing in w_u by using the relevant results from Persson & Tabellini.

$$\begin{aligned} U_i(w_u) - U(w_u) &= \alpha \left[w_m + \frac{1}{2} \tau_i [(y - w_u) + (w_m - w_u)] \right] - \left[w_m + \frac{1}{2} \frac{(y - w_m)}{y} [(y - w_u) + (w_m - w_u)] \right] \\ &= (\alpha - 1) w_m + \frac{1}{2} [(y - w_u) + (w_m - w_u)] \left(\alpha \tau_i - \frac{(y - w_m)}{y} \right) \\ &= \ldots \\ &= \alpha \left[w_m + \frac{1}{2} \frac{(y - w_m)}{y} [(y - w_u) +)(w_m - w_u)] \right] \end{aligned}$$

when $U_i(w_u) - U(w_u)$ is increasing in w_u, all individuals u such that $w_u > (<) w_{mi}$ favour separation (unification) when the median voter mi does. when instead it is decreasing in w_u, the reverse applies. then determining when the median voter favours separation in at least 1 region suffices to see when it occurs in equilibrium.

viii) compute the equilibrium tax rate τ_i^* of region i under separation and write down the individual u's indirect utility function.
the equilibrium tax rate is the one preferred by the median voter

$$\begin{aligned} \tau_i^* &= \frac{y_i - w_{mi}}{y_i} \\ U_i(w_{mi}) &= \alpha \left[w_{mi} + \frac{1}{2} \frac{(y_i - w_{mi})^2}{y_i} \right] \text{ indirect utility of the median voter under separation} \\ U(w_{mi}) &= w_{mi} + \frac{1}{2} \frac{y - w_m}{y} [(y - w_{mi}) + (w_m - w_u)] \text{ indirect utility of the median voter under unification} \end{aligned}$$

ix) compute the transfer/subsidy S that makes an individual indifferent between living in a unified country or a separated region; interpret the effects triggering the separation choice in the expression you get.
the median voter prefers separation if

$$S = U_i(w_{mi}) - U(w_{mi}) = \underbrace{\frac{1}{2} \frac{(w_m - w_{mi})^2}{y}}_{\text{political effect}} + \underbrace{\frac{1}{2} \left[\left(\alpha y_i - \frac{w_{mi}^2}{y} \right) - \left(y - \frac{\alpha w_{mi}^2}{y_i} \right) \right]}_{\text{efficiency and tax base effects}} > 0$$

there are 3 effects determining the separation choice in a region:
i) the political effect: the difference in preferences over fiscal policy between the median voter in region *i* (*mi*) and in the unified country (*m*);
ii) the efficiency effect: a decrease in efficiency by α decreases the benefits from separation S;
iii) the tax base effect: the difference between y and y_i; if $y_i < (>) y$, then separation there is a cost (benefit) from separation arising from the smaller tax base after separation (no longer transfering money to the poorer region).