# The Refoundation of the Symmetric Equilibrium in the Schumpeterian Growth Models*

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**Abstract**

We provide a refoundation of the symmetric growth equilibrium characterizing the research sector of vertical R&D-driven growth models. We argue that the usual assumptions made in this class of models leave the agents indifferent as to where targeting research: hence, the problem of the allocation of R&D investment across sectors is indeterminate. By introducing an "$\varepsilon-$contamination of confidence" in the expected distribution of R&D investment, we prove that the symmetric structure of R&D investment is the unique rational expectations equilibrium compatible with ambiguity-averse agents adopting a maximin strategy.

*Keywords*: R&D-Driven Growth Models, Indeterminacy, Ambiguity, $\varepsilon-$contamination.

JEL Classification: 032, 041, D81.

## 1 Introduction

Most vertical R&D-driven growth models (such as Grossman-Helpman [9], Segerstrom [12], Aghion-Howitt [1]) focus on the symmetric equilibrium in the research sector, that

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is, on that path characterized by an equal size of R&D investment in each industry. In these models the engine of growth is technological progress, which stems from R&D investment decisions taken by profit-maximizing agents. By means of research, each product line can be improved an infinite number of times, and the firms manufacturing the most updated version of a product monopolize the relative market and thus earn positive profits. These profits have a temporary nature since any monopolistic producer is doomed to be displaced by successive improvements in its product line. The level of expected profits together with their expected duration, as compared to the cost of research, determines the profitability of undertaking R&D in each line.

The plausibility of the symmetric equilibrium requires that each R&D industry be equally profitable, so that the agents happen to be indifferent as to where targeting their investment (Grossman and Helpman [9], p.47). The profit-equality requirement implies two different conditions. First, the profit flows deriving from any innovation need to be the same for each industry: this is guaranteed by assuming that all the monopolistic industries share the same cost and demand conditions. Second, the monopolistic position acquired by innovating needs to be expected to last equally long across sectors: this requires that the agents expect the future amount of research to be equally distributed among the different sectors. As is well known to the reader familiar with the neo-Schumpeterian models of growth, future is allowed to affect current (investment) decisions via the forward-looking nature of the Schumpeterian "creative destruction" effect.

Expecting equal future profitability across sectors, however, does not constitute a sufficient condition for each agent to choose a symmetric allocation of R&D efforts: indeed, equal future profitability makes the investor indifferent as to where targeting research. As a result, when symmetric expectations are assumed the allocation problem of investment across product lines is *indeterminate*. First, notice that this indeterminacy in the intersectoral allocation of R&D may have powerful effects on the equilibrium growth rate in this class of models, as recently pointed out by Cozzi [3,4]. Second, indeterminacy does not depend on the focus on the symmetric equilibrium. In a recent paper[1] Giordani and Zamparelli develop an extension of the standard quality-ladder model to an economy with asymmetric fundamentals where the equilibrium allocation of R&D investment turns out to be asymmetric. However, the multiplicity of equilibria still exists, because the source of indeterminacy is *not* the symmetric

[1] P. Giordani and L. Zamparelli (2006), "The Importance of Industrial Policy in the Quality Ladder Growth Models", Mimeo.

structure of the economy but the fact that, in equilibrium, the returns from R&D are equalized, which still characterizes the asymmetric extension and which, once again, makes the agents indifferent in the allocation of R&D efforts.

In this paper we provide a way to eliminate indeterminacy in this class of models. Our reasoning goes as follows: the agents' indifference - arising from the equalization of R&D returns across industries - gives them in principle the possibility of adopting a whatever (even randomly chosen) investment strategy. This makes these agents highly uncertain about the configuration of future R&D investment, since that configuration is the result of a decision problem analogous to the one they are currently facing. To represent uncertainty (or *ambiguity*) and the agents' attitude towards it, we follow the maxmin expected utility (MEU) theory axiomatized by Gilboa and Schmeidler [8]. In representing subjective beliefs this approach suggests to replace the standard single (additive) prior with a closed and convex set of (additive) priors. The choice among alternative acts is determined via a maximin strategy, where the minimization is carried out over the set of priors and is meant to represent the individuals' aversion towards ambiguous scenarios. The plausibility of individuals' *aversion* to ambiguity (or preference for "pure risk") has been first shown by Ellsberg [6] via a thought experiment (then known as the Ellsberg paradox)[2]. In particular, we follow the "$\varepsilon-$contamination of confidence" argument, recently axiomatized by Nishimura and Ozaki [11]. As we will see, a however small "contamination of confidence" in the expectations of the future investment' allocation annihilates the agents' indifference and makes the configuration where R&D returns are equalized across industries emerge as the unique equilibrium.

Importantly, our assumption on the agents' attitude towards uncertainty does not concern any fundamental of the economy and is to be interpreted as a way of treating "extrinsic uncertainty". Moreover, uncertainty does not affect expectations on the aggregate amount of research. In fact, we introduce uncertainty to eliminate indeterminacy arising from situations where agents are indifferent among a set of choices. This is not the case for the total amount of research: if agents expect the equilibrium aggregate amount of research, their choice between consumption and savings, which are channelled to the research sector, is uniquely determined and confirms their expectations; there is no indifference, which is the source of the uncertainty in the agents' beliefs. Hence, in order to develop our argument all we need is the description of the

[2] Abundant experimental evidence supports the idea of the decision-makers' ambiguity aversion. See among the others Heath and Tversky [10], Fox and Tversky [7]. See also Camerer and Weber [2] for a survey.

R&D sector.

The rest of the paper is organized as follows. In Section 2 we briefly describe the basic structure of the R&D sector, with particular reference to the Segerstrom's [12] formalization[3]. In Section 3 we explain the core of our argument, enunciate and prove the proposition.

## 2 The R&D Sector

In this Section we provide a description of the vertical innovation sector, which is basically common to most neo-Schumpeterian growth models. This sector is characterized by the efforts of R&D firms aimed at developing better versions of the existing products in order to displace the current monopolists. We assume a continuum of industries indexed by $\omega$ over the interval $[0,1]$. There are free entry and perfect competition in each R&D race. Firms employ labor and produce, through a constant returns technology, a Poisson arrival rate of innovation in the product line they target. Adopting Segerstrom's [12] notation, any firm $j$ hiring $l_j$ units of labor in industry $\omega$ at time $t$ acquires the instantaneous probability of innovating $Al_j/X(\omega,t)$, where $X(\omega,t)$ is the industry-specific R&D difficulty index.

Since independent Poisson processes are additive, the specification of the innovation process implies that the industry-wide instantaneous probability of innovation is $AL_I(\omega,t)/X(\omega,t) \equiv I(\omega,t)$, where $L_I(\omega,t) = \sum_j l_j(\omega,t)$. The function $X(\omega,t)$ describes the evolution of technology; as in Segerstrom [12], we assume it to evolve in accordance with:

$$\frac{\dot{X}(\omega,t)}{X(\omega,t)} = \mu I(\omega,t)$$

where $\mu$ is a positive constant. Then, by substituting for $I(\omega,t)$ into the expression above and solving the differential equation for $X(\omega,t)$ we get:

$$X(\omega,t) = X(\omega,t_0) + \mu A \int_{t_0}^{t} L_I(\omega,z)dz.$$

Whenever a firm succeeds in innovating, it acquires the uncertain profit flow that accrues to a monopolist, that is, the stock market valuation of the firm: let us denote it by $v(\omega,t)$. Thus, the problem faced by an R&D firm is that of choosing the amount of labor input in order to maximize its expected profits[4]:

[3] Notice however that our argument can be applied, *mutatis mutandis*, to the whole class of models.

[4] As usual, let us consider labor as the numeraire.

$$\max_{l_j}[v(\omega,t)Al_j/X(\omega,t)-l_j],$$

which provides a finite, positive solution for $l_j$ only when the arbitrage equation $v(\omega,t)A/X(\omega,t)=1$ is satisfied. Notice that in this case, though finite, the size of the firm is indeterminate because of the constant returns research technology[5].

The firm's market valuation at a given instant $t$, $v(\omega,t)$, is the expected discounted value of its profit flows from $t$ to $+\infty$:

$$v(\omega,t)=\int_t^{+\infty}\pi(s)\exp\left[-\int_t^s[r(\tau)+I(\omega,\tau)]\,d\tau\right]ds.$$

By plugging $I(\omega,\tau)$ into $v(\omega,t)$, we finally obtain the following expression for $v(\omega,t)$:

$$v(\omega,t)=\int_t^{+\infty}\pi(s)\exp\left\{-\int_t^s\left[r(\tau)+\frac{AL_I(\omega,\tau)}{X(\omega,t_0)+\mu A\int_{t_0}^{\tau}L_I(\omega,z)dz}\right]d\tau\right\}ds. \quad (1)$$

The usual focus on the symmetric growth equilibrium is based on the assumption that the R&D intensity $I(\omega,\tau)$ is the same in all industries $\omega$ and strictly positive. The suggestion of a new rationale for this symmetric behavior is the topic of the next Section.

# 3 The Refoundation of the Symmetric Equilibrium

Assume that the agent is $(1-p)100\%$ sure to face in the future a symmetric configuration of R&D investment, and that with probability $p$ any other possible configuration can occur. We can call this situation a "$p-$contamination of confidence"[6]. Aversion to uncertainty in this context implies that with probability $p$ the agent expects the

[5] In the next Section our focus will be on the *individuals*' investment decisions, the reason being that R&D firms are actually financed by consumers' savings, which are channeled to them through the financial market. Thus the role of these firms is merely that of transforming these savings into research activity.

[6] To avoid confusion let us remark that in the literature this situation is usually called $\varepsilon-$contamination (which is also the phrase used in the Introduction). However, as we will see, in our context $\varepsilon$ stands for the extension of the state space.

worst configuration of future investment, that is, the one which minimizes her expected returns[7]. Since the minimizing configuration is a function of the agent's investment choice, this choice can then be formalized as the result of a "two-player zero-sum game" characterized by:

- the minimizing behavior of a "malevolent Nature", which selects the worst possible configuration of *future* R&D efforts and
- the maximizing behavior of the agent, who selects the best possible configuration of *current* R&D efforts.

We start our analysis at the beginning of time $t = t_0$, and assume that, at this time, all industries share the same difficulty index $X(\omega, t_0) = X(t_0)\ \forall \omega \in [0, 1]$ in order to focus on the role of expectations on the kind of equilibrium that will prevail. Our problem can then be stated as follows. At time $t = t_0$, the agent is asked to allocate a certain amount of R&D investment among all the existing industries: in maximizing her expected pay-off, she will take into account the minimizing strategy that a "malevolent Nature" will be carrying out in choosing the composition of future R&D efforts. We denote with $l_m(\omega, t_0) \equiv l_m(t_0)[1 + \alpha(\omega)]$ the agent's investment in sector $\omega$ at time $t_0$, and with $L_I(\omega, t) \equiv L_I(t)[1 + \varepsilon(\omega)]$ the agent's expectations about the aggregate research in sector $\omega$ at a generic point in time $t$. $l_m(t_0)$ and $L_I(t)$ are, respectively, the agent's average investment per sector at $t_0$ and the expected average research per sector at a generic $t$. $\varepsilon(\cdot)$ and $\alpha(\cdot)$ represent relative deviations from these averages satisfying:

$$\int_0^1 \varepsilon(\omega) d\omega = 0; \qquad \int_0^1 \alpha(\omega) d\omega = 0 \quad \text{and} \quad \varepsilon(\omega) \geq -1; \qquad \alpha(\omega) \geq -1.$$

[7] See the representation theorem (theorem 1) in Nishimura and Ozaki [11] for an axiomatization of the choice behavior assumed here.

The presence of the two functions $\alpha(\cdot)$ and $\varepsilon(\cdot)$ is intended to allow for asymmetry in the agent's current and expected investment[8] [9]. Note that $\alpha(\cdot)$ and $\varepsilon(\cdot)$ are unbounded above because the zero-measure of each sector allows the investment in any of them to be however big, without violating the constraint on the total R&D investment. From now on we will drop the argument $t_0$ in the expression for[10] $l_m(\omega, t_0)$ and enunciate the following:

**Proposition 1** *For a however small probability* $(p)$ *of deviation* $(\varepsilon(\omega))$ *from symmetric expectations on the future R&D investment, decision makers adopting a maxmin strategy to solve their investment allocation problem, choose a symmetric investment strategy, i.e.* $l_m[1+\alpha(\omega)] = l_m \ \forall\omega \in [0,1]$. *The associated distribution of expected R&D efforts among sectors is:* $L_I(t)[1+\varepsilon(\omega)] = L_I(t) \ \forall\omega \in [0,1]$.

**Proof.** If we substitute for $L_I(\omega,t) \equiv L_I(t)[1+\varepsilon(\omega)]$ into (1), and use the condition $\int_0^1 \alpha(\omega)d\omega = 0$, our problem can be stated as:

$$\max_{\alpha(\cdot)} \left\{ (1-p)l_m \frac{A}{X(t_0)} \int_{t_0}^{+\infty} \pi(s) \exp\left[ -\int_{t_0}^{s} \left( r(\tau) + \frac{AL_I(\tau)}{X(t_0)+\mu A\int_{t_0}^{\tau} L_I(z)dz} \right) d\tau \right] ds + \right.$$

$$\left. +p\min_{\varepsilon(\cdot)} \int_0^1 l_m[1+\alpha(\omega)] \frac{A}{X(t_0)} \left\{ \int_{t_0}^{+\infty} \pi(s) \exp\left[ -\int_{t_0}^{s} \left( r(\tau) + \frac{AL_I(\tau)[1+\varepsilon(\omega)]}{X(t_0)+\mu A\int_{t_0}^{\tau} L_I(z)[1+\varepsilon(\omega)]dz} \right) d\tau \right] ds \right\} d\omega \right\}$$

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[8] These definitions imply:
$\int_0^1 L_I(t)[1+\varepsilon(\omega)]d\omega = L_I(t) = L(t)\int_0^1 l_m(t)[1+\alpha(\omega)]d\omega = L(t)l_m(t)$
where $L(t)$ denotes the mass of agents in the economy at time $t$. With reference to Section 2 the following relation between $l_j$ and $l_m$ holds:
$\int_0^1 \sum_j l_j(\omega,t)\, d\omega = L(t)l_m(t)$.

[9] As in the standard quality-ladder models, here the agent is still assumed to be risk-averse, and to able to completely diversify her portfolio - by means of the intermediation of costless financial institutions. In fact, in order to carry out this diversification, it is sufficient to allocate investments in a non-zero measure interval of R&D sectors (and not necessarily in the whole of them), according to a measure that is absolutely continuous with respect to the Lebesgue measure of the sector space. Ambiguity here affects the mean return of the R&D investment and not its volatility, against which the agent has already completely hedged.

[10] As we show below, this does not result in any loss of generality.

$$s.t. \qquad \int_0^1 \varepsilon(\omega)d\omega = 0; \qquad \int_0^1 \alpha(\omega)d\omega = 0;$$

$$\alpha(\omega) \in [-1, \infty); \qquad \varepsilon(\omega) \in [-1, \infty).$$

Notice that the first addend of the maximand is constant with respect to $\alpha(\cdot)$ and $\varepsilon(\cdot)$, so that it does not affect the solution of the problem.

This problem admits the same solution for a however small probability $p$. In order to prove that the unique equilibrium is given by $\alpha(\omega) = \varepsilon(\omega) = 0 \ \forall \omega \in [0, 1]$, we will proceed through the following steps (the reader can refer to Figure 1, where $c_1$,$c_2$,$c_3$,$c_4$ represent the agent's pay-offs).

1. We will first prove that, if the agent plays a symmetric strategy, $\alpha(\omega) = 0$ $\forall \omega \in [0, 1]$, then the worst harm Nature can inflict to the agent is also associated with a symmetric strategy, $\varepsilon(\omega) = 0 \ \forall \omega \in [0, 1]$ (that is, with reference to Figure 1: $c_1 < c_2$).

2. We will then prove that, if Nature chooses $\varepsilon(\omega) = 0 \ \forall \omega \in [0, 1]$, the pay-off the agent will obtain is independent of her investment strategy (that is, $c_1 = c_3$).

3. We will finally show that, if the agent plays an asymmetric strategy, $\alpha(\omega) \neq 0$ in a non-zero measure set, then the worst harm Nature can inflict to the agent is also associated with an asymmetric strategy, $\varepsilon(\omega) \neq 0$ in a non-zero measure set (with reference to Figure 1: $c_4 < c_3$).

Then the configuration given by $\alpha(\omega) = \varepsilon(\omega) = 0 \ \forall \omega \in [0, 1]$ will emerge as the unique equilibrium of the zero-sum game (since $c_2 > c_1 = c_3 > c_4$). Let us proceed step by step.

1. $(c_1 < c_2)$. If $\alpha(\omega) = 0 \ \forall \omega \in [0, 1]$, we first show that the function:

$$\Phi \equiv \int_0^1 l_m \left\{ \int_{t_0}^{+\infty} \pi(s) \exp\left[ -\int_{t_0}^{s} \left( r(\tau) + \frac{AL_I(\tau)[1+\varepsilon(\omega)]}{X(t_0)+\mu A \int_{t_0}^{\tau} L_I(z)[1+\varepsilon(\omega)]dz} \right) d\tau \right] ds \right\} d\omega$$

is a sum over $\omega$ of strictly convex functions in $\varepsilon(\omega)$. In fact, set:

$$f(\varepsilon, \tau) \equiv - \left( r(\tau) + \frac{AL_I(\tau)[1+\varepsilon(\omega)]}{X(t_0)+\mu A \int_{t_0}^{\tau} L_I(z)[1+\varepsilon(\omega)]dz} \right)$$

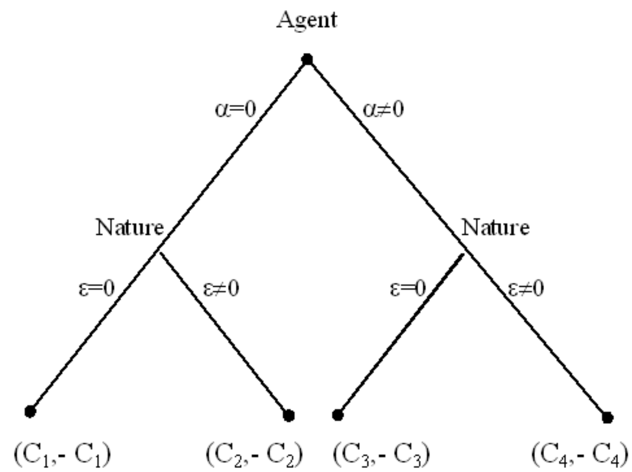


Figure 1: The Game between the Agent and Nature

Since $\dfrac{\partial^2 f(\varepsilon,\tau)}{\partial\varepsilon^2} > 0$, then $f(\varepsilon,\tau)$ is strictly convex[11]. As a result, the function $F(\varepsilon,s) \equiv \int_{t_0}^{s} f(\varepsilon,\tau)d\tau$, as a sum of strictly convex functions, is also strictly convex, that is: $\dfrac{\partial^2 F(\varepsilon,s)}{\partial\varepsilon^2} > 0$. Now, for each $s \in [t_0,+\infty]$, we can define: $H(\varepsilon,s) \equiv l_m\pi(s)\exp\left[F(\varepsilon,s)\right]$. $H(\varepsilon,s)$ is also strictly convex, being a positive transformation of the exponential of a strictly convex function; and so it is the sum of all $H(\varepsilon,s)$ over $t \in [t_0,+\infty)$. Finally, $\Phi(\varepsilon(\omega)) = \int_0^1 \int_{t_0}^{+\infty} H(\varepsilon,s)dsd\omega \equiv \int_0^1 G(\varepsilon(\omega))d\omega$ is a sum over $\omega$ of continuous and strictly convex functions, $G(\cdot)$, of $\varepsilon$. Notice that $\Phi(\cdot)$ is an operator transforming measurable real functions into real numbers, whereas $H(\cdot)$ and $G(\cdot)$ are functions transforming real numbers into real numbers.

Let $0(\cdot)$ be the function that is identically equal to zero, i.e. $0(\omega) = 0$ for all $\omega \in [0,1]$. We want to show that the minimum value of $\Phi(\cdot)$ occurs when $\varepsilon(\cdot) = 0(\cdot)$, that is, when $\varepsilon(\omega) = 0$, $\forall\omega \in [0,1]$.

Let $\omega_1 = \frac{1}{N}$, $\omega_2 = \frac{2}{N}$,..., $\omega_N = 1$, with $N > 0$ being an integer number. By definition of convexity we have:

$$G(\alpha_1\varepsilon(\omega_1) + \alpha_2\varepsilon(\omega_2) + .. + \alpha_N\varepsilon(\omega_N)) \leq \alpha_1 G(\varepsilon(\omega_1)) + \alpha_2 G\left(\varepsilon(\omega_2)\right) + .. + \alpha_N G\left(\varepsilon(\omega_N)\right),$$

[11]It is $\dfrac{\partial^2 f(\varepsilon,\tau)}{\partial\varepsilon^2} = \dfrac{2A^2 L_I(\tau)\mu X(t_0)\int_{t_0}^{\tau} L_I(z)dz}{\left(A\mu(1+\varepsilon(\omega))\int_{t_0}^{\tau} L_I(z)dz + X(t_0)\right)^3} > 0$ as $\varepsilon(\omega) \geq -1$.

with $\sum_{i=1}^{N}\alpha_i = 1$. Let us posit $\alpha_1 = \alpha_2 = .. = \alpha_N = 1/N$, then we have:

$$G\left(\sum_{i=1}^{N}\varepsilon(\omega_i)\frac{1}{N}\right) \leq \frac{1}{N}\sum_{i=1}^{N}G\left(\varepsilon(\omega_i)\right).$$

By the continuity of $G(\cdot)$ and the definition of integral, it is:

$$\lim_{N\to\infty} G\left(\sum_{i=1}^{N}\varepsilon(\omega_i)\frac{1}{N}\right) = G\left(\int_0^1 \varepsilon(\omega)d\omega\right) \leq \lim_{N\to\infty}\frac{1}{N}\sum_{i=1}^{N}G\left(\varepsilon(\omega_i)\right) = \int_0^1 G(\varepsilon(\omega))d\omega.$$

Noting that $\int_0^1 \varepsilon(\omega)d\omega = 0$ and that $\int_0^1 G(0)d\omega = G(0)$ it follows that:

$$\Phi(0(\cdot)) = \int_0^1 G\left(\int_0^1 \varepsilon(\omega)d\omega\right)d\omega = G(0) \leq \int_0^1 G(\varepsilon(\omega))d\omega = \Phi(\varepsilon(\omega))$$

for all measurable functions $\varepsilon(\cdot)$. This implies that $\varepsilon(\cdot) = 0(\cdot)$ is the minimizing configuration of $\varepsilon$ satisfying $\int_0^1 \varepsilon(\omega)d\omega = 0$. The pay-off, obtained by setting $\varepsilon(\omega) = \alpha(\omega) = 0\ \forall\omega \in [0,1]$ in $\Phi$ is then the one that the agent can surely obtain if she plays a symmetric strategy.

2. $(c_1 = c_3)$. If $\varepsilon(\omega) = 0$, $\forall\omega \in [0,1]$, then the agent would be totally indifferent in the allocation of her R&D efforts. In fact, the maximum problem obtained by setting $\varepsilon(\omega) = 0\ \forall\omega \in [0,1]$ is (under the usual constraints):

$$\max_{\alpha(\cdot)}\int_0^1 l_m[1+\alpha(\omega)]\int_{t_0}^{+\infty}\pi(s)\exp\left[-\int_{t_0}^{s}\left(r(\tau)+\frac{AL_I(\tau)}{X(t_0)+\mu A\int_{t_0}^{\tau}L_I(z)dz}\right)d\tau\right]ds\,d\omega,$$

which, since $\int_0^1 \alpha(\omega)d\omega = 0$, always gives the same constant value:

$$l_m\int_{t_0}^{+\infty}\pi(s)\exp\left[-\int_{t_0}^{s}\left(r(\tau)+\frac{AL_I(\tau)}{X(t_0)+\mu A\int_{t_0}^{\tau}L_I(z)dz}\right)d\tau\right]ds.$$

3. $(c_4 < c_3)$. Assume $\alpha(\omega) \neq 0$ for some non zero measure set of $\omega \in [0,1]$. Then the Nature's minimum problem with respect to $\varepsilon(\cdot)$ can be stated as follows:

$$\min_{\varepsilon(\cdot)}\int_0^1 l_m[1+\alpha(\omega)]\left\{\int_{t_0}^{+\infty}\pi(s)\exp\left[-\int_{t_0}^{s}\left(r(\tau)+\frac{AL_I(\tau)[1+\varepsilon(\omega)]}{X(t_0)+\mu A\int_{t_0}^{\tau}L_I(z)[1+\varepsilon(\omega)]dz}\right)d\tau\right]ds\right\}d\omega$$

$$s.t. \quad \int_0^1 \varepsilon(\omega) d\omega = 0$$

The solution to this problem is $\varepsilon[\alpha(\omega)]$, which is the reaction function of Nature, that is, its optimal (minimizing) response to any possible value of $\alpha(\omega)$. We do not need, however, to find it explicitly since our conclusion follows straightforwardly. We can build the Lagrangian and then derive the first-order conditions (f.o.c.):

$$L = \int_0^1 l_m[1+\alpha(\omega)] \left\{ \int_{t_0}^{+\infty} \pi(s) \exp\left[ -\int_{t_0}^{s} \left( r(\tau) + \frac{AL_I(\tau)[1+\varepsilon(\omega)]}{X(t_0)+\mu A \int_{t_0}^{\tau} L_I(z)[1+\varepsilon(\omega)]dz} \right) d\tau \right] ds \right\} d\omega + \zeta \int_0^1 \varepsilon(\omega) d\omega$$

For every $\omega \in [0,1]$, the f.o.c. with respect to $\varepsilon$ are:

$$l_m[1+\alpha(\omega)] \int_{t_0}^{+\infty} \pi(s) \exp\left[ -\int_{t_0}^{s} \left( r(\tau) + \frac{AL_I(\tau)[1+\varepsilon(\omega)]}{X(t_0)+\mu A \int_{t_0}^{\tau} L_I(z)[1+\varepsilon(\omega)]dz} \right) d\tau \right] ds \cdot$$

$$\cdot \left[ -\int_{t_0}^{s} \frac{AL_I(\tau)X(t_0)}{\left( X(t_0)+\mu A(1+\varepsilon(\omega)) \int_{t_0}^{\tau} L_I(z)dz \right)^2} d\tau \right] = -\zeta$$

It results that, if $\alpha(\omega) \neq 0$ for some $\omega \in [0,1]$, and if the constraint $\int_0^1 \alpha(\omega) d\omega = 0$ holds, the necessary conditions for a minimum can never be satisfied if $\varepsilon[\alpha(\omega)] = 0$ $\forall \omega \in [0,1]$[12]. ■

The intuition of the result is as follows. Under symmetric expectations ($\varepsilon = 0$) the agent is indifferent as to where targeting her investment ($c_1 = c_3$); this has been the starting point of our paper. The agent also knows that, when investing symmetrically, the corresponding pay-off ($c_1$) is also the minimum that she can obtain: in fact, if future investment turns out to be asymmetric ($\varepsilon \neq 0$) she will be better off ($c_2 > c_1$) given the convexity of the pay-off function in $\varepsilon$. On the contrary, when allocating investment asymmetrically ($\alpha \neq 0$), even with a slight probability ($p \to 0$) that a future non-symmetric distribution will arise ($\varepsilon \neq 0$), our agent will expect to be targeting above average exactly those sectors that will subsequently experience above average innovative efforts, thus lowering the expected payoff as compared to the symmetric investment case ($c_4 < c_3$). As a result, since the worst that can happen while investing

[12] In fact, consider an economy with only two sectors, $\omega_1$,$\omega_2$. If it were $\varepsilon(\omega_1) = \varepsilon(\omega_2) = 0$, the satisfaction of the f.o.c. and the constraint would require $\alpha(\omega_1) = \alpha(\omega_2)$ and $\alpha(\omega_1) + \alpha(\omega_2) = 0$, which proves that there cannot exist $\omega$ where $\alpha(\omega) \neq 0$.

symmetrically ($c_1$) is always better than the worst that can happen while choosing a (whatever) asymmetric allocation of investment ($c_4$), our "cautious" agent will always strictly prefer the first option.

Notice that the fact that the symmetric equilibrium is being derived at the beginning of time $t = t_0$ does not result in any loss of generality. In fact, this equilibrium guarantees that the difficulty index $X(\omega, t)$ starts growing at the same rate - and is therefore always equal - across sectors. This condition in turn assures that, at any point in time $t$, the agent continuously faces a decision problem equivalent to the one we have analyzed and, hence, continuously finds the same optimal (symmetric) solution. Notice also that our result holds even when the "punishment power" of Nature ($\varepsilon(\omega)$) is restricted to be however small. The proof is straightforward: given $\varepsilon(\omega) \in [-\eta, \eta]$ $\forall \eta \in (0, 1)$, steps 1 and 2 of the proof are clearly unaffected. For step 3 notice that $\varepsilon(\omega) = 0$ is always an inner point of the domain and, hence, the non-fulfillment of the f.o.c. guarantees that it is not a minimum.

We have shown that, even though the agent is "almost sure" ($p \to 0$) of facing a symmetric configuration of future investment (which would leave her in a position of indifference in her current allocation problem), the mere possibility of a slightly different configuration ($\varepsilon \to 0$) makes her strictly prefer to equally allocate her investment across sectors. The symmetric equilibrium then emerges as the unique optimal investment allocation.

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