Do Sunspots Matter under Complete Ignorance?[*]

Guido Cozzi[†] **Paolo E. Giordani**[‡]

Abstract

In a two-period, sunspot, pure-exchange economy we analyze the case in which agents do not assign subjective probabilistic beliefs to the 'sunspot activity'. Two generations, each of which is made up of identical agents, populate this economy. The participation in the Arrow securities market is restricted and the generation, which is allowed to trade in assets, can alternatively face uncertainty via two distribution-free decision rules under 'complete ignorance' (axiomatized by Milnor (1954)): the 'minimax regret criterion' (Savage (1954), ch.9) and the 'maxmin return criterion' (Wald (1950)). When the former is used, then sunspots can matter. In particular we prove that, if the economy admits two Walrasian equilibria, then a unique sunspot equilibrium always exists. We pin down this equilibrium, determine the prices of the Arrow securities and show that, at these prices, no trade in securities takes place. In the same framework we prove that, with agents using the maxmin return criterion, sunspots do not matter.

Keywords: General Equilibrium, Extrinsic Uncertainty, Complete Ignorance.

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[†] University of Rome "La Sapienza", Dipartimento di Economia Pubblica, via del Castro Laurenziano 9, 00161 Roma. Tel. +39-0649766302, E-mail: gcozzi@dep.eco.uniroma1.it

[‡] European University Institute, Robert Schuman Centre for Advanced Studies (RSCAS), via delle fontanelle 20, 50016 Florence (Italy). Tel.: +390554685812. E-Mail: paolo.giordani@iue.it

1 Introduction

In a seminal paper Cass and Shell (1983, JPE) prove that, in a simple general equilibrium model with overlapping generations, 'extrinsic uncertainty' - that is, uncertainty not affecting fundamentals - may play a role in determining the equilibrium allocation. In their model 'sunspots matter' because the overlapping generations structure of the model brings about restricted participation in the Arrow securities market[1].

We develop a two-period pure-exchange general equilibrium model much in the spirit of Cass and Shell (1983). Two generations, each of which is made up of identical agents, populate the economy and the participation in the Arrow securities market is restricted to the one born in the first period.

'Strong uncertainty' seems a promising way to qualify purely 'extrinsic uncertainty', and to represent the possibility of an agent's 'fuzzy perception' of the sunspot activity[2]. In our model the agents trading in assets do not assess subjectively the probability of the realization of the different states of nature generated by 'extrinsic uncertainty'. In this choice scenario of *complete ignorance* (Luce and Raiffa (1958)), these agents can alternatively select their optimal consumption bundle via two 'non-probabilistic' decision rules: the 'minimax regret criterion' (MR henceforth, Savage (1954), ch.9) and the 'maxmin return criterion' (MM, Wald (1950))[3].

We show that, in a world populated by decision makers who care about minimizing their maximum regret, sunspots matter. In particular, we prove that, in an economy admitting two Walrasian equilibria, it is always possible to build a unique sunspot equilibrium. We determine the equilibrium prices of the Arrow securities and show that, at these prices, no trade in securities will take place. In the same framework we prove that, with the agents confronting extrinsic uncertainty via the maxmin return criterion, sunspots do not matter.

The rest of the paper is organized as follows. In the next Section we introduce the two decision rules, the MR and the MM. Section 3 describes our simple pure-exchange economy. In Section 4 we prove our results.

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[1] In the same work the authors also prove that, even though participation is not restricted, sunspots can matter if economic agents have heterogeneous beliefs on the 'sunspot activity'.

[2] A contribution in this direction is Tallon (1998): in his model agents are assumed to be Choquet-expected-utility maximizers (see Schmeidler (1989)).

[3] Both rules have been axiomatized by Milnor (1954). Interest in the maxmin has recently grown since Gilboa and Schmeidler (1989) provided an axiomatic extension of this criterion to a multiple-prior framework. Much in the same spirit Hayashi (2005) has recently axiomatized minimax regret and introduced the concept of 'regret aversion'.

2 The Minimax Regret Criterion (MR) and the Maxmin Return Criterion (MM)

Decision makers (DMs) using the minimax regret criterion and those using the maxmin return criterion have one salient feature in common: they determine their preference order without forming any prior over the set of the states of nature. This characteristic marks an essential difference between these agents and the 'subjective expected utility' maximizers, who always assign probabilistic beliefs to the randomness which they face. Formally there are two alternative sets of axioms that each uniquely determine each of the two criteria (Milnor (1954))[4]. The differences in the two sets lie behind the different choice behavior that each of them prescribes and that we are going to recall briefly in what follows.

A decision problem under uncertainty can be represented through a 'decision table', such as the one below:

States of nature

Actions

	S_1	S_2	...	S_n
A_1	c_{11}	c_{12}	...	c_{1n}
A_2	c_{21}	c_{22}	...	c_{2n}
...	...	...	...	...
A_m	c_{m1}	c_{m2}	...	c_{mn}

Table 1. *The generic decision table.*

where A_i (for $i = 1, ..., m$) represents the generic action, S_j (for $j = 1, ..., n$) represents the generic state of nature and c_{ij} is the consequence (pay-off) associated with act i and state j. The decision process driven by the MR can now be split in three stages. In the first stage, the DM computes the regret associated with any given pair action/state by subtracting the consequence corresponding to that pair from the best consequence that is achievable in the same state, that is:

$$r_{ij} = \max_{l \in [1,m]} \{c_{lj}\} - c_{ij}$$

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[4] Given a set of six common axioms, maxmin builds on the Independence of Irrelevant Alternatives, while minimax regret only satisfies a weaker version of that axiom, the Independence of Never Best Reply, plus an axiom, called by Milnor (1954) Column Linearity, which in turn is not satisfied by maxmin.

The result of this process is the 'matrix of regrets', which by construction has the same dimension as the starting decision table. In the second stage the DM associates with each action the maximum regret over all states of nature, that is:

$$\rho_i = \max_{j \in [1,n]} \{r_{ij}\}$$

Finally she selects the action for which the maximum regret (associated with each action) is the smallest, that is:

$$\text{choose } A_k \text{ such that } \rho_k = \min_{i \in [1,m]} \{\rho_i\} = \min_{i \in [1,m]} \left\{ \max_{j \in [1,n]} \{r_{ij}\} \right\} \qquad \text{(MR)}$$

Hence an MR agent does not care about the outcome per se but about not missing profitable opportunities. Regret exactly measures the loss associated with making the 'wrong choice' for given state of nature: the agent chooses the action which minimizes that loss.

With the maxmin return criterion, the DM for each action identifies the worst possible state (that is, the state associated with the minimum pay-off):

$$s_i = \min_{j \in [1,n]} \{c_{ij}\}$$

where s_i is usually called the 'security level' of action A_i. Then she chooses the action for which this security level is the highest:

$$\text{choose } A_k \text{ such that } s_k = \max_{i \in [1,m]} \{s_i\} = \max_{i \in [1,m]} \left\{ \min_{j \in [1,n]} \{c_{ij}\} \right\} \qquad \text{(MM)}$$

The maximin agent has thus a 'highly conservative' attitude towards uncertainty, and acts as if the worst state of nature were certain to occur. The fear of being punished by a 'malevolent Nature' leads her to choose the best action under the 'worst case' belief. Intutively, while minimax regret is associated with 'aversion to lost opportunities', maxmin is associated with 'extreme pessimism'.

3 The Model

We consider a simple pure-exchange economy lasting two periods, $\tau = 0, 1$ and characterized by l commodities, and two states of nature, $s = \alpha, \beta$. The uncertainty generated by the existence of these two states is 'extrinsic', in the sense that it does not affect any fundamentals (preferences and endowments). There are two distinct generations of identical agents[5], G_0 born in period 0 and living to the end of time, and G_1 born in period 1 and also living to the end of time. The agents of both generations evaluate their consumption bundles via smooth, strictly increasing and strictly concave utility functions $U_h^{\alpha}(\cdot) \equiv U_h^{\beta}(\cdot) \equiv U_h(\cdot)$ for $h = G_0,G_1$. Endowments are represented by the vector $\omega_h(s) \equiv \omega_h$, while consumption bundles by $x_h(s)$, for $h = G_0, G_1$, $s = \alpha, \beta$. We denote the prices of the l commodities as the vector $p_c(s)$.

The timing of the model is the same as in Cass and Shell (1983). After their birth in period 0, the agents of generation G_0 are allowed to trade in Arrow securities, which are contingent upon the realization of the extrinsic random variable. The amount of the s-contingent security bought - sold, if negative - by agent h in G_0 is $b_h(s)$ and its price is $p_b(s)$. At the end of period 0, before the birth of generation G_1, sunspot activity is observed (that is, people realize which state of nature has actually occurred). When both generations are alive in period 1, they trade in spot commodities and, finally, consume their bundles. As is well known, with completely extrinsic uncertainty, if an equilibrium exists in which $x_h(\alpha) \neq x_h(\beta)$ for some h, then sunspots matter.

4 The Search for Sunspot Equilibria under MR and MM

Suppose that, for given fundamentals, the economy described above admits two distinct Walrasian equilibria, and that the 'extrinsic uncertainty' that the agents in G_0 perceive corresponds to these two equilibria. We then index them as equilibrium 'α' with quantities and prices respectively given by $x_h^*(\alpha), p_c^*(\alpha)$, and equilibrium '$\beta$' with quantities and prices respectively given by $x_h^*(\beta), p_c^*(\beta)$, for h in G_0,G_1.

Agents in G_0 do not know the probability distribution over the two states of nature α,β, and their choice under 'complete ignorance' is alternatively driven by the MR and the MM. Let us verify whether any other - sunspot-driven - equilibrium exists in this economy. Since G_1-type agents make their consumption choices after 'extrinsic uncertainty' has been resolved, they simply maximize their utility function under the usual budget constraints. On the contrary, G_0- agents can trade in Arrow securities and, then, must decide whether and, possibly, which amount $b_h(s)$ of assets to buy/sell

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[5] That is to say, the identical agents of one generation are *different* from the identical agents of the other.

in period 0, before sunspot activity is revealed.

Let us define the state-contingent pay-offs among which agent h in G_0 can choose. If agent h selects her optimal amount $b_h(\cdot)$ of Arrow security for a given price vector $p_b(\cdot)$, the pay-off she obtains can be summarized by the following indirect utility functions:

$$v_h^\alpha = v_h\left[p_c(\alpha), p_c(\alpha)\omega_h + b_h(\alpha)\right] \text{ if } \alpha \text{ occurs and:}$$

$$v_h^\beta = v_h\left[p_c(\beta), p_c(\beta)\omega_h + b_h(\beta)\right] \text{ if } \beta \text{ occurs}$$

where:

$$-\omega_h p_c(\alpha) \leq b_h(\alpha) = -b_h(\beta)\frac{p_b(\beta)}{p_b(\alpha)} \leq p_c(\beta)\omega_h \frac{p_b(\beta)}{p_b(\alpha)}$$

In particular, if this agent decides to employ all her income in buying a positive amount of α-contingent security at the price $p_b(\alpha)$, her return is[6]:

$$v_h^{\alpha(\max)} = v_h\left[p_c(\alpha), \omega_h\left(p_c(\alpha) + \frac{p_b(\beta)}{p_b(\alpha)}p_c(\beta)\right)\right]$$

if state α occurs and

$$v_h[p_c(\beta), 0]$$

if state β occurs. Analogously, if she decides to employ all her income in buying a positive amount of β-contingent security at the price $p_b(\beta)$, her return is:

$$v_h^{\beta(\max)} = v_h\left[p_c(\beta), \omega_h\left(p_c(\beta) + \frac{p_b(\alpha)}{p_b(\beta)}p_c(\alpha)\right)\right]$$

if state β occurs and

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[6] These functions are determined by solving, for $s, t = \alpha, \beta$ and $s \neq t$ the following maximum problem:

$$\max_{x_h} U[x_h(s)]$$

$$\text{s.t. } p_c(s)x_h(s) = p_c(s)\omega_h + b_h(s),$$

$$\text{s.t. } b_h(s) = p_c(t)\omega_h \frac{p_b(t)}{p_b(s)}.$$

$$v_h[p_c(\alpha), 0]$$

if state α occurs.

Finally, if agent h in G_0 does not trade in assets, the utilities she gains are those associated with the two deterministic equilibria: $U_h[x_h^*(\alpha)]$ if α occurs and $U_h[x_h^*(\beta)]$ if β occurs.

It is now possible to define the expression $v_h^{s(\max)} - v_h^s$ as the generic regret associated with an amount $b_h(\cdot)$ of Arrow security for agent h when state s has occurred. In a general equilibrium framework the optimization under uncertainty via the MR requires that all the regrets be equalized across all states of nature. With two states the following 'optimum condition' must hold:

$$v_h^{\alpha(\max)} - v_h^{\alpha} = v_h^{\beta(\max)} - v_h^{\beta} \tag{1}$$

Just to give an intution to the equation above, refer to table 2 and suppose that, for a given price, agent h has bought an amount of Arrow security $\tilde{b}_h(\beta) > 0$ such that $v_h^{\alpha(\max)} - \tilde{v}_h^{\alpha} > v_h^{\beta(\max)} - \tilde{v}_h^{\beta}$. Since an increase in utility in state α (β) is inevitably associated with a decrease in state β (α) - for the obvious reason that, in order to buy one asset you must sell the other -, and since what matters under the MR is *the maximum regret across the states*, agent h would find it profitable to start selling that security until the two regrets would converge towards each other. Only when (1) holds exactly, there is no more incentive to trade in assets, as the maximum regret is at its minimum.

	α	β
$b_h^{(\max)}(\alpha)$	$v_h^{\alpha(\max)}$	$v_h[p_c(\beta), 0]$
$b_h^{(\max)}(\beta)$	$v_h[p_c(\alpha), 0]$	$v_h^{\beta(\max)}$
$\tilde{b}_h(\beta)$	$\tilde{v}_h^{\alpha}$	$\tilde{v}_h^{\beta}$
$b_h(\cdot)$	v_h^{α}	v_h^{β}
...	...	...

Table 2. *The choice of Arrow Securities*

Analogously the 'optimum condition' under MM requires that the minima be directly equalized across all states of nature. With two states only, the condition is:

$$v_h^{\alpha} = v_h^{\beta} \tag{2}$$

We can now state the two following propositions.

Proposition 1 *If agents in G_0 make use of the minimax regret criterion, a unique sunspot equilibrium always exists in the economy. The vector of the equilibrium prices of the Arrow securities $[p_b^*(\alpha); p_b^*(\beta)]$ is such that no trade in Arrow securities takes place in equilibrium, i.e. $b_h(s) = 0$. Moreover, the prices of the l commodities, $[p_c^*(\alpha); p_c^*(\beta)]$, and the consumption allocations, $x_h^* = [x_h^*(\alpha); x_h^*(\beta)]$ for h in G_0, G_1, are those corresponding to the two Walrasian equilibria.*

Proof. We prove our result in two stages. In the first we show that, if an equilibrium exists, it must be characterized by no trade in Arrow securities. In the second this equilibrium is pinned down and the equilibrium prices of Arrow securities are found.

1. By definition of Arrow securities, an equilibrium must be characterized by:

$$\sum_{h \in G_0} b_h(s) = 0 \text{ for } s = \alpha, \beta \tag{3}$$

Moreover, since agents in G_0 are identical, then in equilibrium all individuals choose the same unique optimal portfolio. Hence:

$$b_h(s) = \bar{b}(s) \ \forall h \text{ in } G_0 \tag{4}$$

Eq.s (3) and (4) imply $b_h(s) = 0$. Then, if an equilibrium exists, it must be characterized by no trade in Arrow securities.

2. Indeed, the unique consumption allocation compatible with no trade in Arrow securities is the pair $x_h^* = [x_h^*(\alpha); x_h^*(\beta)]$. Now we prove that a vector of Arrow securities' prices $p_b^* = [p_b^*(\alpha); p_b^*(\beta)]$ always exists, which renders the vectors of prices $p^* = [p_c^*(\alpha); p_c^*(\beta); p_b^*(\alpha); p_b^*(\beta)]$ and of allocations $x_h^* = [x_h^*(\alpha); x_h^*(\beta)]$ for h in G_0, G_1 the unique sunspot equilibrium of our economy.

Since agents apply the MR, and since in equilibrium it holds $p_b(\alpha) + p_b(\beta) = 1$, Arrow securities' prices are determined via the following system:

$$\begin{cases} v_h^{\alpha(\max)} - U_h[x_h^*(\alpha)] = v_h^{\beta(\max)} - U_h[x_h^*(\beta)] \\ p_b(\alpha) + p_b(\beta) = 1 \end{cases} \tag{5}$$

The first equation of system (5) equalizes the regret associated with the consumption bundle $x^*(\alpha)$ to the one associated with $x^*(\beta)$. It is a special case of the 'optimum condition' under MR (equation (1)), obtained when $b_h(s) = 0$.

Continuity and monotonicity of the utility function $U_h(\cdot)$, for h in G_0, constitute sufficient conditions for the existence of a solution $0 < p_b^*(s) < 1$, for $s = \alpha, \beta$, in system (5). In fact (for a graphical intuition of this result see figure 1):

$$\lim_{p_b(\alpha)\to 0} v_h^{\alpha(\max)} - U_h[x_h^*(\alpha)] > 0; \lim_{p_b(\alpha)\to 0} v_h^{\beta(\max)} - U_h[x_h^*(\beta)] = 0$$

and:

$$\lim_{p_b(\alpha)\to 1} v_h^{\alpha(\max)} - U_h[x_h^*(\alpha)] = 0; \lim_{p_b(\alpha)\to 1} v_h^{\beta(\max)} - U_h[x_h^*(\beta)] > 0$$

Since it generically holds $x_h^*(\alpha) \neq x_h^*(\beta)$, the equilibrium is characterized by sunspot activity. ■

The theoretical underpinnings of the minimax regret criterion can give explanation for a widespread attitude towards choice encountered in the real consumption markets. As recently pointed out by McFadden (2006), experimental evidence suggests the prevalence of *agoraphobic* consumers, that is, of consumers who "fear markets and find choice troubling". Agoraphobia, literally 'fear of the marketplace', refers exactly to the sense of bewilderment and unease that the consumers experience when confronting several alternatives of choice. This feeling is in fact a major characteristic of minimax-regret agents. Unlike maxmin agents or Bayesians, who *always* weakly prefer that an alternative be added to their set of actions, minimax-regret agents do not: for instance, suppose they confront one action only (and, hence, have no real choice to make). In that case they reach the minimum regret (equal to 0): any other action to be added would weakly worsen them off (*weakly* because adding up a dominated action would leave them indifferent). The reason for their agoraphobia lies in the regret associated with missing (possibly profitable) opportunities: no opportunity means no trouble[7]. In this choice scenario we have proven that purely extrinsic uncertainty may play a role in the final equilibrium allocation, and that the emergence of a *social norm* (sunspot) may drive the *agoraphobic agents*' optimal consumption choices.

Proposition 2 *If agents make use of the maxmin return criterion, then sunspots do*

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[7] According to McFadden, agoraphobic consumers reason well in accord with a Dutch proverb saying: "He who has choice has trouble".

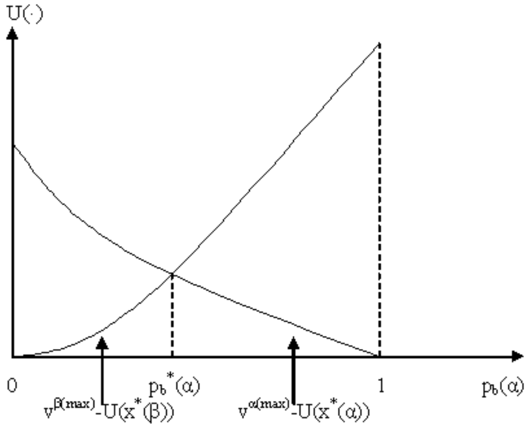

Figure 1:

not matter.

Proof. The first part of the proof is exactly the same as the one in the previous proposition, in which we have shown that in equilibrium no trade in Arrow securities can take place ($b_h(s) = 0$). However, since in general $U[x^*_h(\alpha)] \neq U[x^*_h(\beta)]$, $x^* = [x^*(\alpha); x^*(\beta)]$ cannot be the optimal solution for this decision rule (recall equation (2)). In fact, for every vector of asset prices p'_b, G_0-agents would be better off by buying an amount of securities $b'_h(\alpha) \neq 0$ such that:

$$\begin{cases} v_h\left[p_c(\alpha), p_c(\alpha)\omega_h + b'_h(\alpha)\right] = v_h\left[p_c(\beta), p_c(\beta)\omega_h - b'_h(\alpha)\dfrac{p'_b(\alpha)}{p'_b(\beta)}\right] \\ p_b(\alpha) + p_b(\beta) = 1 \end{cases} \tag{6}$$

This configuration is however not sustainable in equilibrium, since it would imply trade in asset markets, while it must necessarily be $b^*_h(s) = 0$ $\forall h$ in G_0. Hence a sunspot equilibrium does not exist. ■

We have proven that in our economy market equilibria among maxmin agents are sunspot-free. The conservative attitude of these agents in the face of uncertainty and, consequently, their willingness to insure completely are at the root of this immunity result. Hence, extrinsic uncertainty will not affect consumption choices and the final equilibrium allocation.

References

[1] Cass, D. and K. Shell (1983). “Do Sunspots Matter?”. *Journal of Political Economy* 91, 193-227.

[2] Gilboa, I. And D. Schmeidler (1989). “Maxmin Expected Utility with Non-Unique Prior”. *Journal of Mathematical Economics* 18, 141-153.

[3] Hayashi, T. (2005). “Regret Aversion and Opportunity Dependence”. Mimeo.

[4] Luce, D.K. and H. Raiffa (1958). “Games and Decisions". Wiley, New York.

[5] McFadden, D (2006). “Free Markets and Fettered Consumers”. *American Economic Review* 96, 5-29.

[6] Milnor, J. (1954). “Games against Nature”. In R. Thrall, C. Coombs and R. Davis (Eds), *Decision Processes.* London: John Wiley, 49-60.

[7] Savage, L.J. (1954). “The Foundations of Statistics”. Wiley, New York.

[8] Schmeidler, D. (1989). “Subjective Probability and Expected Utility without Additivity”. *Econometrica* 57, 571-587.

[9] Tallon, J.M. (1998). “Do Sunspots Matter When Agents Are Choquet-Expected-Utility Maximizers?”. J*ournal of Economic Dynamics and Control* 22, 357-368.

[10] Wald, A. (1950). “Statistical Decision Functions”. Wiley, New York.