Spatial Competition in Quality and Schumpeterian Growth*

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Abstract

We develop a general equilibrium model of vertical innovation in which multiple firms compete monopolistically in the quality space. The model features many firms that each hold the monopoly to produce a unique quality level of an otherwise homogenous good and consumers who are heterogeneous in their valuation of the good's quality. If the marginal cost of production is convex with respect to quality, multiple firms coexist and their equilibrium markups are determined by the degree of convexity and the density of quality-competition. To endogenize the latter, we nest this industry setup in a Schumpeterian model of endogenous growth. Each firm enters the industry as the technology leader and successively transits through the product cycle as it becomes superseded by further innovations. Our setup does not necessarily feature business "stealing" in the sense that already marginal innovations grant non-negligible profits. Rather, innovators sell only to a set of consumers that was served relatively poorly by pre-existing firms. Thus, the intrinsic reason of why innovation happens in our economy is not one of displacing the incumbent, but rather, innovation is a means to differentiate oneself from existing firms. Never the less, "creative destruction" prevails, albeit of a different form than in the standard Schumpeterian setup: new entrants make the set of available goods more differentiated, thereby exerting a pro-competitive effect on the entire industry.

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1 Introduction

Quality innovation is a major engine of economic growth. This is impressively testified by the extensive literature on endogenous growth in vertically differentiated markets, sparked by Segerstrom et al. (1990), Grossman and Helpman (1991a and 1991b), and Aghion and Howitt (1992).

This literature has, however, ignored the well-known fact that good quality is also an important dimension along which firms differentiate their products[1]. On the contrary, the sizeable theoretical growth literature remains focused on markets in which natural oligopolies prevail, i.e. markets that are dominated by a small number of "market leaders".

This limitation is striking considering that a large number of heterogeneous firms coexist in many of these vertically differentiated industries. For example, recent empirical studies in the field of international trade document that nearly all manufacturing industries are characterized by many firms with very heterogenous profits, and that the profit heterogeneity can to a large extent be explained by underlying heterogeneity in product quality.[2] Yet, there exists no general equilibrium framework to analyze how multiple firms compete monopolistically in the quality space.

The present paper aims to fill this gap. We set up a framework that enables us to analyze how multiple firms compete in the quality space. In particular, we show how firms' innovation decisions determine the density of quality supply, equilibrium markups and profits in a model of endogenous growth.

Our analysis proceeds in three steps. In the first one, we focus on monopolistic competition in the quality space. More precisely, we develop a model that is suitable to analyze the density of competition and firm markups in vertically differentiated markets characterized by a large number of active firms.

Our model explains how a large number of seemingly "inferior" low-quality firms can exist alongside the technological leader. The reason for their survival is that, although the highest quality good is preferred by all consumers, it also carries a higher price tag, which is not worth paying for consumers with relatively low valuation for quality.

We document that firms' market power arises if the marginal cost of production is

[1] See Mussa and Rosen (1978) or Shaked and Sutton (1982 and 1983).

[2] See, in particular Khandelwal (forthcoming) and Kugler and Verhoogen (2010), but also Baldwin and Harrigan (2007), Johnson (2007), Verhoogen (2008), and Hallak and Schott (2009).

convex with respect to quality. Consider, for example, three firms producing $q_1 < q_2 < q_3$ under a marginal cost schedule that is convex in quality (the cost increment per quality between production of q_3 and q_2 is larger than the cost increment between q_1 and q_2). Next, consider the range of consumers whose willingness to pay for additional quality exceeds the first increment but falls short of the second. These consumers receive a surplus by buying q_2 at marginal production cost instead of buying either q_1 or q_3 at any price exceeding the respective marginal production costs. Since the producers of q_1 and q_3 never sell below their marginal cost, the producer of good q_2 enjoys positive market power. In this way, convexity of the marginal cost schedule generates market power of individual firms.[3]

In the second part of our analysis, we endogenize the firms' location choice in the quality space and analyze the resulting degree of competition under constantly growing income and valuation for quality. Firms can incur a fixed cost to improve upon the existing qualities and are granted a perpetual patent to produce the quality level of their choice. Each firm enters the industry as the technology leader and successively transits through the product cycle as it becomes superseded by further innovations. The benefit of entering with a higher quality good and the cost of doing so both grow at constant rates so that all entering firms face a scaled but symmetric entry condition. We prove that in this setup, the conditions that are required for the economy to be on a balanced growth path imply that there is a dynamic equilibrium in which each new entrant chooses a quality that is a constant percentage higher than the incumbent technology leader.[4]

Upon market entry, a firm chooses its quality level. Doing so, it aims to distinguish its quality from those of the incumbents, since such isolation in quality increases market power and profits. Higher qualities, however, come at higher fixed and marginal production costs. While the former effect drives firms to pick 'remote' qualities, the latter one limits quality dispersion.

We also analyze how market size and the underlying technology parameters affect equi-

[3] Shaked and Sutton (1982 and 1983), as well as succesive work, focuses on the case where marginal costs is concave in quality, hence implying that only on firm can survie in equilibrium.

[4] In our setup with a clear ranking along the quality line, there is a unique top quality producer, whose first order condition differs from the first order conditions of the rest of the firms facing two competitors each. The latter fact substantially complicates our analysis and we thus do not consider a simultaneous entry game as in Vogel (2008). In models based on Hotelling (1929) one can avoid such border conditions since one can think of a circle street or the beach surrounding an island. In our setup, however, any attempt to "close the circle" must fail since it would amount to identifying the highest quality good to the lowest quality good.

librium quality spacing, prices and quantities. We find that larger markets induce more frequent firm entry and a higher density of quality supply, because higher sales and profits allow a faster recovery of setup costs. Markups, in turn, are decreasing in the density of supply and are thus decreasing in the market size.

Surprisingly, a proportional increase in the marginal cost of production for all firms in the industry by the same proportion is associated with a more densely supplied market. The reason is that in equilibrium, markups are proportional to costs. Thus, when production costs rise for all firms, profits actually increase for any given quality spacing. Excess profits cannot exist in equilibrium and, consequently, firms must exhibit denser quality spacing at tighter competition.

The third part of the analysis finally nests the above-described economy in a dynamic model of endogenous growth with vertical innovation a là Aghion and Howitt (1992). Our general equilibrium setup unveils a novel channel through which innovation generates "creative destruction." In setups following these standard models of Schumpeterian growth, such creative destruction is driven by the business stealing effect: entrants instantaneously take over the entire market even if they offer only marginally better quality. In our setup, innovators sell only to a set of high-valuation consumers that was served relatively poorly by pre-existing firms. Hence, our model features only very limited business stealing and creative destruction rather works through the destruction of rents. Thus, new entrants make the set of available goods more differentiated, which is shown to reduce the market power of all firms so that in equilibrium firm entry exerts a pro-competitive effect on the entire industry.

To our knowledge, the present paper's model is the first to explain how a large number of seemingly "inferior" low-quality firms can exist alongside a technological leader. By doing so, our paper contributes to two broad literatures. First, it adds to the sizeable literature deriving from Mussa and Rosen (1978) and Shaked and Sutton (1982 and 1983) that focuses on vertically differentiated markets in which natural oligopolies prevail, i.e. the markets that are dominated by a limited number of "market leaders".[5]

Our approach differs from this literature only in the underlying production technology. Existing studies assume that the marginal production costs increase only moderately with

[5] See also Shaked and Sutton (1984) and Sutton (2007) and (2007a) for the case of one firm environments and Champsaur and Rochet (1989) for the duopolistic case.

quality, which enables high quality firms to out-price low quality competitors.[6] Whenever this condition is violated, heterogeneous consumers may differ in their individual ranking of variety-price pairs. Shaked and Sutton (1983) do not analyze this case, which would, given their assumption of costless market entry, imply entry of unaccountably many firms and competitive pricing along a dense set of qualities. In the present paper, we analyze the case where the marginal cost of production does increase sufficiently in quality, while explicitly modeling the firms' quality choice under the standard assumption of costly market entry.

Our model is also relevant to the static and the dynamic aspects of the literature analyzing the product market competition (PMC) and growth nexus. The first principal difference between classical Schumpeterian growth models a là Aghion and Howitt (1992) and our approach is that, whereas the former introduce PMC via exogenous parameters, in our setup the degree of PMC is determined endogenously, arising from the entrant's decision to differentiate its product from existing goods. Further, in the existing literature the rate of innovation is strictly decreasing in PMC, reflecting the profit flow of monopolists. In our approach, this finding is reversed: a high degree of PMC can only arise if entry to this industry is cheap and, therefore, innovation happens frequently.

In this sense, the incentives to innovate in our model are related to the "escape competition" motive for R&D in Aghion et al. (2001) (see also Aghion et al. (1997) and Aghion et al. (2005), as well as the informal discussion in Boldrin and Levine (2004)), where incumbent firms innovate to increase their cost advantage over lagging imitators. However, whereas previous work focuses on cost innovation in a setup featuring given demand parameters, we emphasize how post-innovation demand itself is shaped by the degree to which innovators distinguish their products from existing ones.

In a dynamic sense, our setup has stark implications for the nature of "creative destruction" and for the mechanisms through which innovating firms create aggregate innovation (see Klette and Kortum (2003)). In existing Schumpeterian growth models, innovation happens because it allows entrants to displace the incumbent firm. Our setup does not necessarily feature such a "business stealing" effect in the sense that already marginal innovations grant non-negligible profits. Instead, innovators sell only to a small set of consumers that was served relatively poorly by pre-existing firms. Thus, the intrinsic reason

[6] See Lahmandi-Ayed (2000 and 2004) for an extensive discussion of the conditions on technology that induce natural oligopolies.

of why innovation happens in our economy is not one of displacing the incumbent entrant (i.e. "creative destruction"), but rather, innovation as a means to differentiate oneself from existing firms.

Firm innovation thus creates aggregate growth because innovational efforts are directed towards consumer preferences. The motive for innovation in our setup is thus akin to the one in the literature on the direction of technical change: Acemoglu (1998, 2002, and 2007) argues that growing supply of skilled labor generates incentives to invest in technology directed toward skills, we focus on how the direction of technological advance tracks the evolution of consumer preferences. As consumer valuations grow over time, the market for higher quality goods expands, thus creating technological advances biased towards quality.[7]

Last, our model has potentially important implications for the product life cycle. As mentioned above, our model does not feature the standard business steeling effect. "Creative destruction" still exists in our setup, albeit in a very different form than in the standard Schumpeterian setup: new entrants make the set of available goods more differentiated, thereby exerting a pro-competitive effect on the entire industry.[8]

The remainder of this paper is structured as follows. In Section 2 we motivate the choice of our model's preference structure by discussing the existing literature and further examine the static predictions of our approach in subsection 2.2. We next analyze free entry decisions and the stationary equilibria in 3. Finally, we endogenize the growth rate in Section 4 before concluding in Section 5.

2 Spatial Competition in Quality

Hotelling's classic 'location' paradigm is widely used to reflect generic product characteristics. The well-studied formalism of the spacing model, however, does not apply to competition in quality. By its very definition, quality requires that individuals agree on the ranking of varieties so that, in particular, their individually preferred "ideal variety"

[7] Indeed, in an empirical study, Saha (2007) finds that serving consumer preferences is a major determinant of innovation activities (see also Sutton (1996 and 1998))

[8] The model also features the substitution and complementarily effects of innovation and the product life cycle first analyzed in Young (1993)). In our model, as new innovation happens, the economy grows and consequently, consumer valuations increase. This has two consequences: first, since it raises the average willingness to pay for quality, prices increase. Second, as the support of the valuation distribution grows, its density thins out and any firm serving a fixed range of consumer valuations thus serves less customers.

coincide. When it comes to vertical differentiation – or differentiation in quality – only the higher price tag of the universally preferred higher quality goods makes different consumers buy distinct qualities.

Aware of the spacing model's fundamental misfit to address competition in quality, Shaked and Sutton (1982, 1983) pioneered research on vertically differentiated markets in which natural oligopolies prevail, i.e. the markets that are dominated by a limited number of "market leaders". The authors call this feature, characterizing vertically differentiated markets, the *finiteness property.* Its key element is that the marginal production costs increase only moderately with quality, which enables high quality firms to outprice low quality competitors (see also Shaked and Sutton 1984, Lahmandi-Ayed 2000 and 2004, and Sutton 2007, 2007a).

Whenever this condition is violated, heterogeneous consumers may differ in their individual ranking of variety-price pairs and Shaked and Sutton (1983) observe that the competition in quality is "reminiscent of the 'location' paradigm" by Hotelling. The authors do not analyze this case, which would, given their assumption of costless market entry, imply entry of unaccountably many firms and competitive pricing along a dense set of qualities.

In the present paper, we analyze the case where the marginal cost of production does increase sufficiently in quality, thus violating the *finiteness property*, while explicitly modeling the firms' quality choice under the standard assumption of costly market entry.

Doing so, we necessarily depart form the standard Hotelling (1929) model of product differentiation, since Hotelling's classic location paradigm is inept in quality markets. We adopt the general setup of Shaked and Sutton, to the extend that consumers prefer quality at a linear rate and the same is true for our setup.[9] We depart form their setup, however, by assuming that the price of a good increases steeply in the good's quality, so that lower valuation consumers in equilibrium prefer to buy goods other than the one of the current technological leader. Figure 1 depicts the resulting equilibrium market structure of our approach: higher valuation consumers tend to buy from high quality producers. Each firm has two direct competitors (one for the maximum quality producer) and sells to a range of consumers that on the one hand do value quality enough to buy from the firm in question rather than the direct lower competitor but on the other hand do not value quality enough

[9] See also Mussa and Rosen (1978), and Auer and Chaney (2008 and 2009)

to buy from the higher quality competitor.

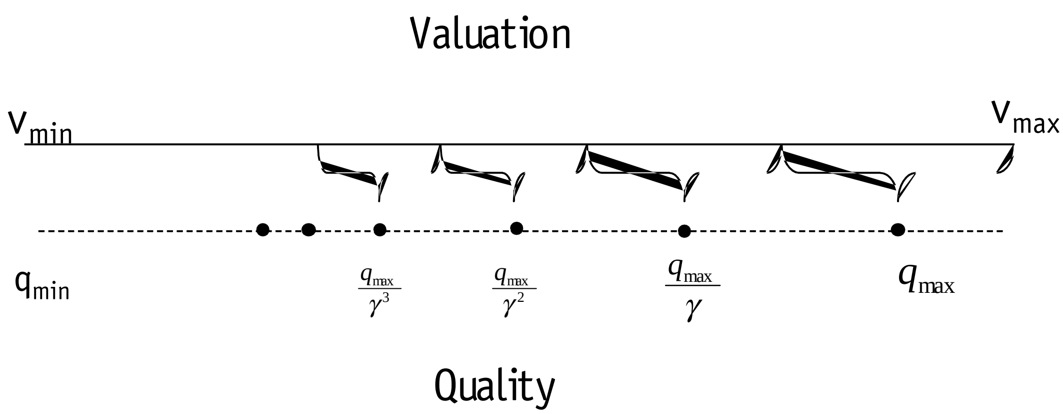

Figure 1: Segmentation of the consumer/valuation space by quality levels.

In the next steps we set up the framework and analyze the static determinants of prices and profits for a given quality spacing.

2.1 General Setup

There is a homogeneous good O, and differentiated goods of total mass one Q_j $(j \in [0,1])$. Each of the differentiated goods comes in a set of different quality levels $\{q_{nj}\}_{n \in S_j}$.

2.1.1 Preferences

For each differentiated good Q_j consumers consume either one unit or none at all. When consuming the amount d of good O and the vector of qualities $\mathbf{q} = \{q_j\}_{j\in[0,1]}$ of the Q_j-goods, an individual derives utility

$$u_v(\mathbf{q}, d) = v \cdot \left[\int_0^1 q_j \ dj\right] + d \tag{1}$$

The higher v, the higher is the individual's desire to consume quality, wherefore in the following we will call the valuation of quality or simply *valuation*. We further assume that valuations are represented by a non-negative and increasing function of income I, i.e. $v(I) \geq 0$ and $v'(I) \geq 0$.

Our formulation of preferences slightly modifies the standard approach from Shaked and Sutton (1982) to a multitude of differentiated goods. The most important implication of our assumption is that a single firm's price does not impact the valuation v, which is therefore exogenous to the firm. This feature simplifies the firm's optimization problem. The preferences specified by (1) imply that, as standard in the literature, income and quality are complementary. The higher a consumer's income, the higher is v and thus her willingness to pay for quality.

We normalize the price of the homogeneous good O to unity and write $p_i(q_i)$ for the price of quality q_i. The mass of individuals totals L. Different individual labor productivities give rise to a non-degenerate income distribution. This distribution of income translates into a cumulative density function for v, the valuations of quality. Instead of specifying the income distribution, we chose to define the corresponding cumulative density function of valuations as

$$G(v) : [v_{\min}, v_{\max}] \to [0, 1] \tag{2}$$

where $0 \le v_{\min} < v_{\max} < 0$.

2.1.2 Production

The O-type good is produced competitively with constant returns to scales and labor as the only factor. Production technologies of the Q-type good exhibit increasing returns to scale and depend on the quality level produced. In the following, we consider a representative Q-industry and drop the index j. Firms that enter the Q-market to produce the quality $q \in (0, \infty)$ need to acquire a blueprint at the fixed cost of

$$F(q, \bar{q}) = \phi f(q/\bar{q}) \bar{q}^{\theta} \tag{3}$$

labor units, where $\bar{q}$ is the maximum quality of the incumbent firms. The function $f(.)$ is differentiable and increasing, while $f(q/\bar{q}) \bar{q}^{\theta}$ is (weakly) decreasing in $\bar{q}$. We thus assume that blueprints of higher qualities are always more expensive but invention of a given quality is less expensive the more advanced the existing quality frontier.

A firm, having acquired a blueprint for quality q, can produce at the constant marginal cost of

$$c(q) = \varphi q^{\theta} \tag{4}$$

labor units. The parameters $\phi, \varphi > 0$ govern the production cost. We assume that both, the fixed cost of entry as well as marginal cost are increasing and convex in quality ($\theta > 1$).

We characterize the equilibrium in which firms enter production at the optimal quality level and subsequently engage in monopolistic pricing. The equilibrium is, of course, solved through backward induction, i.e., we first determine the prices at given quality levels and subsequently analyze entry decisions.

2.2 Optimal Pricing

We begin by characterizing the general pricing solution for an arbitrary distribution of a countable set of qualities. For notational simplicity, we set $p_n = p(q_n)$ and $c_n = c(q_n)$, where q_n is the quality level produced by firm n. We index firms by $n \in \mathcal{N}_0 = \{0, -1, -2, ...\}$ and order firms by their quality level so that firm 0 produces the highest quality level q_0 and all further quality levels satisfy $q_{n-1} < q_n$.[10]

Firms compete in prices, *i.e.* each firm sets the price of its quality to maximize its operating profits, while taking total demand and the other firms' prices as given. Under preferences (1) a consumer with valuation v is indifferent between two goods q_n and q_{n+1} if and only if their prices p_n and p_{n+1} are such that $vq_{n+1} - p_{n+1} = vq_n - p_n$. Thus, given $G(v)$ from (2) and given the prices $\{p_n\}_{n\leq 0}$, the n^{th} firm sells to all consumers with valuations v in the interval $[v_{n-1},\ v_n]$, where[11]

$$v_n = \begin{cases} v_{\max} & \text{if } n = 0 \\ \dfrac{p_n - p_{n-1}}{q_n - q_{n-1}} & \text{if } n < 0 \\ v_{\min} & \text{if } n = n_{\min} - 1 \end{cases} \tag{5}$$

The firms market shares are thus $[v_n, v_{n+1}]$ and the market is partitioned as shown in Figure 1. Since each consumer with valuation $v \in [v_n, v_{n+1}]$ demands one unit of the variety produced by firm n, firm n's serves the mass of $G(v_{n+1}) - G(v_n)$ consumers and solves the maximization problem

$$\max_{p_n} (p_n - c_n) \left[G(v_{n+1}) - G(v_n)\right] L \qquad s.t.\ (5) \tag{6}$$

The optimality conditions of this problem are

$$G(v_{n+1}) - G(v_n) - (p_n - c_n) \left[\frac{G'(v_{n+1})}{q_{n+1} - q_n} + \frac{G'(v_n)}{q_n - q_{n-1}}\right] = 0 \tag{7}$$

[10] Notice that we implicitly assume that the set of firms is countable. By making this assumption we already anticipate that in equilibrium of the later entry game, firms need to recoup their setup cost with monopoly rents. Under Bertrand competition and positive setup cost this implies that firms must be located at positive distance to each other and the number of firms is necessarily countable.

[11] We rule out undercutting, where firm n sets its quality-adjusted price to take the market share of a directly neighboring firm and compete with second-next firms.

where the expressions (5) apply. At $v_{\min}$, $v_{\max}$, the constant limits of the distribution, the derivatives in (7) are set to zero ($G'(v_{\min}) = G'(v_{\max}) = 0$). Firm n's profits are zero at $p_n = c_n$ as well as at

$$\bar{p}_n = \frac{(q_n - q_{n-1})\,p_{n+1} + (q_{n+1} - q_n)\,p_{n-1}}{q_{n+1} - q_{n-1}}$$

since the latter price implies $v_{n+1} = v_n$ and thus zero market share for the n^{th} firm. Finally, as[12]

$$\bar{p} = \frac{(q_n - q_{n-1})\,p_{n+1} + (q_{n+1} - q_n)\,p_{n-1}}{q_{n+1} - q_{n-1}} \geq \frac{(q_n - q_{n-1})\,c_{n+1} + (q_{n+1} - q_n)\,c_{n-1}}{q_{n+1} - q_{n-1}} > c_n$$

and profits are positive for $p_n \in [c_n, \bar{p}]$, there is an interior solution to the profit maximization problem, which necessarily satisfies (7). Generic profits are

$$\pi_n = (p_n - c_n)^2 \left[\frac{G'(v_{n+1})}{q_{n+1} - q_n} + \frac{G'(v_n)}{q_n - q_{n-1}}\right] L \tag{8}$$

With this characterization of prices and operating profits some regularities of equilibrium prices and profits emerge.

Lemma 1 *Let $\{q_n\}_{n\leq 0}$, $\{c_n\}_{n\leq 0}$ and (7) define a system with the prices $\{p_n\}_{n\leq 0}$ and operating profits $\{\pi_n\}_{n\leq 0}$. For any $\chi > 0$ the following statements hold:*
(i) The transformed system defined by $q'_n = \chi q_n$, $c'_n = \chi^\theta c_n$, $v' = \chi^{\theta-1} v$ and corresponding (7) has the solution $\{p'_n\}_{n\leq 0}$ and $\{\pi'_n\}_{n\leq 0}$ satisfying

$$p'_n = \chi^\theta p_n \qquad \text{and} \qquad \pi'_n = \chi^\theta \pi_n \quad \forall n.$$

(ii) The transformed system defined by $q''_n = q_n$, $c''_n = \chi c_n$, $v'' = \chi v$ and corresponding (7) has the solution $\{p''_n\}_{n\leq 0}$ and $\{\pi''_n\}_{n\leq 0}$ satisfying

$$p''_n = \chi p_n \qquad \text{and} \qquad \pi''_n = \chi \pi_n \quad \forall n.$$

Proof. *See Appendix* ■

The first part of the Lemma states that, if quality levels, marginal production costs and valuations increase at the right proportions (according to (3) and (4)), then equilibrium prices and profits are a constant proportion of marginal production costs. Part (ii)

[12] The last inequality holds by convexity of $c(q)$.

of the Lemma states that, if marginal production costs and valuations increase proportionally, while quality levels are constant, then prices and profits are a constant proportion of marginal production costs.

These regularities will lead us to a particularly nice patters of the firms' quality choice – namely proportional spacing. We will turn to this feature next.

3 Endogenous Spacing Under Free Entry

This subsection shows that in a dynamic version of the general setup described above, free entry supports equilibria with equal relative spacing of firms, endogenously generating quality levels that satisfy

$$\gamma q_{n+1} = q_n \qquad \forall\ n. \tag{9}$$

We introduce a dynamic dimension to our model, by assuming that time is continuous and that valuations grow at the constant rate a. The rate a is for now given exogenously, and is endogenized in Section (4) below. Indexing each valuation parameter with time subscripts, we can write $v_t = e^{at} v_o$. Consequently, the distribution G is time dependent and satisfies

$$G_t(v) = G_o(e^{-at} v) \tag{10}$$

where G_o is the distribution at initial date $t = 0$.

Out analysis aims at a stationary equilibrium in which each firm enters the industry as technological leader and successively transits through the product cycle as it becomes superseded by further innovators. The gain such a dynamic entry game is that we need to analyze the entry problem of one firm at a time only. In particular, we avoid the problems that arise in a simultaneous entry game as in Vogel (2008).[13]

In a dynamic game of this type profits of a firm producing q_o evolve as depicted by the bold line in Figure 2. The continuous sections represent the profits when no innovation happens. Innovations occur at regular intervals (depicted by t_1^*, t_2^*...). At these moments,

[13] In fact, the arising complications would be tremendous our setup, because the clear ranking of the quality line hinders us to use the symmetry properties that arise in models based on Hotelling (1929), where one can think of economies formed like a circle street or the beach surrounding an island. In a quality setup, however, any attempt to "close the circle" must fail since it would amount to identifying the highest quality good to the lowest quality good.

the firm's profit drops by a discrete amount, because the new competitor reduces the incumbents' sales and markups.

The dashed line illustrates the general trend. Two opposing forces are at work that explain why this trend may be first increasing and then decreasing over time. First, for given set of firms, the profit flow for top quality firms is increasing as consumer valuations increase over time, driving up market shares and markups. Second, the growing range of consumer valuations also implies that density of consumer valuations constantly thins out. Thus, since firms in the limit converge to serve a fixed interval of valuations while the density of valuation over this range constantly thins out, the profit flow drop to zero in the limit.

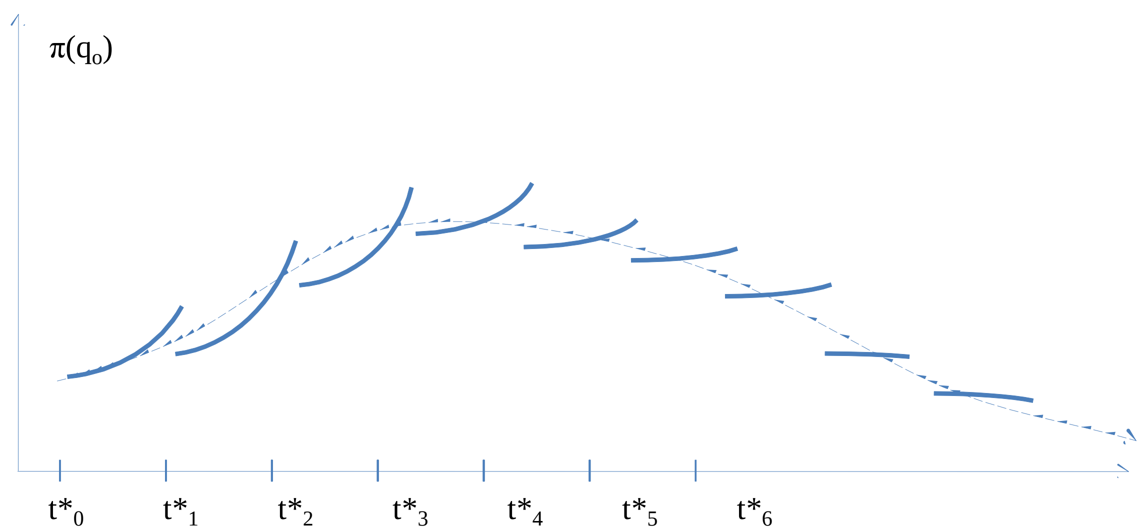

Figure 2: Evolution of firm 0's profits over time.

In the entry game, firms not only decide which quality to aim for but also when to enter the industry. We assume that at each point in time, there is a mass of potential entrants who could pay a fixed cost $F(q)$ to receive a perpetual monopoly to produce the good of quality level q. With this setup, the potential entrants start innovating as soon as innovation generates a profit flow whose net present value is as least as big as the innovation costs.

Initially, the set of active firms is $\{0, -1, -2, ...\}$ and firms are ordered according to ascending qualities, as described in the previous subsections. These initially active firms produce qualities $\{q_n\}_{n\leq 0}$, which satisfy (9). As demand for goods at the top-end of the quality spectrum grows new firms gradually establish at the upper end of the quality spectrum.

We assume that a plant established at quality level q_m automatically holds the blueprints for all qualities between q_{m-1} and q_m, where q_{m-1} is the next-lower quality level. This assumption restricts entry of additional firms to quality levels above the pre-existing ones $(q_{m+1} \geq q_m)$.

Now, for $m \geq 1$ let t_m denote the entry date of the m^{th} *additional* firm (implying $0 \leq t_1 \leq t_2 \leq ..$) and let further q_m stand for its quality level $(q_0 \leq q_1 \leq q_2 \leq ..)$. It will prove convenient to express the quality choice of the m^{th} entrant relative to the highest quality of all incumbents (q_{m-1}) as

$$\gamma_m = q_m/q_{m-1} \qquad\qquad m \geq 1.$$

At time $\tau \in [t_{m+k}, t_{m+k+1})$ the set of quality levels supplied to the market is $\{q_n\}_{n\leq m+k}$. Current prices are determined implicitly by (7) and depend on all currently produced quality levels as well as on all current valuations $v_\tau = e^{a\tau}v$. Consequently, at time $\tau \in [t_{m+k}, t_{m+k+1})$ the operating profits (8) of the m^{th} additional firm are a function of qualities $\{q_n\}_{n\leq m+k}$ and time τ. We can express this time dependence as dependence on the factor $e^{a\tau}$, which multiplies all valuation parameters v. Formally, operating profits of the firm m at time τ are thus

$$\pi_m\left(e^{a\tau}, q_{m+k}, \gamma_{m+k}, \gamma_{m+k-1}, \gamma_{m+k-2}, ..., \gamma_1, \gamma\right) \qquad\qquad \tau \in [t_{m+k}, t_{m+k+1}).$$

Defining now the product

$$\Gamma_{m,k} = \prod\nolimits_{j=1}^{k} \gamma_{m+j} \tag{11}$$

we have $q_{m+k} = \Gamma_{m,k} q_m$ so that at time t_m the present value of the flow of operating profits for a potential entrant is

$$\Pi(\gamma_m, t_m) = \sum_{k\geq 0} \int_{t_{m+k}}^{t_{m+k+1}} e^{-r(\tau - t_m)} \pi_m\left(e^{a\tau}, \Gamma_{m,k}\gamma_m\Gamma_{0,m-1}q_0, \gamma_{m+k}, \gamma_{m+k-1}, ..., \gamma_1, \gamma\right) d\tau. \tag{12}$$

The parameter r is the constant rate at which firms discount future profits.

We are now ready to formulate the entry decision of firms. The m^{th} firm chooses its entry date (t_m) and its location on the quality line (γ_m). With the second choice it maximizes the present value of profits at time t_m (12) net of costs (3). Given the spacing $\gamma_{m-1}, \gamma_{m-2}, ..., \gamma_1, \gamma$, and conditional on the entry date t_m the m^{th} optimal quality choice is

$$\hat{\gamma}_m\left(\gamma_{m-1}, ..., \gamma_1, \gamma\right) = \arg\max_{\tilde{\gamma}\geq 1}\left\{\sum\nolimits_{k\geq 0}\int_{t_{m+k}}^{t_{m+k+1}} e^{-r(\tau-t_m)}\pi_m\left(e^{a\tau}, \tilde{\Gamma}_{m,k}\tilde{\gamma}\Gamma_{0,m-1}q_0, \hat{\gamma}_{n+k}, \hat{\gamma}_{n+k-1}\cdots\right.\right.$$
$$\left.\left. \cdots, \hat{\gamma}_{n+1}, \tilde{\gamma}, \gamma_{n-1}, ..., \gamma_1, \gamma\right) d\tau - F(\tilde{\gamma}, \Gamma_{0,m-1}q_0)\right\} \tag{13}$$

Here, $\tilde{\Gamma}_{m,k}$ stands, similar to (11), for the product of the k future optimal relative spacing parameters, given that the m^{th}-entrant plays $\tilde{\gamma}$:

$$\tilde{\Gamma}_{m,k} = \prod\nolimits_{j=1}^{k}\hat{\gamma}_{m+j}\left(\hat{\gamma}_{m+j-1}, \hat{\gamma}_{m+j-2}, ...\tilde{\gamma}_m, \gamma_{m-1}, .., \gamma_1, \gamma\right).$$

Notice that all future locations choices $\hat{\gamma}_{m+j}$ (and $\tilde{\Gamma}_{m,j}$) as well as future entry dates t_{m+j} are functions of the m^{th} firm's choice. For expositional purposes, however, the arguments $\hat{\gamma}_{m+j}(\tilde{\gamma})$, $\tilde{\Gamma}_{m,j}(\tilde{\gamma})$, $t_{m+j}(\tilde{\gamma})$ are suppressed in (13) and further down. The m^{th} firm's entry date is determined by the free entry condition, *i.e.*, the requirement $\Pi(\gamma_m, t_m) \geq F(\gamma_m, q_{m-1})$. Formally, we write

$$t_m = \inf\left\{t \geq t_{m-1} \;\middle|\; \sup_{\tilde{\gamma}\geq 1}\left[\sum\nolimits_{k\geq 0}\int_{t_{m+k}}^{t_{m+k+1}} e^{-r(\tau-t_m)}\pi_m\left(\tau, \tilde{\Gamma}_{m,k}\tilde{\gamma}\Gamma^*_{0,m-1}q_0, \hat{\gamma}_{m+k}, \hat{\gamma}_{m+k-1}, \cdots\right.\right.\right.$$
$$\left.\left.\left. ..., \hat{\gamma}_{m+1}, \tilde{\gamma}, \gamma^*_{m-1}, \gamma^*_{m-2}..., \gamma^*_1, \gamma\right) d\tau - F(\tilde{\gamma}, \Gamma^*_{0,m-1}q_0)\right] \geq 0\right\} \tag{14}$$

where γ^*_m denotes the equilibrium locations

$$\gamma^*_1 = \hat{\gamma}_1\left(\gamma\right) \qquad \text{and} \qquad \gamma^*_k = \hat{\gamma}_k(\gamma^*_{k-1}, \gamma^*_{k-2}, ..., \gamma^*_1, \gamma) \tag{15}$$

and $\Gamma^*_{0,k}$ is defined parallel to the above definitions as the product of the equilibrium γ^*_j

$$\Gamma^*_{0,k} = \prod\nolimits_{j=1}^{k}\gamma^*_j.$$

Optimal quality choices (13) and the free entry conditions (14) of all entrants ($m \geq 1$) determine the equilibrium of the entry game. The first important result of this section concerns the solution of the system (13) - (14) and is formulated in the following Proposition.

Proposition 1 *For any combination of positive parameters* $(\theta, \phi, \varphi, L, r, a)$ *there exists a* $\bar{\gamma} > 1$ *so that the equilibrium of the entry game (13) - (14) sustains*

$$q_m = \bar{\gamma} q_{m-1} \qquad\qquad m \in \mathbb{Z}.$$

In this equilibrium the time intervals between consecutive entries are constant and equal to

$$\Delta = (\theta - 1)\, a^{-1} \ln(\bar{\gamma}). \tag{16}$$

Proof. See Appendix. ■

For the parameters $(\theta, \phi, \varphi, L, r, a)$ and an according parameter $\bar{\gamma}$ we label the corresponding equilibrium the Equal Relative Spacing Equilibrium (ERSE). Notice that the proposition establishes existence of the ERSE but is silent about uniqueness. We therefore restrict all further considerations to the one ERSE (out of possibly many) with the minimal spacing parameter $\bar{\gamma}$. Now, (as argued in the proof of Proposition 1), under a preexisting spacing parameter equal to one ($\gamma = 1$) the optimal spacing of the first entrant $\gamma^*(\gamma)$ from (15) satisfies $\gamma^*(\gamma) > \gamma$ for all $\gamma \in (1, \bar{\gamma})$ Consequently, we conclude that at the minimal symmetric $\bar{\gamma}$, characterized by $\gamma^*(\bar{\gamma}) = \bar{\gamma}$

$$\left.\frac{d\gamma^*(\gamma)}{d\gamma}\right|_{\gamma=\bar{\gamma}} < 1. \tag{17}$$

holds. For this ERSE with the smallest γ, we can show the following Lemma.

Lemma 2 *Let* $\bar{\gamma}$ *be the spacing parameter of the ERSE. Then,*

(i) $\bar{\gamma}$ *depends on* $\phi/(\varphi L)$ *only and can be written as* $\bar{\gamma}(\phi/(\varphi L))$.

(ii) $\bar{\gamma}$ *is constant under the transformation* $(r, a, L) \to \chi \cdot (r, a, L)$, *where* $\chi > 0$.

Proof. *(i)* Operating profits π are linear in L; setup costs F are linear in ϕ. Thus, when replacing $\phi' = \phi/L$ population L factors out of the slanted brackets in (13) and the square brackets in (14). Consequently, the solution to problem (13) - (15) and thus $\bar{\gamma}$ depends on $\phi' = \phi/L$ only. Similarly, operating profits are, by Lemma 2 (ii), linear in φ under the transformation $v'(t) = v(t)/\varphi$ (or $t' = t - \ln(\varphi^{1/a})$). Hence, replacing $\phi' = \phi/\varphi$ in (13) and (14), $\bar{\gamma}$ depends on ϕ' only.

(ii) Net present profits (12) are constant under the time transformation $t \to \chi t$, given that locations are constant and firm entry dates transform by $t_m \to \chi t_m$. Under this condition, the firm entry remains unchanged. By (13), firm entries are transformed accordingly. Finally, the time transformation is equivalent to $(r, a, L) \to \chi \cdot (r, a, L)$. ■

Technically, the Lemma shows that we can choose the notation $\bar{\gamma}(\phi)$ to reflect the functional dependence of $\bar{\gamma}$ on all three parameters ϕ, φ and L. Economically, it says that the density of quality supply (and of competition) is equally affected by a doubling of the market size or the marginal costs or a reduction of setup cost by half.

It would be premature, however, to infer welfare consequences based on the parameter $\bar{\gamma}$ (and its impact on markups) alone, conjecturing, e.g., that an equal increase of the setup costs ϕ and the operating costs φ leaves the economy unchanged. In fact it does not. Such a change in technology actually postpones innovation (reflected in the time transformation in the Lemma's proof) so that more time elapses until one given quality is on the market. This means that individuals purchase lower qualities because each quality is more expensive and fewer high-quality goods are available on the market. Both effects have obviously adverse impact on consumer surplus. – It is straight forward, however, to show that an increase in setup cost that is entirely offset by an ϕ increase in the size of the workforce L, preserving not only parameter of the relative spacing $\bar{\gamma}$ but also the timing of innovations, and thus leaving the quality spectrum of each point in time unchanged.

This section has derived a novel results about regularity of spacing (Proposition 1) and the relative impact of the model's key parameters (Lemma 2). These findings hold in a relatively general setup, which included, in particular, a non-degenerate distribution of valuations (2). This generality, however, comes at a price. In particular, we were unable to show uniqueness of equilibria – neither of the entry game nor, in fact, of the pricing game (determined by (7)). We solved the first of these uniqueness problems by restricting our analysis on the equilibrium with the highest density of quality and simply ignored the second.[14] In the important case of uniform distribution of valuations, also the second of the ambiguities luckily vanishes. We will turn next to this case.

3.1 Uniform Distribution

We analyze the special case when valuations are distributed uniformly as

$$G_o(v) = U([0, v_{\max}]). \tag{18}$$

[14] In fact, it is easy to remedy this problem by either assuming that economic agents correctly anticipate one stable pricing equilibrium or by introducing expectations, of profits in particular, when realizations of equilibria are identically and independently distributed over time.

In this case, which also appears in Auer and Chaney (2007), the optimality conditions (7) give rise to the system

$$p_n = \begin{cases} \frac{1}{2}\left[c_0 + \left(q_0 - q_{-1}\right) v_{\max} + p_{-1}\right] & \text{if } n = 0 \\ \frac{1}{2}\left[c_n + \frac{q_n - q_{n-1}}{q_{n+1} - q_{n-1}} p_{n+1} + \frac{q_{n+1} - q_n}{q_{n+1} - q_{n-1}} p_{n-1}\right] & \text{if } n < 0 \end{cases} \tag{19}$$

which implies that equilibrium prices are determined as follows.

Proposition 2 *Assume equal relative spacing in quality, i.e. (9) holds. Then prices are*

$$p_n = \left(A \left(\lambda/\gamma^\theta\right)^n + \alpha\right) c_n \qquad \forall \;\; n \leq 0 \tag{20}$$

where

$$\alpha = \frac{\gamma + 1}{2(\gamma + 1) - \gamma^\theta - \gamma^{1-\theta}} \tag{21}$$

$$\lambda = \gamma + 1 + \sqrt{\gamma^2 + \gamma + 1} \tag{22}$$

$$A = \frac{\lambda}{2\lambda - 1}\left(1 - \alpha\left(2 - \gamma^{-\theta}\right) + \frac{\gamma - 1}{\gamma}\frac{q_0 v_{\max}}{c_0}\right). \tag{23}$$

Proof. See Appendix ■

Proposition 2 provides not only a closed form solution but, in addition, establishes uniqueness of the pricing equilibrium.

Notice further that the term α from (20), which is common to firms' markups, might be positive or negative, depending on whether or not $\gamma^\theta + \gamma^{1-\theta} < 2(\gamma + 1)$ holds. Nevertheless, expression (20) defines positive markups in either of the cases provided that the highest quality firm is active in the market. To verify this fact, observe first that[15]

$$\alpha > 0 \qquad \Leftrightarrow \qquad \lambda/\gamma^\theta > 1. \tag{24}$$

With this relation we can distinguish two different cases. First, if $\alpha < 0$ holds, we have $A > 0$ and $\lambda/\gamma^\theta < 1$ so that $A\left(\lambda/\gamma^\theta\right)^n + \alpha > A + \alpha$ for all $n < 0$. Second, in the case $\alpha > 0$ we easily verify that $\alpha > 1$. Therefore, if $A > 0$ all markups are positive. If, instead, $A < 0$ then $A\left(\lambda/\gamma^\theta\right)^n + \alpha > A + \alpha$ holds again for all $n < 0$ by (24).

[15] Equivalence (24) is quickly checked by verifying that both inequalities $2(\gamma + 1) > \gamma^\theta + \gamma^{1-\theta}$ and $\lambda/\gamma^\theta > 1$ hold (are violated) for $\gamma \to 1$ ($\gamma \to \infty$) and that, moreover, $\lambda = \gamma^\theta$ if and only if γ solves $\gamma^\theta + \gamma^{1-\theta} = 2(\gamma + 1)$.

Overall, a sufficient condition for all markups to be positive is thus $A+\alpha>1$. By (23), this condition is satisfied as long as $v_{\max}q_0/c_0$ is large enough. Obviously, it may happen that $v_{\max}q_0/c_0$ is very small. In that case, however, top-quality firms do not sell and produce at all and we can renumber firms, indexing the highest quality firm with *positive output* with $n=0$. This increases q_0/c_0 – and hence A – up to the point where $A+\alpha>1$ holds (and thus $A\left(\lambda/\gamma^{\theta}\right)^n+\alpha>1$ for all $n\leq 0$).

Now, we write for the relative markup of the highest quality firm

$$A+\alpha-1=\frac{\lambda}{2\lambda-1}\left\{\frac{\gamma-1}{\gamma}\frac{q_0v_{\max}}{c_0}+\left(\frac{\alpha}{\gamma^{\theta}}-1\right)-\frac{\alpha-1}{\lambda}\right\}$$

With the explicit formula for the prices (20), the operating profits from (8) are thus

$$\pi_n=\begin{cases}\gamma\left(\gamma-1\right)\left(\dfrac{A+\alpha-1}{\gamma-1}\right)^2\dfrac{c_0^2}{q_0}\dfrac{L}{v_{\max}} & \text{if } n=0\\[2ex] \left(\gamma^2-1\right)\left(\dfrac{A\left(\lambda/\gamma^{\theta}\right)^n+\alpha-1}{\gamma-1}\right)^2\dfrac{c_n^2}{q_n}\dfrac{L}{v_{\max}} & \text{if } n<0\end{cases}\tag{25}$$

Observe with (21) and (23) that the limit

$$\lim_{\gamma\to1}\frac{A+\alpha-1}{\gamma-1}=\frac{1}{\sqrt{3}}\left\{\frac{q_0v_{\max}}{c_0}-\theta\right\}$$

is finite. Thus, in the case of equal relative spacing, the operating profits are, by (21) - (23) and the limit above, continuously differentiable for all $\gamma\geq1$ and satisfy, moreover

$$\pi_n\to0\qquad(\gamma\to1).$$

Moreover, and very importantly, we can sign the slope of the ERSE's location, i.e. the function $\bar{\gamma}(\phi)$.

Proposition 3 *Let $0<\underline{\phi}<\bar{\phi}<\infty$. Then, the following statements hold.*

(i) There is a $r_0>0$ so that $\bar{\gamma}(\phi)$ is weakly increasing on $[\underline{\phi},\bar{\phi}]$ for all $r\in[0,r_0]$.

(ii) There is a $\phi_0\geq0$ so that $\bar{\gamma}(\phi)$ is constant if and only if $\phi\leq\phi_0$.

Proof. See Appendix. ■

The proposition shows that higher setup costs increase the relative spacing between quality levels. Intuitively, firms compensate increases in setup costs by increasing profits.

These latter are brought about by larger market shares as well as higher markups and, ultimately, by a wider spacing parameter $\bar{\gamma}$.

Together with Lemma 2, the proposition also determines the impact of market size (L) and marginal production costs (φ) on the spacing $\bar{\gamma}$ of the ERSE. In particular, increases in L and φ have a similar effect on $\bar{\gamma}$ as reductions in setup costs – all of them decreasing the equilibrium spacing $\bar{\gamma}$. Clearly, a larger market induces, *certeris paribus*, higher profits and allows firms to generate more profits. At constant setup costs, larger markets therefore experience more frequent entry of firms at closer distances – the competitive pressure among firms rises. Conversely, productivity growth at the margin (a decrease in marginal production costs φ) *increases* relative spacing and *reduces* the toughness of competition.

This adverse effect of marginal productivity growth on competitive pressure may appear somewhat puzzling. To understand the forces operating to its effect, observe that the preference specification generates, just as ordinary CES preferences, relative firm markups $p_n/c_n - 1$ that are independent of costs (see prices (20)). Put differently, at given relative spacing, operating profits constitute a constant share of revenues. Hence, when quality levels are constant, an increase in marginal productivity (a drop in marginal costs) tends to curb revenues and thereby depresses operating profits.[16] As firms must cover their setup costs, however, the productivity gains that curb profits per consumer must come about with increases in market share, *i.e.*, with a wider equilibrium spacing. This widening of relative spacing does, at the same time, increase relative markups. Hence, competitive pressure decreases as marginal productivity grows.

Notice that for this effect to operate a crucial role comes to the assumption that demand does not react along an intensive margin. In particular, consumers do not react to price changes by consuming more or less but by switching to other firms.

4 Endogenous Growth

Up to this point we treated the valuation growth (10) and interest rates as exogenous, and neglected, moreover, resource constraints. In this section, we repair this shortcoming by postulating spillover effects of innovation and solve for endogenous growth rates. Doing

[16] This does not, of course, mean that each single firm can raise its profit by decreasing its productivity. The firm's profits would, instead, rise under a drop in productivity that affects all firms uniformly, given constant spacing.

so, we show that the partial equilibrium model above is compatible balanced growth under consumer optimization.

We aim to identify a path of balanced growth, characterized by constant and equal growth rates of income and, respectively, expenditure on consumption and innovation, which gives rise to a constant interest rate r and valuation growth according to (10). To this purpose, we return to the notation of many identical industries, indexed by $j \in [0, 1]$. Within industry j the set of qualities S_j is produced, where S_j expands over time.

Technology Spillovers. The structure of the model implies that, in addition to endogenous innovation through market entry, more general efficiency gains must be generated in order to sustain positive growth. More precisely, as an increasing number of high-quality goods are produced at higher unit labor requirements, the effective labor supply must increase at a corresponding pace. We therefore define time-dependent labor productivity, which multiplies raw labor supply, by B_t and postulate that B_t positively depends on all qualities produced up to time t:

$$B_t = \int_0^1 \sum\nolimits_{n \in S_j} [q_{nj}]^\mu \, dj$$

Identifying a variety with the date at which it has been invented we write q_t and compute B_t as the integral

$$B_t = \int_{-\infty}^{t} (q_\tau)^\mu \;\; \Psi(\tau) \; d\tau.$$

where Ψ is the density or rate at which the q are invented.

We assume that innovation takes place at different times in different industries in such a way that the average rate of innovation is constant over time. Put differently, in any two time-intervals of equal length, the same number of additional innovations occur. Therefore, the rate at which qualities appear on the time-line is constant: each dt there are $\Delta^{-1}dt$ qualities invented (Δ from (16)) and the density Ψ of qualities is Δ^{-1}.

Further, the maximal q must increase at a constant rate, λ, so that we have $q_t = q_o e^{\lambda t}$. To determine λ, notice that within a time interval of length Δ each industry experiences exactly one innovation and the maximal q must therefore increase by the factor γ. Hence, $e^{\lambda\Delta} = \gamma$ or $\lambda = a/(\theta - 1)$. We thus have

$$B_t = \frac{q_t^\mu}{\Delta} \int_{-\infty}^{t} e^{-\mu a/(\theta-1)(t-\tau)} \; d\tau = \frac{1}{\mu \ln(\bar{\gamma})} q_t^\mu.$$

As q_t grows at rate $a/(\theta-1)$, the growth rate of labor productivity $g = \dot{B}/B$ is

$$g = \frac{a}{\theta-1}\mu.$$

We now set the valuations v to be proportional to a power of income

$$v = I^{\sigma}.$$

On the balanced growth path expenditure is proportional to income, which, in turn, is proportional to labor productivity B. Thus, valuation growth a must equal σg. With g from above this implies that, in order to guarantee balanced growth, the relation

$$\mu\sigma = \theta - 1 \tag{26}$$

holds.

Innovation Turning to R&D decisions, we observe that the cost of innovation (3) is proportional to q^{θ}. Innovation costs are covered by savings and the savings rate is constant under balanced growth. Therefore, as income is proportional to labor productivity B, we have $g = \theta g_q$, where $g_q = a/(\theta-1)$ is the growth rate of the maximal q. Combining these conditions renders $\mu = \theta$ and $\sigma = (\theta-1)/\theta$ and hence

$$v = I^{(\theta-1)/\theta} \qquad \text{and} \qquad B_t = \frac{1}{\theta \ln(\bar{\gamma})} q_t^{\theta} \tag{27}$$

Equations (26) and (27) reflect two essential requirements need to be fulfilled for growth to be balanced. First, at a given rate of innovation, income growth (generated by spillovers) needs to be just strong enough so that total savings finance the growing costs of innovation. Second, income growth needs to generate higher expenditure, so that demand for quality grows, thus creating markets for new qualities at the top end of the quality spectrum. While the first effect reflects firm credit, the second effect captures the demand firm faces and thus its revenues.

Under the knife-edge condition that production costs, preferences and spillover effects are governed by θ in the way specified by (1), (3), (4) and (27), and if individuals save at constant rates, the model generates balanced growth. We next propose a setup of individual optimization that generates this feature.

Consumer Optimization Nesting the paper's model in a standard dynamic setting with infinitely lived consumers and dynamic optimization is tricky for the following reason.

The somewhat peculiar instantaneous utility (1) implies that u is a non-linear function of expenditure. More precisely, the relative price between the quality aggregate and the homogeneous good is not constant in expenditure. A consumer internalizes these price effect when trading off a marginal increase of consumption today against consumption tomorrow. This distorts the standard optimality conditions. But worse even, the price effect of a marginal quality upgrading of the bundle (1) is different for rich and for poor consumers, due to the varying markups from (20). We cannot hope to bring the resulting heterogeneous savings rates to a constant aggregate one while preserving the expenditure patterns that generate valuations (10) and (18).

To resolve these difficulties, we turn to a setting of overlapping generations, assuming that at each infinitesimal time interval dt, the constant mass $L/T \cdot dt$ of individuals is born, which constitutes a fraction of a continuum of overlapping generations. Individuals live for T time units so that, at each point in time, L individuals populate the economy. An individuals born at t is endowed with l_{it} labor units, where l_{it} is distributed as

$$l_{it}^{(\theta-1)/\theta} \sim G$$

(G from (2)). Individuals save their labor income in order to consume at the end of their life. To avoid the difficulties of intertemporal expenditure allocation sketched above, the final consumption period has length zero. As lifetime wealth W is proportional to labor productivity l_{it}. This implies that $v = W^{(\theta-1)/\theta}$ and hence valuations v are distributed according to (2).

Aggregate Savings Consider the cohort born at t_0, which is endowed with labor L/T. At time $t \in (t_0, t_0 + T)$ the cohort earns $B_t L/T$ from labor income, which it saves. Consequently, at period t of its life, its wealth amounts to (we use that B_t grows at rate g, keeping in mind that $g = a\theta/(\theta - 1)$)

$$w_{t,t_0} = \int_{t_0-t}^{0} e^{-r\tau} B_{t+\tau} L/T \, d\tau = \frac{1 - e^{(g-r)(t_0-t)}}{g-r} \frac{B_t L}{T}$$

Hence, the wealth of the oldest living cohort, which equals aggregate consumption E_t, is

$$E_t = w_{t,t-T} = \frac{e^{(r-g)T} - 1}{r-g} \frac{B_t L}{T}. \tag{28}$$

Total wealth of all living cohorts, expressed in terms of E_t, is

$$W_t = \int_0^T w_{t,t-\tau} d\tau = \frac{E_t - B_t L}{r-g}. \tag{29}$$

Investment. Total investment goes to the invention of blueprints. Since each Δ time units the qualities of all industries are upgraded exactly once (see (16)), $\Delta^{-1} \cdot dt$ new blueprints appear each infinitesimal time interval dt, generating the flow of investment costs

$$IN_t = \Delta^{-1} \phi q_t^{\theta}. \tag{30}$$

(We have set $f(.) \equiv 1$ here.)

Resource Constraint. We write Y_t for the value of total output produced at time t. As every dollar produced ultimately ends up in the pockets of individuals, and individual income consists of returns to savings plus labor income, we have

$$Y_t = rW_t + B_t L \tag{31}$$

In the firms' books, the total value of output appears as the wage bill plus the flow of operating profits. Aggregating over all firms this implies

$$Y_t = B_t L + P_t \tag{32}$$

where we have set $P_t = \int_0^1 \sum_{n \in S_j} \pi_{nj} \, dj$.[17] With these equations, we are ready to pin down the evolution of the economy and the interest rate.

Proposition 4 *The growth and interest rates are uniquely determined by*

$$\frac{1}{r-g}\left[\frac{e^{(r-g)T}-1}{(r-g)\,T}-1\right] = \frac{\phi}{L} \tag{33}$$

$$r \cdot W_t = P_t \tag{34}$$

Proof. *Capital market clearing requires that total investment equals output minus consumption expenditure. Combining equations (30) and (31) and collecting E_t and W_t from (28) and (29), renders (33), pinning down the difference between interest rate and effective growth rate, $r-g$. Hence, expenditure and consumption (28) and total wealth (29) are determined. Finally, combine (31) and (32) to obtain (34). Writing this condition in per capita terms renders*

$$r \cdot \frac{W_t}{L} = \frac{P_t}{L}.$$

[17] We avoid the greek Π in order not to confound total instantaneous profits with discounted flow of profits

Consider now a transformation $(r, a, L) \to (\chi r, \chi a, \chi L)$. By Lemma 2 (ii), this transformation leaves relative spacing γ constant so that per capita profits on the right of (34) are unchanged. The expression on the left is linear in r, thus pinning down the interest rate r and g through (33). Finally, valuations grow at rate $a = g(\theta - \theta)/\theta$. ■

By Proposition 3 we have closed the general equilibrium model, determining all variables of a standard growth model.

5 Conclusion

The "Schumpeterian" class of endogenous growth models has focused almost exclusively on industries where the technological leader takes over the entire market[18]. A major shortcoming of this modeling strategy is that only one firm is active at a time so that the degree of product market competition has to be introduced via exogenous parameters. In particular, the toughness of product market competition is not arising from firms' decisions to differentiate their products from those of their competitors.

In this paper, we address this shortcoming by developing a new model that is suitable to analyze the competition, innovation, and growth nexus in vertically differentiated markets featuring a large number of firms and an endogenous degree of product differentiation. Our model helps to understand how firms profits, firm innovation and the toughness of competition emerge endogenously. This understanding enables us to analyze how market characteristics influence product market competition and how, in turn, the toughness of competition affects investments in innovation and economic growth.

Regarding the dynamics, our work points out that creative destruction may indeed generate more pro-competitive effects and less business stealing than the existing literature suggest. In our setup, business stealing effects of new entrants very limited in the sense that innovators only sell to a set of consumers whose demand was relatively poorly matched by supply of pre-existing firms. Consequently, it is not warranted that innovation occurs too often in the decentralized economy. On the contrary, new entrants make the set of available goods more differentiated, which is shown to exert a pro-competitive effect to *all*

[18] Aghion et al. (1997) and Aghion et al. (2005) analyze an economy with two firms in a setup where demand parameters are fixed but firms can innovate repeatedly to "escape" their competition. In this paper, we focus on one-time innovation decisions and examine how the demand parameters themselves are shaped by the degree to which innovators distinguish their products from existing ones.

firms, leading to a reduction of firm markups. This reduction is strongest for those firms close to the technological frontier.

Finally, our model exhibits a monopoly distortion that is new to the endogenous growth literature: positive markups lead consumers to choose the quality that differs from the socially optimal one: in equilibrium, each consumer compares the increase in good quality to the increase in the goods price. Since markups are generally increasing along the quality dimension, the increase in the price from one good to the other is higher than the cost increase. Consumers, therefore, tend to choose a too low quality.

A Appendix

Lemma 3 ***Lemma 1.*** *(i) Under $q_n' = \chi q_n$, $v' = \chi^{\theta-1} v$ and $p_n' = \chi^{\theta} p_n$, the cutoffs from (5) become*

$$v_n' = \chi^{\theta-1} v_n \qquad \textit{and} \qquad v_{n+1}' = \chi^{\theta-1} v_{n+1}.$$

Now, based on (2), the transformation $v' = \chi^{\theta-1} v$ induces a new cdf $\tilde{G}$ with

$$\tilde{G}(v') = G(v). \tag{35}$$

With this identity and $v' = \chi^{\theta-1} v$, compute

$$\tilde{G}'(v') = \lim_{\delta \to 0} \frac{\tilde{G}(v') - \tilde{G}(v' - \delta)}{\delta} = \lim_{\delta \to 0} \frac{G(v) - G(v - \chi^{1-\theta}\delta)}{\delta} = \chi^{1-\theta} G'(v)$$

Hence, (7) is satisfied, since

$$\tilde{G}\left(v_{n+1}'\right) - \tilde{G}\left(v_n'\right) = G\left(v_{n+1}\right) - G\left(v_n\right)$$

and

$$\left(p_n' - c_n'\right)\left[\tilde{G}'\left(v_{n+1}'\right)\frac{dv_{n+1}'}{dp_n'} - \tilde{G}'\left(v_n'\right)\frac{dv_n'}{dp_n'}\right] = \left(p_n - c_n\right)\left[G'\left(v_{n+1}\right)\frac{dv_{n+1}}{dp_n} - G'\left(v_n\right)\frac{dv_n}{dp_n}\right]$$

where $dv_n'/dp_n' = \chi^{-1} dv_n/dp_n$ and $dv_{n+1}'/dp_n' = \chi^{-1} dv_{n+1}/dp_n$ has been used. This shows that $p_n' = \chi^{\theta} p_n$ solves the transformed pricing system. By (4), (8) and (35) $\pi_n' = \chi^{\theta} \pi_n$ follows, completing the proof of (i).
(ii) Under $q_n'' = q_n$, $v'' = \chi v$ and $p_n'' = \chi p_n$ the cutoffs from (5) become

$$v_n'' = \chi v_n \qquad \textit{and} \qquad \overline{v}_n'' = \chi \overline{v}_n.$$

As in (i), the transformation $v'' = \chi v$ induces a new cdf $\hat{G}$ with

$$\hat{G}(v'') = G(v) \qquad \textit{and} \qquad \hat{G}'(v'') = \chi^{-1} G'(v)$$

This implies that (7) is satisfied, since

$$\hat{G}\left(v_{n+1}''\right) - \hat{G}\left(v_n''\right) = G\left(v_{n+1}\right) - G\left(v_n\right)$$

holds and with $c_n'' = \chi c_n$ and the above

$$\left(p_n'' - c_n''\right)\left[\frac{d\hat{G}}{dv''}\left(v_{n+1}''\right)\frac{d\overline{v}_n''}{dp_n''} - \frac{dG}{dv''}\left(v_n''\right)\frac{d\underline{v}_n''}{dp_n''}\right] = \chi\left(p_n - c_n\right)\chi^{-1}\left[\frac{dG}{dv}\left(v_{n+1}\right)\frac{d\overline{v}_n}{dp_n} - \frac{dG}{dv}\left(v_n\right)\frac{d\underline{v}_n}{dp_n}\right]$$

where $dv''_n/dp''_n = dv_n/dp_n$ *and* $dv''_{n+1}/dp''_n = dv_{n+1}/dp_n$ *has been used. This shows that* $p''_n = \chi p_n$ *solves the transformed pricing system. By (8) and* $\hat{G}(v'') = G(v)$, $\pi''_n = \chi\pi_n$ *follows, completing the proof of (ii).* ■

Proof: Proposition 1. Consider the location choice of the first entrant ($n = 1$), given $\{q_n\}_{n\leq 0}$ satisfying (9) with prevailing γ. Observe that $\gamma_1 = 1$ is not optimal since Bertrand competition would imply $\Pi_{t_1}(\gamma_1) = 0$ (regardless of t_1) in this case, thus violating the free entry condition. Hence, $\gamma = 1$ implies $\gamma_1 > \gamma$. At the same time, γ_1 is trivially finite and $\gamma_1 < \gamma$ holds for γ large enough. Consequently, continuity implies that there is a $\gamma > 1$ so that $\gamma_1 = \gamma$. Denote this by $\bar{\gamma}$. Then, at $\gamma = \bar{\gamma}$, the firm $n = 1$ locates in the quality space, extending equal relative spacing (9) to all $n \leq 1$.
Take this case of $\gamma = \gamma_1 = \bar{\gamma}$ and call the spacing problem of the remaining additional firms ($n = 2, 3, ...$) the *residual spacing problem*. With the notation

$$\gamma'_n = \gamma_{n+1} \qquad (n \geq 1) \qquad q'_0 = \bar{\gamma}q_0 = q_1 \qquad \text{and} \qquad \tau' = \tau + a^{-1}(\theta - 1)\ln\bar{\gamma} \qquad \text{(A1)}$$

the residual spacing problem solves the corresponding system (13) - (15) above, where now all state and choice variables bear a prime (q', v', γ'). Apply Lemma 1 (i), (10) and (A1) to verify that

$$\pi_n\left(e^{a\tau}, \tilde{\Gamma}'_{n,k}\hat{\gamma}'\Gamma'^{(*)}_{0,n-1}q'_0, \gamma'_m, \gamma'_{m-1}, ..., \gamma'_1, \bar{\gamma}\right) = \bar{\gamma}^{\theta}\pi_n\left(e^{a\tau'}, \tilde{\Gamma}'_{n,k}\hat{\gamma}'\Gamma'^{(*)}_{0,n-1}q_0, \gamma'_m, \gamma'_{m-1}, ..., \gamma'_1, \bar{\gamma}\right)$$

Notice that the setup cost (3) satisfies $F(\Gamma'^{(*)}_{0,n-1}, q'_0) = F(\Gamma'^{(*)}_{0,n-1}\bar{\gamma}, q_0) = \gamma^{\theta}F(\Gamma'^{(*)}_{0,n-1}, q_0)$. Hence, γ^{θ} factors out of the right hand side of (13) and of the square brackets in (14). Consequently, the solution of the residual spacing problem coincides with the original problem, implying $\gamma'_1 = \gamma_2 = \gamma_1 = \bar{\gamma}$. A simple induction argument completes the proof that $\gamma_n \equiv \bar{\gamma}$ for all $n \geq 1$.
Finally, (10) and the transformation (A1) show that two consecutive entries occur at dates satisfying $v_{\max}(t_n) = \bar{\gamma}^{\theta-1}v_{\max}(t_{n+1})$. With (10), this is $e^{a(t_{n+1}-t_n)} = \bar{\gamma}^{\theta-1}$ and proves the second statement. ■

Proposition 2. Substitution $u_n = p_n - \alpha c_n$ and recursive formulation (19) of the prices gives

$$2\left[u_n + \alpha c_n\right] = c_n + \frac{1}{\gamma+1}\left[u_{n+1} + \alpha c_{n+1}\right] + \frac{\gamma}{\gamma+1}\left[u_{n-1} + \alpha c_{n-1}\right]$$

for $n > 1$. With $\alpha = (1+\gamma)/\left[2(1+\gamma) - \gamma^{\theta} - \gamma^{1-\theta}\right]$ this is $2(\gamma+1)u_n = u_{n+1} + \gamma u_{n-1}$. The equation

$$X^2 - 2(\gamma+1)X + \gamma = 0 \tag{36}$$

has two roots, $\lambda = \left[\gamma + 1 + \sqrt{\gamma^2+\gamma+1}\right]$ larger than unity and $\mu = \left[\gamma + 1 - \sqrt{\gamma^2+\gamma+1}\right]$, smaller than unity. The general solution to the recursive series is thus

$$p_n = \tilde{A}\lambda^n + \tilde{B}\mu^n + \alpha c_n \tag{37}$$

where $\tilde{B} = 0$ because of $\mu < 1$ and the transversality condition $\lim_{n\to-\infty} p_n = 0$. Equation (19) for $n = 0$ is $2p_0 = c_0 + (q_0 - q_{-1})\, v_{\max} + p_{-1}$ and implies

$$2\left[\tilde{A} + \alpha c_0\right] = c_0 + q_0\,(1 - 1/\gamma)\, v_{\max} + \tilde{A}/\lambda + \alpha c_{-1}.$$

Solving for $\tilde{A}$ and replacing $A = \tilde{A}/c_0$ proves (23). ■

Proof: Proposition 3. As a preparatory step, define net profits of the first entrant as a function of existing spacing γ, setup costs ϕ, entry date t and location choice $\hat{\gamma}$, while suppressing dependence of Ψ on parameters other than ϕ and normalizing $q_0 = 1$:

$$\Psi(\gamma, t, \hat{\gamma}, \phi) = \Pi(\hat{\gamma}, t) - \phi f\left(\hat{\gamma}\right)$$

Free entry implies that equilibrium entry date and location $t^*(\gamma)$ and $\gamma^*(\gamma)$ satisfy

$$\Psi(\gamma, t^*(\gamma), \gamma^*(\gamma), \phi) = 0. \tag{38}$$

for all γ and ϕ and optimal location choice implies

$$\Psi_{\gamma^*}(\gamma, t^*(\gamma), \gamma^*(\gamma), \phi) = 0. \tag{39}$$

Taking derivatives of (38) w.r.t. γ and using (39) yields

$$\Psi_{\gamma} + \Psi_{t^*}\frac{dt^*}{d\gamma} = 0. \tag{40}$$

At the ERSE, (38) is $\Psi(\bar{\gamma}(\phi), t^*(\bar{\gamma}(\phi), \phi), \bar{\gamma}(\phi), \phi) = 0$. Taking derivatives w.r.t. ϕ yields

$$0 = \left[\Psi_{\gamma} + \Psi_{t^*}\frac{dt^*}{d\gamma}\right]\frac{d\bar{\gamma}}{d\phi} + \Psi_{t^*}\frac{\partial t^*}{\partial\phi} + \Psi_{\phi} = \Psi_{t^*}\frac{\partial t^*}{\partial\phi} + \Psi_{\phi}$$

where equation (40) has been used. With $\Psi_{\phi} = -f(\bar{\gamma})$ this implies

$$\frac{\partial t^*}{\partial\phi} = \frac{f(\bar{\gamma})}{\Psi_{t^*}}. \tag{41}$$

Now, taking derivatives of (39) w.r.t. γ leads to

$$0 = \Psi_{\gamma^*\gamma} + \Psi_{\gamma^* t^*} \frac{dt^*}{d\gamma} + \Psi_{\gamma^*\gamma^*} \frac{d\gamma^*}{d\gamma}. \tag{42}$$

At the ERSE, (39) is $\Psi_{\gamma^*}(\bar{\gamma}(\phi), t^*(\bar{\gamma}(\phi), \phi), \bar{\gamma}(\phi), \phi) = 0$. Taking derivatives w.r.t. ϕ yields

$$0 = \left[\Psi_{\gamma^*\gamma} + \Psi_{\gamma^* t^*} \frac{dt^*}{d\gamma} + \Psi_{\gamma^*\gamma^*}\right] \frac{d\bar{\gamma}}{d\phi} + \Psi_{\gamma^* t^*} \frac{\partial t^*}{\partial \phi} + \Psi_{\gamma^*\phi}$$

or, with (42),

$$\Psi_{\gamma^*\gamma^*} \left[1 - \frac{d\gamma^*}{d\gamma}\right] \frac{d\bar{\gamma}}{d\phi} = -\Psi_{\gamma^* t^*} \frac{\partial t^*}{\partial \phi} - \Psi_{\gamma^*\phi}$$

Equations (3) and (12) imply $\Psi_{t^*}|_{\gamma=\bar{\gamma}} = r\phi f(\bar{\gamma}) - \pi(t^*)$ and $\Psi_{t^*\gamma^*}|_{\gamma=\bar{\gamma}} = r\phi f'(\bar{\gamma}) - \pi_{\gamma^*}(t^*)$ and $\Psi_{\gamma^*\phi} = -f'(\gamma^*)$ so that we have with (41)

$$\Psi_{\gamma^*\gamma^*} \left[1 - \frac{d\gamma^*}{d\gamma}\right] \frac{d\bar{\gamma}}{d\phi} = \left\{\pi_{\gamma^*}(t^*) - \pi(t^*) \frac{f'(\bar{\gamma})}{f(\bar{\gamma})}\right\} \frac{f(\bar{\gamma})}{\Psi_{t^*}} \tag{43}$$

The second order condition of (43) and the firm's optimization yields $\Psi_{\gamma^*\gamma^*} < 0$, while (17) implies that the term in the square brackets is positive. Moreover, by definition of t^*, $\Psi_{t^*} > 0$ holds. Consequently, $\bar{\gamma}(\phi)$ is increasing (constant) in ϕ if and only if the expression in the slanted brackets on the right is negative (zero).

(i) At $\pi(t^*) = 0$ the expression on the right is zero and thus $\bar{\gamma}$ is constant in ϕ. We thus need to show that at the ERSE $d\ln(\pi(t^*)/f(\gamma^*))/d\gamma^* < 0$ holds for $\pi(t^*) > 0$. To this aim, recall that $f(.)$ from (3) was assumed to satisfy $d\left[f(q/\bar{q})\bar{q}^{\theta}\right]/d\bar{q} \leq 0$, or, with $\gamma = q/\bar{q}$, $\theta \leq \gamma f'(\bar{\gamma})/f(\bar{\gamma})$. It is thus suffices to show (remember $q_0 = 1$ so that $q_1 = \gamma^*$)

$$\frac{d}{dq_1} \ln(\pi(t^*)) < \theta / q_1$$

With profits $\pi_1 = (p_1 - c_1)(v_{\max} - v_1)L$ and $v_1 = (p_1 - p_0)/(q_1 - q_0)$ (compare (5) and (6), shifting up indices) and the envelope theorem the condition above is equivalent to

$$\frac{-\dot{c}_1}{p_1 - c_1} - \frac{1}{v_{\max} - v_0}\left(\frac{-\dot{p}_0}{q_1 - q_0} - \frac{p_1 - p_0}{(q_1 - q_0)^2}\right) < \theta$$

where $\dot{x} \equiv dx/dq_1$. With (5) and (19) (shifting up indices) this condition is

$$\dot{p}_0 + v_1 - \dot{c}_1 < \theta\,(p_1 - c_1) \tag{44}$$

To compute $\dot{p}_0$ write the system (19) as

$$\begin{pmatrix} 2 & -1 & 0 & 0 \\ -(q_0 - q_{-1}) & 2(q_1 - q_{-1}) & -(q_1 - q_0) & 0 \\ 0 & -(q_{-1} - q_{-2}) & 2(q_0 - q_{-2}) & ... \\ ... & 0 & ... & ... \end{pmatrix} p = \begin{pmatrix} c_1 + (q_1 - q_0)v_{\max} \\ (q_1 - q_{-1})c_0 \\ (q_0 - q_{-2})c_{-1} \\ ... \end{pmatrix} \tag{45}$$

where $p \equiv (p_1, p_0, ...)^t$. Taking derivatives w.r.t. q_1 yields

$$\begin{pmatrix} 2 & -1 & 0 & 0 \\ -(q_0-q_{-1}) & 2(q_1-q_{-1}) & -(q_1-q_0) & 0 \\ 0 & -(q_{-1}-q_{-2}) & 2(q_0-q_{-2}) & ... \\ ... & 0 & ... & ... \end{pmatrix} \dot{p} + \begin{pmatrix} 0 & 0 & 0 & 0 & ... \\ 0 & 2 & -1 & 0 & \\ 0 & 0 & 0 & 0 & \\ ... & & & & ... \end{pmatrix} p = \begin{pmatrix} \dot{c}_1 + v_{\max} \\ c_0 \\ 0 \\ ... \end{pmatrix}$$

and evaluating at the ERSE leads to

$$\begin{pmatrix} 2 & -1 & 0 & 0 \\ -1 & 2(\gamma+1) & -\gamma & 0 \\ 0 & -1 & 2(\gamma+1) & -\gamma \\ ... & 0 & ... & ... \end{pmatrix} \dot{p} = \begin{pmatrix} \dot{c}_0 + v_{\max} \\ \gamma \frac{-2p_{-1}+p_{-2}+c_{-1}}{q_1-q_0} \\ 0 \\ ... \end{pmatrix} \tag{46}$$

Replicating the proof of Proposition 2, we obtain that $\dot{p}_n$ satisfies $\dot{p}_n = \lambda^n \dot{p}_0$ with $\lambda = \gamma + 1 + \sqrt{\gamma^2 + \gamma + 1}$ for $n \leq 0$. The the second row of (46) thus becomes

$$-\dot{p}_1 + [2(\gamma+1) - \gamma/\lambda]\, \dot{p}_0 = -\dot{p}_1 + \lambda \dot{p}_0 = -\gamma \frac{2p_0 - p_{-1} - c_0}{q_1 - q_0}$$

where we have used that λ solves (36). Combining this equation with the first row of (46) ($2\dot{p}_1 - \dot{p}_0 = \dot{c}_1 + v_{\max}$) leads to

$$[2\lambda - 1]\, \dot{p}_0 = \dot{c}_1 + v_{\max} - 2\gamma \frac{2p_0 - p_{-1} - c_0}{q_1 - q_0} = \dot{c}_0 + v_{\max} - 2\frac{p_1 - 2p_0 + c_0}{q_1 - q_0}$$

where we used is the second row of (45) in the last step. With $v_1 = (p_1 - p_0)/(q_1 - q_0)$ and $v_{\max} - v_1 = (p_1 - c_1)/(q_1 - q_0)$ (compare (5) and (19)), we have

$$\dot{p}_0 = \frac{1}{2\lambda - 1} \left\{ 3v_{\max} - 6v_1 + \dot{c}_1 + 2\frac{c_1 - c_0}{q_1 - q_0} \right\}$$

and hence

$$\dot{p}_0 + v_1 - \dot{c}_1 = \frac{1}{2\lambda - 1} \left\{ 3\,(v_{\max} - v_1) - 2\left(\dot{c}_1 - \frac{c_1 - c_0}{q_1 - q_0}\right) - [2\lambda - 4]\,(\dot{c}_1 - v_1) \right\} \tag{47}$$

We show next that (44) holds for all $t \in [t^{**}, t^{**} + \delta]$ with $\delta > 0$ small enough and t^{**} defined as the date where $v_{\max} = v_1$ holds. At this date we have $p_1 = c_1$ by (19) so that

$$\dot{p}_0 + v_1 - \dot{c}_1 = \frac{-2}{2\lambda - 1} \left[\left(\dot{c}_1 - \frac{c_1 - c_0}{q_1 - q_0}\right) + [\lambda - 2]\,(\dot{c}_1 - v_1) \right] < 0$$

where the last inequality holds by $\lambda > 2$ and

$$v_1 = \frac{c_1 - p_0}{q_1 - q_0} < \frac{c_1 - c_0}{q_1 - q_0} < \dot{c}_1$$

for all $\gamma > 1$, showing (44). By continuity, there is a $\delta > 0$ so that (44) holds for all $t \in [t^{**}, t^{**} + \delta]$.

Now, since π_1 from (25) is increasing in t, there is an $\varepsilon > 0$ so that $t \in (t^{**}, t^{**} + \delta)$ holds whenever $\pi_1 < \varepsilon$. This last condition holds for $r > 0$ small enough as $\Psi_{t^*} > 0$ implies $0 < -\pi_1 + r\Pi$ or

$$\pi_1 < r\phi f(\bar{\gamma}) \tag{48}$$

Finally, we restrict the pair of parameters (r, ϕ) to the compact set $[0, r_1] \times [\underline{\phi}, \bar{\phi}]$. Hence, there are $\gamma_{\min}$ and $\gamma_{\max}$ with $1 < \gamma_{\min} < \gamma_{\max} < \infty$ so that $\bar{\gamma}$ is restricted to the compact set $[\gamma_{\min}, \gamma_{\max}]$. Consequently, there is a uniform $r_0 \leq r_1$ so that for all $(r, \phi) \in [0, r_1] \times [\underline{\phi}, \bar{\phi}]$ we have $\pi_1 < \varepsilon$ and (44) holds uniformly. This proves the statement.

(ii) First notice with (43) that $\bar{\gamma}$ is constant if $\pi(t^*) = 0$. If $\pi(t^*) = 0$, (40) and (41) imply

$$\frac{dt^*(\bar{\gamma}(\phi), \phi)}{d\phi} = \frac{\partial t^*}{\partial \phi} = -\frac{\Psi_\phi}{\Psi_{t^*}} = \frac{f(\bar{\gamma})}{r\Pi} = \frac{1}{r\phi}$$

and we have $t^*(\phi) = const + \ln(\phi)$. As profits from (25) are increasing in t, this implies that, if $\pi(t^*)|_{\phi=\phi_1} = 0$ for $\phi_1 > 0$ decreases in ϕ leave $\bar{\gamma}$ unchanged and decrease t^*. Consequently, $\pi(t^*) = 0$ holds for all $\phi < \phi_1$.

Using $t^*(\phi) = const + \ln(\phi)$ and rescaling time we can write $t^*(\phi) = \ln(\phi)$. But by (25) there is a $\tilde{v}_{\max} \in (0, \infty)$ so that $A + \alpha - 1 = 0$, implying that $\pi_1 = 0$ marginally, and $\pi_1 > 0$ if $v_{\max} > \tilde{v}_{\max}$. Therefore, at entry cost $\tilde{\phi} \equiv (\tilde{v}_{\max}/v_{\max}(0))^{1/a}$ we have $t^*(\tilde{\phi}) = a^{-1} \ln(\tilde{v}_{\max}/v_{\max}(0))$ and

$$v_{\max}(t^*(\tilde{\phi})) = \tilde{v}_{\max}$$

Hence, $\pi_1 = 0$ holds for all $\phi \leq \tilde{\phi}$ and $\pi_1 > 0$ else. Together with (i) and (43) this proves the statement. ■

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