Helmut Lütkepohl: Time Series Econometrics, 2009

Exercise 1

Given the AR(2) process $y_t = 1 + 1.3y_{t-1} - 0.4y_{t-2} + u_t$, $t \in \mathbb{Z}$, with $u_t \sim N(0, 1)$.

(a) Is y_t stationary?
(b) Determine $E(y_t)$.
(c) Determine $\text{Var}(y_t)$.
(d) Determine $\rho_1 = Corr(y_t, y_{t-1})$ and $\rho_2 = Corr(y_t, y_{t-2})$.

Exercise 2

Given the MA(2) process $y_t = 1 + u_t - 1.3u_{t-1} + 0.4u_{t-2}$, $t \in \mathbb{Z}$, with $u_t \sim N(0, 1)$.

(a) Is y_t stationary?
(b) Determine $E(y_t)$.
(c) Determine $\text{Var}(y_t)$.
(d) Determine $\rho_1 = Corr(y_t, y_{t-1})$ and $\rho_2 = Corr(y_t, y_{t-2})$.

Exercise 3

(a) Consider the AR(1) process $y_t = \alpha_1 y_{t-1} + u_t$, $t = 1, 2, \ldots$, with $u_t \sim WN(0, \sigma_u^2)$. Specify y_0 such that the process y_t is stationary.
(b) Consider the AR(2) process $y_t = 1.3y_{t-1} - 0.4y_{t-2} + u_t$, $t = 1, 2, \ldots$, with $u_t \sim WN(0, \sigma_u^2)$. Specify (y_{-1}, y_0) such that the process y_t is stationary.

Exercise 4

Consider the AR(2) process from Exercise 1.

(a) Suppose $y_T = 1.0$, $y_{T-1} = 1.2$. Forecast y_{T+1}, y_{T+2} and y_{T+3}.
(b) Determine the forecast error variances $\sigma_y^2(1)$, $\sigma_y^2(2)$ and $\sigma_y^2(3)$.
(c) Compute 95% interval forecasts for y_{T+1}, y_{T+2} and y_{T+3}.

Exercise 5

Write the AR(3) process $y_t = 1.5y_{t-1} - 0.3y_{t-2} - 0.2y_{t-3} + u_t$ in ADF form $\Delta y_t = \phi y_{t-1} + \alpha_1^* \Delta y_{t-1} + \alpha_2^* \Delta y_{t-2} + u_t$. Determine the values of ϕ, α_1^*, α_2^*.

Exercise 6

Analyze the U.S. investment series.

(a) Find a model for the data generation process.
(b) Check the model carefully.
(c) Determine forecasts for the next two quarters and compute also 90% and 95% forecast intervals.

Exercise 7

Analyze the German consumption series.

(a) Find a model for the data generation process.
(b) Check the model carefully.
(c) Determine forecasts for the next two quarters and compute also 90% and 95% forecast intervals.

Exercise 8

(a) Suppose $H \sim U(-\pi, \pi)$. Show that $E\cos(H) = E\sin(H) = 0$ and $E[\cos(H)\sin(H)] = 0$.
(b) Let A and H be independent random variables with $A \sim (0, \sigma_A^2)$ and $H \sim U(-\pi, \pi)$ and define the stochastic process

$$y_t = A\cos(\lambda t + H), \qquad t = 0, \pm 1, \pm 2, \ldots,$$

where $\lambda \in (-\pi, \pi)$ is a fixed real number. Use the relation

$$\cos(a)\cos(b) = \frac{1}{2}[\cos(a+b) + \cos(a-b)]$$

to show that

$$\begin{aligned} E(y_t y_{t+j}) &= \frac{\sigma_A^2}{2}\left[\int_{-\pi}^{\pi} \frac{1}{2\pi}\cos(2\lambda t + \lambda j + 2h)dh + \cos(\lambda j)\int_{-\pi}^{\pi} \frac{1}{2\pi} dh\right] \\ &= \frac{\sigma_A^2}{2}\cos(\lambda j). \end{aligned}$$

Exercise 9

Suppose y_t is a stationary stochastic process with autocovariances γ_j, $j = 0, \pm 1, \pm 2, \ldots$ and spectral density $f_y(\lambda)$. Show that

$$\gamma_j = \int_{-\pi}^{\pi} e^{i\lambda j} f_y(\lambda) d\lambda, \qquad j = 0, \pm 1, \pm 2, \ldots.$$

Exercise 10

Determine the spectral density function of the MA(2) process $y_t = 1 + u_t - 1.3u_{t-1} + 0.4u_{t-2}$, $t \in \mathbb{Z}$. Plot and interpret the function.

Exercise 11

Analyze the German income series (y_t) form file E4.

(a) Estimate the spectrum of $\log y_t$ using the Bartlett window with different window widths. Consider also the log spectrum and interpret the results.
(b) Estimate the spectrum of $\Delta \log y_t$ using the Bartlett window with different window widths. Consider also the periodogram and log periodogram and interpret the results.
(c) Estimate the spectrum of $\Delta_4 \log y_t$ using the Bartlett window with different window widths. Consider also the log spectrum, the periodogram and log periodogram and interpret the results.

Exercise 12

Given the dynamic regression model $y_t = 1.3y_{t-1} - 0.4y_{t-2} + x_t + 1.2x_{t-1} + u_t$, where u_t is white noise.

(a) Determine the impact multiplier of a change in x_t.
(b) Determine the long-run effect of a change in x_t.
(c) If u_t were not white noise but an MA(1) process, how would you estimate the parameters?

Exercise 13

Consider the following VAR(2) model:

$$\begin{bmatrix} y_{1t} \\ y_{2t} \\ y_{3t} \end{bmatrix} = \begin{bmatrix} 2 \\ 1 \\ 0 \end{bmatrix} + \begin{bmatrix} .7 & .1 & 0 \\ 0 & .4 & .1 \\ .9 & 0 & .8 \end{bmatrix} \begin{bmatrix} y_{1,t-1} \\ y_{2,t-1} \\ y_{3,t-1} \end{bmatrix} + \begin{bmatrix} -.2 & 0 & 0 \\ 0 & .1 & .1 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} y_{1,t-2} \\ y_{2,t-2} \\ y_{3,t-2} \end{bmatrix} + \begin{bmatrix} u_{1t} \\ u_{2t} \\ u_{3t} \end{bmatrix},$$

$$\Sigma_u = \begin{bmatrix} .26 & .03 & 0 \\ .03 & .09 & 0 \\ 0 & 0 & .81 \end{bmatrix}.$$

(a) Is y_2 Granger-causal for (y_1, y_3)?
(b) Is y_3 Granger-causal for (y_1, y_2)?
(c) Is there instantaneous causality between y_2 and (y_1, y_3)?
(d) Is there instantaneous causality between y_3 and (y_1, y_2)?

Exercise 14

Consider the VAR(1) process

$$y_t = \begin{bmatrix} -.2 & .8 \\ .6 & .6 \end{bmatrix} y_{t-1} + u_t, \qquad \Sigma_u = \begin{bmatrix} .26 & .03 \\ .03 & .09 \end{bmatrix} = PP', \qquad P = \begin{bmatrix} .5 & .1 \\ 0 & .3 \end{bmatrix}.$$

(a) Determine the forecast error impulse responses Φ_1 and Φ_2.
(b) Determine the orthogonalized impulse responses Θ_0, Θ_1 and Θ_2.
(c) Suppose $y_t = (1, 2)'$. Compute forecasts for y_{T+1} and y_{T+2}.
(d) Determine the forecast error covariance matrices $\Sigma_y(1)$ and $\Sigma_y(2)$.
(e) Suppose that the process is started in period $t = 1$. Specify y_0 such that the process is stationary.

Exercise 15

Analyze the bivariate series $y_t = (y_{1t}, y_{2t})'$ of first differences of the U.S. investment data in File E2.

(a) Find a VAR model for the data generation process.
(b) Check the model carefully.
(c) Is y_{1t} Granger-causal for y_{2t}? Is y_{2t} Granger-causal for y_{1t}? Perform suitable tests.
(d) Determine forecast error and orthogonalized impulse responses.
(e) Perform a forecast error variance decomposition.

Exercise 16
Consider the process

$$y_t = \begin{bmatrix} 1 & 0 \\ 0 & \psi \end{bmatrix} y_{t-1} + u_t.$$

(a) What is the cointegrating rank of the process?
(b) Write the process in VECM form.

Exercise 17
Determine the roots of $\det(I_2 - A_1 z)$ and, if applicable, the cointegrating rank of the VAR(1) process

$$y_t = \begin{bmatrix} -.2 & .8 \\ .6 & .6 \end{bmatrix} y_{t-1} + u_t.$$

Can you write the process in VECM form?

Exercise 18
What is the maximum possible cointegrating rank of a three-dimensional process $y_t = (y_{1t}, y_{2t}, y_{3t})'$,

(a) if y_{1t}, y_{2t} are $I(0)$ and y_{3t} is $I(1)$?
(b) if y_{1t}, y_{2t}, and y_{3t} are $I(1)$ and y_{1t} and y_{2t} are not cointegrated in a bivariate system?
(c) if y_{1t}, y_{2t}, and y_{3t} are $I(1)$ and $(y_{1t}, y_{2t})'$ and $(y_{2t}, y_{3t})'$ are not cointegrated as bivariate systems?

Exercise 19
Consider the VECM

$$\Delta y_t = \begin{bmatrix} -0.1 \\ 0.1 \end{bmatrix} (1, -1) y_{t-1} + u_t.$$

(a) Rewrite the process in VAR form.
(b) Determine the roots of $\det(I_2 - A_1 z)$.
(c) Determine the MA coefficient matrices Φ_1, Φ_2, Φ_3.
(d) Determine $\lim_{i\to\infty} \Phi_i$. What does this imply for the impulse responses?
(e) Has a forecast error impulse in y_{1t} a permanent impact on y_{2t}? Has a forecast error impulse in y_{2t} a permanent impact on y_{1t}?

Exercise 20
Analyze the U.S. data in File E3.

(a) Find a VAR or VEC model for the data generation process as appropriate.
(b) Check the model carefully.
(c) Determine forecast error and orthogonalized impulse responses.
(d) Perform a forecast error variance decomposition.