

$$\sqrt{T} \text{vec}(\widehat{\Phi}_i - \Phi_i) \xrightarrow{d} \mathcal{N}(0, G_i \Sigma_{\widehat{\alpha}} G_i'), \quad i = 1, 2, \dots, \quad (3.7.5)$$

where

$$G_i := \frac{\partial \text{vec}(\Phi_i)}{\partial \alpha'} = \sum_{m=0}^{i-1} J(\mathbf{A}')^{i-1-m} \otimes \Phi_m.$$

$$\sqrt{T} \text{vec}(\widehat{\Psi}_n - \Psi_n) \xrightarrow{d} \mathcal{N}(0, F_n \Sigma_{\widehat{\alpha}} F_n'), \quad n = 1, 2, \dots, \quad (3.7.6)$$

where $F_n := G_1 + \dots + G_n$.

If $(I_K - A_1 - \dots - A_p)$ is nonsingular,

$$\sqrt{T} \text{vec}(\widehat{\Psi}_\infty - \Psi_\infty) \xrightarrow{d} \mathcal{N}(0, F_\infty \Sigma_{\widehat{\alpha}} F_\infty'), \quad (3.7.7)$$

where $F_\infty := \underbrace{(\Psi'_\infty, \dots, \Psi'_\infty)}_{p \text{ times}} \otimes \Psi_\infty$.

$$\sqrt{T} \text{vec}(\widehat{\Theta}_i - \Theta_i) \xrightarrow{d} \mathcal{N}(0, C_i \Sigma_{\widehat{\alpha}} C_i' + \bar{C}_i \Sigma_{\widehat{\sigma}} \bar{C}_i'), \quad i = 0, 1, 2, \dots, \quad (3.7.8)$$

where

$$C_0 := 0, \quad C_i := (P' \otimes I_K) G_i, \quad i = 1, 2, \dots, \quad \bar{C}_i := (I_K \otimes \Phi_i) H, \quad i = 0, 1, \dots,$$

and

$$\begin{aligned} H &:= \frac{\partial \text{vec}(P)}{\partial \sigma'} = \mathbf{L}'_K \{ \mathbf{L}_K [(I_K \otimes P) \mathbf{K}_{KK} + (P \otimes I_K)] \mathbf{L}'_K \}^{-1} \\ &= \mathbf{L}'_K \{ \mathbf{L}_K (I_{K^2} + \mathbf{K}_{KK}) (P \otimes I_K) \mathbf{L}'_K \}^{-1}. \end{aligned}$$

$$\sqrt{T} \text{vec}(\widehat{\Xi}_n - \Xi_n) \xrightarrow{d} \mathcal{N}(0, B_n \Sigma_{\widehat{\alpha}} B_n' + \bar{B}_n \Sigma_{\widehat{\sigma}} \bar{B}_n'), \quad (3.7.9)$$

where $B_n := (P' \otimes I_K) F_n$ and $\bar{B}_n := (I_K \otimes \Psi_n) H$.

If $(I_K - A_1 - \dots - A_p)$ is nonsingular,

$$\sqrt{T} \text{vec}(\widehat{\Xi}_\infty - \Xi_\infty) \xrightarrow{d} \mathcal{N}(0, B_\infty \Sigma_{\widehat{\alpha}} B_\infty' + \bar{B}_\infty \Sigma_{\widehat{\sigma}} \bar{B}_\infty'), \quad (3.7.10)$$

where $B_\infty := (P' \otimes I_K) F_\infty$ and $\bar{B}_\infty := (I_K \otimes \Psi_\infty) H$.

Finally,

$$\sqrt{T} (\widehat{\omega}_{jk,h} - \omega_{jk,h}) \xrightarrow{d} \mathcal{N}(0, d_{jk,h} \Sigma_{\widehat{\alpha}} d_{jk,h}' + \bar{d}_{jk,h} \Sigma_{\widehat{\sigma}} \bar{d}_{jk,h}') \quad (3.7.11)$$

$$j, k = 1, \dots, K, \quad h = 1, 2, \dots,$$

where

$$\begin{aligned} d_{jk,h} &:= \frac{2}{\text{MSE}_j(h)^2} \sum_{i=0}^{h-1} \left[\text{MSE}_j(h) (e'_j \Phi_i P e_k) (e'_k P' \otimes e'_j) G_i \right. \\ &\quad \left. - (e'_j \Phi_i P e_k)^2 \sum_{m=0}^{h-1} (e'_j \Phi_m \Sigma_u \otimes e'_j) G_m \right] \end{aligned}$$