As we have seen in Remark 2, the matrix operator $\boldsymbol{\Theta}(z)$ can be decomposed as

$$\boldsymbol{\Theta}(z) = \boldsymbol{\Theta}(1) + (1-z)\boldsymbol{\Theta}^*(z).$$

Hence, we get from (6.3.20),

$$z_t = \boldsymbol{\Theta}(1)u_t + \boldsymbol{\Theta}^*(L)\Delta u_t = B(1)^{-1}u_t + \boldsymbol{\Theta}^*(L)\Delta u_t. \tag{6.3.21}$$

Using

$$y_t = Q^{-1}Qy_t = [\bar{\beta} : \beta_\perp] \begin{bmatrix} \beta' y_t \\ \bar{\beta}'_\perp y_t \end{bmatrix} = \bar{\beta}\beta' y_t + \beta_\perp \bar{\beta}'_\perp y_t$$

and, hence,

$$\Delta y_t = \bar{\beta}\beta' \Delta y_t + \beta_\perp \bar{\beta}'_\perp \Delta y_t,$$

it follows from (6.3.17) and (6.3.18) that $\Delta y_t = z_t - \bar{\beta}\beta' z_{t-1}$. Thus,

$$\beta_\perp \bar{\beta}'_\perp \Delta y_t = \beta_\perp \bar{\beta}'_\perp z_t.$$

Substituting the expression from (6.3.21) for z_t gives

$$\begin{aligned} \Delta y_t &= \beta_\perp \bar{\beta}'_\perp z_t + \bar{\beta}\beta' \Delta y_t \\ &= \beta_\perp \bar{\beta}'_\perp B(1)^{-1} u_t + \beta_\perp \bar{\beta}'_\perp \boldsymbol{\Theta}^*(L)\Delta u_t + \bar{\beta}\beta' \Delta y_t := w_t. \end{aligned}$$

Solving for $y_t = y_{t-1} + w_t$ results in

$$\begin{aligned} y_t &= y_0 + \sum_{i=1}^{t} w_i \\ &= y_0 + \beta_\perp \bar{\beta}'_\perp B(1)^{-1} \sum_{i=1}^{t} u_i + \beta_\perp \bar{\beta}'_\perp \boldsymbol{\Theta}^*(L) \sum_{i=1}^{t} \Delta u_i + \bar{\beta}\beta' \sum_{i=1}^{t} \Delta y_i \\ &= y_0 + \beta_\perp \bar{\beta}'_\perp B(1)^{-1} \sum_{i=1}^{t} u_i + \beta_\perp \bar{\beta}'_\perp \boldsymbol{\Theta}^*(L)(u_t - u_0) + \bar{\beta}\beta'(y_t - y_0) \\ &= \beta_\perp \bar{\beta}'_\perp B(1)^{-1} \sum_{i=1}^{t} u_i + \beta_\perp \bar{\beta}'_\perp \boldsymbol{\Theta}^*(L)u_t + \bar{\beta}\beta' y_t + y_0^*, \end{aligned} \tag{6.3.22}$$

where $y_0^* := y_0 - \beta_\perp \bar{\beta}'_\perp \boldsymbol{\Theta}^*(L)u_0 - \bar{\beta}\beta' y_0$. Using $\beta' y_t = \beta' z_t$, the term $\bar{\beta}\beta' y_t = \bar{\beta}\beta' z_t$ is seen to have a representation

$$\bar{\beta}\beta' z_t = \bar{\beta}\beta' \boldsymbol{\Theta}(L)u_t$$

and, thus, $\beta_\perp \bar{\beta}'_\perp \boldsymbol{\Theta}^*(L)u_t + \bar{\beta}\beta' y_t$ has an MA representation

$$\boldsymbol{\Xi}^*(L)u_t = [\beta_\perp \bar{\beta}'_\perp \boldsymbol{\Theta}^*(L) + \bar{\beta}\beta' \boldsymbol{\Theta}(L)]u_t.$$