Forecast Bias and MSFE Encompassing*

Massimiliano Marcellino
Istituto di Economia Politica, Università Bocconi
and I.G.I.E.R.

This Version: January 2000

Abstract

We show that the standard condition for MSFE encompassing is no longer valid when the forecasts to be compared are biased. We propose a simple modification of such a condition and of tests for its validity. The theoretical results are illustrated by an empirical example on inflation and deficit forecasts, key variables for the formulation of monetary and fiscal policy.

J.E.L. Classification: C52, C53

*I would like to thank Mike Artis, Anindya Banerjee, Clive Granger and a referee for helpful comments on an earlier draft. The usual disclaimers apply.

1. Introduction

Several different forecasts of the same phenomenon are often available. The usual approach in this case is either to choose one of them on the basis of a certain criterion, such as minimum mean squared forecast error (MSFE) or mean absolute error (MAE), or to combine them in a pooled forecast, again with the aim of minimizing a certain loss function, see e.g. Clemen (1989).

An alternative approach is based on the so-called encompassing principle, see e.g. Marcellino and Mizon (2000), which was originally developed for in-sample model comparison, and later on extended to forecast comparison, starting with the seminal paper by Chong and Hendry (1986). Forecast encompassing requires one model to be able to correctly predict the forecasts of the competing models, i.e. the models that produce the alternative forecasts, or at least their implied MSFEs. In this case the competing models become redundant, and all the relevant information is contained in the encompassing model.

The evaluation of forecast encompassing requires knowledge of the forecasting models, or at least of the relationship among the competing forecasts, while for MSFE encompassing only the forecasts and the realizations are necessary. The larger information set is often not available when comparing, for example, forecasts from international organizations such as the IMF or the OECD, or forecasts from econometric models with consensus forecasts. Therefore, we will focus on MSFE encompassing. MSFE encompassing tests, namely tests for checking whether one model can correctly predict the MSFE arising from a competing model, were originally proposed by Chong and Hendry (1986), as mentioned, and were further analysed and extended by Lu and Mizon (1991), Ericsson (1992), Ericsson and Marquez (1993).

In this paper we show that the standard conditions and tests for MSFE encompassing are no longer valid when the forecasts under comparison are biased, and modify them properly. We think that this is an important extension because forecast unbiasedness is often rejected in practice, which can be also justified theoretically if the loss function of the forecaster is asymmetric (e.g., Granger and Newbold (1986, Ch. 4)), if other goals rather than accuracy are important, e.g. publicity (Laster *et al.* (1997)), or if there are unaccounted structural changes over the forecast period (e.g. Clements and Hendry(1997)). In Section 2 we define formally MSFE encompassing, and present tests for its validity. In Section 3 we apply the tests to compare the IMF inflation and deficit forecasts for the G7 countries with random walk based forecasts. Section 4 summarizes and concludes.

2. MSFE Encompassing

2.1. Definitions

We assume that only two univariate forecasting models are available, M_1 and M_2, their parameters are known and constant over the forecast period, the variable to be forecast, y, is stationary, and the forecast horizon is one period. These hypotheses are useful for focusing on the main topics, they can be easily relaxed (see Marcellino (1998)). The forecasts for period t made at time $t-1$ by M_1 and M_2 are labelled $\widehat{y}_{1t}$ and $\widehat{y}_{2t}$, with $t = T+1, T+2, ..., T+N$.

For the reasons exposited in the Introduction, we will not require $\widehat{y}_{1t}$ and $\widehat{y}_{2t}$ to be unbiased, namely,

$$\begin{array}{lllll} M_1 \Rightarrow & y_t = a + b\widehat{y}_{1t} + u_{1t}, & u_{1t} \sim i.i.d.(0, \sigma_{1u}), & cov(u_{1t}, \widehat{y}_{1t}) = 0 & i \geq 0, \\ M_2 \Rightarrow & y_t = c + d\widehat{y}_{2t} + u_{2t}, & u_{2t} \sim i.i.d.(0, \sigma_{2u}), & cov(u_{2t}, \widehat{y}_{2t}) = 0 & i \geq 0. \end{array} \tag{2.1}$$

Unbiasedness requires $a = (1-b)E(\widehat{y}_{1t})$ and/or $c = (1-d)E(\widehat{y}_{2t})$, as pointed out by Holden and Peel (1990). A sufficient condition for unbiasedness is the well-known restriction $(a = 0, b = 1)$ and/or $(c = 0, d = 1)$. Under such a restriction, u_{it} coincides with the forecast error $e_{it} = y_t - \widehat{y}_{it}$, $i = 1, 2$; otherwise they will be different.

From (2.1) it follows that

$$MSFE_2 = MSFE_1 + E(\widehat{y}_{1t} - \widehat{y}_{2t})^2 + 2E[(y_t - \widehat{y}_{1t})(\widehat{y}_{1t} - \widehat{y}_{2t})]. \tag{2.2}$$

The prediction of M_1 for $MSFE_2$, $MSFE_{21}$, is

$$MSFE_{21} = MSFE_1 + E(\widehat{y}_{1t} - \widehat{y}_{2t})^2 + 2E[(a + (b-1)\widehat{y}_{1t})(\widehat{y}_{1t} - \widehat{y}_{2t})]. \tag{2.3}$$

M_1 encompasses M_2 with respect to the MSFE if and only if $MSFE_{21} = MSFE_2$.

If $(a = 0, b = 1)$, the last term in the right hand side of (2.3) is equal to zero, and MSFE encompassing requires lack of correlation between the forecast error from one model and the difference of the two forecasts, i.e.,

$$E[(y_t - \widehat{y}_{1t})(\widehat{y}_{1t} - \widehat{y}_{2t})] = 0. \tag{2.4}$$

If the sufficient condition for unbiasedness does not hold, it is still possible to have MSFE encompassing but the requirement becomes

$$E[(y_t - \widehat{y}_{1t})(\widehat{y}_{1t} - \widehat{y}_{2t})] = E[(a + (b-1)\widehat{y}_{1t})(\widehat{y}_{1t} - \widehat{y}_{2t})].$$

This can be rewritten as

$$E[u_{1t}(\widehat{y}_{1t} - \widehat{y}_{2t})] = 0, \tag{2.5}$$

namely, the error u_{1t} (no longer the forecast error $y_t - \widehat{y}_{1t}$) must be uncorrelated with the difference of the forecasts. Notice that this is the appropriate condition to be verified not only when the forecasts are biased, but also when they are unbiased but $(a = 0, b = 1)$ is not satisifed.

It is now worth analysing the relationship between MSFE dominance of M_1 ($\text{MSFE}_1 < \text{MSFE}_2$) and MSFE encompassing. When $(a = 0, b = 1)$, MSFE dominance is only a necessary condition for MSFE encompassing, and selecting a model according to this criterion does not ensure that the resulting forecast errors cannot be explained by the alternative forecasts. Instead, MSFE encompassing is a sufficient condition for MSFE dominance. Both propositions can be easily derived from a comparison of (2.2) and (2.3).

When $\widehat{y}_{1t}$ is biased, or if it is unbiased but $(a = 0, b = 1)$ does not hold, MSFE dominance is no longer necessary for MSFE encompassing, and the latter is not sufficient for the former. Actually, from (2.3), it can be $MSFE_{21} = MSFE_2$, i.e. M_1 MSFE encompasses M_2, but

$$E(\widehat{y}_{1t} - \widehat{y}_{2t})^2 + 2E[(a + (b-1)\widehat{y}_{1t})(\widehat{y}_{1t} - \widehat{y}_{2t})] < 0,$$

which implies $MSFE_2 < MSFE_1$.

As an example, let us supplement (2.1) with a description of the relationship between the two forecasts,

$$\widehat{y}_{2t} = \alpha + \beta\widehat{y}_{1t} + \eta_t, \quad \eta_t \sim i.i.d.(0, \sigma_\eta), \tag{2.6}$$

with $cov(u_{1t}, \eta_t) = 0$. It is

$$E[(y_t - \widehat{y}_{1t})(\widehat{y}_{1t} - \widehat{y}_{2t})] = E[(a + (b-1)\widehat{y}_{1t})(\widehat{y}_{1t} - \widehat{y}_{2t})],$$

so that $MSFE_{21} = MSFE_2$, and M_1 MSFE encompasses M_2.[1] Notice that using the condition (2.4) to verify encompassing, we would reject it because it is $E[(y_t - \widehat{y}_{1t})(\widehat{y}_{1t} - \widehat{y}_{2t})] \neq 0$. Then we have,

$$\begin{aligned} MSFE_2 - MSFE_1 &= E(\widehat{y}_{1t})^2(1-\beta)(2b-1-\beta) + 2E(\widehat{y}_{1t})[a(1-\beta) - \alpha(b-\beta)] + \\ &\quad + \alpha^2 + \sigma_\eta - 2a\alpha. \end{aligned}$$

This quantity can be larger or smaller than zero, e.g., it is $MSFE_2 < MSFE_1$ for $2b - 1 < \beta < 1$, $\alpha = a(1-\beta)/(b-\beta)$, $\sigma_\eta < \alpha(2a - \alpha)$.

To conclude, it can be worth stressing that MSFE encompassing provides a measure of the relative performance of a model, not of its overall goodness in forecasting, so that the latter should be separately assessed.

2.2. Univariate tests

One of the earliest attempts to provide a statistical tool for choosing a forecasting formula, Hoel (1947), is based on the significance of the regressor in the model

$$y_t - \widehat{y}_{1t} = \phi(\widehat{y}_{2t} - \widehat{y}_{1t}) + u_t, \quad u_t \sim i.i.d.(0, \sigma_u). \tag{2.7}$$

The underlying idea is that when $\phi = 0$ the forecast error from M_1 cannot be explained by M_2, which is therefore redundant. From (2.4), a test for $\phi = 0$ is also a MSFE encompassing test, as noticed by Ericsson (1992) and Clements and Hendry

[1] Instead, $E[(y_t - \widehat{y}_{2t})(\widehat{y}_{2t} - \widehat{y}_{1t})] \neq E[(c + (d-1)\widehat{y}_{2t})(\widehat{y}_{2t} - \widehat{y}_{1t})]$, so that M_2 does not MSFE encompass M_1.

(1993). Yet, from (2.5), this statement is correct only if $(a = 0, b = 1)$ in (2.1). When this hypothesis does not hold, in order to construct a MSFE encompassing test based on the hypothesis $\phi = 0$, the regression should be modified into

$$\widehat{u}_{1t} = \phi(\widehat{y}_{2t} - \widehat{y}_{1t}) + u_t, \quad u_t \sim i.i.d.(0, \sigma_u), \tag{2.8}$$

where $\widehat{u}_{1t}$ is the estimated counterpart of u_{1t}.

Chong and Hendry (1986) instead considered an equation similar to (2.7), namely,

$$y_t - \widehat{y}_{1t} = \xi \widehat{y}_{2t} + u_t, \tag{2.9}$$

and proposed to test for $\xi = 0$ for MSFE encompassing. They showed that the t-statistic for $\xi = 0$ has a $N(0, 1)$ distribution for large T and N. This is also the suggestion by Ericsson (1992) in the case of stationary variables. Yet, again, this is a MSFE encompassing test only if $(a = 0, b = 1)$. Otherwise, the dependent variable should be substituted by $\widehat{u}_{1t}$.

Continuing the example in the previous section, when the forecasts are biased, even under MSFE encompassing of M_1 for M_2 both the OLS estimator of ϕ in (2.7) and that of ξ in (2.9) converge to non zero values, because $E[(y_t - \widehat{y}_{1t})(\widehat{y}_{1t} - \widehat{y}_{2t})]$ and $E[(y_t - \widehat{y}_{1t})\widehat{y}_{2t}]$ are different from zero. Instead, using $\widehat{u}_{1t}$ as the dependent variable in either (2.7) or (2.9), the OLS estimator of the coefficient of the regressor converges to zero, correctly indicating that M_1 MSFE encompasses M_2.

Some of the aforementioned drawbacks of standard MSFE encompassing tests can be also avoided by adopting the model

$$y_t = e + f\widehat{y}_{1t} + g\widehat{y}_{2t} + u_t, \tag{2.10}$$

which nests both equations in (2.1). Equation (2.10) has been often employed in the literature on forecast pooling, see, e.g., Clemen (1989), because the estimates of the parameters provide the optimal weights in the sense of minimising the MSFE for the pooled forecast. The hypothesis $g = 0$ corresponds to M_1 encompasses M_2 with respect to MSFE. When $e = 0$, $f = 1$ the forecasts from M_1 are also unbiased. This restriction leads to (2.9), while $e = 0$, $f + g = 1$ (which holds when at least one forecast is unbiased) leads to (2.7). It can be easily verified that in the case of the example in the previous section the OLS estimators of e, f, and g converge, respectively, to a, b, and zero.

2.3. Multivariate tests

We now assume that x is a $n \times 1$ vector of variables whose forecasts by M_1 and M_2 are the $n \times 1$ vectors $\widehat{x}_{1t}$ and $\widehat{x}_{2t}$, with

$$\begin{array}{lllll} M_1 \Rightarrow & x_t = a + B\widehat{x}_{1t} + u_{1t}, & u_{1t} \sim i.i.d.(0, \Omega_{1u}), & cov(u_{1t}, \widehat{x}_{1t-i}) = 0 & i \geq 0, \\ M_2 \Rightarrow & x_t = c + D\widehat{x}_{2t} + u_{2t}, & u_{2t} \sim i.i.d.(0, \Omega_{2u}), & cov(u_{2t}, \widehat{x}_{2t-i}) = 0 & i \geq 0, \end{array} \tag{2.11}$$

where a, c are $n \times 1$ vectors while B, D are $n \times n$ matrices. To start with, we wish to discuss alternative tests for trace MSFE encompassing, where the MSFE matrix implied by the two models is

$$\Phi^i = \begin{pmatrix} E(e_{i1t})^2 & E(e_{i1t}^{'}e_{i2t}) & ... & E(e_{i1t}^{'}e_{int}) \\ E(e_{i1t}^{'}e_{i2t}) & E(e_{i2t})^2 & ... & E(e_{i2t}^{'}e_{int}) \\ ... & & & \\ E(e_{i1t}^{'}e_{int}) & E(e_{i2t}^{'}e_{int}) & ... & E(e_{int})^2 \end{pmatrix},$$

$e_{ijt} = x_{jt} - \widehat{x}_{ijt}$, $j = 1, ..., n$, and $i = 1, 2$.

Trace MSFE encompassing of M_1 for M_2 requires $tr(\Phi_2) = tr(\Phi_{21})$, where Φ_{21}, the prediction of M_1 for Φ_2, is

$$\Phi_{21} = \Phi_1 + E(\widehat{x}_{1t} - \widehat{x}_{2t})(\widehat{x}_{1t} - \widehat{x}_{2t})^{'} + 2E[(a + (B - I)\widehat{x}_{1t})(\widehat{x}_{1t} - \widehat{x}_{2t})^{'}], \qquad (2.12)$$

while Φ_2 can be written as

$$\Phi_2 = \Phi_1 + E(\widehat{x}_{1t} - \widehat{x}_{2t})(\widehat{x}_{1t} - \widehat{x}_{2t})^{'} + 2E(x_t - \widehat{x}_{1t})(\widehat{x}_{1t} - \widehat{x}_{2t})^{'}. \qquad (2.13)$$

Hence, to have trace MSFE it must be

$$tr(E(x_t - \widehat{x}_{1t})(\widehat{x}_{1t} - \widehat{x}_{2t})^{'}) = tr(E[(a + (B - I)\widehat{x}_{1t})(\widehat{x}_{1t} - \widehat{x}_{2t})^{'}]). \qquad (2.14)$$

It immediately follows that when $(a = 0, B = I)$, the condition simplifies to

$$tr(E(x_t - \widehat{x}_{1t})(\widehat{x}_{1t} - \widehat{x}_{2t})^{'}) = 0. \qquad (2.15)$$

Trace MSFE dominance is often employed as a tool for selecting a multivariate forecasting model. However, when $(a = 0, B = I)$, it is necessary but not sufficient for trace MSFE encompassing. The latter implies trace MSFE dominance, because $E(\widehat{x}_{1t} - \widehat{x}_{2t})(\widehat{x}_{1t} - \widehat{x}_{2t})^{'}$ is a symmetric matrix so that its trace is non negative. As in the univariate case, these relationships no longer necessarily hold when the forecasts are biased or the condition $(a = 0, B = I)$ is not satisfied.

Notice also that while variable by variable MSFE dominance and encompassing imply trace MSFE dominance and encompassing, the converse is not necessarily true. As an example of this, let us assume that $x_t = (y_t, z_t)^{'}$, and that the competing models and relationships among the forecasts are such that

$$\begin{array}{c} y_t = \widehat{y}_{1t} + u_{yt}, \quad \widehat{y}_{1t} = \widehat{y}_{2t} + e_{yt}, \\ z_t = \widehat{z}_{2t} + u_{zt}, \quad \widehat{z}_{2t} = \widehat{z}_{1t} + e_{zt}, \\ \begin{pmatrix} u_{yt} \\ e_{yt} \end{pmatrix} \sim i.i.d. \left(\begin{pmatrix} 0 \\ 0 \end{pmatrix}, \begin{pmatrix} \sigma_{yu} & \sigma_{yue} \\ \sigma_{yue} & \sigma_{ye} \end{pmatrix} \right), \\ \begin{pmatrix} u_{zt} \\ e_{zt} \end{pmatrix} \sim i.i.d. \left(\begin{pmatrix} 0 \\ 0 \end{pmatrix}, \begin{pmatrix} \sigma_{zu} & \sigma_{zue} \\ \sigma_{zue} & \sigma_{ze} \end{pmatrix} \right), \\ cov(u_{yt}, u_{zt}) = cov(u_{yt}, e_{zt}) = cov(e_{yt}, u_{zt}) = cov(e_{yt}, e_{zt}) = 0. \end{array} \qquad (2.16)$$

We have

$$\Phi_1 = \begin{pmatrix} \sigma_{yu} & 0 \\ 0 & \sigma_{zu} + \sigma_{ze} + 2\sigma_{zue} \end{pmatrix}, \quad \Phi_2 = \begin{pmatrix} \sigma_{yu} + \sigma_{ye} + 2\sigma_{yue} & 0 \\ 0 & \sigma_{zu} \end{pmatrix},$$

$$\begin{aligned} E(x_t - \widehat{x}_{1t})(\widehat{x}_{1t} - \widehat{x}_{2t})' &= \begin{pmatrix} \sigma_{yue} & 0 \\ 0 & -\sigma_{zue} - \sigma_{ze} \end{pmatrix}, \\ \Phi_{21} &= \begin{pmatrix} \sigma_{yu} + \sigma_{ye} & 0 \\ 0 & \sigma_{zu} - 2\sigma_{zue} - 2\sigma_{ze} \end{pmatrix}. \end{aligned}$$

Hence, for $\sigma_{yue} = \sigma_{zue} + \sigma_{ze}$, we have trace MSFE encompassing but not equation by equation MSFE encompassing. Under this condition, it is also $tr(\Phi_1) - tr(\Phi_2) = -\sigma_{ye} - \sigma_{ze} < 0$, so that trace MSFE dominance holds, but not necessarily MSFE dominance equation by equation.

In order to test for trace MSFE encompassing, a first possibility is to run n type-(2.10) regressions

$$x_{jt} = e + f\widehat{x}_{jt}^1 + g\widehat{x}_{jt}^2 + u_{jt}, \quad j = 1, ..., n, \tag{2.17}$$

and test the hypothesis $g = 0$ in each of them. In order to have an overall size of α, the size of each test should be equal to α/n. (2.17) is a system of seemingly unrelated regression equations, and Nelson (1972) proposed to estimate it by means of GLS in order to obtain more efficient estimators than OLS. Trace MSFE encompassing is accepted when the hypothesis $g = 0$ is accepted in each equation. Yet, as mentioned, this is a sufficient but not necessary condition for trace MSFE encompassing, so that in particular cases the latter could be wrongly rejected.

In order to avoid this problem, the n variables and forecasts can be stacked into the $nN \times 1$ vectors X, $\widehat{X}_1$, $\widehat{X}_2$. These are then used in the regression

$$X_l = e + f\widehat{X}_{1l} + g\widehat{X}_{2l} + U_l, \quad l = 1, ..., nN, \tag{2.18}$$

and a t-test for $g = 0$ is performed. In general, the error term in (2.18) is heteroskedastic. The varying variance can be estimated from the system in (2.17), so that GLS estimation is feasible. Under the null hypothesis, using (2.18) instead of (2.17) can be also advantageous because of the larger number of available observations. Yet, we are imposing that the values of the constant and of the coefficient of the first forecast are equal for all variables, an hypothesis that can be relaxed by inserting appropriate dummies in the regression, at the cost of losing degrees of freedom. Individual properties such as forecast unbiasedness and efficiency could then also be tested in this framework.

As far as efficiency is concerned, in (2.18) and in (2.17) we are assuming that the forecasts of other variables from the two models are not relevant explanatory variables. This hypothesis can be also relaxed, considering the system of equations

$$x_t = c + \Xi\widehat{x}_{1t} + H\widehat{x}_{2t} + v_t, \tag{2.19}$$

where Ξ and H are $n \times n$ matrices of parameters and c is an $n \times 1$ vector of constants. (2.19) boils down to (2.17) when Ξ and H are diagonal. The joint hypothesis of efficiency and encompassing requires therefore $\Xi = diag$, $H = 0$ while if $c = 0$, $\Xi = I$ the forecasts from M_1 are also unbiased.[2] Notice that in this case encompassing is with respect to Φ_2, the MSFE matrix for M_2, which implies trace MSFE encompassing.

When $\Xi + H = I$, which holds for example when $\widehat{x}_{1t}$ is unbiased, (2.19) can be re-written as

$$x_t - \widehat{x}_{1t} = c + H(\widehat{x}_{2t} - \widehat{x}_{1t}) + v_t. \tag{2.20}$$

A test for $H = 0$ in (2.20) is invariant to isomorphic dynamic transformations of the underlying system for the x variables and corresponding forecasts, while this is not true in (2.19). This property follows from invariance of the 1-step ahead forecast errors to these transformations, and it also holds for the univariate case, see Clements and Hendry (1993). Under the null hypothesis, the test is also invariant to contemporaneous linear transformations (Clements and Hendry (1998a, Ch. 10.3)).

3. An empirical example

To illustrate some of the previous theoretical results. We analyse the (yearly) deficit to gdp ratio and the inflation forecasts from the IMF for the G7 countries, over the period 1975-1994. These are two important variables for fiscal and monetary policy, and it seems therefore important to evaluate how accurate their forecasts are. We consider current year forecasts, namely those for period t published in the May issue of year t of the *World Economic Outlook*. The actual data are the first released values, which appear in the May issue of year $t+1$ of the *Outlook*.[3] A comparison with the OECD and EC deficit forecasts is presented in Artis and Marcellino (1998, 1999). Here we use as alternative forecasts those based on a random walk model for the variables, which therefore coincide with the actual values for year $t-1$ (which become available at the same time as the IMF forecasts). Hence, $\widehat{y}_{1t} = \widehat{y}_t^{IMF}$, $\widehat{y}_{2t} = y_{t-1}$. Notwithstanding the likely misspecification of the random walk model (the persistence of most variables is rather low), and the larger information set embodied in the IMF forecasts, the alternative naive forecasts seem to perform rather well in some cases. We argue that this is due to the robustifying role of differencing in the presence of structural changes (see e.g. Clements and Hendry (1998b)).

We directly jointly analyze the deficit to gdp ratio and inflation forecasts.[4] The second and third columns of Table 3.1 report the ratio of the trace and determinant of the MSFE matrix, Φ, for the IMF to those for the RW. The IMF does better in

[2] As in the univariate case, this condition is only sufficient for unbiasedness, the necessary and sufficient condition being $c = (I - \Xi)E(\widehat{x}_{1t})$.

[3] See, e.g., Gallo and Marcellino (1999) for issues of data revision.

[4] All the calculations were performed with PcGive and PcFiml 9.01, see Hendry and Doornik (1997), Doornik and Hendry (1997). Univariate analyses are presented in Marcellino (1998). Detailed results are available upon request.

Table 3.1: Deficit and Inflation forecasts, joint analysis.

	$tr(\Phi)$ IMF/RW	$\|\Phi\|$ IMF/RW	*Eff.*	*Eff+Enc.* IMF	*Eff+Enc.* RW	*Eff.+Enc.+Unb.*
US	0.24	0.08	7.46	10.1	**197**	**39.6**
CAN	0.47	0.61	7.95	11.0	**54.8**	**22.9**
JAP	0.49	0.99	1.85	**21.8**	**14.2**	–
GER	0.40	0.28	7.62	**13.9**	**103**	–
FR	0.54	0.36	4.64	9.04	**49.7**	13.4
IT	0.72	0.76	8.04	9.19	**176**	**49.8**
UK	0.17	0.09	4.99	5.11	**95.0**	12.3

Eff.: Wald test ($\chi^2(4)$) for Ξ and H diagonal in (2.19).
Eff.+Enc. IMF: Wald test ($\chi^2(6)$) for Eff. and $H = 0$ in (2.19).
Eff.+Enc. RW: Wald test ($\chi^2(6)$) for Eff. and $\Xi = 0$ in (2.19).
Eff.+Enc.+Unb.: Wald test ($\chi^2(10)$) for $c = 0$, $\Xi = I$, $H = 0$ in (2.19).
Significant values are reported in boldface.

all cases; the worse performance in some deficit forecasts (for Japan, Germany and Italy) is more than compensated by the good one in forecasting inflation. The fourth column reports the test for efficiency, i.e., for the irrelevance of the deficit forecasts in explaining inflation, and viceversa. Formally, it is a Wald test for Ξ and H diagonal in (2.19), an hypothesis that is accepted for all countries

The fifth and sixth columns present the joint test for efficiency and Φ encompassing. The IMF Φ encompasses the RW in all cases ($H = 0$ in (2.19)), apart from Japan and Germany, and therefore it also encompasses RW with respect to the trace and determinant of Φ. The RW never encompasses the IMF. The last column of Table 3.1 contains the test for the joint hypothesis of efficiency, encompassing, and unbiasedness of both IMF forecasts. It is accepted for France and UK.[5]

4. Conclusions

MSFE encompassing is an important and easily testable property. Hence, testing for its validity should become a first step in forecast comparison exercises. We have

[5]Notice that as a consequence of efficiency the system in (2.19) is made up of two equations like (2.10) for the deficit ratio and inflation. Yet, the results on encompassing and unbiasedness from the joint analysis differ from those from the univariate analysis. In particular, encompassing was rejected for the US and Canada in the case of inflation forecasts, as well as unbiasedness of the French IMF deficit ratio forecasts. Strictly speaking, the results of the univariate and multivariate tests cannot be compared because of the different null hypotheses and distributions of the statistics. Yet, in our case, the partly mismatching conclusions are likely due to the validity of only some of the components of the joint hypotheses under analysis. Overall, the univariate and multivariate approaches can be considered as complementary rather than substitute.

extended the standard definitions and testing procedures to let the forecasts under comparison be biased, that is quite common in practice. The empirical analysis supports the practical usefulness of these generalizations.

References

[1] Artis, M. and Marcellino, M., (1998), "Fiscal Solvency and Fiscal Forecasting in Europe", CEPR W.P. 1836.

[2] Artis, M. and Marcellino, M., (1999), "Fiscal Forecasting: The Track Record of IMF, OECD and EC", CEPR W.P. 2206.

[3] Chong, Y.Y. and Hendry, D.F. (1986), "Econometric evaluation of linear macroeconomic models", *Review of Economic Studies*, 53, 671-690.

[4] Clemen, R.T. (1989), "Combining Forecasts: A Review and Annotated Bibliography", *Journal of Forecasting*, 5, 559-583.

[5] Clements, M.P. and Hendry, D.F. (1993), "On the Limitations of Comparing Mean Square Forecast Errors", *Journal of Forecasting*, 12, 617-637.

[6] Clements, M.P. and Hendry, D.F. (1997), "Intercept corrections and structural change", *Journal of Applied Econometrics*, 11, 475-494.

[7] Clements, M.P. and Hendry, D.F. (1998a), *The Marshall Lectures on Economic Forecasting*, Cambridge University Press.

[8] Clements, M.P. and Hendry, D.F. (1998b), *The Zeuthen Lectures on Forecasting*, MIT Press.

[9] Doornik, J.A. and Hendry, D.F. (1997), *Modelling Dynamic Systems using Pc-Fiml 9 for Windows*, London: International Thomson Business Press.

[10] Ericsson, N.R. (1992), "Parameter constancy, mean square forecast errors, and measuring forecast performance: An exposition, extensions, and illustration", *Journal of Policy Modeling*, 14, 465-495.

[11] Ericsson, N.R. and Marquez, J. (1993), "Encompassing the Forecasts of US Trade-Balance Models", *Review of Economics and Statistics*, 75, 19-31.

[12] Gallo, G.M., and Marcellino, M. (1999), "Ex-Post and Ex-ante Analysis of Provisional Data", *Journal of Forecasting* (*forthcoming*).

[13] Godfrey, L.G. (1988), *Misspecification Tests in Econometrics*, Cambridge: Cambridge University Press.

[14] Granger, C.W.J. and Newbold, P. (1986), *Forecasting economic time series*, 2nd edn., New York: Academic Press.

[15] Hendry, D.F. and Doornik, J.A. (1997), *Empirical Econometric Modelling using PcGive 9 for Windows*, London: International Thomson Business Press.

[16] Hoel, P.G. (1947), "On the Choice of Forecasting Formulas", *Journal of the American Statistical Association*, 42, 605-611.

[17] Holden, K. and Peel, D.A. (1990), "On Testing for Unbiasedness and Efficiency of Forecasts", *The Manchester School*, 58(2), 120-127.

[18] Laster, D., Bennett, P., and Geoum, I.S. (1997), "Rational Bias in Macroeconomic Forecasts", Federal Reserve bank of New York, Staff Reports n. 21.

[19] Lu, M. and Mizon, G.E. (1991), "Forecast encompassing and model evaluation", in Hackl, P. and Westlund, A.H. (eds.), *Economic structural change, Analysis and Forecasting*, 123-138, Berlin: Springer-Verlag.

[20] Marcellino, M. (1998), "Further results on MSFE encompassing", IGIER W.P. 143.

[21] Marcellino, M., and Mizon, G.E. (2000), *Progressive Modelling: The Encompassing Principle and Non-nested Testing*, Oxford: Oxford University Press (*forthcoming*).

[22] Nelson, C.R. (1972), "The Prediction Performance of the F.R.B.-MIT-PENN Model of the US Economy", *American Economic Review*, 62, 902-917.