Markov-Perfect Optimal Taxation

Salvador Ortigueira*
Department of Economics
European University Institute

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Abstract

In this paper we study optimal taxation in a dynamic game played by a sequence of governments and the private sector. We focus on the Markov-Perfect equilibrium of this game under two different assumptions on the extent of government's intra-period commitment, which in turn define two within-period timing of actions. Our results show that the extent of government's intra-period commitment has important quantitative implications for policies, welfare, and macroeconomic variables, and consequently that it must be explicitly stated as one of the givens of the economy, alongside preferences, markets and technology. We see this as an important result, since most of the previous literature on Markovian optimal taxation has assumed, either interchangeably or unnoticeably, different degrees of government's intra-period commitment.

Keywords: Markov-Perfect Optimal Taxation; Time-Consistent Policies; Instantaneous and Non-Instantaneous Commitment; Numerical Methods.

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1 Introduction

This paper analyzes time-consistent, optimal fiscal policy in a simple extension of the neoclassical model of capital accumulation. Since we will be assuming that the government is unable to commit to *future* policies, the Ramsey equilibrium becomes an invalid equilibrium concept as it delivers time-inconsistent policies. Instead, we study the Markov-Perfect equilibria of a dynamic game played by a sequence of governments (one for each time period), and the private sector, under two alternative assumptions on the within-period timing of actions. Our main aim is to understand and quantify the effects of these two timings on the Markov-Perfect equilibrium of this game.

As noted by Cohen and Michel (1988), the literature on Markovian optimal policies has made use of two notions of time consistency involving two different within-period timings of actions. More importantly, they also note that the two notions have been used either interchangeably or unnoticeably, thus hindering the assessment of the results. Under the first notion of time consistency both the private sector and the period-t government choose their actions simultaneously. Hence, the government does not need to express any commitment to its policy since there is no time lag between the tax and the consumption decisions. This notion of time consistency bears the strongest sense of lack of commitment. Under the second notion the period-t government moves first to set taxes before the household chooses consumption. This enables the government to anticipate the private sector's response to taxes, which confers the government with instantaneous leadership. Consequently, intra-period or instantaneous commitment is needed, so that the private sector will not expect a change in taxes. This second notion of time consistency bears a weaker sense of lack of commitment.

Analyses of Markovian optimal taxes where one or the other of these notions of time consistency is adopted can be found in the literature: Turnovsky and Brock (1980) and Judd (1998, Ch. 16) present a simple model with valued government expenditure, and derive Markovian optimal taxes under the assumption that the government and the private sector move simultaneously. Klein, Ríos-Rull and Krusell (2003) study optimal taxes in a similar model assuming that the government moves first.

Nevertheless, the extent to which the choice of the within-period timing of actions affects the level of optimal taxes and other macroeconomic variables is not well understood. Even though the distinction between the two scenarios is subtle, there is no reason to believe that it has only negligible effects on optimal policies and macroeconomic variables. In this paper we ask the following questions: How do Markovian optimal taxes change with the timing of actions? Are the effects of the timing of actions quantitatively important? Does the government attain higher welfare when granted with instantaneous commitment and moves first? The only attempt we are aware of to answer these questions is in Cohen and Michel (1988). Although their analysis sheds new light on how to calculate time-consistent policies, it presents important limitations to address the abovementioned questions. Their model is not a fully-fledged macroeconomic model. They assume a quadratic loss function and a linear law of motion for the state variable, so that the model can be solved analytically.

In this paper we answer the questions posed above within the context of a neoclassical model of capital accumulation that includes valued government expenditure and income taxation. The model comprises the main tensions arising in the determination of optimal taxes, and it has become the standard framework in the recent literature of optimal taxation [besides the papers cited above, see also Phelan and Stachetti (2001) for an analysis of optimal taxation under sequential equilibria]. The current paper builds on Judd (1998) and Klein, Krusell and Ríos-Rull (2003). In particular, our definition of a simultaneous-move Markov-Perfect equilibrium corresponds to Judd's (1998) timing of actions. Likewise, our definition of a government-moves-first Markov-Perfect equilibrium follows Klein, Krusell and Ríos-Rull (2003).

The main result in this paper shows that the within-period timing of actions has sizable quantitative implications in terms of optimal income taxes. For our benchmark economy, income taxes set by the Markovian government under simultaneous moves are about two-thirds the tax rates set under within-period leadership. This result does not hinge on any specific feature of the model. As will become clearer below, the result springs from the differential ability of the two Markovian governments to internalize the distortionary effects of income taxation. A Markovian government with within-period leadership can anticipate the response of consumption to income taxes. Hence, when assessing the value of a marginal increase in taxes, the government weighs the reduction in consumption that will absorb part of the increase, thus diminishing the amount of investment crowded out. This, added to the fact that Markovian governments can not internalize the distortionary effects of current taxation on past investment, leads to high income taxes and high public-good consumption. On the other hand, a Markovian government with simultaneous moves can not assess the household's consumption response, and it thus perceives income taxation as being relatively more distortionary on savings. This leads to lower taxes and lower public-good consumption.

The welfare consequences of the timing of actions are computed as the compensation in private-good consumption that leaves the representative household indifferent between the two timings. Since Markovian governments are prone to overtax (they do not internalize the effect of taxation on past investment), it turns out that the timing with simultaneous moves helps curb that tendency and, therefore, renders higher welfare than under leadership. Indeed, for our benchmark economy, government's leadership bears a welfare cost of 2% of consumption. Our results unambiguously show that the timing of actions is a key ingredient of the environment in the analysis of Markovian optimal taxes, and that, contrary to previous studies in this area, it must be explicitly stated alongside other ingredients such as preferences, markets and technology. Furthermore, our analysis also sheds light on the issue of optimal fiscal constitutions. Our quantitative exercise shows that the optimal fiscal constitution must limit the government's ability to commit, if the alternative is to grant intra-period commitment.

Our numerical approach to compute Markov-Perfect equilibria is based on a standard projection method. We compute these equilibria for a wide range of the state space, which allows us not only to derive its global properties, but also to carry out the welfare analysis. Projection methods [see Judd (1992),(1998) and Miranda and Fackler (2002)] are a natural and efficient way to solve functional equation problems, and, more specifically, to solve dynamic games. The application of these methods has proved extremely useful to solve dynamic games arising in models of industrial organization. In a recent paper, Venedov and Miranda (2001) apply a projection method to a dynamic duopoly game in which competing firms can invest in physical capital. A similar method is applied by Miranda and Rui (1996) to models of world commodity markets where two countries compete using strategic storage. Our model in this paper has, however, remarkable differences with respect to models of dynamic duopoly games. On the one hand, one of our players is the private sector which is made up of a continuum of households, and "plays" through the market. On the other hand, we cannot assume symmetric strategies, as is usually done in duopoly games.[1]

The paper is organized as follows. Section 2 presents the model and a formal definition of the Markov-Perfect equilibrium under the two timings of actions. In Section 3 the model is parameterized and calibrated, and the equilibria are computed. Section 4 presents an extension of the model with endogenous leisure-labor choice. The Markov-Perfect equilibrium is presented, and the macroeconomic and welfare consequences of the timing of actions are discussed. Section 5 presents our computational approach, Section 6 concludes,

[1]An exception to this can be found in Pakes and McGuire (1994), where the Markov-Perfect equilibrium of a model of industry dynamics with heterogeneous firms is analyzed and computed.

and Section 7 contains the Appendixes.

2 The Model and Equilibrium Definitions

In this section we present a simple model of capital accumulation with valued government expenditure. The government uses taxes on total income to finance its expenditures.

Households

There is a measure one of identical households. Household's lifetime utility is given by

$$\sum_{t=0}^{\infty} \beta^t U(c_t, g_t), \tag{2.1}$$

where c_t denotes the level of private-good consumption, and g_t is the level of public-good consumption. The instantaneous utility function, U, is assumed to be increasing in both arguments, jointly concave and continuously differentiable. The household supplies one unit of labor time inelastically, whereby it receives a wage rate ω_t. It also owns a stock of capital, k_t, which rents at a rate r_t, and whose rate of depreciation is denoted by $0 < \delta \leq 1$. Total income is subject to taxation. The household's budget constraint, in combination with the law of motion for capital, yields,

$$c_t + k_{t+1} = k_t + (1 - \tau_t)[w_t + (r_t - \delta)k_t], \tag{2.2}$$

where τ_t is the tax rate on total income net of capital depreciation.

The Firms' Sector

The firms' sector employs capital and labor to produce the aggregate good. Total production is given by,

$$Y_t = F(K_t, 1) = f(K_t), \tag{2.3}$$

where K_t is the aggregate stock of capital, and f is a neoclassical production function. Firms are competitive, and therefore,

$$r_t = f_K(K_t), \tag{2.4}$$

$$w_t = f(K_t) - r_t K_t. \tag{2.5}$$

Government

The government is assumed to run a balanced budget, which implies that at every period total expenditure must equal total income from taxes,

$$g_t = \tau_t[\omega_t + (r_t - \delta)K_t] = \tau_t[f(K_t) - \delta K_t], \tag{2.6}$$

where the second equality follows from the zero-profit condition in the firms' sector.

2.1 Markov-Perfect Optimal Taxation

Among all equilibrium concepts delivering time-consistent policies, the Markov-Perfect equilibrium stands out for the simplicity with which it embeds rational behavior, and for its power to enhance the model's predictability by yielding a lower number of equilibria than alternative equilibrium concepts[2]. In contrast to the Ramsey equilibrium, the government is now assumed to act sequentially. Thus, we can think of a sequence of governments, each foreseeing how its successors will behave, that chooses the tax rate on total income, conditioning on the aggregate stock of capital, to maximize social welfare. Our analysis of the Markov-Perfect equilibria under the two timings of actions described above will be restricted to the case of differentiable Markovian strategies. This is a standard restriction in the literature of Markovian optimal policies.[3]

2.1.1 The Simultaneous-Move Markov-Perfect Equilibrium

In this section we characterize Markovian optimal taxes when the period-t government and households choose their actions simultaneously. Each household decides how much to consume and save given its stock of capital, the economy-wide stock of capital, and the expected tax policy for the current and future governments. The current government chooses the tax rate for the current period given the economy-wide stock of capital and the expected tax policy of future governments. Before providing a definition of a simultaneous-move Markovian equilibrium, let us present the problem of a household, and of the current government in more detail.

The problem of the household

Let k denote the household's stock of capital, and K the economy-wide stock of capital. If the household expects that the current and future governments will set taxes according to the policy ψ, where ψ is a function that maps the economy-wide stock of capital into

[2] For a more detailed discussion of the virtues of the Markov-Perfect equilibrium see Maskin and Tirole (2001).

[3] It has been recently shown [see Sorger (1998) for an application to resource games, and Krusell and Smith (2003) for an application to models with quasi-geometric discounting] that the Markov-Perfect equilibrium may be indeterminate if continuity and differentiability are not imposed.

a tax rate, its maximization problem can be compactly written as,

$$v(k, K; \psi) = max_{c,k'} \left\{ U(c, g) + \beta v(k', K'; \psi) \right\}, \tag{2.7}$$

subject to the budget constraint,

$$c + k' = k + (1 - \psi(K))[w(K) + (r(K) - \delta)k], \tag{2.8}$$

where the economy-wide stock of capital is expected to evolve according to the law, say, $K' = H(K)$. In (2.7), $v(k, K; \psi)$ is the value to a household with a stock of capital k, when the economy-wide stock of capital is K, and the tax policy adopted by the current and future governments is ψ.

If we use the assumption of identical households, $k = K$, and the government's budget constraint $g = \psi(K)[w(K) + (r(K) - \delta)K]$, the consumption function in a competitive equilibrium under tax policy ψ is a function of K alone, say $C(K)$, which satisfies the following Euler equation,

$$U_c\left(C(K),\, G(K)\right) = \beta U_c\left(C(K'),\, G(K')\right)\left[1 + (1 - \psi(K'))[f_K(K') - \delta]\right], \tag{2.9}$$

where K' in equilibrium is given by,

$$K' = K + (1 - \psi(K))[f(K) - \delta K] - C(K), \tag{2.10}$$

and public expenditure is,

$$G(K) = \psi(K)[f(K) - \delta K]. \tag{2.11}$$

The problem of the government

The government's objective function is to maximize social welfare. The period-t government chooses the period-t tax rate, τ, taking as given the tax policy followed by future governments. Since the government and the household move simultaneously, the government cannot anticipate the household's consumption response to different tax rates. The only trade-off in the government's problem is the amount of the public good to provide, and next period's capital stock.[4] More specifically, the problem of the period-t government is,

$$V(K) = max_{\tau} \left\{ U\left(C(K),\, \mathcal{G}(K, \tau)\right) + \beta V(K') \right\}, \tag{2.12}$$

[4]The alternative scenario where households set investment and the government trades off private-good consumption for public-good consumption is discussed below.

where,

$$K' = K + (1-\tau)[f(K) - \delta K] - C(K), \tag{2.13}$$

and

$$\mathcal{G}(K,\tau) = \tau[f(K) - \delta K]. \tag{2.14}$$

Proposition 1: *The tax policy that solves the government's problem is the solution to the following Generalized Euler Equation:*

$$U_g = \beta \left[U_c' C_K' + U_g' \cdot \left(f_K' + 1 - \delta - C_K' \right) \right], \tag{2.15}$$

where a subscript denotes the variable with respect to which the derivative is taken, and a prime indicates the function is evaluated at next-period's values.

The proof of this proposition is presented in Appendix I. Functions' arguments in (2.15) have been dropped for expositional clarity, since there is no risk of ambiguity. The interpretation of the Generalized Euler Equation is reminiscent of that for the household's Euler equation. The government will set the tax rate so that the marginal utility of taxation equates the marginal utility of investment in physical capital. In this simultaneous-move economy the government computes these two marginal utilities as follows. The marginal utility of taxation is the increase in government expenditure, $\mathcal{G}_\tau$, times the marginal utility of consuming the public good, U_g, per unit of investment crowded-out, which is exactly $\mathcal{G}_\tau$, since an increase in government expenditure translates into a one-to-one drop in investment (this follows from the government's inability to anticipate the consumption response to taxes). Hence, today's marginal utility of taxation is U_g, the left-hand side of (2.15). Seeing that the right-hand side of equation (2.15) is the marginal utility of investing in capital is equally straightforward. An extra unit of investment today brings about an increase in tomorrow's resources of $f_K' + 1 - \delta$. The value of this increase can be broken down as: (i) consumption of the private good increases by C_K', which yields $U_c' C_K'$; (ii) the increase in government expenditure derived from the increase in the tax base is $\mathcal{G}_K'$, which yields $U_g' \mathcal{G}_K'$; and (iii) the remaining $f_K' + 1 - \delta - C_K' - \mathcal{G}_K'$ is the increase in taxation, and since tomorrow's marginal utility of taxation is U_g', this yields $U_g' \cdot (f_K' + 1 - \delta - C_K' - \mathcal{G}_K')$. The right-hand side of the Generalized Euler Equation results from adding up all these values and discounting.

We can now define a simultaneous-move Markov-perfect equilibrium for this economy.

Definition: A simultaneous-move Markov-Perfect equilibrium (**SME**) is a triplet of functions $C(K)$, $\psi(K)$ and $V(K)$, such that: (i) $C(K)$ solves the first-order condition of the household sector [eqs. (2.9)-(2.11)]; (ii) $\psi(K)$ solves the Euler equation of the government [eqs. (2.13)-(2.15)]; and (iii) $V(K)$ is the value function of the government, that is, $V(K) = U[C(K), \mathcal{G}(K, \psi(K))] + \beta V(K')$.

2.1.2 The Government-Moves-First Markov-Perfect Equilibrium

The period-t government is now assumed to have instantaneous leadership in the sense that it sets the period-t tax rate before the household decides on consumption and savings. Thus, when choosing among feasible tax rates, the government is able to incorporate the household's response to taxes.

The problem of the household

When the household decides on consumption and savings, the period-t government has already set the period-t tax rate. Then, the problem of a household with k units of capital that has to pay taxes on current income at rate τ, and expects future governments to follow the tax policy γ, is

$$\nu(k, K; \tau; \gamma) = max_{c,k'} \left\{ U(c, g) + \beta \tilde{\nu}(k', K'; \gamma) \right\}, \tag{2.16}$$

subject to the budget constraint,

$$c + k' = k + (1 - \tau)[w(K) + (r(K) - \delta)k], \tag{2.17}$$

where the economy-wide stock of capital is expected to evolve according to the law, say, $K' = \mathcal{H}(K, \tau)$. Then, from (2.16)-(2.17), from the representative household assumption $k = K$, and the government's budget constraint $g = \tau[w(K)+(r(K)-\delta)K]$, it follows that the consumption function in a competitive equilibrium when today's tax rate is τ, and future taxes are given by policy γ, is a function of K and τ, which satisfies the following Euler equation,

$$U_c \Big(\mathcal{C}(K, \tau),\, \mathcal{G}(K, \tau)\Big) = \beta U_c \Big(\mathcal{C}(K', \tau'),\, \mathcal{G}(K', \tau')\Big) \Big[\, 1 + (1 - \tau')[f_K(K') - \delta]\Big], \tag{2.18}$$

where $\tau' = \gamma(K')$, and K' in equilibrium is given by,

$$K' \;\; = \;\; K + (1 - \tau)[f(K) - \delta K] - \mathcal{C}(K, \tau), \tag{2.19}$$

and,

$$\mathcal{G}(K, \tau) = \tau[f(K) - \delta K]. \tag{2.20}$$

Furthermore, in equilibrium $\nu(K, K; \gamma(K); \gamma) = \tilde{\nu}(K, K; \gamma)$.

The problem of the government

As in the simultaneous-move equilibrium, the current government chooses the current tax rate, τ, taking as given the tax policy followed by future governments. Now, however, government's leadership implies that it internalizes the effect of τ on the level of consumption, as given by the consumption function that solves (2.18). More specifically, the problem of the government is,

$$\mathcal{V}(K) = max_{\tau} \left\{ U\left(\mathcal{C}(K,\tau),\, \mathcal{G}(K,\tau)\right) + \mathcal{V}(K')\right\} \tag{2.21}$$

where K' and $\mathcal{G}(K,\tau)$ are given by (2.19) and (2.20), respectively.

Proposition 2: *The tax policy that solves the government's problem is the solution to the following Generalized Euler Equation:*

$$\frac{U_c\mathcal{C}_\tau + U_g\mathcal{G}_\tau}{f - \delta K + \mathcal{C}_\tau} = \beta\left[U_c'\mathcal{C}_K' + U_g'\mathcal{G}_K' + \left(\frac{U_c'\mathcal{C}_\tau' + U_g'\mathcal{G}_\tau'}{f' - \delta K' + \mathcal{C}_\tau'}\right)\left(f_K' + 1 - \delta - \mathcal{C}_K' - \mathcal{G}_K'\right)\right], \tag{2.22}$$

where a subscript denotes the variable with respect to which the derivative is taken, and a prime indicates the function is evaluated at next period's values.

The proof of this proposition is presented in Appendix I. Again, functions' arguments have been dropped for expositional clarity.

This Generalized Euler Equation is necessarily more involved, as the current government has now more trade-offs to take care of. Yet, its interpretation is equally straightforward: the optimal tax rate equates the marginal utility of taxation to the marginal utility of investing in capital. The only difference with respect to the case of simultaneous moves relies on the determination of the marginal utility of taxation. Now, the government assesses the response of consumption to current taxes, and, therefore, an increase in government expenditure is not seen as causing a one-to-one decrease in investment.

The left-hand side of (2.22) is today's marginal utility of taxation, which is made up of three values: the change in utility from the consumption of the private good, $U_c\mathcal{C}_\tau$, the change in utility from the consumption of the public good, $U_g\mathcal{G}_\tau$, and the lost in investment, $f - \delta K + \mathcal{C}_\tau$. That is, the marginal value of taxation is the net gain in utility per unit of investment crowded-out. The right-hand side of (2.22) is the marginal utility of investing in capital. An extra unit of investment today brings about an increase in tomorrow's resources of $f_K' + 1 - \delta$, whose value is broken down as: (i) C_K' is the increase in consumption of the private good, which yields $U_c'C_K'$; (ii) $\mathcal{G}_K'$ is the increase in government expenditure from the increase in the tax base, which yields $U_g'\mathcal{G}_K'$; and (iii) the remaining $f_K' + 1 - \delta - C_K' - \mathcal{G}_K'$ is in taxation, whose marginal utility has been calculated above. The right-hand side of the Generalized Euler Equation results from adding up all these values and discounting.

We define now a government-moves-first Markov-Perfect equilibrium for this economy.

Definition: A government-moves-first Markov-Perfect equilibrium (**GMFE**) is a triplet of functions $\mathcal{C}(K, \tau)$, $\gamma(K)$ and $\mathcal{V}(K)$, such that: (i) $\mathcal{C}(K, \tau)$ solves the first-order condition of the household sector [eqs. (2.18)-(2.20)]; (ii) $\gamma(K)$ solves the Euler equation of the government [eqs. (2.19), (2.20) and (2.22)]; and (iii) $\mathcal{V}(K)$ is the value function of the government, that is, $\mathcal{V}(K) = U[\mathcal{C}(K, \gamma(K)), \mathcal{G}(K, \gamma(K))] + \beta\mathcal{V}(K')$.

The two Markov-Perfect equilibrium definitions stated above —SME and GMFE— embed two different within-period timings of actions, and hence two different notions of time consistency. Arguing which equilibrium definition fits better the timing of actions in real economies is beyond the scope of this paper. If the "true" model is one where the government makes decisions at discrete times but consumers and producers are making decisions continuously, then one could argue that the GMFE is a better approximation to this true model. Even so, studying optimal taxation under the two equilibrium definitions has value added because (i) it highlights the consequences of choosing one over the other, and (ii) it allows us to derive normative implications in terms of fiscal constitutions.

Since we study the macroeconomic and welfare implications of the two notions of time consistency in the next section, we find it convenient to close this section with the characterization of the efficient equilibrium with lump-sum taxes (**LSTE**). With lump-sum taxes, the household's Euler equation is undistorted, and the government will set taxes so that the marginal utility of consuming the public good equals the marginal utility of consuming the private good. That is, in the efficient equilibrium with lump-sum taxes, the Generalized Euler Equation is replaced by the condition $U_g = U_c$.

Finally, we also compute steady-state optimal taxes in the Ramsey equilibrium, i.e., when the time-zero government has full commitment to future taxes. The maximization problem solved by the Ramsey government is the standard one and, therefore, it will not be discussed here.

3 Macroeconomic and Welfare Implications

In this section we parameterize our economic model, assign values to its parameters, and compute the equilibria defined above: the two Markov-Perfect equilibria (SME and GMFE), the equilibrium with lump-sum taxes, LSTE, and the Ramsey equilibrium.

Then, we present and discuss the macroeconomic and welfare effects of the two timings of actions embedded in the two Markov equilibrium definitions. The functional forms for the production technology and preferences are the standard Cobb-Douglas production function, and the CES utility function, respectively. If we use α to denote the capital's share of income, the production function is,

$$f(K_t) = AK_t^{\alpha}, \tag{3.1}$$

where $A > 0$ is a constant. Likewise, if we use $1/\sigma$ to denote the elasticity of intertemporal substitution of the composite good $c_t g_t^{\theta}$, the instantaneous utility function is,

$$u(c_t, g_t) = \frac{\left(c_t g_t^{\theta}\right)^{1-\sigma} - 1}{1-\sigma}, \tag{3.2}$$

where $\theta > 0$ is a constant. In the limiting case where $\sigma = 1$, this function is $log\ c_t + \theta\ log\ g_t$.

Regarding parameter values, we choose our benchmark economy as follows: the values for A and σ are set arbitrarily at 1; the value for α is set at 0.36, which is the capital's share of income in the US economy; δ is set at 0.09; the value for β is 0.96; finally, the value for θ is set at 0.2 so that government spending as a share of income falls within the range of 10-20 per cent, for all equilibrium concepts studied in this paper. Thus, our benchmark economy is:

$$A = \sigma = 1,\ \alpha = 0.36,\ \delta = 0.09,\ \beta = 0.96, \text{ and } \theta = 0.2.$$

The numerical approach to compute the Markov-Perfect equilibria is explained in detail in Section 5. As we have already advanced in the introductory section, our approach is a version of a standard projection method with Chebyshev polynomials. Our computations are confined to values of the capital stock in the interval $[3.75, 5.2]$, which contains the steady-state capital stocks of the SME, GMFE, the efficient equilibrium with lump-sum taxes and the Ramsey equilibrium. The interval is sufficiently large to capture the properties of the Markov-Perfect equilibria along the transitional dynamics, and small enough so that arbitrarily low computational errors can be attained without having to resort to "too" high-order Chebyshev polynomials.

The results are presented in Table 1 and in Figures 1 to 4. Table 1 presents steady-state values for optimal taxes and macroeconomic variables in the efficient, Ramsey and Markovian equilibria. In Figure 1 we plot tax policies, net savings, consumption of the private good and public expenditure in the simultaneous-move equilibrium, the government-moves-first equilibrium, and the efficient equilibrium with lump-sum taxes. First, the main properties of the two Markov equilibria, and the differences between them, can be summed up as follows: *i)* Tax policies in the SME and GMFE, $\psi(K)$ and $\gamma(K)$, respectively, are increasing functions of K. Taxes in the government-moves-first equilibrium are roughly 50% higher than those in the simultaneous-move equilibrium. *ii)* The government-moves-first equilibrium yields lower savings, lower consumption of the private good, and higher consumption of the public good than the simultaneous-move equilibrium. *iii)* The government-moves-first equilibrium yields a lower steady-state capital stock, 4.16, as compared to 4.39 in the simultaneous-move equilibrium. Both steady-state capital stocks are below the level of the efficient equilibrium with lump-sum taxes.

Second, the two Markov-Perfect equilibria compare with the efficient equilibrium as follows. When the Markovian government is endowed with instantaneous commitment (moves first), there is underconsumption of the private good, and overconsumption of the public good, yielding a ratio of private-to-public consumption below the efficient level. When the Markovian government lacks instantaneous commitment (simultaneous moves) there is overconsumption of the private good, and underconsumption of the public good, yielding a ratio of private-to-public consumption above the efficient level.

Table 1

Optimal Income Taxes: Steady State. Exogenous Labor

			Markovian	
	Efficient	Ramsey	SME	GMFE
Y	1.7608	1.7010	1.7042	1.6710
K	4.8143	4.3741	4.3971	4.1632
C	1.1062	1.0894	1.1007	0.9911
G	0.2212	0.2178	0.2077	0.3052
τ		0.1666	0.1588	0.2354

Notes: Steady-state optimal income taxes for the efficient equilibrium with lump-sum taxes, the Ramsey equilibrium, and the two Markov-Perfect equilibria: SME and GMFE.

To understand why a Markovian government that moves first sets a higher income tax than under simultaneous moves, we must look at the corresponding Generalized Euler Equations. In short, the key difference is that by moving first, the government can take into account the response of consumption to the current income tax. For given capital and expectations on next-period government's taxation policy, current consumption decreases with current taxes. Therefore, the amount of investment crowded out by a marginal increase in the tax rate is lower from the standpoint of the government with within-period leadership, and then it will set a higher income tax. In Appendix II we present a simple version of our model economy with a closed-form solution, where taxes under the two timings of actions can be solved for analytically. The two-period economy presented in the Appendix helps understand why income taxes are higher when the government moves first.

Figure 2 plots the residuals of Chebyshev collocation for the Euler Equation, the Generalized Euler Equation, and the Bellman Equation in the three computed equilibria: SME, GMFE, and LSTE.

We conduct a sensitivity analysis with respect to σ. Figure 3 displays the SME for $\sigma = 1$, $\sigma = 5$ and $\sigma = 10$. The effects of increases in σ are the expected ones. As the elasticity of intertemporal substitution, $1/\sigma$, decreases, the tax policy and consumption become flatter with respect to K. The steady-state capital stock is, by contrast, highly insensitive to changes in σ. Figure 4 depicts the GMFE for the three values of σ. The results in terms of consumption and steady-state capital are qualitatively the same as those in the SME. Regarding the tax policy, taxes shift upwards in the considered state space, but are relatively less sensitive to increases in σ, as compared to the SME.

3.1 Welfare Implications

Value functions for the SME, GMFE and the efficient equilibrium with lump-sum taxes for $\sigma = 1, \sigma = 5$, and $\sigma = 10$ are plotted in Figure 5. The GMFE delivers the lowest level of welfare. In order to provide a measure of the welfare consequences of the timing of actions, we adopt an approach which has been first proposed to measure the welfare cost of fluctuations, and then applied to measure the welfare cost of taxation. In our model, it consists in computing the percentage increase in consumption of the private good that would leave the representative household indifferent between the two timings of actions. Since we do not restrict the initial capital stock, the welfare cost of endowing the government with within-period leadership can be expressed as the following function of K,

$$\lambda(K) = \left(\frac{1 + (1-\beta)(1-\sigma)V(K)}{1 + (1-\beta)(1-\sigma)\mathcal{V}(K)} \right)^{\frac{1}{1-\sigma}} - 1. \tag{3.3}$$

Equation (3.3) follows from equating lifetime utility under both timings of actions when the instantaneous utility function is given by (3.2). In the limiting case where $\sigma = 1$, the welfare cost of government's leadership is,

$$\lambda(K) = exp\Big((1-\beta)\left[V(K) - \mathcal{V}(K)\right]\Big) - 1. \tag{3.4}$$

In the bottom-right chart of Figure 5 we plot $\lambda(K)$ for the three values of σ. The welfare cost of instantaneous commitment in our calibrated economy is around 2% of consumption of the private good, and there is no much variation with respect to the value of σ. A cost of 2% of consumption is non-negligible; it is about the same order of magnitude as the welfare cost of economic fluctuations, as computed in the business cycle literature. It should be noted that our result that welfare is higher in the SME than in the GMFE is not generic, and it depends crucially on the economy's horizon and the discount factor. A short temporal horizon or a low discount factor may reverse the result by dwarfing the cross-period gains of the low taxes arising in the simultaneous-move equilibrium. In Appendix II we present a two-period economy and show that in such an economy welfare is higher in the government-moves-first equilibrium.

The model studied in this section assumes that labor is inelastically supplied, and it thus fails to comprise the effects of the timing of actions *via* distortions in labor supply. In the next section we extend the model by introducing leisure in the utility function.

4 The Model with Endogenous Labor Supply

In this section we study an extended version of the previous model that includes an endogenous leisure-labor choice in the household problem. Household's instantaneous utility is now given by $U(c_t, g_t, l_t)$, where l_t stands for leisure, and the function U is assumed to have all the standard properties. Production is now represented by $F(K_t, N_t)$, where N_t denotes labor. The normalization of the total endowment of time per period implies that $l_t = 1 - N_t$, and therefore, only a new function needs to be solved for in equilibrium, which we denote by $N(K)$ in the simultaneous-move equilibrium, and by $\mathcal{N}(K, \tau)$ in the government-moves-first equilibrium.

The Simultaneous-Move Equilibrium

When households expect the current and future governments to set taxes according to the tax policy γ, first-order conditions for leisure and consumption in a competitive equilibrium under tax policy γ, are, respectively,

$$
\begin{aligned}
U_l\Big(C(K),\, L(K), G(K)\Big) &= U_c\Big(C(K),\, L(K), G(K)\Big)\,(1-\psi(K))F_N(K, N(K)) \\
U_c\Big(C(K),\, L(K), G(K)\Big) &= \beta U_c\Big(C(K'),\, L(K'), G(K')\Big) \times \\
&\qquad \Big(1 + (1-\psi(K'))[F_K(K', N(K') - \delta]\Big),
\end{aligned}
$$

where $L(K) = 1 - N(K)$, and K' and $G(K)$ are given, respectively, by (2.10) and (2.11) after substituting $f(K)$ by $F(K, N(K))$.

Likewise, following arguments similar to those in Section 2, the problem of the government yields the following Generalized Euler Equation,

$$
U_g = \beta\left[\, U_c' C_K' - U_l' N_K' + U_g' \cdot \Big(\, F_K' + F_N' N_K' + 1 - \delta - C_K'\Big)\right], \tag{4.1}
$$

where, as in Section 2, we have omitted functions' arguments, and primes indicate that the function is evaluated at next period's values. The interpretation of this Generalized Euler Equation is identical to the case of inelastic labor supply, and therefore, we will not repeat it here. For the same reason, we also skip the formal definition of the Markov equilibrium with simultaneous moves.

The Government-Moves-First Equilibrium

When the government moves first household's first-order conditions for leisure and consumption are, respectively,

$$
\begin{aligned}
U_l\Big(\mathcal{C}(K,\tau),\, \mathcal{L}(K,\tau), \mathcal{G}(K,\tau)\Big) &= U_c\Big(\mathcal{C}(K,\tau),\, \mathcal{L}(K,\tau), \mathcal{G}(K,\tau)\Big)\,(1-\tau)F_N(K, \mathcal{N}(K,\tau)) \\
U_c\Big(\mathcal{C}(K,\tau),\, \mathcal{L}(K,\tau), \mathcal{G}(K,\tau)\Big) &= \beta U_c\Big(\mathcal{C}(K',\tau'),\, \mathcal{L}(K',\tau'), \mathcal{G}(K',\tau')\Big) \times \\
&\qquad \Big(1 + (1-\tau')[F_K(K', \mathcal{N}(K',\tau')) - \delta]\Big),
\end{aligned}
$$

where $\mathcal{L}(K,\tau) = 1 - \mathcal{N}(K,\tau)$, and $\tau' = \gamma(K')$. As before, K' and $\mathcal{G}(K,\tau)$ are given, respectively, by (2.19) and (2.20) after substituting $f(K)$ by $F(K, \mathcal{N}(K,\tau))$.

The government's Euler equation is given by,

$$
\begin{aligned}
\frac{U_c\mathcal{C}_\tau - U_l\mathcal{N}_\tau + U_g\mathcal{G}_\tau}{F - \delta K - (1-\tau)F_N\mathcal{N}_\tau + \mathcal{C}_\tau} &= \beta\Big[U_c'\mathcal{C}_K' - U_l'\mathcal{N}_K' + U_g'\mathcal{G}_K' \;+ \\
&\left(\frac{U_c'\mathcal{C}_\tau' - U_l'\mathcal{N}_\tau' + U_g'\mathcal{G}_\tau'}{F' - \delta K' - (1-\gamma')F_N'\mathcal{N}_\tau' + \mathcal{C}_\tau'}\right)\Big(F_K' + F_N'\mathcal{N}_K' + 1 - \delta - \mathcal{G}_K' - \mathcal{C}_K'\Big)\;\Big].
\end{aligned}
$$

It is worth noting how the endogenous leisure-labor choice affects the Generalized Euler Equation in the GMFE. The marginal value of taxation —both the change in current utility and the amount of investment crowded out by a marginal increase in the tax rate—depends now on the labor response to taxes (see the numerator and denominator of the left-hand side). Likewise, the marginal utility of investment (the right-hand side of the equation) depends on how tomorrow's labor supply responds to changes in capital. When leisure is a normal good, total income taxation may decrease today's consumption of leisure, which contributes to decrease the marginal value of taxation.

A Parameterized Economy with Endogenous Labor

The functional forms for the production and utility functions are the natural extensions of the functions presented above. That is, the production function is Cobb-Douglas,

$$F(K_t, N_t) = AK_t^{\alpha} N_t^{1-\alpha}, \tag{4.2}$$

where $A > 0$, and $0 < \alpha < 1$ is the capital's share of income. And instantaneous utility is the CES function,

$$u(c_t, g_t, l_t) = \frac{\left(c_t g_t^{\theta} l_t^{\eta}\right)^{1-\sigma} - 1}{1-\sigma}, \tag{4.3}$$

where $\eta > 0$ is a constant parameter. In the limiting case where $\sigma = 1$ this function is $log\ c_t + \theta\ log\ g_t + \eta\ logl_t$.

We adopt the benchmark economy presented in the previous section, along with a value for η equal to 0.5. Steady-state optimal income taxes are presented in Table 2.

Table 2

Optimal Income Taxes: Steady State. Endogenous Labor

			Markovian	
	Efficient	Ramsey	SME	GMFE
Y	1.1810	1.0628	1.0636	1.0509
K	3.2288	2.7331	2.7381	2.6575
C	0.7420	0.6807	0.6831	0.6458
G	0.1484	0.1361	0.1340	0.1658
N	0.6707	0.6248	0.6249	0.6236
τ		0.1666	0.1640	0.2043

Notes: Steady-state optimal income taxes for the efficient equilibrium with lump-sum taxes, the Ramsey equilibrium, and the two Markov-Perfect equilibria: SME and GMFE.

Figures 6 and 7 show the simultaneous-move, the govenment-moves-first and the efficient equilibrium for the benchmark economy with endogenous labor. The results here are not qualitatively different from those in Section 3: taxes are higher when the government moves first; there is underinvestment, underconsumption of the private good, and overconsumption of the public good. When the government and the private sector move simultaneously, there is underinvestment, overconsumption of the private good, and underconsumption of the public good. Regarding the new endogenous variable in this version of the model, labor, both Markov-Perfect equilibria render underemployment.

These results are, however, quantitatively different from those under inelastic labor. Here, taxes in the GMFE are 20% higher than in the SME, as compared to the 50% in the model with inelastic labor supply. The effects on welfare are also substantially lower in this version of the model. The bottom-left chart in Figure 7 shows the welfare cost associated with government's instantaneous leadership for all values of K in our state space. The proportion of private-good consumption that the representative household gives up ranges now from 0.65% to 0.7%.

4.1 Capital Income Taxation

In this section we abstract from labor income taxation and compute optimal capital income taxes. Table 3 presents steady-state optimal income taxes for the considered equilibrium concepts.

Table 3

Optimal Capital Income Taxes: Steady State. Endogenous Labor

			Markovian	
	Efficient	Ramsey	SME	GMFE
Y	1.1810	0.9501	0.9221	0.9158
K	3.2288	1.9257	1.7647	1.7301
C	0.7420	0.6883	0.6636	0.6582
G	0.1484	0.0884	0.0996	0.1019
N	0.6707	0.6385	0.6400	0.6404
τ		0.5244	0.5753	0.5857

Notes: Steady-state optimal capital income taxes for the efficient equilibrium with lump-sum taxes, the Ramsey equilibrium, and the two Markov-Perfect equilibria: SME and GMFE.

The main results remain qualitatively unchanged: The Markovian government sets a higher tax rate when endowed with instantaneous leadership. Quantitatively, optimal capital income taxes are above those found for total income taxation. This is a consequence of the small tax base available to a government taxing only capital income.

4.2 Labor Income Taxation

We compute now optimal labor income taxes when capital taxation is not available. Table 4 presents steady-state optimal taxes on labor income.

Table 4
Optimal Labor Income Taxes: Steady State. Endogenous Labor

			Markovian	
	Efficient	Ramsey	SME	GMFE
Y	1.1810	1.0931	1.0492	1.0934
K	3.2288	2.9889	2.5836	2.9896
C	0.7420	0.6868	0.6294	0.6889
G	0.1484	0.1373	0.1309	0.1354
N	0.6707	0.6208	0.6319	0.6209
τ		0.1963	0.1950	0.1934

Notes: Steady-state optimal labor income taxes for the efficient equilibrium with lump-sum taxes, the Ramsey equilibrium, and the two Markov-Perfect equilibria: SME and GMFE.

The main result is that labor income taxes are roughly constant across the four equilibrium concepts. Indeed, a Markovian government with instantaneous leadership sets a slightly lower tax on labor than under simultaneous moves. The main reason behind this similarity of labor taxes across the different equilibria is that the determination of labor in the household sector solves an intratemporal first-order condition. That is, labor income taxation does not embed the main intertemporal distortions associated with total and capital income taxation. These latter distortions are internalized differently by governments with different degrees of leadership, which explains the differential in total and capital income taxes found above. On the contrary, the distortions associated with labor taxation are predominantly intratemporal, and, therefore, the timing of actions plays a lesser role in the determination of the optimal labor tax.

We close this section by briefly considering a variation of our economy with simultaneous moves where households choose investment rather than consumption. In this scenario, the investment function depends only on aggregate capital, and leaves private-good consumption as the variable the time-t government trades off against public-good consumption. Then, the time-t government sets the tax rate so that $U_c = U_g$. Two points are worth noticing. First, this solution does not reach the efficient allocation as taxes are distortionary. Second, contrary to the case where households choose consumption, now the government can not anticipate the investment's response to taxes and understands that private-good consumption will fully absorb a marginal increase in taxes. Consequently, the government in the economy with simultaneous moves sets higher taxes than in the economy with within-period leadership. In Appendix III we make use of a two-period economy with closed-form solutions to further illustrate this result.

5 Numerical Strategy

In this section, we present the strategy adopted for the computation of the Markov equilibria. We restrict the discussion to the case where the government moves first, since the computation of the simultaneous-move equilibrium is substantially simpler, and it can be easily formulated as a particular case of the former. Also, we confine the presentation to the model with exogenous labor.

Computation of the Markov-Perfect Equilibrium when the Government Moves First.

We need to compute three functions, $\mathcal{C}(K, \tau)$, $\gamma(K)$ and $\mathcal{V}(K)$, that solve the two Euler equations (2.18)-(2.20) and (2.22), and the Bellman equation (2.21). First, we solve for $\mathcal{C}(K, \tau)$ and $\gamma(K)$, and then we compute $\mathcal{V}(K)$. We follow a standard projection method with Chebyshev polynomials and Chebyshev collocation. The basic idea is to approximate the two unknown functions using Chebyshev polynomials, which are a basis for the space of continuous functions, and then to compute the coefficients of the Chebyshev polynomials so that these approximations exactly satisfy the two Euler equations at a number of pre-established points. These latter points are the zeros of the Chebyshev polynomial whose order is equal to the number of coefficients to be computed. Finally, and in order to verify the quality of the approximation, we must check the errors at all points other than the zeros of the Chebyshev polynomial. Even though the application of this method to our problem at hand is quite straightforward, there are some specific features which need to be embedded in this general algorithm. One of these features is the endogeneity of taxes, which contributes to increase the non-linearity of the problem, and adds a new set of restrictions.

The consumption function, $\mathcal{C}(K,\tau)$, is approximated by,

$$\tilde{\mathcal{C}}(K,\tau,a)=\sum_{i=1}^{n_1}\sum_{j=1}^{n_2}a_{ij}\phi_{ij}(K,\tau), \tag{5.1}$$

where the two-dimensional Chebyshev polynomials, $\phi_{ij}(K,\tau)$, are the tensor products of the one-dimensional polynomials, that is, $\phi_{ij}(K,\tau)=\phi_{i-1}[2(K-K_{min})/(K_{max}-K_{min})-1]\phi_{j-1}[2(\tau-\tau_{min})/(\tau_{max}-\tau_{min})-1]$. The one-dimensional Chebyshev polynomials are evaluated by recursion: $\phi_0(x)=1,\ \phi_1(x)=x,\ \phi_{i+1}(x)=2x\phi_i(x)-\phi_{i-1}(x)$. The matrix $a=(a_{ij})$, $i=1,...,n_1;\ j=1,...,n_2$, is the matrix of unknown coefficients in the consumption function. The values $K_{min},K_{max},\tau_{min}$ and τ_{max} are used to transform the original variables, since the Chebyshev polynomials are defined in the interval $[-1,1]$. Notice that the election of K_{min} and K_{max} is free, and it simply indicates that K is confined to the interval $[K_{min},K_{max}]$. We will set these two values so that the steady-state capital stock lies within this interval. Since there is no way the steady-state equilibrium can be known *a priori*, some experimentation is necessary in order to pin down an interval that contains the steady-state capital stock. Regarding τ_{min} and τ_{max}, it should be noted that the tax rate is a choice variable and, therefore, the election of the interval must be consistent with the optimal policy rule. We will discuss this issue below.

The policy rule, $\gamma(K)$, is approximated by,

$$\tilde{\gamma}(K,b)=\sum_{i=1}^{n_3}b_i\phi_{i-1}[2(K-K_{min})/(K_{max}-K_{min})-1], \tag{5.2}$$

where the vector $b=(b_1,...,b_{n_3})$, is the vector of unknown coefficients in the policy rule.

The total number of coefficients to be determined is $n_1\times n_2+n_3$. These coefficients are fixed by imposing that $\tilde{\mathcal{C}}(K,\tau,a)$ and $\tilde{\gamma}(K,b)$ satisfy the two Euler equations at $(n_1\times n_2+n_3)/2$ collocation points. These points are the $(n_1\times n_2+n_3)/2$ roots of the Chebyshev polynomial of order $(n_1\times n_2+n_3)/2$. Thus, the problem reduces to solving a system of non-linear equations formed by the household's Euler equation,

$$U_C\left[\tilde{\mathcal{C}}(K_i,\tilde{\gamma}(K_i,b),a),\ \mathcal{G}(K_i,\tilde{\gamma}(K_i,b))\right]-$$
$$\beta U_C\left[\tilde{\mathcal{C}}(K_i',\tilde{\gamma}(K_i',b),a),\ \mathcal{G}(K_i',\tilde{\gamma}(K_i',b))\right]\left(1+(1-\tilde{\gamma}(K_i',b))[f_K(K_i')-\delta]\right)=0, \tag{5.3}$$

and the government's Euler equation,

$$\frac{U_C(K_i,a,b)\tilde{\mathcal{C}}_\tau(K_i,a,b)+U_g(K_i,a,b)\mathcal{G}_\tau(K_i,b)}{f(K_i)-\delta K_i+\tilde{\mathcal{C}}_\tau(K_i,a,b)}-\beta\Big\{U_C(K_i',a,b)\tilde{\mathcal{C}}_K(K_i',a,b)+$$

$$U_g(K_i',a,b)\mathcal{G}_K(K_i',b)+\left(\frac{U_C(K_i',a,b)\tilde{\mathcal{C}}_\tau(K_i',a,b)+U_g(K_i',a,b)\mathcal{G}_\tau(K_i',b)}{f(K_i')-\delta K_i'+\tilde{\mathcal{C}}_\tau(K_i',a,b)}\right)\times$$

$$\Big(1+(1-\tilde{\gamma}(K_i',b))[f_K(K_i')-\delta]-\tilde{\mathcal{C}}_K(K_i',a,b)\Big)\Big\}=0, \tag{5.4}$$

evaluated at the collocation points K_i, for $i=1,...,(n_1\times n_2+n_3)/2$; where K_i' is

$$K_i'=K_i+(1-\tilde{\gamma}(K_i,b))\Big[f(K_i)-\delta K_i\Big]-\tilde{\mathcal{C}}\Big(K_i,\ \tilde{\gamma}(K_i,b),a\Big). \tag{5.5}$$

The compact notation in (5.4) corresponds to,

$$\begin{aligned}
U_X(K_i,a,b) &= U_X\Big(\tilde{\mathcal{C}}(K_i,\tilde{\gamma}(K_i,b),a),\ \mathcal{G}(K_i,\tilde{\gamma}(K_i,b))\Big)\\
U_X(K_i',a,b) &= U_X\Big(\tilde{\mathcal{C}}(K_i',\tilde{\gamma}(K_i',b),a),\ \mathcal{G}(K_i',\tilde{\gamma}(K_i',b))\Big)\\
\tilde{\mathcal{C}}_Y(K_i,a,b) &= \tilde{\mathcal{C}}_Y(K_i,\tilde{\gamma}(K_i,b),a)\\
\tilde{\mathcal{C}}_Y(K_i',a,b) &= \tilde{\mathcal{C}}_Y(K_i',\tilde{\gamma}(K_i',b),a)\\
G_Y(K_i,b) &= G_Y(K_i,\tilde{\gamma}(K_i,b))\\
G_Y(K_i',b) &= G_Y(K_i',\tilde{\gamma}(K_i',b))
\end{aligned}$$

for $X=c,g$ and $Y=K,\tau$.

Since taxes are endogenous variables, we must guarantee that they lie in the interval $[\tau_{min},\tau_{max}]$. In order to do so, we set τ_{min} and τ_{max} as,

$$\tau_{min}=min\{\tilde{\gamma}(K_i,b)\}, \tag{5.6}$$

$$\tau_{max}=max\{\tilde{\gamma}(K_i,b)\}. \tag{5.7}$$

The way we implement these two latter restrictions is the following: (i) we impose $\tau_{min}=\tilde{\gamma}(K_{min},b)$ and $\tau_{max}=\tilde{\gamma}(K_{max},b)$, and then look for solutions to (5.3)-(5.5); (ii) then, we impose $\tau_{min}=\tilde{\gamma}(K_{max},b)$ and $\tau_{max}=\tilde{\gamma}(K_{min},b)$ and look for solutions to (5.3)-(5.5); (iii) finally, we look for Markov-Perfect equilibria by imposing that the minimum or maximum tax rates, or both, are in the interior of $[K_{min},K_{max}]$.

One of the main difficulties in implementing the projection method is to come up with a good initial guess for unknown coefficients so that a solution to the system of

Euler equations, evaluated at the collocation points, can be found. This is a problem that typically arises when the system of equation is highly non-linear. In our model, the problem is aggravated by the fact that taxes are endogenous. There are mainly two ways to cope with this situation. The first one is to use the solution of degenerate cases, or solutions of alternative, lower-quality methods as the initial the guess. The second one is to use the solution of the least squares method, consisting in minimizing the sum of the squared residuals, as the initial guess. This minimization problem always yield a solution, and is extremely easy to implement.[5] In this paper, we use the latter alternative to obtain initial guesses. Some remarks are in order. Since uniqueness of the Markov-Perfect equilibrium can not be guaranteed, and since the problem of minimizing the sum of the squared residuals can find a local minimum, some additional work is needed. What we have done here is to use the solution of the minimization problem, and some conveniently perturbed versions of this solution, as initial guesses. For our parameterization of the model, and all sets of parameter values considered in this paper, we found a unique solution to the system (5.3)-(5.5) rendering a stable equilibrium.

Finally, the computation of the value function is straightforward. Using the solutions for $\tilde{\mathcal{C}}(K_i, \tau, a)$ and $\tilde{\gamma}(K_i, b)$, the value function, $\mathcal{V}(K)$, is approximated by,

$$\tilde{\mathcal{V}}(K, d) = \sum_{i=1}^{n_4} d_i \phi_{i-1}[2(K - K_{min})/(K_{max} - K_{min}) - 1],$$

where the vector $d = (d_1, ..., d_{n_4})$, is the vector of unknown coefficients in the value function. These coefficients are the solutions to the following system of n_4 equations.

$$\tilde{\mathcal{V}}(K_i, d) = U\left[\tilde{\mathcal{C}}(K_i, \tilde{\gamma}(K_i, b), a),\ \mathcal{G}(K_i, \tilde{\gamma}(K_i, b))\right] + \tilde{\mathcal{V}}(K_i', d),$$

at the collocation points $K_1, ..., K_{n_4}$, and where K_i', for $i = 1, ..., n_4$, are given by equation (5.5).

6 Conclusions

Since the seminal paper by Kydland and Prescott (1977), several authors have followed different strategies to characterize time-consistent optimal fiscal policies. In this paper we have studied Markovian optimal taxation under two alternative scenarios regarding the within-period timing of actions. Our main motivation to bring the timing of actions to the

[5]For instance, Mathematica uses the Levenberg-Marquardt algorithm, which is a modification of the Gauss-Newton algorithm, and is especially useful to solve non-linear least squares problems.

forefront in the analysis of Markovian taxation is the manifest lack of concurrence found in the literature when it comes to specify the order of movements, and the consequent lack of knowledge of the macroeconomic implications of such order. This paper shows that the effect of the timing of actions on optimal policies is large. Hence, we draw a clear implication from our results: In models of optimal taxation without government's full commitment to future taxes, the within-period timing of actions must be explicitly stated as one of the givens of the economy, alongside preferences, markets and technology. A second implication that emerges from our results is normative: The fiscal constitution should not grant the government with instantaneous leadership, i.e., social welfare is maximized when the government and households choose taxes and savings simultaneously.

Our analysis is carried out within a simple model of consumption and savings, augmented to include valued government expenditure. The model is completely standard in the literature of optimal taxation. We use dynamic programming techniques to characterize the Markov-Perfect equilibria of the game played by a sequence of governments and the private sector. And then we use a version of a projection method to solve the two functional equations (the Euler Equation of the household and the Euler equation of the government) to find optimal taxes and savings. Our computational strategy allows us to compute Markov-Perfect equilibria for a wide range of the state space, and not only steady states. This turns out to be critical for our purposes in this paper since it makes possible to carry out a welfare analysis.

Our findings in this paper prompt the analysis of a new set of questions regarding the optimal determination of taxes. Allowing for different tax rates on capital and labor income seems a natural extension of our analysis. It is not entirely obvious whether the result found in this paper —i.e., that the timing of actions characterized by simultaneous moves is welfare maximizing— would still apply to both tax rates. Another extension worth investigating is to allow the government to issue debt. By doing so, the government would have a tool to spread the burden of taxation over time, whose effects on the optimal within-period timing of actions are also far from being obvious *a priori*. The computational method used in this paper can be easily adjusted to address these, and other, new questions.

7 Appendix

Appendix I

Proof of Proposition 1: The proof is straightforward. It simply combines the first-order condition to problem (2.7)-(2.8), and the envelope condition. The first-order condition is,

$$U_g\mathcal{G}_\tau - \beta V'_K \cdot \Big(f - \delta K \Big) = 0 \tag{7.1}$$

The envelope condition is

$$\begin{aligned} V_K = U_c C_K \quad &+ \quad U_g \cdot (\mathcal{G}_K + \mathcal{G}_\tau \psi_K) \\ &+\beta V'_K \cdot \Big(1 + [f_K - \delta](1-\psi) - [f - \delta K]\psi_K - C_K \Big) \end{aligned} \tag{7.2}$$

which, after collecting terms and using the first-order condition, yields,

$$V_K = U_c C_K + U_g \cdot \Big(f_K + 1 - \delta - C_K \Big) \tag{7.3}$$

Now, updating this equation one period ahead, and using the first-order condition it yields equation (2.15) in Proposition 1.

Proof of Proposition 2: The proof follows the same arguments as in Proposition 1. The first-order condition to the government's problem, (2.19)-(2.21), is

$$U_c\mathcal{C}_\tau + U_g\mathcal{G}_\tau - \beta \mathcal{V}'_K \cdot \Big(f - \delta K + \mathcal{C}_\tau \Big) = 0 \tag{7.4}$$

and the envelope condition, after using the first-order condition and rearranging terms, yields,

$$\mathcal{V}_K = U_c\mathcal{C}_K + U_g\mathcal{G}_K + \beta \mathcal{V}'_K \cdot \Big(1 + [f_K - \delta](1-\gamma) - \mathcal{C}_K \Big) \tag{7.5}$$

Now, updating one period ahead, and using again the first-order condition, gives the expression presented in Proposition 2.

Appendix II

A Two-Period Economy with Full Capital Depreciation

We compute optimal income taxes in a two-period version of our model in Section 2 for the two timings of actions. In order to reach closed-form solutions we assume full depreciation of capital and remove the tax depreciation allowance. Furthermore, we assume logarithmic preferences and a Cobb-Douglas production function.

In a two-period economy, it is straightforward to show that households' consumption in periods 0 and 1 are given, respectively, by,

$$c_0 = \frac{1-\tau_0}{1+\alpha\beta} K_0^\alpha \tag{7.6}$$

$$c_1 = (1 - \tau_1)K_1^\alpha \tag{7.7}$$

In the simultaneous-move equilibrium (SME), the household does not know the tax rate at the time it chooses consumption and must therefore forecast the government's taxation policy, say τ_0^e and τ_1^e. Thus, the consumption function can be written as $C_t(K_t, \tau_t^e)$, for $t = 0, 1$. In the government-moves-first equilibrium (GMFE), the household knows the tax rate when it chooses consumption. The consumption function is written then as $\mathcal{C}_t(K_t, \tau_t)$, for $t = 0, 1$.

The problem of the government in period 1 (the last period) is trivial. Since $K_2 = 0$ the period-1 government sets $\tau_1 = \theta/(1 + \theta)$ regardless of the assumed within-period timing of actions. In period 0, however, the tax rate depends on the within-period timing of actions as follows.

In the SME, the time-0 government's problem is,

$$V_0(K_0) = max_{\tau_0} \Big\{ \log(C_0(K_0, \tau_0^e) + \theta \log \tau_0 K_0^\alpha + \beta \log(1 - \tau_1)K_1^\alpha + \beta\theta \log \tau_1 K_1^\alpha \Big\} \tag{7.8}$$

where

$$C_0(K_0, \tau_0^e) + K_1 = (1 - \tau_0)K_0^\alpha, \tag{7.9}$$

K_0 is given and τ_0^e is taken as given by the time-0 government. The first-order condition to this problem is then,

$$\frac{\theta}{\tau_0} = (1 + \theta)\beta\alpha \frac{K_0^\alpha}{(1 - \tau_0)K_0^\alpha - C_0(K_0, \tau_0^e)}, \tag{7.10}$$

which, after imposing the equilibrium condition $\tau_0 = \tau_0^e$ and using the consumption function given by (7.6)-(7.7), yields,

$$\tau_0^{SME} = \frac{\theta}{(1 + \theta)(1 + \alpha\beta) + \theta}. \tag{7.11}$$

In the GMFE, the time-0 government's problem is,

$$\mathcal{V}_0(K_0) = max_{\tau_0} \Big\{ \log(\mathcal{C}_0(K_0, \tau_0) + \theta \log \tau_0 K_0^\alpha + \beta \log(1 - \tau_1)K_1^\alpha + \beta\theta \log \tau_1 K_1^\alpha \Big\}, \tag{7.12}$$

where

$$\mathcal{C}_0(K_0, \tau_0) + K_1 = (1 - \tau_0)K_0^\alpha, \tag{7.13}$$

where K_0 is given. The first-order condition to this problem is then,

$$\frac{\mathcal{C}_{\tau_0}(K_0, \tau_0)}{\mathcal{C}_0(K_0, \tau_0)} + \frac{\theta}{\tau_0} = (1 + \theta)\beta\alpha \frac{K_0^\alpha + \mathcal{C}_{\tau_0}(K_0, \tau_0)}{(1 - \tau_0)K_0^\alpha - \mathcal{C}_0(K_0, \tau_0)}, \tag{7.14}$$

which, after using the consumption function, yields,

$$\tau_0^{GMFE} = \frac{\theta}{(1+\theta)(1+\beta\alpha)}. \tag{7.15}$$

From a simple inspection of (7.11) and (7.15) it follows that $\tau_0^{SME} < \tau_0^{GMFE}$.

Welfare implications of the within-period timing of actions in the two-period economy

In this simple, two-period economy the welfare consequences of the timing of actions can be easily assessed by comparing the corresponding lifetime utilities. By evaluating (7.8) and (7.12) at equilibrium solutions we obtain,

$$V_0(K_0) - \mathcal{V}_0(K_0) = (1+\theta)(1+\beta\alpha)\log\frac{(1+\theta)(1+\beta\alpha)}{(1+\theta)(1+\beta\alpha)+\theta} + [1+(1+\theta)\beta\alpha]\log\frac{(1+\theta)(1+\beta\alpha)}{1+(1+\theta)\beta\alpha}.$$

For the values of α, β and θ in our benchmark economy of Section 3 the Markov-Perfect equilibrium with government leadership renders higher lifetime utility than the simultaneous-move equilibrium. In the two-period economy the cross-period gains of lower tax rates in the SME are not big enough to compensate the within-period losses associated with the lack of government leadership.

Appendix III

In this Appendix we make use of the two-period economy shown above to study a version of the game where the private sector chooses investment rather than consumption. It is obvious that this new feature will have consequences on taxes and allocations in the simultaneous-move equilibrium but not in the equilibrium where the government moves first.

In a two-period economy it is straightforward to show that households' investment in period 0 is,

$$K_1 = \frac{(1-\tau_0)\alpha\beta}{1+\alpha\beta}K_0^{\alpha}, \tag{7.16}$$

and zero in period 1.

In the simultaneous-move equilibrium (SME), the household chooses investment before knowing the tax rate and must therefore forecast the government's taxation policy, say τ_0^e and τ_1^e.

The problem of the government in period 1 (the last period) is trivial. Since $K_2 = 0$ the period-1 government sets $\tau_1 = \theta/(1+\theta)$. The time-0 government's problem is,

$$V_0(K_0) = max_{\tau_0}\left\{ \log C_0 + \theta\log\tau_0 K_0^{\alpha} + \beta\log(1-\tau_1)K_1^{\alpha} + \beta\theta\log\tau_1 K_1^{\alpha}\right\} \tag{7.17}$$

where

$$C_0 + K_1 = (1 - \tau_0)K_0^\alpha, \tag{7.18}$$

and

$$K_1 = \frac{(1 - \tau_0^e)\alpha\beta}{1 + \alpha\beta} K_0^\alpha \tag{7.19}$$

K_0 is given and τ_0^e is taken as given by the time-0 government. The first-order condition to this problem is then,

$$\frac{1}{(1 - \tau_0^e)K_0^\alpha - K_1} = \frac{\theta}{\tau_0 K_0^\alpha}, \tag{7.20}$$

which, after imposing the equilibrium condition $\tau_0 = \tau_0^e$ yields,

$$\tau_0^{SME} = \frac{\theta}{1 + \alpha\beta + \theta}. \tag{7.21}$$

Since the time-0 optimal tax in the GMFE is the same as in the case of households choosing consumption, it thus follows that $\tau_0^{GMFE} < \tau_0^{SME}$, as expected from our discussion above.

References

[1] Cohen, D. and P. Michel, (1988). "How Should Control Theory Be Used to Calculate a Time-Consistent Government Policy?," *Review of Economic Studies,* LV, 263–274.

[2] Judd, K., (1992). "Projection Methods for Solving Aggregate Growth Models," *Journal of Economic Theory,* 58, 410–452.

[3] Judd, K., (1998). Numerical Methods in Economics, The MIT Press, Cambridge, London.

[4] Klein, P., Krusell, P. and J.-V. Ríos-Rull, (2003). "Time-Consistent Public Expenditures," manuscript.

[5] Kydland, F. E. and E. C. Prescott, (1977). "Rules Rather than Discretion: The Inconsistency of Optimal Plans," *Journal of Political Economy,* 85, 473–491.

[6] Krusell, P. and A. Smith, (2003). "Consumption-Savings Decisions with Quasi-Geometric Discounting," *Econometrica,* 71, 365–375.

[7] Maskin, E. and J. Tirole, (2001). "Markov Perfect Equilibrium I. Observable Actions," *Journal of Economic Theory,* 100, 191–219.

[8] Miranda, M. and P.L. Fackler (2002). Applied Computational Economics and Finance, The MIT Press, Cambridge, London.

[9] Miranda, M. and X. Rui, (1996). "Solving Nonlinear Dynamic Games via Orthogonal Collocation: An Application to International Commodity Markets," *Annals of Operations Research,* 68, 89–108.

[10] Pakes, A. and P. McGuire, (1994). "Computing Markov-Perfect Nash Equilibria: Numerical Implications of a Dynamic Differentiated Product Model," *RAND Journal of Economics,* 25, pp. 555–589.

[11] Phelan, C. and E. Stachetti, (2001). "Sequential Equilibria in a Ramsey Tax Model," *Econometrica,* 69, 1191–1218.

[12] Sorger, G., (1998). "Markov-Perfect Nash Equilibria in a Class of Resource Games," *Economic Theory,* 11, 79–100.

[13] Turnovsky, S. J. and W. A. Brock, (1980). "Time Consistency and Optimal Government Policies in Perfect Foresight Equilibrium," *Journal of Public Economics,* 13, 183–212.

[14] Venedov, D. V. and M. Miranda, (2001). "Numerical Solution of Dynamic Oligopoly Games with Capital Accumulation," *Economic Theory,* 18, 237–261.

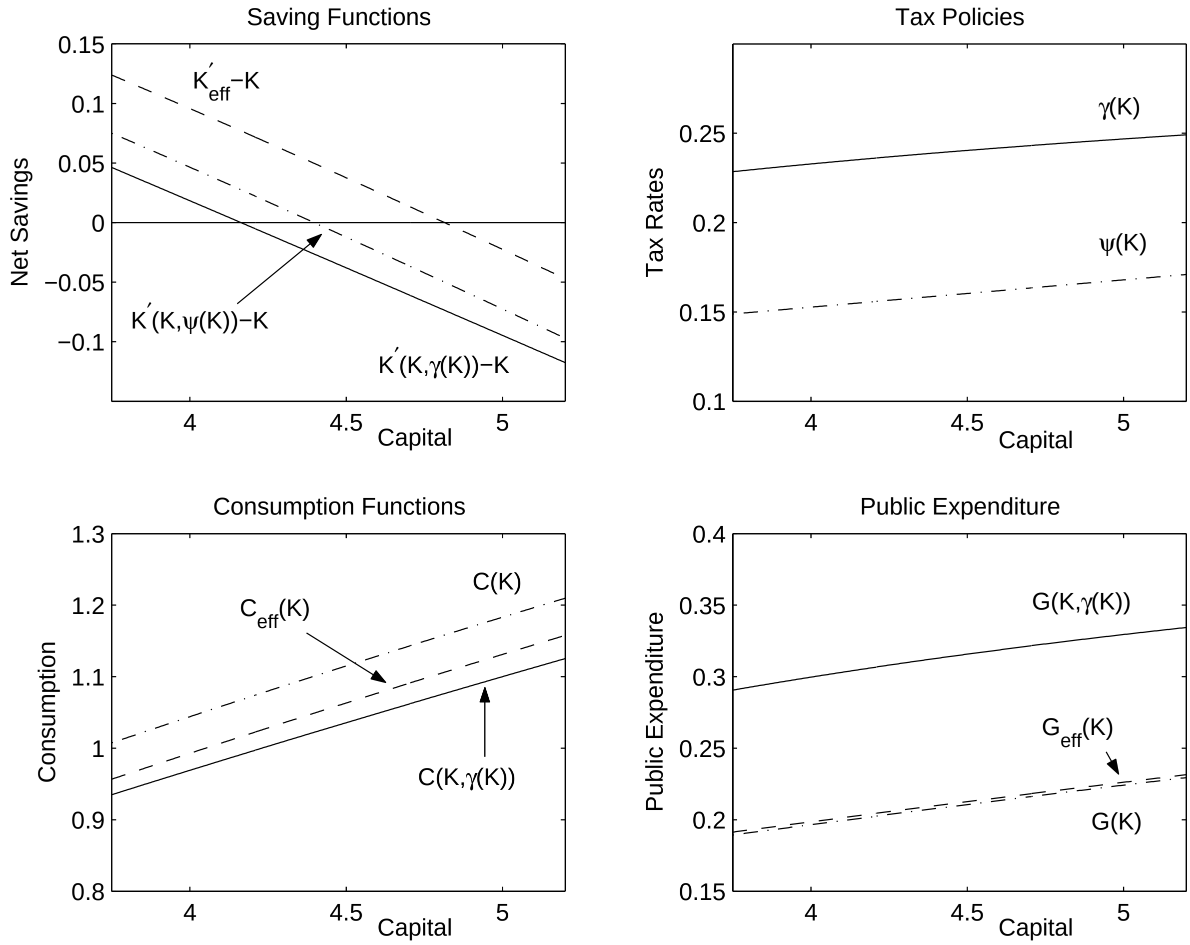

Figure 1: This figure displays net savings, tax rates, consumption of the private good, and public expenditure under simultaneous-move taxation (dash-dot lines), government-moves-first taxation (solid lines), and under lump-sum taxation (dashed lines).

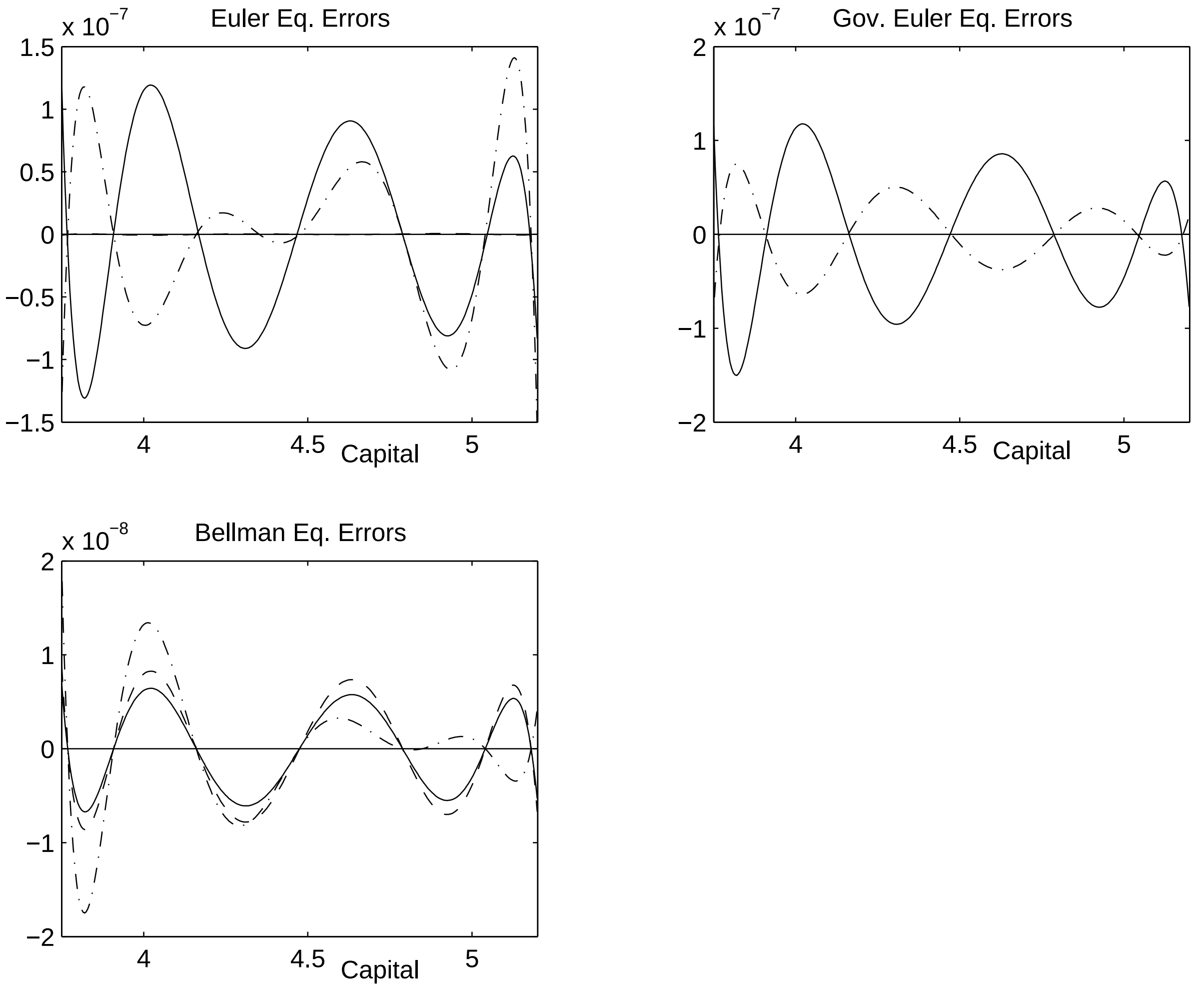

Figure 2: This figure displays the errors for the household's Euler equation, the government's Euler equation, and the government's Bellman equation for the simultaneous-move equilibrium (dashed-dot lines); the government-moves-first equilibrium (solid lines); and the efficient equilibrium with lump-sum taxes (dashed lines).

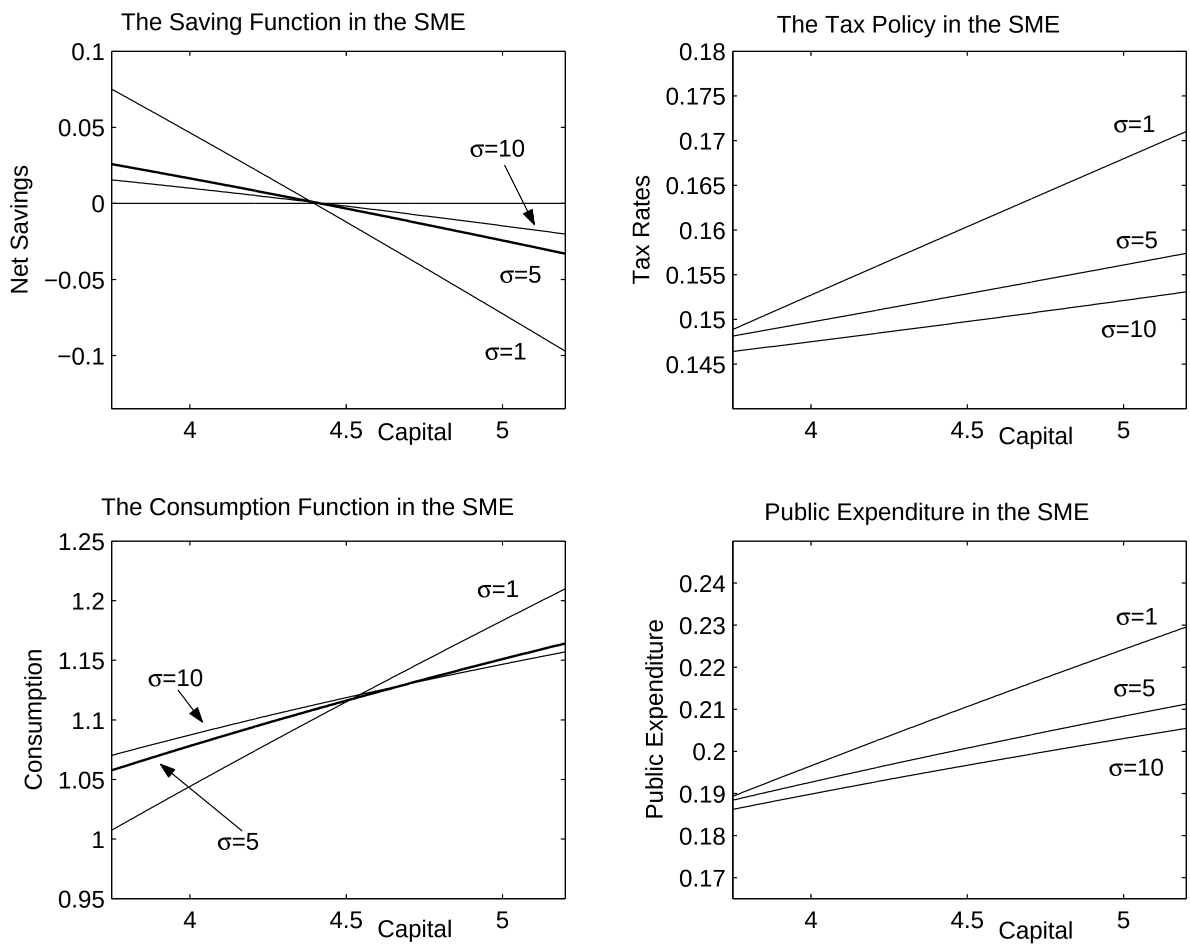

Figure 3: This figure displays the Simultaneous-Move Markov-Perfect Equilibrium (SME) for $\sigma = 1$, $\sigma = 5$ and $\sigma = 10$.

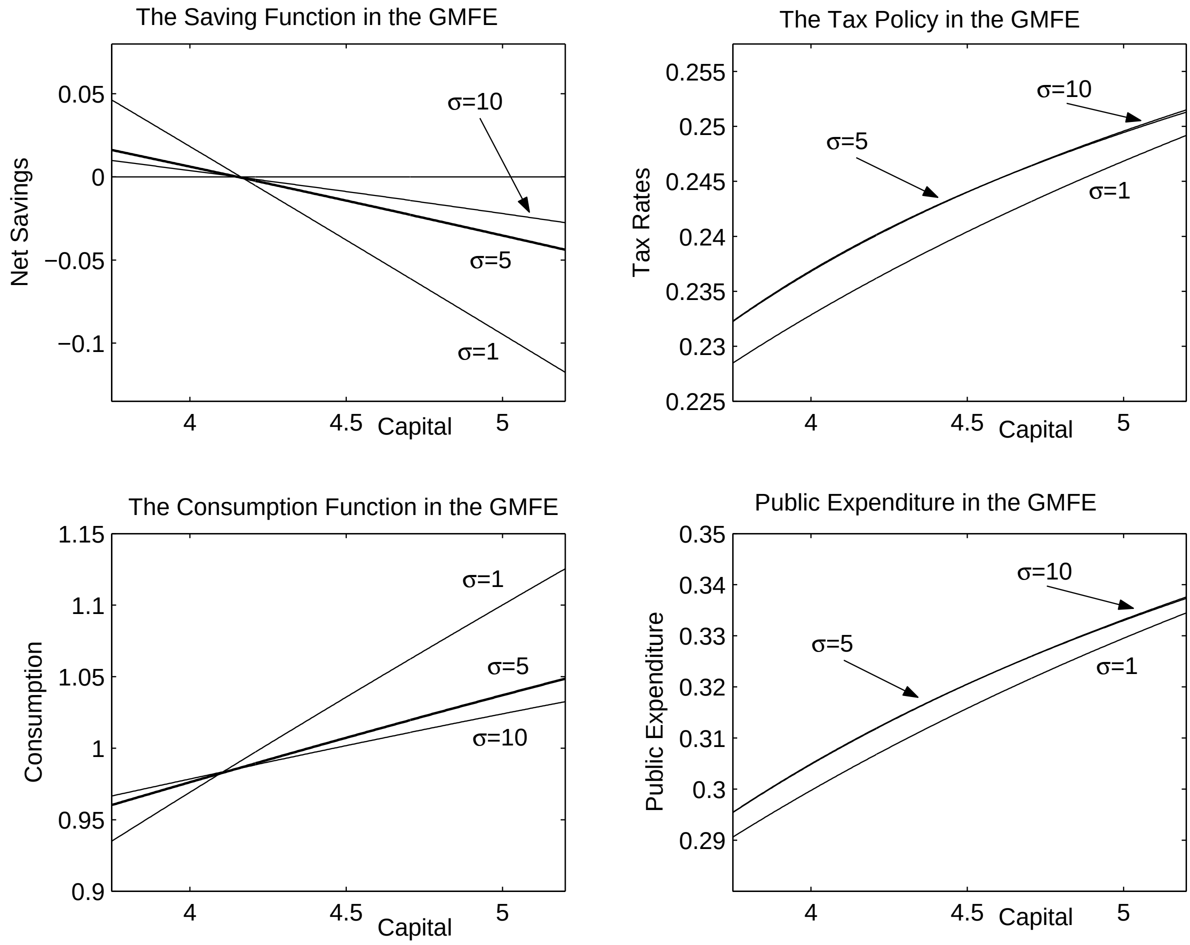

Figure 4: This figure displays the Government-Moves-First Markov-Perfect Equilibrium (GMFE) for $\sigma = 1$, $\sigma = 5$ and $\sigma = 10$.

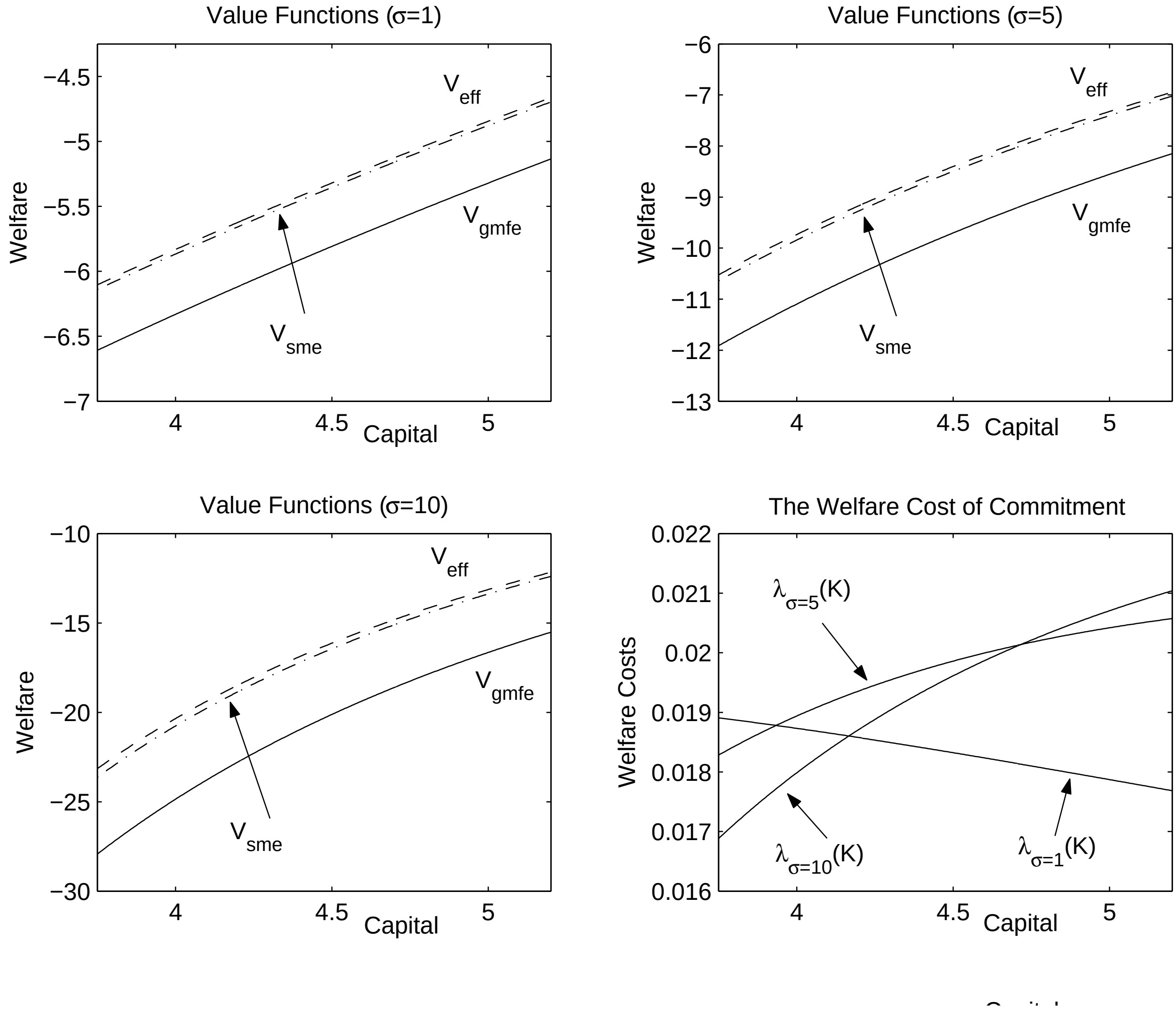

Figure 5: This figure displays the value functions for the simultaneous-move equilibrium, $V_{sme}(K)$, the government-moves-first equilibrium, $V_{gmfe}(K)$, and the efficient equilibrium with lump-sum taxes, $V_{eff}(K)$, for $\sigma = 1, \sigma = 5$, and $\sigma = 10$. The chart at the bottom right plots the welfare cost of commitment, $\lambda(K)$, for the three values of σ.

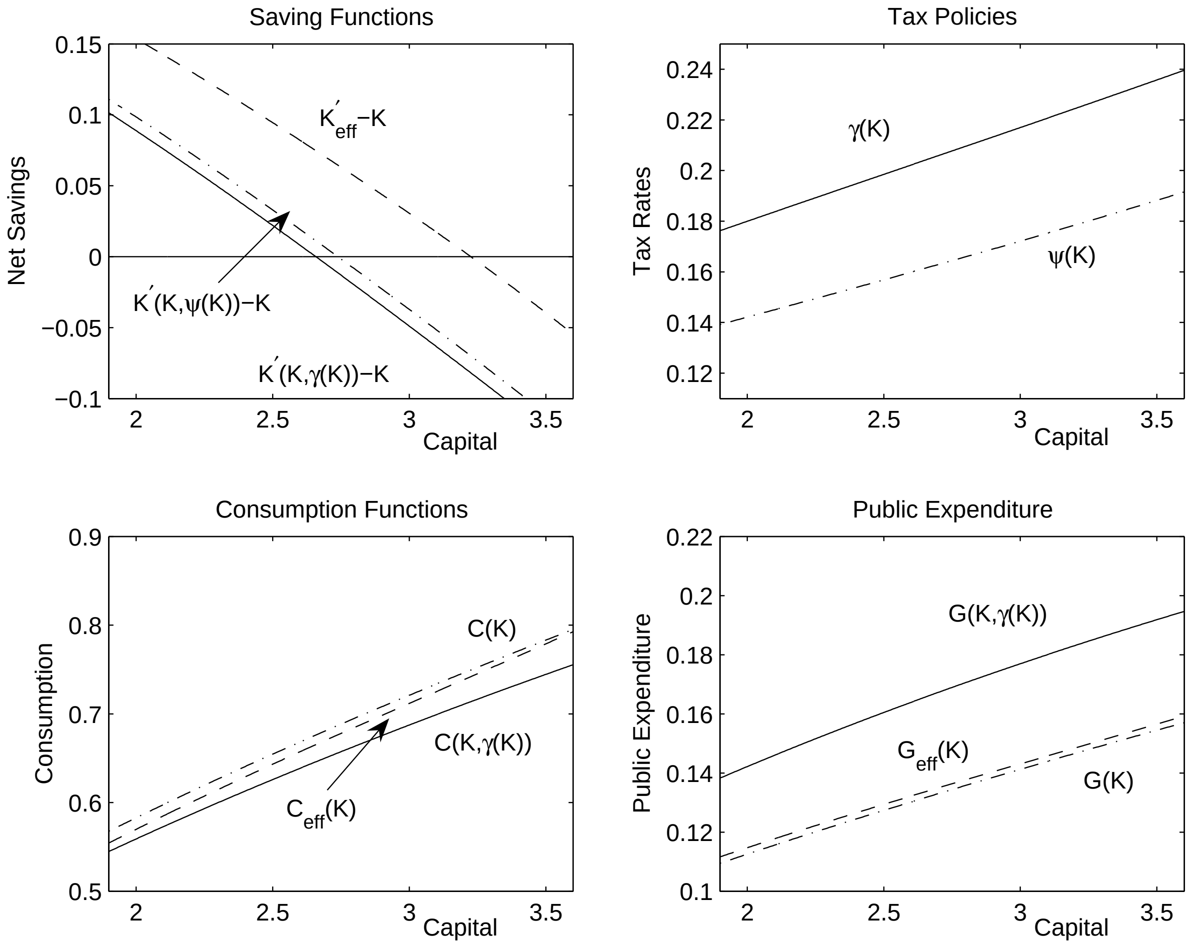

Figure 6: This figure displays net savings, tax rates, consumption of the private good, and public expenditure under simultaneous-move taxation (dash-dot lines), government-moves-first taxation (solid lines), and under lump-sum taxation (dashed lines) in the model with endogenous labor.

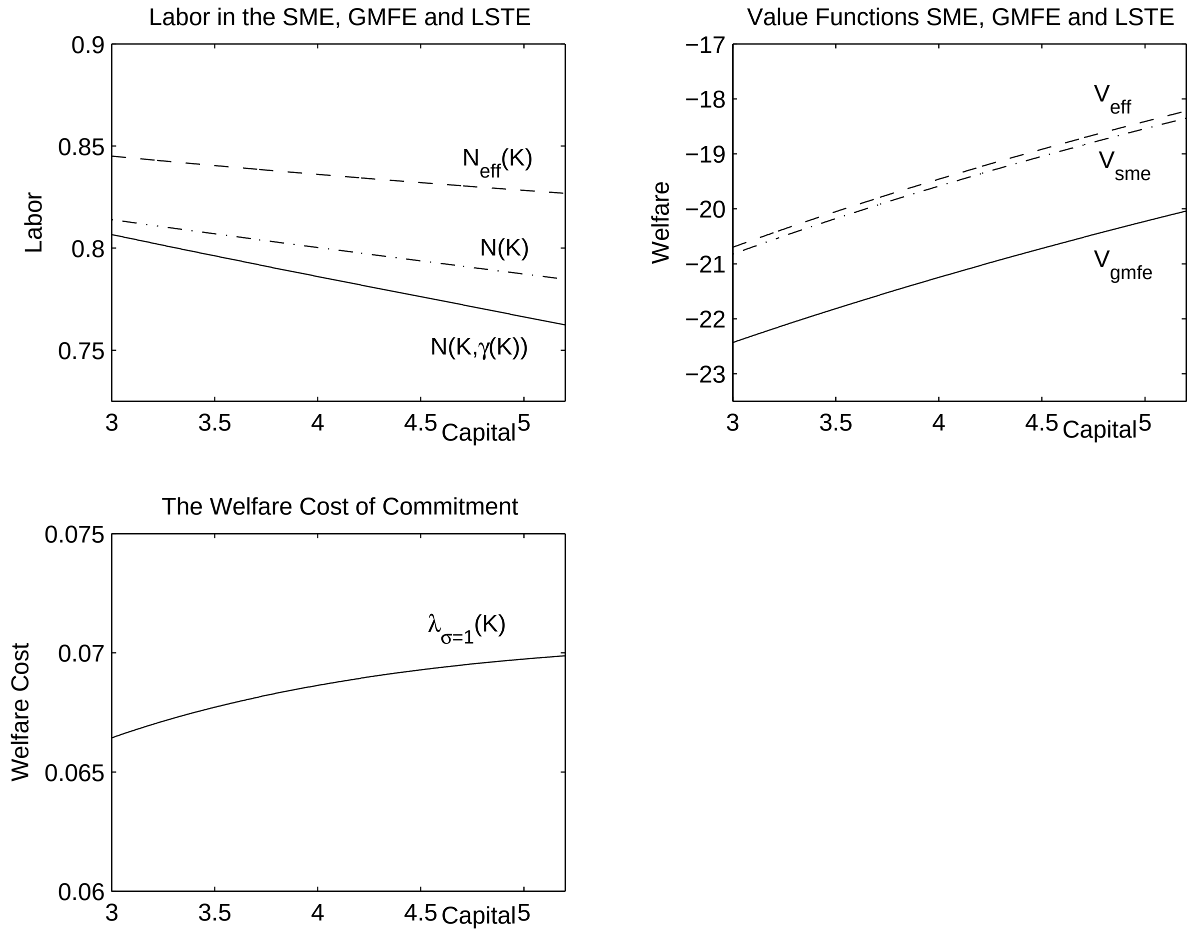

Figure 7: The top two charts in this figure display labor, and the value functions in the model with endogenous labor in the simultaneous-move equilibrium (dash-dot lines), government-moves-first equilibrium (solid lines), and the efficient equilibrium with lump-sum taxes (dashed lines). The chart at the bottom displays the welfare cost, as a percentage of consumption of the private good, of having a government with instantaneous commitment.