Endogenous Business Cycles with Frictional Labour Markets*

Nigar Hashimzade† and Salvador Ortigueira‡

September 20, 2004

Abstract

In this paper we present a version of the neoclassical model of capital accumulation with frictional labour markets. We show that under standard parameter values the equilibrium of the model is indeterminate, and that it consequently displays expectations-driven business cycles —the so-called endogenous business cycles. We study the properties of such business cycles, and find that the model predicts the high autocorrelation in output growth and the hump-shaped impulse response of output found in US data —important business cycle features that existing endogenous-business-cycle models, as well as real business cycle models fail to explain. The indeterminacy of the equilibrium in our economy stems from the job search externalities, and does not rely on increasing returns to scale as in most models in the literature of endogenous business cycles.

Keywords: Job Search; Endogenous Cycles; Output Dynamics.

JEL Codes: E32, J22, J23.

*We are grateful to two anonymous referees and to one editor for their helpful comments on an early version of this paper. We also thank audiences at Cornell University and the Penn State/Cornell macro workshop.

†E-mail: n.hashimzade@exeter.ac.uk.

‡E-mail: Salvador.Ortigueira@iue.it.

1 Introduction

In this paper, we present a business cycle model in which fluctuations in economic activity are driven solely by revisions in expectations. The main idea underlying this type of model is that fluctuations in economic variables do not stem from shocks to fundamentals, but from shocks to expectations, thus placing the emphasis on extrinsic rather than on intrinsic uncertainty. The possibility of constructing such models under plausible economic assumptions, and their ability to account for key observations about business cycles, has been assessed by a number of authors [see, e.g., Benhabib and Farmer (1994), Farmer and Guo (1994), Schmitt-Grohe (2000), Thomas (2004)].

Although these models unambiguously succeed in accounting for volatilities and comovements of various series over the business cycle, less agreement is found regarding their ability to explain the positive serial correlation of output growth and the hump-shaped impulse response function to innovations. These two latter stylized facts (see Cogley and Nason (1995) for a detailed analysis of US output dynamics), have challenged the business cycle literature, including the real business cycle model (RBC), and the expectations-driven model. More specifically, most business cycle models cannot replicate those facts, because of their weak internal propagation mechanisms.

The contribution of our paper to this literature is twofold. First, we show that by extending the neoclassical model of capital accumulation to include a frictional labour market *à la* Pissarides (1990), we open an avenue to expectations-driven fluctuations. Contrary to most models in the literature, we do not need to rely on increasing returns to scale. Equilibrium indeterminacy in our model results from job search externalities, which are an inherent feature in the matching process. Second, we calibrate our model economy to match average values for the US economy, and evaluate its performance along two dimensions: (i) volatility of output, consumption, investment, wages, labor productivity, and the vacancies-unemployment ratio over the business cycle, and their comovement with output; (ii) volatilities relative to output growth, comovements of growth rates, and autocorrelation of output growth. Our results show that the model explains acceptably well the standard second moments in (i), and that it accounts for the relative volatilities and the positive autocorrelation of output growth. Our model thus outperforms previous models of endogenous business cycles, as they fail to explain the positive short-run autocorrelation in output growth. The explanation for the relative success of our model must be found in the increased internal propagation of shocks brought about by the frictions in the labour market. As today's workers and firms' search accrues only to tomorrow's employment, labour market frictions introduce a lag in the determination of employment that increases the propagation of shocks.

Notwithstanding the relative success of our model, we refrain from drawing any conclusion regarding the source of economic fluctuations in actual economies. Our analysis is not meant to contribute to the debate on whether observed fluctuations stem from revisions in expectations or from real shocks. The aim of our analysis is to show that endogenous-business-cycle models can predict the positive first-order autocorrelation observed in US output growth, a result that stands in sharp contrast with the view expressed recently by some authors.[1] Indeed, persistence in growth rates does not seem to be related to whether shocks come from expectations or from fundamentals, but to the strength of the internal propagation of shocks. We show that a simple version of the neoclassical growth model with frictions in the labour market generates both endogenous cycles and the needed internal propagation mechanism.

Our paper builds on two literatures. We follow the neoclassical model of capital accumulation to model both consumption-saving decisions and the economy's production sector. We then follow the literature on frictional unemployment to model the labour market. We assume that both workers and firms search in the labour market, some get matched and negotiate the wage. The externalities springing from the search-matching process yield endogenous fluctuations in economic variables. Our model is close to Andolfato (1996) and Merz (1996) in the modelling of saving decisions and the labour market, but it departs from them in a crucial aspect. While these two authors focus on the efficient solution, disregarding thus the above-

[1] Schmitt-Grohe (2000) writes: "Therefore, I conclude that the endogenous-business-cycle model fails to explain the autocorrelation function of output growth at conventional significance levels even when business cycles are driven jointly by technology and sunspot shocks. [...] The results suggest that endogenous fluctuations do not provide the dynamic element that is missing in existing real business cycle models.

mentioned externalities and the possibility of endogenous fluctuations, we will look at the market equilibrium. By doing so, we uncover the equilibrium effects of frictional labour markets on macroeconomic aggregates. Our findings show that there exist substantial differences between equilibrium and efficient allocations.

The paper is organized as follows. In Section 2 we present the model, define the search equilibrium, and characterize both the steady-state equilibrium and local dynamics. In Section 3 we compute the equilibrium and study the properties of expectations-driven fluctuations. Section 4 presents the main conclusions and Section 5 contains two appendixes.

2 The Model

The Labour Market

The labour market is frictional. There is a matching technology that determines the number of job matches, M_t, as a function of vacancies and workers' search effort, i.e.,

$$M_t = \mathcal{M}(v_t, u_t s_t) \tag{2.1}$$

where v_t denotes the number of vacancies open at time t, and workers' search effort, $u_t s_t$, is the product of unemployed workers, u_t, and their search intensity s_t. Function $\mathcal{M}$ is assumed to be increasing in both arguments, jointly concave and linearly homogeneous. Vacancies are filled at rate M_t/v_t, and unemployed workers switch to employment at rate $M_t/(u_t s_t)$.

We now embed this model of frictional unemployment into a standard model of capital accumulation, in which the number of unemployed workers, search intensity and vacancies are determined by optimizing households and firms.

Households

The household sector is represented by a continuum of infinitely-lived, identical households with measure one. The representative household derives utility from consumption and leisure, and maximizes lifetime discounted utility,

$$\sum_{t=0}^{\infty} \beta^t U(c_t, l_t). \tag{2.2}$$

The instantaneous utility function, $U(\cdot,\cdot)$, is assumed to be strictly jointly concave and twice continuously differentiable. Household members are engaged in only one of the following three activities: working in the production sector, n_t; searching for jobs (unemployment), u_t; or enjoying leisure, l_t.

Household's total income at period t —capital income, wage income, and profits from the production sector— is allocated to consumption c_t, investment i_t, and to finance the cost of job search intensity, $p_t s_t$; where s_t is the intensity of search, and p_t is its cost per unit. The budget constraint is thus,

$$c_t + i_t + p_t s_t = r_t k_t + \omega_t n_t + \pi_t \tag{2.3}$$

where r_t, ω_t and π_t are the rental price of capital, the wage rate and time-t profits, respectively. Physical capital depreciates at rate $0 < \delta < 1$, and, therefore, its law of motion is given by,

$$k_{t+1} = i_t + (1-\delta)k_t. \tag{2.4}$$

The law of motion for employment within the household is,

$$n_{t+1} = n_t + m_t u_t s_t - \theta n_t, \tag{2.5}$$

where m_t is the perceived probability that an unemployed worker be matched with a vacancy per unit of efficient search, and $\theta \geq 0$ is the exogenous rate of job destruction.

The problem of the household is to maximize (2.2) subject to (2.3), (2.4), (2.5) and $n_t + s_t + l_t = 1$. The optimal allocation of income to savings must satisfy the standard Euler equation,

$$U_c(c_t, l_t) = \beta U_c(c_{t+1}, l_{t+1}) \left[1 + r_{t+1} - \delta\right], \tag{2.6}$$

where $U_c(\cdot,\cdot)$ denotes the derivative of the utility function with respect to consumption. The optimal level of search intensity satisfies,

$$p_t = W_t m_t u_t, \tag{2.7}$$

which equates the marginal income cost of search intensity to its marginal benefit, where W_t denotes the income value of employment to the household.

The optimal allocation of family members across leisure and job search satisfies,

$$\frac{U_l(c_t, l_t)}{U_c(c_t, l_t)} = W_t m_t s_t, \tag{2.8}$$

which equates the income value of leisure to the expected net income from finding a job.

Finally, the income value of employment, W_t, satisfies the following arbitrage equation,

$$(r_{t+1} - \delta)W_t + \frac{U_l(c_{t+1}, l_{t+1})}{U_c(c_{t+1}, l_{t+1})} = \omega_{t+1} + W_{t+1} - W_t - \theta W_{t+1}, \tag{2.9}$$

which equates the income cost of holding the job —the capital cost plus the leisure income cost—, to the wage rate plus capital gains, net of the risk of losing the job.

Firms

The production sector is made up of identical competitive firms. There is a representative firm which uses capital and labor to produce the aggregate good. The production technology is represented by $F(k_t, a_t n_t)$, where a_t is the level of labour-augmenting technology which follows the deterministic process $a_{t+1} = g a_t$, where $g > 1$ is the gross rate of technological progress. F is assumed to be a neoclassical production function —i.e., is strictly jointly concave, twice continuously differentiable, increasing in both arguments, and satisfies the standard Inada-type conditions.

Since the labour market is frictional, the law of motion of employment to firms is,

$$n_{t+1} = n_t + \mu_t v_t - \theta n_t, \tag{2.10}$$

where μ_t is the perceived probability (matching rate) that a vacancy be matched with an unemployed worker.

The firm hires capital and open vacancies to maximize the present value of cash flows,

$$\sum_{t=0}^{\infty} \frac{1}{\Pi_{\tau=0}^{t} R_\tau} \left[F(k_t, a_t n_t) - r_t k_t - \omega_t n_t - q_t v_t \right]. \tag{2.11}$$

subject to (2.10), where $R_\tau = r_\tau + 1 - \delta$ is the gross rate of return, and $q_t v_t$ denotes the cost of opening v_t vacancies. Both the cost per vacancy, q_t, and the unit cost of search intensity, p_t, are assumed to growth at the same rate as technology.

The firm's demand for capital obeys the standard optimality condition,

$$r_t = F_k(k_t, a_t n_t), \tag{2.12}$$

where F_k denotes the marginal productivity of capital. The condition that determines the optimal number of vacancies at period t is given by,

$$q_t = \mu_t J_t, \tag{2.13}$$

where J_t is the income value of employment to the firm. This latter value satisfies the following arbitrage condition,

$$(r_{t+1} - \delta)J_t = F_n(k_{t+1}, a_{t+1}n_{t+1}) - \omega_{t+1} + J_{t+1} - J_t - \theta J_{t+1}, \tag{2.14}$$

where F_n denotes the marginal productivity of labor. This arbitrage equation establishes that the capital cost of the job, $(r_{t+1} - \delta)J_t$, must equal the job's yields, $F_n(k_{t+1}, a_{t+1}n_{t+1}) - \omega_{t+1}$, plus capital gains, $J_{t+1} - J_t$, minus the risk of losing the job, θJ_{t+1}.

Wage determination

We follow the standard literature on frictional unemployment and assume that wages are the solution to a Nash-bargaining problem between the household and the firm. The Nash solution maximizes the weighted product of the household's and the firm's income values of employment. Hence, if we use λ to denote the worker's bargaining power, the wage rate is,

$$\omega_t = \arg\max \left\{ W_t^{\lambda} J_t^{1-\lambda} \right\}. \tag{2.15}$$

The first-order condition to this maximization problem is $W_t = \lambda(W_t + J_t)$, which states that the worker will get a share λ of the total income generated by the match. It then follows from (2.9) and (2.14) that the wage rate is given by,

$$\omega_t = \lambda F_n(k_t, a_t n_t) + (1 - \lambda)\frac{U_l(c_t, l_t)}{U_c(c_t, l_t)}. \tag{2.16}$$

Equation (2.16) establishes that the wage rate is a linear combination of the marginal productivity of labor and the income value of leisure, which is the worker's reservation wage.

Equilibrium

We can now define a search equilibrium for this economy as a set of infinity sequences for quantities $\{k_t, n_t, c_t, u_t, s_t, l_t, v_t\}_{t=0}^{\infty}$, job matching rates $\{m_t, \mu_t\}_{t=0}^{\infty}$, and prices $\{r_t, \omega_t\}_{t=0}^{\infty}$, such that:

- Given matching rates and prices, $\{k_t, n_t, c_t, u_t, s_t, l_t\}_{t=0}^{\infty}$ solve the household's optimization problem.
- Given matching rates and prices, $\{k_t, n_t, v_t\}_{t=0}^{\infty}$ solve the firm's maximization problem.
- Matching rates are given by the matching function: $m_t u_t s_t = \mu_t v_t = M(v_t, u_t s_t)$.
- Wages are the solution to Nash-bargaining problems.

Functional Forms for Matching, Preferences and Technology

We now choose functional forms for the matching function, instantaneous utility and the production technology. We adopt functional forms which are standard in the literatures of job matching and business cycles.

The matching technology is assumed to take the following form,

$$M_t = \mathcal{M}(v_t, u_t s_t) = \frac{v_t \cdot (u_t s_t)}{\left[v_t^{\eta} + (u_t s_t)^{\eta}\right]^{1/\eta}}, \tag{2.17}$$

where η is a parameter. It should be noted that this matching function has constant returns to scale in vacancies and efficiency units of search, $u_t s_t$. One of the main advantages of using this matching function, as explained by Den Haan, Ramey and Watson (2000), is that it yields matching probabilities between zero and one, for all admissible values of v_t and $u_t s_t$.

Regarding preferences, we adopt the standard utility function in the growth and business cycles literatures,

$$U(c_t, l_t) = \frac{\left(c_t^{\gamma} l_t^{1-\gamma}\right)^{1-\sigma} - 1}{1-\sigma}, \tag{2.18}$$

where $0 < \gamma \leq 1$ and $\sigma > 0$ are parameters. For $\sigma = 1$ the utility function is logarithmic in consumption and leisure.

The production function is assumed to be a constant returns to scale, Cobb-Douglas function,

$$F(k_t, a_t n_t) = A k_t^{\alpha}(a_t n_t)^{1-\alpha}, \tag{2.19}$$

where $A > 0$ is a parameter, and $0 < \alpha < 1$ is the elasticity of output with respect to capital.

2.1 The Steady-State Equilibrium and Endogenous Cycles

We first study the long-run equilibrium of the model in the absence of revisions in expectations, and then show how such revisions generate economic fluctuations.

In the long-run equilibrium, output, physical capital, consumption and investment all grow at the rate of labor-augmenting technological progress. Hence, we can redefine these variables in units of efficiency, and derive a steady-state equilibrium where all variables are stationary. Thus, if we use a tilde to denote variables in efficiency units, the steady-state equilibrium is characterized by the following system of equations,

$$\frac{c}{y} + \frac{ps + pv}{y} - (1 - \delta - g)\frac{k}{y} = 1 \tag{2.20}$$

$$r + 1 - \delta = \frac{g^{1-(1-\sigma)\gamma}}{\beta} \tag{2.21}$$

$$r = \alpha \left(\frac{\tilde{k}}{n}\right)^{\alpha - 1} \tag{2.22}$$

$$\theta n = \mu v \tag{2.23}$$

$$\tilde{p}\frac{s}{u} = \frac{1-\gamma}{\gamma}\frac{c}{l} \tag{2.24}$$

$$(1 - \lambda)\tilde{p}s = \lambda \tilde{q} v \tag{2.25}$$

$$\Big(r + 1 - \delta - (1 - \theta)g\Big)\frac{\tilde{p}s}{g\theta} = \tilde{\omega} n - \frac{1-\gamma}{\gamma}\frac{\tilde{c}}{l}n, \tag{2.26}$$

where $l = 1 - n - u$, and $\tilde{\omega}$ is given by (2.16). Equation (2.20) is the feasibility condition, equation (2.21) is the Euler equation, and (2.22) is the demand function for capital. Equations (2.23) and (2.24) are, respectively, the Beveridge curve and the first-order condition for search intensity. Finally, equations (2.25) and (2.26) are the sharing rule, and the value of employment to the household. It can be easily shown that if a stationary equilibrium exists, then it is unique (for a proof of uniqueness, see Appendix A).

Calibration

To assign values to the parameters in the model economy we follow the standard calibration exercise. We use two sources of information. First, some parameters are set using *a priori* information. Second, remaining parameter values are set so that the steady-state equilibrium mimics some average values observed in post-WWII US economy. We assume that a time period in our model corresponds to a quarter.

Parameter values set using *a priori* information are: (i) θ is set at 0.1, as calculated by Andolfato (1996); (ii) η is set at 1.27 following the value in den Haan *et al.* (2000); (iii) the growth rate of technical progress is set so that per capita output grows at 1.6% per year. Remaining parameter values are chosen to match the following averages in the post-war US economy: (i) an unemployment rate of 6%, (ii) a labor market participation rate of 65%; (iii) a consumption-output ratio of 0.67; (iv) a capital-output ratio of 10; (v) an average duration of a vacancy of 53 days, as reported in Andolfato (1996). The only parameter we set arbitrarily is the workers' bargaining power. We focus on the symmetric equilibrium and, therefore, assume $\lambda = 1/2$. Finally, the standard deviation of the sunspot shock is set at 0.0047, in order to match the standard deviation of output over the business cycle.

Our calibrated parameter values are presented in Table 1 below.

Table 1

Calibrated parameter values

Discount factor	β	0.99
Gross growth rate of technical progress	g	1.004
Depreciation rate of capital	δ	0.022
Intertemporal elasticity of substitution	$1/\sigma$	0.44
Share parameter for leisure in utility	γ	0.65
Workers' bargaining power	λ	0.50
Job destruction rate	θ	0.10
Parameter in the matching function	η	1.27
Unit cost of search intensity	p	0.016
Unit cost of vacancies	q	1.226

Endogenous Cycles

To study the local properties of the search equilibrium we log-linearize the system of difference equations describing equilibrium allocations. For most configurations of parameter values, including our benchmark economy in Table 1, the number of eigenvalues of the matrix in the log-linearized system within the unit circle is greater than the number of state variables and, therefore, the equilibrium is indeterminate. That is, for a given pair of initial conditions, (k_0, n_0), there is a continuum of sequences solving the system of equilibrium conditions. Contrary to most models in the literature, equilibrium indeterminacy in our model is not a consequence of increasing returns to scale. In our model, equilibrium indeterminacy is the result of search externalities, which spring from the effect that each individual's search exerts both on the matching rate of other searchers and of vacancies. While a new job searcher decreases the matching probability of other searchers, it increases the probability that a vacancy be filled. Only under a set of parameter values of measure zero —the Hosios' (1990) knife-edge condition— both effects cancel out, and the search equilibrium becomes efficient, yielding thus a steady state with saddle-path stability.[2] For all other configurations of parameter values, search externalities

[2]Andolfato (1996) and Merz (1996) restrict the analysis of their job-search models to the planner's solution, eliminating thus the possibility of endogenous fluctuations due to revisions in expectations.

are present and equilibrium indeterminacy arises for a fairly large set of parameter values.

In models with a continuum of equilibria, revisions in expectations give rise to economic fluctuations. Roughly, the implied mechanism is that, at a given time period, expectations on the future path of the economy ultimately pick an equilibrium in the set of equilibria. We model the effect of this revision in expectations as a one-period ahead forecasting error in investment, and assume that this error is an i.i.d. sunspot shock. Since this is the only shock present in the model, we are therefore abstracting from real shocks to technology and/or aggregate demand. Thus, if we use a hat to denote the percentage deviation of a variable from its steady state, and use X_t to denote the vector $[\hat{k}_t \; \hat{n}_t \; \hat{i}_t \; \hat{u}_t]'$, equilibrium allocations are the solution to the following system,

$$X_{t+1} = QX_t + S_t, \quad t = 0, 1, 2... \tag{2.27}$$

where $S_t = [0 \; 0 \; \epsilon_t \; 0]'$ and ϵ_t is the realization of an i.i.d. random variable with normal distribution of zero mean and variance σ_ϵ^2. Values for $\hat{s}_t$ and $\hat{v}_t$ are obtained from two contemporaneous equations of X_t [eqs. (2.7) and (2.8)]. Initial conditions $\hat{k}_0$ and $\hat{n}_0$ are given.

3 Predicted Business Cycles and Output Dynamics

In this section we study the properties of the economic fluctuations and output growth generated by our model. We start out by solving the model and computing the cyclical component of our endogenous variables. We generate artificial variables for 200 quarters, and for each variable we compute 1,000 draws. For each draw we use the Hodrick-Prescott filter (HP) with smoothing parameter equal to 1,600 for all variables but vacancies and unemployment. These latter variables are detrended using a smoothing parameter equal to 10^5, so that their cyclical properties can be compared with US facts, as reported in Shimer (2003). We then compute the volatility of endogenous variables over the business cycle, and the comovements of the main variables with output. Then, we average these second moments over all draws. We compare our results with the corresponding moments in US data. In the second part of our exercise, we explore the implications of our model in terms of output growth, and confront them with the evidence found for the US economy. To carry out this second part of the exercise, we compute population moments directly from the log-linearized dynamical system (see Appendix B for a brief description of the procedure). Our results show that the first-order autocorrelation in output growth is positive, and it falls within the confidence interval of estimated US first-order autocorrelation.

The Cyclical Behaviour of HP-filtered artificial variables

The properties of the HP business cycle components of post-WWII US economic variables have been reported by a number of authors[3]. For the sake of expositional clarity, we summarize here the main defining features of the US business cycle:

- Consumption of non-durables is less volatile than output.
- Investment is three times more volatile than output.
- Labor productivity is less volatile than output.
- The real wage is less volatile than output.
- Unemployment and vacancies are highly volatile: the vacancy-unemployment ratio is 20 times as volatile as average productivity.
- Consumption, investment and employment are highly procyclical.
- The vacancies-unemployment ratio is highly procyclical.
- Unemployment is highly countercyclical.
- The cyclical components of all macroeconomic variables are highly persistent.

In table 2 below we present the relative volatilities predicted by our model and confront them both with US data and with the predictions of the standard real business cycle model. Our model's predictions are acceptable, in line with the predictions of the RBC model. In terms of unemployment and vacancies, our model exhibits high volatility, as observed in the data, even though it overstates unemployment volatility and understates vacancies-unemployment volatility.

[3] See, for instance, Cooley and Prescott (1995), King and Rebelo (2000) and Shimer (2003)

Table 2

US Economy and Predicted Standard Deviations Relative to Output of HP-Detrended Variables

	σ_x/σ_y						
	c_t	n_t	i_t	y_t/n_t	ω_t	ur_t	v_t/ur_t
U.S. data	0.74	1	2.93	0.56	0.38	6.11	19
RBC model	0.44	0.48	2.95	0.54	0.54	-	-
Our model	0.54	1.45	4.36	0.65	0.8	12	6.54

Notes: σ_x/σ_y is the standard deviation of variable x relative to y (output). Variables are the cyclical components obtained using the HP filter. Moments for US data are taken from King and Rebelo (2000) and Shimer (2003).

Table 3 shows the contemporaneous correlation of our economic variables with output over the business cycle. Our model predicts observed correlations fairly well, and by no means worse than the real business cycle model. As far as labour market variables is concerned, our model predicts the high countercyclicality of unemployment, and the high procyclicality of the vacancies-unemployment ratio.

Table 3

US Economy and Predicted Contemporaneous Correlations with Output of HP-Detrended Variables

	$\rho(x_t, y_t)$					
	c_t	n_t	i_t	ω_t	ur_t	v_t/ur_t
US data	0.88	0.88	0.8	0.68	-0.90	-
RBC model	0.94	0.97	0.99	0.98	-	-
Our model	0.89	0.99	0.72	0.76	-0.74	0.84

Notes: $\rho(x_t, y_t)$ is the correlation coefficient between variable x_t and y_t (output). Variables are the cyclical components obtained using the HP filter.

Table 4 presents first-order autocorrelations over the business cycle. Our model also predicts the high persistence of economic variables over the business cycle.

Table 4

First-Order Autocorrelation of HP-Detrended Variables

	$\rho(x_t, x_{t-1})$						
	y_t	c_t	n_t	i_t	ω_t	ur_t	v_t/ur_t
US data	0.84	0.80	0.88	0.87	0.66	0.89	-
RBC model	0.72	0.79	0.71	0.71	0.76	-	-
Our model	0.64	0.59	0.54	0.62	0.57	0.72	0.58

Notes: $\rho(x_t, x_{t-1})$ is the first-order autocorrelation coefficient of variable x_t. Variables are the cyclical components obtained using the HP filter.

Predicted Output Dynamics

We turn now our attention to output dynamics, especially to the evolution of output growth. Thus, instead of looking at the cyclical component of our economic variables, we look at the properties of the predicted rates of growth. We compare our results both with US data and with the predictions of the real business cycle model and the endogenous-business-cycle model of Schitt-Grohe (2000) (SSG in Tables 5 and 6 below).

In Table 5 we present the relative volatilities and the contemporaneous correlation of selected variables with output growth. It should be noted that our model predicts the positive contemporaneous correlation between consumption and output growth. Schmitt-Grohe's (2000) prediction of this correlation coefficient is -0.95.

Table 5

Standard Deviations Relative to Output Growth and

Contemporaneous Correlations in Growth Rates

	$\frac{\sigma_{\Delta x}}{\sigma_{\Delta y}}$			$\rho(\Delta x_t, \Delta y_t)$		
	Δc_t	Δn_t	Δi_t	Δc_t	Δn_t	Δi_t
US data	0.5	0.94	2.53	0.48	0.77	0.68
RBC model	0.53	0.49	2.47	0.99	0.98	0.99
SSG model	0.47	1.45	5.39	-0.95	0.99	0.99
Our model	0.38	1.56	6.42	0.76	0.44	0.72

Notes: Δx_t denotes the growth rate of non-detrended variable X.
SSG model refers to Schmitt-Grohe's (2000) model.

Finally, in Table 6 we present our results for first-order autocorrelations. Our model predicts positive autocorrelations in growth rates. The predicted first-order autocorrelation in output growth is 0.41, which falls inside the confidence interval for the estimated value in the US economy. Both the real business cycle model and the expectations-driven fluctuations model of Schmitt-Grohe (2000) fail to predict the positive autocorrelation. The real business cycle model, as is well-known, lacks the needed internal propagation mechanism, and predicts no autocorrelation in growth rates.

Table 6

First-Order Autocorrelations in Growth Rates

	$\rho(\Delta x_t, \Delta x_{t-1})$			
	Δy_t	Δc_t	Δn_t	Δi_t
US data	0.37	0.21	0.65	0.49
RBC model	0.02	0.08	-0.04	-0.03
SSG model	0.18*	0.19	0.16	0.16
Our model	0.41	0.5	0.41	0.33

Notes: Δx_t denotes the growth rate of non-detrended variable X. SSG model refers to Schmitt-Grohe's (2000) model. * this prediction lies below the lower bound of the confidence interval, $(0.23, 0.51)$, for the estimated first-order autocorrelation for the US economy.

4 Conclusions

In this paper we have addressed the following question: can a model of endogenous business cycles account for the standard second moments over the business cycle, and the high first-order autocorrelation of output growth? This question has become meaningful after the work of Cogley and Nason (1995), where it is shown that real business cycle models fail to generate positive autocorrelation in output growth . Schmitt-Grohe (2000) has extended the challenge to endogenous business cycle models and concluded that they also fail to explain output dynamics. In this paper we challenge this view by presenting a model of endogenous business cycles that accounts for the above-mentioned positive autocorrelation.

Our model is completely standard and builds on the neoclassical growth model with labour market frictions. We assume no real shocks, and only revisions in expectations cause economic fluctuations. Such revisions are modeled as an i.i.d. sunspot shock. For a calibrated version of our model that matches standard averages for the post-war US economy, the equilibrium of the model displays endogenous cycles. Predicted business cycles and output dynamics match the main empirical observations found in US data.

Besides showing the success of endogenous-business-cycle models to explain US output dynamics, we also raise a methodological issue. We argue that the ability of an economic model to explain output dynamics does not depend on the source of shocks, but on the strength of its internal propagation mechanism. Thus, assessing whether actual fluctuations stem from shocks to expectations or to technology by looking at the two models' performance is an ill-posed exercise. Both models can display, under plausible assumptions, a strong internal propagation of shocks, and generate positive autocorrelation in output growth. Consequently, those interested in finding support for the expectations-driven fluctuations theory must attempt new ways of tracing back the source of economic fluctuations.

5 Appendix

Appendix A. *Uniqueness of the steady-state equilibrium*

In this appendix we show that the system of steady-state equations, (2.20)-(2.26), has at most one solution. The proof of the uniqueness of the steady-state equilibrium becomes a mechanical exercise after noticing that the system of steady-state equations is block-recursive. Thus k, c and v can be expressed in terms of remaining endogenous variables from equations (2.22), (2.24) and (2.25). Hence, remaining equations, after substituting $\tilde{\omega}$ from (2.17), can be rewritten as,

$$\frac{s}{\theta n}\left[\frac{1}{\lambda}+\frac{\gamma}{1-\gamma}\frac{1-n-u}{u}\right] = \phi_1 \tag{5.1}$$

$$\frac{s}{\theta n}\frac{u}{\left[1+(\varphi u)^{\eta}\right]^{1/\eta}} = 1 \tag{5.2}$$

$$\frac{s}{\theta n}\left[1-\beta g^{(1-\sigma)\gamma}\left(1-\theta-\lambda\frac{\theta n}{u}\right)\right] = \phi_2 \tag{5.3}$$

where ϕ_1 and ϕ_2 depend only on parameter values and are given by,

$$\phi_1 \equiv \frac{1}{\tilde{p}\theta}\left[\rho+(1-\delta-g)\rho^{1/\alpha}\right]$$

$$\phi_2 \equiv \frac{\lambda}{\tilde{p}}(1-\alpha)\rho\beta g^{\gamma(1-\sigma)}$$

and where $\varphi \equiv \tilde{q}\lambda/(\tilde{p}(1-\lambda))$, $\rho \equiv r/\alpha$ and r is determined by (2.21).

Now, we can solve for n and s as functions of u from (5.2) and (5.3). Plugging these expressions in (5.1) we derive the following equation for u,

$$\frac{1}{\lambda}+\frac{\gamma}{1-\gamma}\left(\frac{1}{u}-1\right) = \phi_3+\phi_4\frac{u}{\left[1+(\varphi u)^{\eta}\right]^{1/\eta}} \tag{5.4}$$

where

$$\phi_3 \equiv -\frac{\gamma}{1-\gamma}\frac{1-\beta g^{(1-\sigma)\gamma}(1-\theta)}{\lambda\beta\theta g^{\gamma(1-\sigma)}}$$

$$\phi_4 \equiv \phi_1+\frac{\gamma}{1-\gamma}\frac{\phi_2}{\lambda\beta\theta g^{(1-\sigma)\gamma}}$$

Since both ϕ_1 and ϕ_2 are positive, it follows that ϕ_4 is also positive. Then, the left-hand side of equation (5.4) is a strictly decreasing function of u, whereas the right-hand side is a strictly increasing function of u. Therefore, (5.4) has at most one solution for u. Solutions for remaining variables can be found by direct substitution, and it turns out straightforward to see that these values will also be unique. We conclude that if the steady-state equilibrium exists, then it is unique.

Appendix B. *Derivation of Population Moments for Variables in Growth Rates*

The log-linearized system of equations for the dynamic equilibrium renders a system of four deterministic first-order difference equations, and a system of two contemporaneous equations. This system of difference and contemporaneous equations can be written in matrix form as,

$$X_{t+1} = QX_t \tag{5.5}$$

$$Z_t = PX_t \tag{5.6}$$

where $X_t = [\hat{k}_t\ \hat{n}_t\ \hat{i}_t\ \hat{u}_t]'$ and $Z_t = [\hat{s}_t\ \hat{v}_t]'$ are the vectors of relative deviations of the endogenous variables; Q is the 4×4 transition matrix, and P is the 2×4 matrix of contemporaneous equations. Vector X_t contains two state and two control variables. The elimination of explosive solutions due to the eigenvalue outside the unit circle calls for a linear restriction in the set of solutions. Hence, one control variable in X_t, say $\hat{u}_t$, can be expressed as a linear function of $X_t^{(1)}$, which is the remaining component of X_t. This linear relation is determined by the eigenvectors of matrix Q. The dynamic system is then described by the following matrix equations:

$$X_{t+1}^{(1)} = Q_1 X_t^{(1)} \tag{5.7}$$

$$\hat{u}_t = RX_t^{(1)} \tag{5.8}$$

$$Z_t = P_1 X_t^{(1)}, \tag{5.9}$$

where Q_1 is a 3×3 matrix, R is 1×3, and P_1 is 2×3. These three matrix equations can be combined into,

$$\xi_{t+1} = T\xi_t$$

where

$$\xi_t = \begin{bmatrix} X_t^{(1)} \\ \hat{u}_t \\ Z_t \end{bmatrix} \text{ and } T = \begin{bmatrix} Q_1 & 0 \\ R & 0 \\ P_1 & 0 \end{bmatrix}.$$

Since all the eigenvalues of Q_1 lie within the unit circle by construction, the solution is indeterminate. When we allow a sunspot shock to select the equilibrium, we add a stochastic shock, ϵ_t, to the remaining control variable in $X_t^{(1)}$, i.e., investment. This shock is i.i.d. with a normal distribution of mean zero and variance σ_ϵ^2.

The stochastic dynamic equilibrium can then be described by

$$\xi_{t+1} = T\xi_t + \tilde{\epsilon}_t, \tag{5.10}$$

where $\tilde{\epsilon}_t = [S_t \;\; 0 \;\; 0]'$, for S_t as defined in Section 2. $\tilde{\epsilon}_t$ has mean zero and variance-covariance matrix

$$\tilde{\Sigma}_\epsilon = \begin{bmatrix} \Sigma_\epsilon & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}.$$

where the only non-zero element in Σ_ϵ is the variance of the sunspot shock. Given the properties of T and $\tilde{\epsilon}_t$, the stochastic process $\{\xi_t\}$ is stationary with mean zero and variance-covariance matrix Ω. From (5.10) it is easy to derive an equation for the variance-covariance matrix:

$$\Omega = T\Omega T' + \tilde{\Sigma}_\epsilon. \tag{5.11}$$

Taking *vec* on both sides of equation (5.11), and using $vec(ABC) = (A \otimes C')vec(B)$, we get

$$vec(\Omega) = (T \otimes T)vec(\Omega) + vec(\tilde{\Sigma}_\epsilon)$$

and, therefore,

$$vec(\Omega) = [I - (T \otimes T)]^{-1} vec(\tilde{\Sigma}_\epsilon), \tag{5.12}$$

provided that the matrix in square brackets is non-singular. Thus, knowing the transition matrix and assuming the variance of the sunspot shock, we can compute the population moments of the variables in the model economy. The matlab code to solve our model is available upon request from the authors.

University of Exeter

European University Institute

References

[1] Andolfato, D. (1996). Business cycles and labor-market search. **The American Economic Review 86**(1), 112–132.

[2] Benhabib, J. and R. E. Farmer (1994). Indeterminacy and increasing returns. **Journal of Economic Theory 63**(1), 19–41.

[3] Cooley, T. F. and E. C. Prescott (1995). **Frontiers of Business Cycle Research**. Princeton University Press, Princeton, New Jersey.

[4] Cogley, T. and J.M. Nason (1995). Output dynamics in real business cycle models. **American Economic Review 85**(3), 492–511.

[5] Den Haan, W.J., G. Ramey, and J. Watson (2000). Job destruction and propagation of shocks. **American Economic Review 90**(3), 482–498.

[6] Farmer, R.E. and T.-T. Guo (1994). Real usiness cycles and the animal spirits. **Journal of Economic Theory 63**(1), 42–72.

[7] Hosios, A. J. (1990). On the efficiency of matching and related models of search and unemployment. **Review of Economic Studies 57**(2), 279–298. pp. 279-98.

[8] King, R. and S. Rebelo (2000). Resuscitating Real Business Cycles. NBER Working Paper 7534.

[9] Merz, M. (1996). Search in the labor market and the real business cycle model. **Journal of Monetary Economics 36**(2), 269–300.

[10] Pissarides, C. A. (1990). **Equilibrium Unemployment Theory.** Oxford: Blackwell.

[11] Schmitt-Grohe, S. (2000). Endogenous business cycles. **American Economic Review 90**(5), 1136–1159.

[12] Shimer, R. (2003). The cyclical behaviour of equilibrium unemployment and vacancies: Evidence and theory. NBER Working paper 9536.

[13] Thomas, J. (2004). Do Sunspots Produce Business Cycles? Mimeo.