Skills, Search and the Persitence of High Unemployment

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Abstract

The persistence of high unemployment rates in Europe has fueled theories advocating the existence of multiple natural rates of unemployment. Labor market institutions and increasing returns to scale have been singled out as the main causes of multiplicity and, therefore, of high unemployment traps. The contribution of this paper is both to expand the set of mechanisms leading to multiple natural rates of unemployment and to establish a minimum set of assumptions under which such multiplicity may arise. To this aim, a search-matching model is presented where households allocate time to market and non-market activities, and invest both in physical and human capital It is shown that under the standard assumption of concavity in production and matching such a model yields multiple long-run equilibria with different rates of unemployment. This result does not rely on labor market institutions or increasing returns to scale. Multiplicity in our model arises from differences in the intensity of use of human capital across time-consuming activities.

Keywords: Frictional Unemployment; Economic Growth; Multiple Long-Run Equilibria; Unemployment Persistence.

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1. Introduction

The evolution of unemployment rates in Europe since the 1960's poses a number of questions that have challenged most existing theories of unemployment determination. In the mid 1970's European unemployment started a transition from rates in the order of 1-2% to rates in the order of 10-15% in the 1990's [see Figure 1]. This persistent increase in unemployment has been interpreted by some authors as a shift from a low-unemployment steady-state equilibrium to another with high unemployment. Furthermore, this shift does not seem to be related to policy changes, but to the shocks that hit most developed economies in the mid 1970's. According to this interpretation, a temporary shock may have persistent effects on unemployment, as it may set off a transition from a low-unemployment equilibrium to a high-unemployment equilibrium.

The literature has provided several explanations as to why multiple steady-state equilibria with different rates of unemployment can arise. Blanchard and Summers (1986) emphasize the role played by the wage determination mechanism in generating multiple equilibria. In particular, they use a model where wages are bargained with insiders, and show how multiple equilibria arise. Explanations based on the existence of increasing returns in production or in matching have also been proposed [see Mortensen (1999), and Mortensen (1989) for a review of this literature]. Pissarides (1992) singles out the loss of worker's skills during the unemployment spell as the source of multiple equilibria. The explanation advanced by Saint-Paul (1995) stresses labor market distortions, more specifically firing restrictions. Blanchard and Summers (1987) and den Haan (2003) show that the combination of income taxes and a policy of unemployment benefits may give rise to "fiscal increasing returns", capable of generating a multiplicity of steady states. Finally, Acemoglu (2000) shows how the existence of credit market imperfections can yield multiple natural rates of unemployment.

In this paper we put forward a new mechanism for the multiplicity of long-run rates of unemployment. We show that in an economy with frictions in the labor market and with endogenous formation of skills, the standard household's occupational choice problem leads to multiple long-run equilibria with different rates of growth and unemployment. A striking different between our mechanism and those proposed in the literature is that ours relies neither on increasing returns to scale nor on labor market institutions. On the one hand, mechanisms relying on increasing returns to scale must cope with empirical evidence supporting constant returns to scale, both in production and matching. On the other hand, mechanisms relying on policy distortions, market imperfections and labor market institutions suggest that the removal of such distortions, imperfections and institutions would eliminate the possibility of hysteresis in unemployment. However, our result shows that the seed of hysteresis lies in a deeper layer of the economy.

In order to illustrate our mechanism for multiplicity, a standard, representative-household model with physical and human capital accumulation and frictional unemployment is presented. The model's main ingredients are: *i)* an endogenous determination of savings and of the interest rate; *ii)* an endogenous labor force: agents allocate time to market and non-market activities; *iii)* an endogenous accumulation of human capital. Following the spirit of the early literature on human capital accumulation in macroeconomic models [e.g. Uzawa (1965) and Lucas (1988)], we assume that human capital accumulation is the result of time spent in the education sector. Likewise, it is also assumed that human capital is a state variable attached to a worker while producing but that it does not affect the worker's productivity when he is engaged in non-production activities. More specifically, education and labor are defined in efficiency units, whereas leisure and job search are defined in raw time.

A key consequence of the assumptions stated above is that human capital affects the productivity of time allocated to production and non-production activities asymmetrically. Thus, the current stock of human capital sets an intersectoral productivity differential which shapes the current allocation of time across activities. Since the current allocation of time, through its effect on the future stock of human capital, determines, in turn, the magnitude of the future productivity differential, there is room for multiple long-run equilibria. In other words, the asymmetric effect of human capital across activities creates a complementarity between the current stock of human capital and long-run productivity. Hence, depending upon the initial composition of wealth —physical capital, human capital and employment— the economy will converge either to a long-run equilibrium with high growth and low unemployment, or to an equilibrium with low growth and high unemployment. In our model this equilibrium configuration with two stable long-run equilibria arises under plausible parameter values. In short, our mechanism for multiplicity hinges on the existing feedback between unemployment and growth. So far this feedback had been overlooked as a source of hysteresis in unemployment.

Although the main aim in this paper is to present a new mechanism leading to multiple long-run equilibria in models of frictional unemployment, we also argue that the model is suitable to assess the role played by labor market institutions in the increase and persistence of European unemployment. In models with two stable long-run equilibria like ours, an exogenous, temporary shock to employment may result in a shift from the "good" long-run equilibrium to the "bad" one. Labor market institutions, such as unemployment benefits, firing costs and minimum wages, are bound to shape the economy's response to such a shock. More specifically, institutions are likely to change the fragility of the good equilibrium, as measured by the magnitude of the minimum shock needed to yield a shift to the bad equilibrium. Therefore, equilibrium fragility provides an assessment of the effect of institutions on the economy's vulnerability to shocks. This contributes to the growing literature that, following the empirical findings of Blanchard and Wolfers (2000), accounts for the differential in unemployment dynamics in Europe and the US by examining the interaction of shocks and institutions. [For a recent contribution along these lines see, e.g., den Haan (2003).]

The paper is organized as follows. In Section 2 the model is presented and a search equilibrium is defined. In Section 3 we study the long-run equilibrium and show the existence of multiple balanced growth paths. Section 4 presents the conclusions, and Section 5 contains the Appendix.

2. The Model

The economy builds on the standard search-matching model of frictional unemployment, and Lucas' (1988) model of human capital accumulation. There is a matching technology that determines total job matches as a function of vacancies and job search. We adopt the common assumption of a concave, linearly homogeneous and increasing matching function [see Blanchard and Diamond (1989) and Pissarides (1986)]. In particular, we assume the following matching technology,

$$M(t) = Mv(t)^{\eta}u(t)^{1-\eta} \tag{1}$$

where $M(t)$ is the number of matches, $v(t)$ denotes vacancies, and $u(t)$ is job search (unemployment); $M > 0$ and $0 < \eta < 1$ are parameters. This technology implies that unemployed workers switch to employment according to a Poisson process with rate $M(\frac{v(t)}{u(t)})^{\eta}$, and vacancies are filled according to a Poisson process with rate $M(\frac{v(t)}{u(t)})^{\eta-1}$. The ratio of vacancies to unemployment, $\frac{v(t)}{u(t)}$, is the measure of tightness in the labor market.

This model of frictional unemployment is now embedded into a standard endogenous growth model in which job search is determined by optimizing households.

Households

There is a continuum of identical households in the economy. Each household has one unit of time per period that can be allocated to the following activities: leisure, home production, job search, work in the firms sector, and education.[1] Households derive utility from consumption and leisure according to the following utility function,

$$\int_0^{\infty} e^{-\rho t}\{\gamma \log C(t) + (1-\gamma)\log L(t)\}dt, \tag{2}$$

where $L(t)$ is leisure, and $C(t)$ is a consumption composite made up of market goods consumption, $c(t)$, and home-produced goods consumption, $c_{hp}(t)$. Furthermore, following estimates by Eichenbaum and Hansen (1990), we will assume perfect substitutability between market and home-produced consumption.

The technology in the home-production sector is assumed to be linearly homogeneous in qualified labor, that is, $c_{hp}(t) = A_2 l(t)h(t)$, where $l(t)$ is the fraction of time (family members) engaged in home production, $h(t)$ is the stock of human capital, and $A_2 > 0$ is a constant.

The household supplies the firm sector with physical capital, $k(t)$, and labor services. Labor services are measured in efficiency units, defined as the product of the fraction of time devoted to work, $n(t)$, and the stock of human capital or skills accumulated by the household, that is, $n(t)h(t)$. The budget constraint of the family implies that,

$$c(t) + \dot{k}(t) = r(t)k(t) + \omega(t)n(t)h(t) + \pi(t), \tag{3}$$

[1] Instead of thinking of the household as splitting one unit of time across different activities, one could alternatively think of a household with a continuum of members who are assigned to different activities.

where $r(t)$, $\omega(t)$ and $\pi(t)$ denote the rental price of capital, the wage rate and the flow of profits, respectively. The change in household's employment depends on job creation and job destruction. The rate at which an unemployed worker is matched with a vacancy is denoted by $m(t)$. Then the total flow of job matches is given by $m(t)u(t)$, where $u(t)$, as stated above, denotes job search. The rate of job destruction is exogenous and denoted by λ. Thus, the evolution of employment within the household is given by

$$\dot{n}(t) = m(t)u(t) - \lambda n(t). \tag{4}$$

Investment in human capital is possible through education. Thus, if $e(t)$ is the fraction of time allocated to education, the law of motion for the stock of human capital is

$$\dot{h}(t) = Be(t)h(t), \qquad B > 0. \tag{5}$$

The household's problem is therefore to choose sequences for $c(t)$, $c_{hp}(t)$, $L(t)$, $l(t)$, $u(t)$ and $e(t)$ in order to maximize the lifetime utility, (2), subject to the law of motion for $k(t)$, $h(t)$ and $n(t)$; and subject to $c_{hp}(t) = A_2 l(t)h(t)$ and $L(t) + l(t) + n(t) + u(t) + e(t) = 1$. The representative household takes the job matching rate as given.

The first-order conditions for the household problem are presented in the Appendix. These conditions establish the optimal allocation of time across competing activities, and the allocation of income to consumption and investment.

The income value of employment to the household is denoted by $W(t)$. It follows then from a standard non-arbitrage condition that,

$$r(t)W(t) + A_2h(t) = \omega(t)h(t) + \dot{W}(t) - \lambda W(t). \tag{6}$$

[See the Appendix for a derivation of this non-arbitrage condition.] The interpretation of equation (6) is straightforward. The income cost of employment (capital cost plus forgone home production) must equal the job's yield, which is made up of the wage rate plus capital gains net of the risk of losing the job.

Firms

Firms hire physical capital and labor services from the household sector, and produce according to the constant returns to scale, CES production function,

$$F[k(t), n(t)h(t)] = A\left(\alpha k(t)^{\sigma} + (1-\alpha)(n(t)h(t))^{\sigma}\right)^{\frac{1}{\sigma}}, \tag{7}$$

where $A > 0$ is a constant,[2] and $0 < \alpha < 1$; since the elasticity of substitution is given by $\frac{1}{1-\sigma}$, we impose $\sigma < 1$. In order to contact unemployed workers the firm must open vacancies. The rate at which a vacancy is matched with an unemployed worker is denoted by $\mu(t)$, and the number of vacancies by $v(t)$, then $\mu(t)v(t)$ is the total flow of matches. In order to simplify the analysis and to highlight that the

[2] We are thus implicitly assuming that all increases in productivity come through human capital accumulation. By letting A grow at an exogenous rate it would only amount to a re-normalization of our variables, without affecting our results.

source of aggregate dynamic non-convexities leading to multiple long-run equilibria lies in the household problem, we assume that firms open a constant and exogenous number of vacancies per period, say $\hat{v}$. It is important to notice that the assumption of an exogenous number of vacancies does not affect the mechanism behind our results.

Given the exogenous rate of job destruction, λ, the evolution of employment within the firm sector is,

$$\dot{n}(t) = \mu(t)\hat{v} - \lambda n(t). \tag{8}$$

The current flow of profits is given by,

$$\pi(t) = F[k(t), n(t)h(t)] - (r(t) + \delta)k(t) - \omega(t)n(t)h(t) - a(t)\hat{v}. \tag{9}$$

where $a(t)$ is the cost of creating a vacancy. This cost is assumed to grow at the same rate as the economy's long-run productivity, i.e., $a(t) = ae^{gt}$, where $a > 0$ is a parameter, and g is the long-run productivity growth rate, which is determined endogenously.

The objective of the firm is to maximize the present discounted value of cash flows. Hence, the firm demands physical capital in order to maximize,

$$\int_0^\infty e^{-\int_0^t r(z)dz}\pi(t)dt. \tag{10}$$

The first-order condition for profits maximization is then

$$r(t) + \delta = F_k[k(t), n(t)h(t)], \tag{11}$$

where $F_k(\cdot,\cdot)$ denotes the derivative of the production function with respect to k.

The income value of a filled job to the firm is denoted by $J(t)$. It follows then from a standard non-arbitrage condition that,

$$r(t)J(t) = \Big(F_{nh}[k(t), n(t)h(t)] \; - \omega(t)\Big)\, h(t) + \dot{J}(t) - \lambda J(t), \tag{12}$$

where $F_{nh}(\cdot,\cdot)$ denotes the derivative of the production function with respect to labor in efficiency units, nh.

Wage determination

Regarding wage determination we follow the standard assumption in the literature of frictional unemployment. A realized job match yields pure economic rents which have to be divided between the firm and the worker. The wage rate is then the Nash solution to a bargaining problem. The Nash solution maximizes the weighted product of the worker's and the firm's return from the job match. If we denote the weight of the worker's return by p, then the wage rate is,

$$w(t) = \arg\max \left\{W(t)^p J(t)^{1-p}\right\}. \tag{13}$$

Typically, p is interpreted as the worker's bargaining power, and it is associated with labor market regulations. The first-order condition to problem (13) is

$$pJ(t) = (1-p)W(t). \tag{14}$$

Using equations (6), (12) and (14), and the fact that wages are renegotiated every period, the wage rate is given by

$$\omega(t) = pF_{nh}[k(t), n(t)h(t)] + (1-p)A_2. \tag{15}$$

If $p = 1$, the worker gets the total income generated by the match: F_{nh}; if $p = 0$ the firm pays the worker's reservation wage: A_2. The constancy of the reservation wage arises from the technology in the home production sector. This property of the reservation wage will not affect any of our results, but will simplify the analysis considerably.

Equilibrium

A search equilibrium is defined as a set of sequences for quantities, $\{c(t),\ c_{hp}(t),\ k(t),\ h(t),\ L(t),\ l(t),\ u(t),\ n(t),\ e(t)\}$, prices $\{r(t),\ \omega(t)\}$, profits $\{\pi(t)\}$, and matching rates $\{m(t),\ \mu(t)\}$, such that:

(i) Taking prices, profits and the matching rate $m(t)$ as given, the tuple of quantities solves the household problem.

(ii) Taking prices as given, $\{k(t)\}$ solves the problem of the firm.

(iii) Markets clear, and the wage rate is given by equation (15).

(iv) The matching rates are given by the matching technology, that is $m(t) = M(\frac{\hat{v}}{u(t)})^{\eta}$ and $\mu(t) = M(\frac{\hat{v}}{u(t)})^{\eta-1}$.

As is well known, in the search-matching framework there exist search externalities which render the market equilibrium non-optimal. These externalities spring from the effect that each individual's search exert on the matching rate of other searchers. We show below that these search externalities play no role in our mechanism for multiplicity.

3. Unemployment and Growth in the Long-Run

A balanced growth equilibrium is defined as an equilibrium solution along which $k(t), h(t)$, $c(t)$ and $c_{hp}(t)$ grow at constant rates, and time allocation variables remain constant. It follows from the law of motion for $k(t)$ and the technology in the home production sector that if such an equilibrium exists, then the two capital stocks and the two consumption levels must grow at a common rate, that is, $\frac{\dot{k}(t)}{k(t)} = \frac{\dot{h}(t)}{h(t)} = \frac{\dot{c}(t)}{c(t)} = \frac{\dot{c}_{hp}(t)}{c_{hp}(t)} = g$. When $g > 0$ the balanced growth path is said to be interior, if $g = 0$ it is said to be non-interior.

In this section, we show that the proposed model may display multiple balanced growth paths featuring different rates of growth and unemployment. As was stated above, the key feature of the model that generates the multiplicity result is the asymmetric effect of human capital across time-consuming activities. While the productivity of time devoted to production (home production, human capital and output production) increases with the current stock of human capital, the productivity of time devoted to activities where production does not take place (leisure and job search) is independent of human capital. Hence, the current allocation of time across activities is unambiguously affected by the relative amount of human capital in the economy. Since human capital is a state variable that evolves over time depending on schooling time, there exists a complementarity between the current level of human capital and long-run productivity. Thus, if for a given level of employment, there is a high initial level of human capital, the household is relatively more efficient in the schooling sector, as compared to search and leisure. Therefore, more time will be allocated to production of new human capital. As a result, unemployment will be initially low and the economy will converge to a balanced growth path with high economic growth and low unemployment. On the contrary, if the initial stock of human capital is low, the household is relatively more efficient in job search and leisure. More time will be allocated to search and less to education. Thus, the equilibrium will converge to a balanced growth path with no income growth and high unemployment. It should be noticed that the multiplicity of balanced growth paths in our model arises under concave matching and production functions. Unlike other models of unemployment hysteresis [e.g., Mortensen (1999), Hart (1982) and Howitt and McAfee (1987)] we do not rely on increasing returns to matching or production.

In order to gain further insight on how the asymmetric effect of human capital across occupations can lead to multiple balanced growth paths, let us focus temporarily on the planner's problem in this economy. It is well known that under a certain condition on parameter values —a Hosios (1990)-type condition— search externalities cancel out exactly, and the search equilibrium delivers socially optimal allocations. (The focus on this particular case is made for the sake of expositional clarity, but it should be clear that the argument goes equally through in the decentralized economy.) The main explanation for the multiplicity of long-run solutions is the following. As human capital is used more intensively in some occupations than in others, an increase in its stock expands the production possibility set relatively more along the intensive sectors. Furthermore, the unevenness of the expansion depends on the stock of human capital. Thus, the planner's problem displays aggregate dynamic non-convexities which, depending on parameter values, may yield multiple long-run solutions. [For models with aggregate dynamic non-convexities springing either from start-up costs or human capital asymmetries see Ciccone and Matsuyama (1999) and Ladron-de-Guevara *et al.* (1999).]

The planner's problem, after defining the variables, $h_l = lh,\ h_L = Lh,\ h_u = uh$ and $h_e = eh$, can then be written as,

$$\max \quad \int_0^\infty e^{-\rho t}\left(\gamma \log(c + A_2 h_l) + (1-\gamma)\log(\frac{h_L}{h})\right) dt$$

$$s.t. \quad \dot{k} = F(k, nh) - \delta k - c - a\hat{v} \tag{16}$$

$$\dot{h} = B h_e \tag{17}$$

$$\dot{n} = M \hat{v}^{\eta} h_u^{1-\eta} h^{\eta-1} - \lambda n \tag{18}$$

$$h_l + h_L + h_u + h_e + nh = h \tag{19}$$

$$k_0, h_0 \text{ and } n_0 \text{ given.} \tag{20}$$

Since $log\left(\frac{h_L}{h}\right)$ and $M\hat{v}^{\eta}h_u^{1-\eta}h^{\eta-1}$ are convex functions in h, and since h is an endogenous variable which evolves over time, the concavity of the planner's maximization problem is not guaranteed and, consequently, there may exist multiple balanced growth paths.[3] It is thus clear from the planner's problem that aggregate dynamic non-convexities spring from the asymmetry in the intensity of use of human capital across time-consuming activities.

As will become more apparent below, it can be already anticipated from the arguments above that if the balanced growth path is not unique, there must be an odd number of such paths. More precisely, given the nature of the aggregate dynamic non-convexities in our model the maximum number of stationary solutions is three. Among these, one must be non-interior and one of the interior equilibria must be unstable. Under uniqueness, the stationary solution can be either interior or non-interior.

We turn now to the analysis of the search equilibrium defined in the previous subsection. We begin by imposing balanced growth path conditions on the set of equations that characterize a search equilibrium. After defining $x \equiv \frac{nh}{k}$ and $q \equiv \frac{c}{k}$, interior balanced growth paths are characterized by the following conditions.

The relationship between employment and market tightness, the Beveridge curve, is

$$\lambda n = M_0 \hat{v}^{\eta} u^{1-\eta}. \tag{21}$$

The wage rate is given by,

$$\omega = p(1-\alpha)A\left[\alpha x^{-\sigma} + (1-\alpha)\right]^{\frac{1-\sigma}{\sigma}} + (1-p)A_2. \tag{22}$$

From the law of motion for physical and human capital we obtain, respectively,

$$g = A\left[\alpha + (1-\alpha)x^{\sigma}\right]^{\frac{1}{\sigma}} - \delta - q - a\hat{v}\frac{x}{n} \tag{23}$$

$$g = B(1 - L - l - u - n). \tag{24}$$

The Euler equation yields,

$$\rho + g = \alpha A\left[\alpha + (1-\alpha)x^{\sigma}\right]^{\frac{1-\sigma}{\sigma}} - \delta. \tag{25}$$

Finally, the allocation of time across activities gives the following conditions,

$$M_0 \hat{v}^{\eta} u^{-\eta}\left[\frac{\omega}{A_2} - 1\right] = \rho + \lambda \tag{26}$$

$$\frac{B}{A_2}\omega n = \rho - Bl \tag{27}$$

$$q = A_2\left[\frac{\gamma}{1-\gamma}L - l\right]\frac{x}{n}. \tag{28}$$

[3] The planner's problem can alternatively be written as,

$$f(k,h,n,\dot{k},\dot{h},\dot{n}) = \max_{c,h_l,h_L,h_u,h_e}\left(\gamma \log(c + A_2 h_l) + (1-\gamma)\log(\frac{h_L}{h})\right)$$

subject to (16), (17), (18) and (19). Then, $\{k(t), h(t), n(t)\}$ is an optimal solution if and only if it solves, $V(k(0), h(0), n(0)) = max \int_0^{\infty} e^{-\rho t} f(k,h,n,\dot{k},\dot{h},\dot{n})dt$, subject to (20). Since f is non-concave, the uniqueness of the solution is not guaranteed [see, e.g., Fleming and Rishel (1975) for a systematic treatment of existence and uniqueness of solutions to optimal variational problems].

The system of equations (21)-(28) is not block-recursive and, therefore, all variables must be solved simultaneously.

The strategy to illustrate that this system may yield more than one interior solution consists in expressing L, l, u and n as functions of g, and then to make use of equation (24) to solve for g. To do so, we first use (25) to write x in terms of g. Then, from (22) and (26) u can be written as a function of g, and from (21) n as a function of g as well. From (23) and (28) we write L in terms of g. From (27) we express l as a function of g. By plugging these expressions for L, l, u and n into (24), we obtain an expression in g, say $\Gamma(g) = 0$. Hence, the number of solutions to (21)-(28) is given by the number of positive solutions to $\Gamma(g) = 0$, provided that the remaining variables fall within their feasible range. It is shown below that for realistic parameter values $\Gamma(g)$ yields two interior solutions conforming two interior balanced growth paths.

Before exploring the existence of multiple interior solutions quantitatively, Proposition 1 provides some results on the behavior of our economic variables across interior balanced growth paths.

Proposition 1: *Assume that there are two interior balanced growth paths, then:*

(1) The balanced growth path with a higher growth rate has a lower unemployment rate.

(2) The balanced growth path with a higher unemployment rate has a lower labor-capital ratio.

Proof: See the Appendix.

In addition to interior balanced growth paths, our model economy can also yield a non-interior steady-state equilibrium where households decide not to allocate time to education, and, therefore, there is no productivity growth. The characterization of such a steady-state equilibrium is presented in the Appendix.

In order to assess the multiplicity of long-run equilibria quantitatively parameter values are set following the standard procedure. Some parameter values will be set using *a priori* information. Other parameters will be set so that the interior, stable balanced growth path of the model matches some selected average values for the US economy. However, this is not a proper calibration exercise, as some parameter values will be left free so as to generate the multiplicity result. We think this is the right strategy to assess our mechanism for multiple long-run equilibria quantitatively. It should be noted that the main aim in this paper is to propose a new mechanism for multiplicity and not to explain actual labor market variables.

The de-trended cost per vacancy, a, the scale parameter in the firm's production function, A, and in the matching function, M, are all normalized to one. Parameter σ in the production function is set at 0.1, which yields an elasticity of substitution between capital and labor of 1.1 (this is the value estimated by Papageorgiou and Duffy (2000) when the labor input is adjusted by human capital). The matching elasticity with respect to vacancies is set at 0.6, which is within the range of estimates of the matching function. The quarterly rate of job destruction is set at 0.1, which is the value reported by Andolfato (1996).

A second set of parameter values are chosen so that the interior, stable balanced growth path matches the following average values: (*i*) an average quarterly growth rate of 0.4 per cent; (*ii*) a capital-output ratio equal to 9; (*iii*) an output-consumption ratio of 1.38; (*iv*) a quarterly rate of return to capital of 2.5 per cent. To complete the exercise, we impose the existence of two interior balanced growth paths, and one non-interior steady-state equilibrium with no growth in income.

Our baseline economy is presented in Table 1 below.

[INSERT TABLE 1 HERE]

Table 2 below presents the values of our economic variables in the three long-run equilibria. The interior balanced growth paths are denoted by BGP2 and BGP3, being BGP3 saddle-path stable, and BGP2 unstable. The non-interior, saddle-path-stable steady state is denoted by BGP1. The non-stability of BGP2 was to be expected, as it must separate the stable manifolds of balanced growth path BGP3 and of the steady-state equilibrium BGP1.

[INSERT TABLE 2 HERE]

3.1. Discussion

The long-run equilibrium configuration displayed in Table 3 —with two stable equilibria separated by an unstable equilibrium— has unambiguous implications in terms of unemployment (and growth) determination, and on the effects of temporary shocks. Let us assume that an economy is initially growing along the stable, interior balanced growth path, BGP3, and that it is hit by, say, an employment shock. Our model's prediction is that if the shock is small its effects will be only temporary, and the economy will return to the original balanced growth path. If the shock is large[4] our model predicts a persistent increase in unemployment, as the economy will converge to the non-interior balanced path, BGP1. Unemployment will not decrease until a new shock or series of shocks, place the economy back into the former stable manifold. The fragility of BGP3 to shocks is measured by the size of the minimum shock needed to place such an economy into the stable manifold of BGP1. Technically, this amounts to computing the distance from BGP1 to the frontier of its stable manifold.

This type of argument has been recently used to account for the permanent increase in European unemployment after the shocks of the mid 1970's. Even though the model presented in this highly stylized and does not aim to explain European unemployment, the relationship between our economic variables across the two stable long-run equilibria, BGP3 and BGP1, is qualitatively consistent with the evolution observed in Europe since the mid 1970's. For instance, Blanchard (1997) and Daveri and Tabellini (2000) document that the increase in unemployment after the mid 1970's was accompanied by an increase in capital's share of income, and by a decrease both in growth rates and in the labor-capital ratio. Hence, our

[4] Note that by "large" we mean that it places the economy into the stable manifold of BGP1.

model captures the main trade-offs that help explain the joint determination of key macroeconomic and labor-market aggregates.

Our model constitutes a suitable framework to assess the role played by labor market institutions in the increase in European unemployment. According to Blanchard and Wolfers (2000) and Balakrishnan and Michelacci (2001), the differential in unemployment dynamics between Europe and the US is explained by a different reaction to the same shocks, rather than to the impact of different shocks. Moreover, according to these authors labor market institutions seem to lie at the heart of this differential in the response to common shocks. The type of questions that emerge from this line of research are: Do unemployment benefits increase the vulnerability of the economy to shocks? Why did the shocks of the mid 1970's not result in the US in a switch to an equilibrium with high unemployment as happened in Europe? In light of our results above, our model is a natural framework to address questions of this kind. An analysis of equilibrium fragility in an extension of our model with labor market institutions would bring new insights on the relationship between shocks, institutions and equilibrium unemployment.

4. Conclusions

In this paper we put forward a new mechanism leading to multiple long-run rates of growth and unemployment. In a simple search-matching model with endogenous human capital accumulation and endogenous labor market participation, we show that the problem solved by the household displays aggregate dynamic non-convexities, thus yielding multiple long-run equilibria. Such non-convexities result from the fact that some time-consuming activities are more intensive in human capital than others, which creates a complementarity between the current stock of human capital and long-run productivity. Thus, economies with a high initial stock of human capital will allocate more time to skill-intensive activities (production and education), setting off a transition to a long-run equilibrium with income growth and low unemployment. On the other hand, economies with a low initial stock of human capital will allocate more time to leisure and job search, thus setting off a transition to a long-run equilibrium with no growth in income and high unemployment.

[INSERT FIGURE]

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5. Appendix

The first-order conditions for the household problem are:

$$\frac{\gamma}{C(t)} = \nu_2(t) \tag{A.1}$$

$$\nu_1(t)m(t) = \nu_3(t)Bh(t) \tag{A.2}$$

$$\frac{\gamma A_2}{C(t)} = \nu_3(t)B \tag{A.3}$$

$$\frac{1-\gamma}{L(t)} = \nu_3(t)Bh(t), \tag{A.4}$$

where $\nu_1(t)$, $\nu_2(t)$ and $\nu_3(t)$ denote the shadow prices for employment, physical and human capital, respectively.

Equation (A.1) is the Euler equation, which determines the optimal allocation of income to consumption and investment; equations (A.2), (A.3) and (A.4) give the optimal allocation of time to job search, home production, leisure and education.

From the non-arbitrage conditions, shadow prices for employment, physical and human capital satisfy,

$$\dot{\nu}_1(t) = \nu_1(t)[\rho + \lambda + m(t)] - \nu_2(t)\omega(t)h(t) \tag{A.5}$$

$$\dot{\nu}_2(t) = \nu_2(t)[\rho - r(t)] \tag{A.6}$$

$$\dot{\nu}_3(t) = \nu_3(t)[\rho - Bl(t) - Be(t)] - \nu_2(t)\omega(t)n(t). \tag{A.7}$$

The income value of an occupied job for the household, $W(t)$, is given by the ratio $\frac{\nu_1(t)}{\nu_2(t)}$, which, after taking the derivative with respect to time, gives equation (6) in Section 2.

The non-interior steady-state equilibrium. First-order conditions along a non-interior path are given by (A.1) and,

$$\nu_1 m(t) > \nu_3(t)Bh(t) \tag{A.8}$$

$$\frac{1-\gamma}{L(t)} = \nu_1 m(t), \tag{A.9}$$

where the non-arbitrage condition implies now that

$$\dot{\nu}_3(t) = \nu_3(t)\rho - \nu_2(t)\omega(t)n(t). \tag{A.10}$$

Proof of Proposition 1: Assume that the system of equations (21)-(28) contains two interior balanced growth paths with $\{g_1, ur_1, ud_1, x_1\}$ and $\{g_2, ur_2, ud_2, x_2\}$. Where ur is used to denote the unemployment rate. We show that if $g_1 > g_2$, then $ur_1 < ur_2$, $ud_1 > ud_2$ and $x_1 > x_2$. 1) Let us write the unemployment rate as a function of the growth rate. From equations (22), (25) and (26) we can write u as a function of g, which we denote by $u(g)$. Similarly, from equation (21) we obtain n as a function of g, say $n(g)$. The unemployment rate is then

$$ur(g) = \frac{u(g)}{n(g) + u(g)}.$$

Taking derivatives with respect to g, it yields,

$$\frac{dur}{dg} = \frac{du}{dg}\left[M_0 \hat{v}^{\eta} u^{1-\eta} \frac{\eta}{\lambda}\right].$$

Since $\frac{du}{dg} < 0$, it follows that $\frac{dur}{dg} < 0$.

2) The labor-capital ratio is $\frac{nh}{k}$, which has been denoted by x. Differentiating both sides of equation (25) with respect to g, we obtain

$$\frac{dx}{dg} = \frac{(\alpha + (1-\alpha)x^{\sigma})^{\frac{2\sigma-1}{\sigma}} x^{1-\sigma}}{(1-\sigma)(1-\alpha)\alpha A} > 0.$$

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Table 1. Baseline parameter values

Matching elasticity	$\eta = 0.6$	Rate of capital deprec.	$\delta = 0.025$
Discount factor	$\rho = 0.02$	Job destruction rate	$\lambda = 0.1$
Vacancies	$\hat{v} = 0.023$	Elastic. of subs.	$(1-\sigma)^{-1} = 1.1$
Worker's share	$p = 0.35$	Consumption share	$\gamma = 0.45$

Scale parameters	$A = 1; M = 1; A_2 = 1.55$; and $B = 0.062$
Weight of capital in produc. func.	$\alpha = 0.345$
Detrended cost per vacancy	$a = 1$

Table 2. Long-run equilibria for the baseline economy

	BGP1 (non-interior)	BGP2 (unstable)	BGP3 (interior)
Growth rate (quarterly)	0.0000	0.00714	0.0040
Unemployment rate	0.1214	0.0998	0.1032
Labor-capital ratio	0.0247	0.0327	0.0294
Capital-output ratio	9.6138	8.1851	9.0000
Output-consumption ratio	1.3054	1.4026	1.3831
Capital's share of income	0.4326	0.4057	0.4083

Notes. Balanced growth paths for our baseline economy. Balanced growth path BGP1 is non-interior, and BGP2 and BGP3 are interior. BGP2 is unstable.