Fiscal Consolidation in a Disinflationary Environment: Price- vs. Quantity-Based Measures

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Online Appendix

1 Derivations

1.1 Household's maximisation problem

The first order conditions are:

[wrt c_t]

$$\lambda_{c,t}\left(1+\tau_c\right) = c_t^{-\eta} \tag{1}$$

[wrt i_t^p]

$$\lambda_{c,t}-\lambda_{k,t}\left\{1-\frac{\omega}{2}\left(\frac{i_t^p}{i_{t-1}^p}-1\right)^2-\omega\left(\frac{i_t^p}{i_{t-1}^p}-1\right)\frac{i_t^p}{i_{t-1}^p}\right\} = \beta\lambda_{k,t+1}\omega\left(\frac{i_{t+1}^p}{i_t^p}-1\right)\left(\frac{i_{t+1}^p}{i_t^p}\right)^2 \tag{2}$$

[wrt k_{t+1}^p]

$$\lambda_{k,t} = \beta\left\{\lambda_{k,t+1}\left(1-\delta^p\right)+\lambda_{k,t+1}\left[r_{t+1}^p-\tau_k\left(r_{t+1}^p-\delta^p\right)\right]\right\} \tag{3}$$

[wrt $b_{g,t+1}$]

$$1 = \Lambda_{t,t+1}r_t \tag{4}$$

[wrt $b_{f,t+1}$]

$$1 = \Lambda_{t,t+1}\frac{e_{t+1}}{e_t}r_{f,t} \tag{5}$$

[wrt u_t]

$$\Phi l_t^{\varphi} = \lambda_{c,t}\mathrm{b}+\lambda_{n^p,t}\psi_t^{hp}\left(1-s_t\right)+\lambda_{n^g,t}\psi_t^{hg}s_t \tag{6}$$

[wrt s_t]

$$\lambda_{n^g,t}\psi_t^{hg} \;\; = \;\; \lambda_{n^p,t}\psi_t^{hp} \tag{7}$$

$\left[\text{wrt } n_{t+1}^p\right]$

$$\lambda_{n^p,t} \;\; = \;\; \beta\left[\lambda_{n^p,t+1}\left(1-\sigma^p\right)+\lambda_{c,t+1}\left(1-\tau_n\right)w_{t+1}^p x_{t+1}-\Phi l_{t+1}^{\varphi}\right] \tag{8}$$

$\left[\text{wrt } n_{t+1}^g\right]$

$$\lambda_{n^g,t} \;\; = \;\; \beta\left[\lambda_{n^g,t+1}\left(1-\sigma^g\right)+\lambda_{c,t+1}\left(1-\tau_n\right)w_{t+1}^g-\Phi l_{t+1}^{\varphi}\right] \tag{9}$$

[wrt x_t]

$$\Upsilon x_t^{\xi} \;\; = \;\; \lambda_{c,t}\left(1-\tau_n\right)w_t^p n_t^p \tag{10}$$

where $\lambda_{c,t}$, $\lambda_{n^p,t}$, $\lambda_{n^g,t}$, $\lambda_{k,t}$, are the multipliers on the budget constraint, on the private and public laws of motion of employment, and on the law of motion of capital, respectively, and $\Lambda_{t,t+1}$ is the ratio of marginal utilities of consumption

$$\Lambda_{t-1,t} \;\; = \beta \;\; \frac{\lambda_{c,t}}{\lambda_{c,t-1}} \tag{11}$$

Equations (1)-(5)are standard and include the arbitrage conditions for the returns to private consumption, private capital and bonds. Equations (8) and (9) relate the expected marginal value from being employed in each sector to the after-tax wage, the utility loss from the reduction in leisure, and the continuation value, which depends on the separation probability. Equation (6)states that the marginal utility of the unemployment benefit, minus the marginal utility from leisure should equal the expected marginal values of being employed, given the share of unemployed searching in each sector. Equation (7) is an arbitrage condition according to which the choice of the share, s_t, is such that the expected marginal values of being employed are equal across the two sectors.

We can define the marginal value to the household of having an additional member employed in the private sector, as follows:

$$\begin{aligned} V^h_{n^p t} \equiv \frac{\partial \mathcal{L}}{\partial n^p_t} &= \lambda_{ct} w^p_t x_t (1-\tau_n) - \Phi l_t^{-\varphi} + (1-\sigma^p)\lambda_{n^p t} \\ &= \lambda_{ct} w^p_t x_t (1-\tau_n) - \Phi l_t^{-\varphi} + (1-\sigma^p)\beta E_t(V^h_{n^p t+1}) \end{aligned} \tag{12}$$

where the second equalities come from equation (8).

1.2 Derivation of the private wage

The Nash bargaining problem is to maximize the weighted sum of log surpluses:

$$\max_{w^p_t} \left\{ (1-\vartheta) \ln V^h_{n^p t} + \vartheta \ln V^f_{n^p t} \right\}$$

where $V^h_{n^j t}$ and $V^f_{n^j t}$ are defined as:

$$V^h_{n^p t} \equiv \frac{\partial \mathcal{L}}{\partial n^p_t} = \lambda_{ct} w^p_t x_t (1-\tau^n_t) - \Phi l_t^{-\varphi} + (1-\sigma^p)\lambda_{n^p t} \tag{13}$$

$$V^F_{n^p t} \equiv \frac{\partial Q^p}{\partial n^p_t} = p_{x,t}(1-\phi)\frac{y^p_t}{n^p_t} - w^p_t x_t + \frac{(1-\sigma^p)\kappa}{\psi^{fp}_t} \tag{14}$$

The first order conditions of this optimization problem is:

$$\vartheta V^h_{n^p t} = (1-\vartheta)\lambda_{ct}(1-\tau^n_t) V^f_{n^p t} \tag{15}$$

Plugging the expressions for the value functions into the FOC, we can rearrange to find the expression for the private wage. Using (13),(14) and (15) we obtain:

$$w_t^p x_t = (1-\vartheta)[p_{x,t}(1-\phi)\frac{y_t^p}{n_t^p} + \frac{(1-\sigma^p)\kappa}{\psi_t^{fp}}] + \frac{\vartheta}{(1-\tau_n)\lambda_{c,t}}(\Phi l_t^{-\varphi} - (1-\sigma^p)\lambda_{n^p t}) \quad (16)$$

Finally, taking the time t expectation of15 evaluated at time $t+1$, and using the FOCs of the household and firm, we obtain

$$\vartheta\lambda_{n^p t} = (1-\vartheta)\lambda_{ct}(1-\tau_t^n)\frac{\kappa}{\psi_t^{fp}}$$

which allows us to simplify 16 to obtain the final expression for the private wage

$$w_t^p x_t = (1-\vartheta)p_{x,t}(1-\phi)\frac{y_t^p}{n_t^p} + \frac{\vartheta}{(1-\tau_n)\lambda_{c,t}}\Phi l_t^{-\varphi} \quad (17)$$

2 Calibration Strategy

2.1 Labour market variables

We calibrate $e = 1$, such that it does not effect the rest of the steady state. We calibrate the labour-force participation rate, the unemployment rate, and the share of public employment in total employment to match the observed average values from the Italian data ($1-l = 0.65$, $u^{rate} = \frac{u}{1-l} = 0.1$, $\frac{n^g}{n} = 0.18$). Then we get u, n, $\frac{n^p}{n}$, n^p, n^g as follows:

$$u = u^{rate}(1-l)$$

$$n = 1 - l - u$$

$$\frac{n^p}{n} = 1 - \frac{n^g}{n}$$

$$n^j = \frac{n^j}{n}n$$

We set the following values for the separation rates, $\sigma^p = 0.063$ and $\sigma^g = 0.06$. Then we get m^j from the steady state version of the law of motion of employment:

$$m^j = \sigma^j n^j$$

We calibrate the ratio of unemployed searching in two sectors as ${}^{u^p}\!/_{u^g} = 4$. Then, it holds by definition:

$$u^p = \frac{u}{1 + {}^{u^p}\!/_{u^g}}$$

$$u^g = u - u^p$$

$$\psi^{hj} = \frac{m^j}{u^j}$$

Since there is no exact estimate for the value of the private vacancy-filling probability, ψ^{fp}, in the literature, we use what is considered as standard by setting it equal to 0.1 and then we assume that $\psi^{fp} = \psi^{fg}$. Hence, we get:

$$\upsilon^j = \frac{m^j}{\psi^{fj}}$$

The elasticity in the matching functions, α, is set equal to 0.5. Then the efficiency parameter for private matches, ρ_m^p, is given by inverting the matching function:

$$\rho_m^j = \frac{m^j}{(\upsilon^j)^\alpha (u^j)^{1-\alpha}}$$

2.2 Production

We set the capital depreciation rates, δ^j, equal to 0.02. Following the literature, we set the discount factor, β, equal to 0.99. The tax rates on capital and labour income are calibrated to 30%. Next, we get r^p and R from (3) and (5), respectively:

$$r^p = \frac{1}{(1-\tau_k)} \left(\frac{1}{\beta} - 1 \right) + \delta^p$$

$$R = \frac{1}{\beta}$$

The elasticity of demand for intermediate goods, ϵ, is set equal to 10. The price of the final good is normalized to one, and we assume a steady state subsidy offsets the markup, so that $p_x = 1$.

We set the capital share in the production function of the private good equal to 0.36. Then we obtain $\frac{y^p}{k^p}$ from the firm's FOC with respect to capital,:

$$\frac{y^p}{k^p} = \frac{r^p}{\phi}$$

We set the shares of public capital in public production, μ, equal to 0.36, of the public good in private production, ν, equal to 0.05. Further, using data from Kamps (2006) we set $\frac{k^g}{k^p} = 0.31$, close to the mean value for 1970-2002. Since we restrict our case to a deterministic steady state, we normalize A_t to one. Then from the production function of the private and public good, k^p is determined by:

$$k^p = \left[\frac{y^p}{k^p} \left(n^p\right)^{-(1-\phi)} (n^g)^{\mu\nu - \nu} \left(\frac{k^g}{k^p} \right)^{-\mu\nu} \right]^{\frac{1}{\phi + \mu\nu - 1}}$$

and then we get $y^p$and $k^g$by definition:

$$y^p = \frac{y^p}{k^p} k^p, \; k^g = \frac{k^g}{k^p} k^p$$

and i^p and i^g from the law of motion of private and public capital at steady state:

$$i^p = \delta^p k^p, \; i^g = \delta^g k^g$$

and y^g from the public production function:

$$y^g = (n^g)^{1-\mu} (k^g)^{\mu}$$

Following Hagedorn and Manovskii (2008), Galí (2011), and Bruckner and Pappa (2012), we calibrate the cost of posting a vacancy, κ, by targeting vacancy costs per filled job as a fraction of the real private wage, $\frac{\kappa}{w^p}$, choosing 0.045 as a target as in Galí (2011). Also, we set the replacement rate, $\frac{b}{w^p}$, equal to 0.4 (in accordance with the range [0.2, 0.4] in Petrongolo and Pissarides, 2001). Then, we can get w^p from the firm's FOC with respect to private vacancies:

$$w^p = (1-\phi)\frac{y^p}{n^p}\left(1+\frac{\sigma^p}{\psi^{fp}}\frac{\kappa}{w^p}\right)^{-1}$$

and it follows that κ and b are given by:

$$\kappa = \frac{\kappa}{w^p}w^p$$

$$b = \frac{b}{w^p}w^p$$

2.3 Households

We derive private consumption from the resource constraint

$$c = y^p - i^p$$

We set the consumption tax rate to 15%, the intertemporal elasticity of substitution, $\frac{1}{\eta}$, equal to 1, the Frisch elasticity of labour supply, $\frac{1}{\psi}$, equal to 0.25 (in the range of Domeij and Floden, 2006). We derive the two Lagrange multipliers from the household's first order conditions, (1) and (8) for $j = p$, respectively:

$$\lambda_c = (1+\tau^c)^{-1}c^{-\eta}$$

$$\lambda_{n^p} = \frac{\beta\lambda_c(w^p(1-\tau_n)-b)}{1-\beta(1-\sigma^p)+\beta\psi^{hp}}$$

This allows us to derive Φ from (6), after substituting in (7):

$$\Phi = l^{\varphi}(\lambda_c b + \psi^{hp}\lambda_{n^p})$$

and the firm's bargaining power from the solution of the wage bargaining problem:

$$\vartheta = \frac{(1-\phi)(y^p/n^p)+(1-\sigma^p)\kappa/\psi^{fp}-w^p}{(1-\phi)(y^p/n^p)+(1-\sigma^p)\kappa/\psi^{fp}-(\Phi l^{-\varphi}-(1-\sigma^p)\lambda_{n^p})/\lambda_c(1-\tau_n)}$$

Following Neiss and Pappa (2011) we set $\varphi_2 = 0.5$, and we use (10) to calibrate Υ such that $e = 1$:

$$\Upsilon = -\lambda_c w^p n^p$$

Finally, we derive the Lagrange multiplier from (7), and the public wage from (8) and (9):

$$\lambda_{n^g} = \frac{\lambda_{n^p}\psi^{hp}}{\psi^{hg}}$$

$$w^g = \frac{\Phi l^{-\varphi}+\lambda_{n^g}(R-1+\sigma^g)}{\lambda_c(1-\tau_n)}$$

This also allows us to define total output, $rgdp = y^p + w^g n^g$.

2.4 Fiscal Policy

We set the steady-state annual debt-to-GDP ratio equal to 50%, so that by definition:

$$B = (0.5 * 4)rgdp$$

and using the government's budget constraint in steady state, we have:

$$DF = (\beta - 1)B$$

Next, we calibrate the steady state value for lump-sum transfers, T, from the definition of the deficit:

$$T = i^g + w^g n^g + \varpi u + \kappa \upsilon^g - \tau_k(r^p - \delta^p)k^p - \tau_n(w^p n^p + w^g n^g) - \tau_c c - DF$$

2.5 Other parameters

Finally, the model's steady state is independent of the degree of price rigidities, of the monetary policy rule, the debt-targeting rule for lump-sum taxes, and of the size of the capital adjustment costs. We set the probability that a firm does not change its price within a given period, χ, equal to 0.75, the Taylor rule coefficient, ζ_π, equal to 2.5, and the adjustment costs parameter, ω, equal to 0.5. Finally, we set the parameters for the persistence of the debt-target shock,ρ_1 and ρ_2, equal to 0.85 and 0.0001, respectively.