# Appendix: Housework and fiscal expansions 

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#### Abstract

We build an otherwise-standard business cycle model with housework, calibrated consistently with data on time use, in order to discipline complementarity between consumption and hours worked and relate its strength to the size of fiscal multipliers. Evidence on the substitutability between home and market goods confirms that complementarity is an empirically relevant driver of fiscal multipliers. However, we also find that in a housework model substantial complementarity can be generated without imposing a low wealth effect, which contradicts the microeconomic evidence. Also, explicitly modeling housework matters for assessing the welfare effects of government spending, which are understated by theories that neglect substitutability between home-produced and market goods.


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## 1. The Model

### 1.1. Households' Problem and first-order conditions

In this section we lay out the households' problem and the respective first order conditions. Given initial values of the capital stock $K_{0}$ and assets $B_{0}$, and 5 all prices and policies, households maximize their lifetime utility by choosing a state-contingent sequence $\left\{C_{m, t}, C_{n, t}, h_{m, t}, h_{n, t}, K_{m, t}, K_{n, t}, K_{t+1}, B_{t}\right\}_{t=0}^{\infty}$, i.e.,

$$
\max E_{0} \sum_{t=0}^{\infty} \beta^{t} U\left(C_{t}, l_{t}\right)
$$

subject to

$$
\begin{gather*}
C_{t}=\left[\alpha_{1}\left(C_{m, t}\right)^{b_{1}}+\left(1-\alpha_{1}\right)\left(C_{n, t}\right)^{b_{1}}\right]^{\frac{1}{b_{1}}}, \quad \alpha_{1} \in[0,1] \quad b_{1}<1  \tag{1}\\
C_{n, t}=\left(K_{n, t}\right)^{\alpha_{2}}\left(h_{n, t}\right)^{1-\alpha_{2}} \\
l_{t}=1-h_{t}
\end{gather*}
$$

$$
\begin{gathered}
C_{n, t}=\left(K_{n, t}\right)^{\alpha_{2}}\left(h_{n, t}\right)^{1-\alpha_{2}} \\
l_{t}=1-h_{t}, \\
h_{t}=h_{m, t}+h_{n, t} \\
K_{t}=K_{m, t}+K_{n, t} \\
E_{t}\left\{Q_{t, t+1} B_{t+1}\right\}+P_{t} C_{m, t}+P_{t}\left[K_{t+1}-(1-\delta) K_{t}+\frac{\xi}{2}\left(\frac{K_{t+1}}{K_{t}}-1\right)^{2}\right](2) \\
\leq \quad B_{t}+W_{t} P_{t} h_{m, t}+r_{t}^{k} P_{t} K_{m, t}+T_{t} .
\end{gathered}
$$

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where the utility function is specified as in Section 4.2 in the main text:

$$
U\left(C_{t}, l_{t}\right)=\frac{\left[\left(C_{t}\right)^{b}\left(l_{t}\right)^{1-b}\right]^{1-\sigma}-1}{1-\sigma}, \quad b \in(0,1), \quad \sigma \geq 1
$$

Then, the Lagrangian is defined as follows:

$$
\begin{array}{r}
\mathcal{L}=E_{0} \sum_{t=0}^{\infty} \beta^{t} \frac{\left[\left[\alpha_{1}\left(C_{m, t}\right)^{b_{1}}+\left(1-\alpha_{1}\right)\left(C_{n, t}\right)^{b_{1}}\right]^{\frac{b}{b_{1}}}\left(1-h_{n, t}-h_{m, t}\right)^{1-b}\right]^{1-\sigma}-1}{1-\sigma}+\mu_{t}\left[\left(K_{n, t}\right)^{\alpha_{2}}\left(h_{n, t}\right)^{1-\alpha_{2}}-C_{n, t}\right] \\
+\gamma_{t}\left[K_{t}-K_{m, t}-K_{n, t}\right] \\
+\frac{\lambda_{t}}{P_{t}}\left\{B_{t}+W_{t} P_{t} h_{m, t}+r_{t}^{k} P_{t} K_{m, t}+T_{t}-E_{t}\left\{Q_{t, t+1} B_{t+1}\right\}-P_{t} C_{m, t}\right. \\
\left.-P_{t}\left[K_{t+1}+(1-\delta) K_{t}+\frac{\xi}{2}\left(\frac{K_{t+1}}{K_{t}}-1\right)^{2}\right]\right\}
\end{array}
$$

We divide the budget constraint by $P_{t}$, which is given to households, so that we can interpret $\lambda_{t}$ as the marginal utility of wealth directly. By optimality the marginal utility of wealth coincides with the marginal utility of market consumption. The corresponding first-order conditions are:

$$
\begin{align*}
& \begin{aligned}
\left\{C_{m, t}\right\}: \lambda_{t} & =U_{C}\left(C_{t}, l_{t}\right) \alpha_{1}\left(\frac{C_{m, t}}{C_{t}}\right)^{b_{1}-1} \\
& =b \alpha_{1}\left(1-h_{n, t}-h_{m, t}\right)^{(1-b)(1-\sigma)} C_{m, t}^{b_{1}-1}\left(C_{t}\right)^{b(1-\sigma)-b_{1}}
\end{aligned}  \tag{3}\\
& \begin{aligned}
\left\{C_{n, t}\right\}: \mu_{t}= & U_{C}\left(C_{t}, l_{t}\right)\left(1-\alpha_{1}\right)\left(\frac{C_{n, t}}{C_{t}}\right)^{b_{1}-1} \\
= & b\left(1-\alpha_{1}\right)\left(1-h_{n, t}-h_{m, t}\right)^{(1-b)(1-\sigma)} C_{n, t}^{b_{1}-1}\left(C_{t}\right)^{b(1-\sigma)-b_{1}}
\end{aligned} \\
& \begin{aligned}
\left\{h_{m, t}\right\}: \quad \lambda_{t} W_{t}= & U_{l}\left(C_{t}, l_{t}\right)=(1-b)\left(C_{t}\right)^{b(1-\sigma)}\left(1-h_{n, t}-h_{m, t}\right)^{(1-b)(1-\sigma)-1}
\end{aligned}  \tag{4}\\
& \qquad\left\{h_{n, t}\right\}: U_{l}\left(C_{t}, l_{t}\right)=\mu_{t}\left(1-\alpha_{2}\right)\left(\frac{C_{n, t}}{h_{n, t}}\right) \\
& (1-b)\left(C_{t}\right)^{b(1-\sigma)}\left(1-h_{n, t}-h_{m, t}\right)^{(1-b)(1-\sigma)-1}=\mu_{t}\left(1-\alpha_{2}\right)\left(\frac{C_{n, t}}{h_{n, t}}\right) \tag{5}
\end{align*}
$$

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$$
\begin{equation*}
\left\{K_{m, t}\right\}: \quad \gamma_{t}=\lambda_{t} r_{t}^{k} \tag{7}
\end{equation*}
$$

$$
\begin{gather*}
\left\{K_{n, t}\right\}: \quad \gamma_{t}=\mu_{t} \alpha_{2}\left(\frac{C_{n, t}}{K_{n, t}}\right)  \tag{8}\\
\left\{K_{t+1}\right\}: \beta E_{t}\left\{\frac{\lambda_{t+1}}{\lambda_{t}}\left[1+\frac{\xi}{K_{t}}\left(\frac{K_{t+1}}{K_{t}}-1\right)\right]^{-1}\right.  \tag{9}\\
\left.\left[1-\delta+\xi\left(\frac{K_{t+2}}{K_{t+1}}-1\right)\left(\frac{K_{t+2}}{K_{t+1}^{2}}\right)\right]\right\}+\beta E_{t}\left\{\gamma_{t+1}\right\}=1 \\
\left\{B_{t+1}\right\}: \lambda_{t} E_{t}\left\{Q_{t, t+1}\right\}=\beta E_{t}\left\{\frac{\lambda_{t+1}}{\pi_{t+1}}\right\} \tag{10}
\end{gather*}
$$

Equations (3) and (5) correspond to equations (10) and (11) in the main text, respectively. Combining (4) and (6) yields equation (12),

$$
\begin{equation*}
\frac{U_{l}\left(C_{t}, l_{t}\right)}{\left(1-\alpha_{1}\right) U_{C}\left(C_{t}, l_{t}\right)}\left(\frac{C_{n, t}}{C_{t}}\right)^{1-b_{1}}=\frac{\left(1-\alpha_{2}\right) C_{n, t}}{h_{n, t}} . \tag{11}
\end{equation*}
$$

Similarly, combining (3), (4), (7) and (8) yields equation (13) in the main text

$$
\begin{equation*}
\frac{\alpha_{1}}{1-\alpha_{1}}\left[\frac{C_{m, t}}{C_{n, t}}\right]^{b_{1}-1}=\frac{\alpha_{2} C_{n, t}}{r_{t}^{k} K_{n, t}} . \tag{12}
\end{equation*}
$$

Equation (14) results from the first-order conditions with respect to market capital, (7), and with respect to the next-period total capital stock, (9),

$$
\begin{array}{r}
\beta E_{t}\left\{\frac{\lambda_{t+1}}{\lambda_{t}}\left[1+\frac{\xi}{K_{t}}\left(\frac{K_{t+1}}{K_{t}}-1\right)\right]^{-1}\right.  \tag{13}\\
\left.\left[1-\delta+r_{t+1}^{k}+\xi\left(\frac{K_{t+2}}{K_{t+1}}-1\right)\left(\frac{K_{t+2}}{K_{t+1}^{2}}\right)\right]\right\}=1
\end{array}
$$

Finally, by a no-arbitrage argument $E_{t}\left\{Q_{t, t+1}\right\}=\left(1+R_{t}\right)^{-1}$, such that the firstorder condition with respect to the portfolio of state-contingent assets yields the respective standard Euler equation, equation (15) in the main text

$$
\begin{equation*}
\beta E_{t}\left\{\frac{\lambda_{t+1}}{\lambda_{t}}\left(1+R_{t}\right) \Pi_{t+1}^{-1}\right\}=1 . \tag{14}
\end{equation*}
$$

### 1.2. Firms

In the economy, there are infinitely many monopolistically competitive firms indexed by $i \in[0,1]$. Each firm buys market capital and hours worked on
${ }_{35}$ perfectly competitive markets in order to produce a variety $i$ of the market good, according to the following production function:

$$
\begin{equation*}
Y_{t}(i)=\left(K_{m, t}(i)\right)^{\alpha_{3}}\left(h_{m, t}(i)\right)^{1-\alpha_{3}}, \quad \alpha_{3} \in[0,1] . \tag{15}
\end{equation*}
$$

Cost minimization yields

$$
\begin{align*}
\alpha_{3} R M C_{t}\left(\frac{Y_{t}(i)}{K_{m, t}(i)}\right) & =r_{t}^{k},  \tag{16}\\
\left(1-\alpha_{3}\right) R M C_{t}\left(\frac{Y_{t}(i)}{h_{m, t}(i)}\right) & =W_{t} . \tag{17}
\end{align*}
$$

The real marginal cost, $R M C_{t}$, is constant across firms because of constant returns to scale in production and perfect competition on factor markets. We
${ }_{40}$ follow Calvo 5 and we assume that in any given period each firm resets its price $P_{t}(i)$ with a constant probability $(1-\theta)$. At a given price $P_{t}(i)$, production has to satisfy demand:

$$
\begin{equation*}
Y_{t}(i)=\left[\frac{P_{t}(i)}{P_{t}}\right]^{-\varepsilon} Y_{t}^{d} . \tag{18}
\end{equation*}
$$

where aggregate demand, $Y_{t}^{d}$, is taken as given. We assume that production is subsidized by the government, which pays a fraction $\tau$ of the cost per unit of
${ }_{45}$ production. Maximization of profits

$$
\begin{equation*}
E_{t}\left\{\sum_{j=0}^{\infty} \theta^{j} Q_{t, t+j}\left[P_{t}(i) Y_{t+j}(i)-P_{t+j}(1-\tau) R M C_{t+j} Y_{t+j}(i)\right]\right\} \tag{19}
\end{equation*}
$$

subject to constraint 18) yields the following first-order condition for any firm $i$ that is allowed to re-optimize in period $t$ :

$$
\begin{equation*}
E_{t}\left\{\sum_{j=0}^{\infty} \theta^{j} Q_{t, t+j} Y_{t+j}(i)\left[\frac{P_{t}^{*}}{P_{t}}-\frac{\varepsilon(1-\tau)}{\varepsilon-1} R M C_{t+j} \Pi_{t, t+j}\right]\right\}=0 . \tag{20}
\end{equation*}
$$

$P_{t}^{*}$ is the optimal price, $Q_{t, t+j}$ denotes the stochastic discount factor in period $t$ for nominal profits $j$ periods ahead and it is such that

$$
\begin{equation*}
Q_{t, t+j}=\beta^{j} E_{t}\left\{\frac{\lambda_{t+j}}{\lambda_{t}} \Pi_{t, t+j}^{-1}\right\}, \tag{21}
\end{equation*}
$$

${ }_{50}$ while $\Pi_{t, t+j} \equiv\left(P_{t+j} / P_{t}\right)$. Calvo pricing implies the following conventional relation between inflation and the relative price charged by re-optimizing firms:

$$
\begin{equation*}
\frac{P_{t}^{*}}{P_{t}}=\left(\frac{1-\theta \Pi_{t}^{\varepsilon-1}}{1-\theta}\right)^{\frac{1}{1-\varepsilon}} \tag{22}
\end{equation*}
$$

The necessary condition for profit maximization can easily be rewritten as

$$
\begin{equation*}
\frac{P_{t}^{*}}{P_{t}}=\frac{x_{1, t}}{x_{2, t}}, \tag{23}
\end{equation*}
$$

${ }_{55}$ where the auxiliary variables $x_{1, t}$ and $x_{2, t}$ are recursively defined by

$$
\begin{array}{r}
x_{1, t}=\left[C_{m, t}+I_{t}+G_{t}\right]\left(\frac{\varepsilon(1-\tau)}{\varepsilon-1}\right) R M C_{t}+ \\
\beta \theta E_{t}\left\{\frac{\lambda_{t+1}}{\lambda_{t}} \Pi_{t+1}^{\varepsilon} x_{1, t+1}\right\}, \\
x_{2, t}=\left[C_{m, t}+I_{t}+G_{t}\right]+\beta \theta E_{t}\left\{\frac{\lambda_{t+1}}{\lambda_{t}} \Pi_{t+1}^{\varepsilon-1} x_{2, t+1}\right\} . \tag{25}
\end{array}
$$

### 1.3. Aggregation and Market Clearing

After defining aggregate production

$$
\begin{equation*}
Y_{t}=\left[\int_{0}^{1}\left(Y_{t}(i)\right)^{\frac{\varepsilon-1}{\varepsilon}} d i\right]^{\frac{\varepsilon}{\varepsilon-1}} \tag{26}
\end{equation*}
$$

the clearing of the goods market implies

$$
\begin{equation*}
Y_{t}=C_{m, t}+I_{t}+G_{t} . \tag{27}
\end{equation*}
$$

Define the market capital-labor ratio, $k_{t} \equiv\left(K_{m, t}(i)\right) /\left(h_{m, t}(i)\right)$. By equations 60 (16) and (17), the ratio is constant across firms and satisfies

$$
\begin{equation*}
k_{t}=\frac{\alpha_{3} W_{t}}{\left(1-\alpha_{3}\right) r_{t}} \tag{28}
\end{equation*}
$$

By the clearing of the labor market,

$$
\begin{equation*}
h_{m, t}=\int_{0}^{1} h_{m, t}(i) d i \tag{29}
\end{equation*}
$$

Integrating equation (15) over all firms $i$ yields

$$
\begin{equation*}
Y_{t}=\Delta_{t}^{-1} k_{t}^{\alpha_{3}} h_{m, t} \tag{30}
\end{equation*}
$$

where $\Delta_{t}$ denotes relative price dispersion

$$
\begin{equation*}
\Delta_{t} \equiv \int_{0}^{1}\left(\frac{P_{t}(i)}{P_{t}}\right)^{-\varepsilon} d i \tag{31}
\end{equation*}
$$

and evolves according to

$$
\begin{equation*}
\Delta_{t}=(1-\theta)\left(\frac{P_{t}^{*}}{P_{t}}\right)^{-\varepsilon}+\theta \Pi_{t}^{\varepsilon} \Delta_{t-1} \tag{32}
\end{equation*}
$$

${ }_{65}$ It is well known that $\log \left(\Delta_{t}\right)$ is a second-order term and can thus be neglected when the model is approximated to the first order around the non-stochastic steady state. By the clearing of the capital rental market,

$$
\begin{equation*}
K_{m, t}=\int_{0}^{1} K_{m, t}(i) d i \tag{33}
\end{equation*}
$$

which implies

$$
\begin{equation*}
K_{m, t}=k_{t} h_{m, t} \tag{34}
\end{equation*}
$$

Finally, by using (34) into (30), one can obtain the aggregate production function

$$
\begin{equation*}
Y_{t}=\Delta_{t}^{-1}\left(K_{m, t}\right)^{\alpha_{3}}\left(h_{m, t}\right)^{1-\alpha_{3}} \tag{35}
\end{equation*}
$$

as well as the aggregate counterparts of equations 16 and 17 :

$$
\begin{align*}
\alpha_{3} R M C_{t}\left(\frac{\Delta_{t} Y_{t}}{K_{m, t}}\right) & =r_{t}  \tag{36}\\
\left(1-\alpha_{3}\right) R M C_{t}\left(\frac{\Delta_{t} Y_{t}}{h_{m, t}}\right) & =W_{t} \tag{37}
\end{align*}
$$

### 1.4. Equilibrium Definition

In this section we define the equilibrium for a housework model with a KPR utility function, as specified in Section 4.2 of the main text. The equilibrium of
${ }_{75}$ the model is a set of state-contingent plans for variables $C_{t}, C_{m, t}, C_{n, t}, K_{m, t}$, $K_{n, t}, K_{t}, h_{m, t}, h_{n, t}, I_{t}, \lambda_{t}, Y_{t}, \Pi_{t}, \Delta_{t}, \frac{P_{t}^{*}}{P_{t}}, x_{1, t}, x_{2, t}, R M C_{t}, R_{t}, W_{t}$ and $r_{t}^{k}$ that satisfy the following system of equations

$$
\begin{equation*}
C_{t}=\left[\alpha_{1}\left(C_{m, t}\right)^{b_{1}}+\left(1-\alpha_{1}\right)\left(C_{n, t}\right)^{b_{1}}\right]^{\frac{1}{b_{1}}} \tag{38}
\end{equation*}
$$

$$
\begin{gather*}
C_{n, t}=\left(K_{n, t}\right)^{\alpha_{2}}\left(h_{n, t}\right)^{1-\alpha_{2}}  \tag{39}\\
K_{t}=K_{m, t}+K_{n, t}  \tag{40}\\
K_{t+1}=(1-\delta) K_{t}+I_{t}-\frac{\xi}{2}\left(\frac{K_{t+1}}{K_{t}}-1\right)^{2}  \tag{41}\\
\left(\frac{\alpha_{1}}{1-\alpha_{1}}\right)\left(\frac{C_{m, t}}{C_{n, t}}\right)^{b_{1}-1}=\left(\frac{1-\alpha_{2}}{W_{t}}\right)\left(\frac{C_{n, t}}{h_{n, t}}\right)  \tag{42}\\
\left(\frac{\alpha_{1}}{1-\alpha_{1}}\right)\left(\frac{C_{m, t}}{C_{n, t}}\right)^{b_{1}-1}=\left(\frac{\alpha_{2}}{r_{t}^{k}}\right)\left(\frac{C_{n, t}}{K_{n, t}}\right)  \tag{43}\\
\beta E_{t}\left\{\frac{\lambda_{t+1}}{\lambda_{t}}\left[1+\frac{\xi}{K_{t}}\left(\frac{K_{t+1}}{K_{t}}-1\right)\right]^{-1}\right.  \tag{44}\\
\left.W_{n, t}-h_{m, t}\right)=\left(\frac{1-b}{b \alpha_{1}}\right) C_{m, t}^{1-b_{1}} C_{t}^{b_{1}}  \tag{45}\\
\beta E_{t}\left\{\frac{\lambda_{t+1}}{\lambda_{t}}\left(1+R_{t}\right) \Pi_{t+1}^{-1}\right\}=1 \\
\left.\left.r_{t+1}^{k}+\xi\left(\frac{K_{t+2}}{K_{t+1}}-1\right)\left(\frac{K_{t+2}}{K_{t+1}^{2}}\right)\right]\right\}=1,  \tag{46}\\
\frac{P_{t}^{*}}{P_{t}}=\left(\frac{1-\theta \Pi_{t}^{\varepsilon-1}}{1-\theta}\right)^{\frac{1}{1-\varepsilon}}  \tag{47}\\
\frac{P_{t}^{*}}{P_{t}}=\frac{x_{1, t}}{x_{2, t}} \tag{48}
\end{gather*}
$$

$$
x_{1, t}=\left[C_{m, t}+I_{t}+G_{t}\right]\left(\frac{\varepsilon(1-\tau)}{\varepsilon-1}\right) R M C_{t}+\beta \theta E_{t}\left\{\frac{\lambda_{t+1}}{\lambda_{t}} \Pi_{t+1}^{\varepsilon} x_{1, t+1}\right\}
$$

$$
\begin{gather*}
x_{2, t}=\left[C_{m, t}+I_{t}+G_{t}\right]+\beta \theta E_{t}\left\{\frac{\lambda_{t+1}}{\lambda_{t}} \Pi_{t+1}^{\varepsilon-1} x_{2, t+1}\right\}  \tag{50}\\
Y_{t}=C_{m, t}+I_{t}+G_{t}  \tag{51}\\
Y_{t}=\Delta_{t}^{-1}\left(K_{m, t}\right)^{\alpha_{3}}\left(h_{m, t}\right)^{1-\alpha_{3}}  \tag{52}\\
\alpha_{3} R M C_{t}\left(\frac{\Delta_{t} Y_{t}}{K_{m, t}}\right)=r_{t}^{k}  \tag{53}\\
\left(1-\alpha_{3}\right) R M C_{t}\left(\frac{\Delta_{t} Y_{t}}{h_{m, t}}\right)=W_{t}  \tag{54}\\
\Delta_{t}=(1-\theta)\left(\frac{P_{t}^{*}}{P_{t}}\right)^{-\varepsilon}+\theta \Pi_{t}^{\varepsilon} \Delta_{t-1} \tag{55}
\end{gather*}
$$

for all $t$, for given government expenditure. To close the equilibrium definition, we further need a specification for monetary policy and a law of motion for government expenditure.

### 1.5. Frisch System

Following Frisch [9, we define the Frisch system of our housework model with a KPR utility function, as specified in Section 4.2 of the main text. Six equations define the Frisch system:

$$
\begin{align*}
f 1 & =b \alpha_{1}\left(1-h_{n, t}-h_{m, t}\right)^{(1-b)(1-\sigma)} C_{m, t}^{b_{1}-1} C_{t}^{b(1-\sigma)-b_{1}}-\lambda_{t}  \tag{56}\\
f 2 & =(1-b) C_{t}^{b(1-\sigma)}\left(1-h_{n, t}-h_{m, t}\right)^{(1-b)(1-\sigma)-1}-\lambda_{t} W_{t}  \tag{57}\\
f 3 & =\left(1-h_{n, t}-h_{m, t}\right) b\left(1-\alpha_{1}\right)\left(1-\alpha_{2}\right) \frac{C_{n, t}^{b_{1}}}{h_{n, t}}-(1-b) C_{t}^{b_{1}}  \tag{58}\\
f 4 & =b\left(1-\alpha_{1}\right) \alpha_{2}\left(1-h_{n, t}-h_{m, t}\right)^{(1-b)(1-\sigma)} C_{t}^{b(1-\sigma)-b_{1}} \frac{C_{n, t}^{b_{1}}}{K_{n, t}}-r_{t}^{k} \lambda_{t}  \tag{59}\\
f 5 & =\left[\alpha_{1}\left(C_{m, t}\right)^{b_{1}}+\left(1-\alpha_{1}\right)\left(C_{n, t}\right)^{b_{1}}\right]^{\frac{1}{b_{1}}}-C_{t}  \tag{60}\\
f 6 & =\left(K_{n, t}\right)^{\alpha_{2}}\left(h_{n, t}\right)^{1-\alpha_{2}}-C_{n, t} \tag{61}
\end{align*}
$$

where the choice variables are

$$
\begin{equation*}
y_{t}=\left\{C_{m, t}, C_{n, t}, C_{t}, h_{m, t}, h_{n, t}, K_{n, t}\right\} \tag{62}
\end{equation*}
$$

whereas $x_{t}=\left\{\lambda_{t}, W_{t}, r_{t}^{k}\right\}$ are taken as given. Equation $f 1$ coincides with 3 ) and defines the marginal utility of market consumption and $f 2$ is the first-order optimality condition with respect to hours worked on the market (equation (5). Combining the first-order conditions with respect to home goods and hours are interested in

$$
Z_{y, x}=\left(\begin{array}{ccc}
\frac{\partial C_{m, t}}{\partial \lambda_{t}} & \frac{\partial C_{m, t}}{\partial W_{t}} & \frac{\partial C_{m, t}}{\partial r_{t}^{k}}  \tag{63}\\
\frac{\partial C_{n, t}}{\partial \lambda_{t}} & \frac{\partial C_{n, t}}{\partial W_{t}} & \frac{\partial C_{n, t}}{\partial r_{t}^{k}} \\
\frac{\partial C_{t}}{\partial \lambda_{t}} & \frac{\partial C_{t}}{\partial W_{t}} & \frac{\partial C_{t}}{\partial r_{t}^{k}} \\
\frac{\partial h_{m, t}}{\partial \lambda_{t}} & \frac{\partial h_{m, t}}{\partial W_{t}} & \frac{\partial h_{m, t}}{\partial r_{t}^{k}} \\
\frac{\partial h_{n, t}}{\partial \lambda_{t}} & \frac{\partial h_{n, t}}{\partial W_{t}} & \frac{\partial h_{n, t}}{\partial r_{t}^{k}} \\
\frac{\partial K_{n, t}}{\partial \lambda_{t}} & \frac{\partial K_{n, t}}{\partial W_{t}} & \frac{\partial K_{n, t}}{\partial r_{t}^{k}}
\end{array}\right)
$$

We then solve the following system for matrix $Z_{y, x}$

$$
\begin{equation*}
\underset{6 * 6}{J_{y}} \underset{6 * 3}{Z_{y, x}}+\underset{6 * 3}{J}=0 \tag{64}
\end{equation*}
$$

where $J_{y}$ is the Jacobian matrix of function $f$ with respect to the control variables, and $J_{x}$ is the Jacobian matrix of function $f$ with respect to the state variables.

## 2. Simplified model without capital and $G=0$

### 2.1. Algebra of complementarity

Assume $\alpha_{2}=\alpha_{3}=0$ and $K_{n}=K_{m}=G=0$, where variables without time subscript denote a steady state. Hence, $C_{n}=h_{n}$ and the household's optimality
condition 11 becomes

$$
\begin{equation*}
\frac{U_{l}\left(C_{t}, l_{t}\right)}{\left(1-\alpha_{1}\right) U_{C}\left(C_{t}, l_{t}\right)}\left(\frac{C_{n, t}}{C_{t}}\right)^{1-b_{1}}=1 \tag{65}
\end{equation*}
$$

which, combined with the definition of $\lambda,(3)$, can be rewritten as

$$
\begin{equation*}
\frac{U_{l}\left(C_{t}, l_{t}\right)}{\lambda_{t}} \frac{\alpha_{1}}{1-\alpha_{1}}\left(\frac{C_{m, t}}{C_{n, t}}\right)^{b_{1}-1}=1 . \tag{66}
\end{equation*}
$$

The system of equations (3), (5) and (66), together with feasibility constraints (11), fully describes households' intra-temporal optimality. At a zero-inflation steady state $\Pi=1$ and $\Delta=1$, by firms' optimality and by the clearing of all markets $W=1$ and $C_{m}=h_{m}$, which imply, together with equations (1), (3), (5) and (66), all evaluated at the steady state, relations

$$
\begin{gather*}
\lambda=U_{l}(C, l), \quad U_{l}(C, l)=U_{C}(C, l)\left(1-\alpha_{1}\right)\left(\frac{C_{n}}{C}\right)^{b_{1}-1}  \tag{67}\\
U_{l}(C, l)=U_{C}(C, l) \alpha_{1}\left(\frac{C_{m}}{C}\right)^{b_{1}-1}, \quad \alpha_{1}\left(\frac{C_{m}}{C}\right)^{b_{1}}=\frac{h_{m}}{h} \\
\quad\left(1-\alpha_{1}\right)\left(\frac{C_{n}}{C}\right)^{b_{1}}=\frac{h_{n}}{h}, \quad U_{l}(C, l)=U_{C}(C, l) \frac{C}{h}
\end{gather*}
$$

Log-linearization of (3), (5) and (66), respectively, around the steady state yields

$$
\begin{gather*}
\widehat{\lambda}_{t}=(\nu-\gamma) \widehat{C}_{t}+\frac{\nu l}{h} \widehat{l}_{t}+\left(1-b_{1}\right)\left(\widehat{C}_{t}-\widehat{C}_{m, t}\right)  \tag{68}\\
\widehat{W}_{t}=\nu \widehat{C}_{t}+\frac{(\nu-\varphi) l}{h} \widehat{l}_{t}-\widehat{\lambda}_{t} \\
\widehat{\lambda}_{t}=\nu \widehat{C}_{t}+\frac{(\nu-\varphi) l}{h} \widehat{l}_{t}+\left(1-b_{1}\right)\left(\widehat{C}_{n, t}-\widehat{C}_{m, t}\right)
\end{gather*}
$$

where $\widehat{\text { denote log-deviations from the steady state and parameters are defined }}$ as follows:

$$
\begin{array}{r}
\gamma \equiv-\frac{U_{C, C} C}{U_{C}}+\frac{U_{C, l} C}{U_{l}}  \tag{69}\\
\varphi \equiv-\frac{U_{l, l} h}{U_{l}}+\frac{U_{C, l} h}{U_{C}} \\
\nu \equiv \frac{U_{C, l} h}{U_{C}}
\end{array}
$$

Definitions (69) imply the following relations

$$
\begin{equation*}
\frac{U_{C, C} C}{U_{C}}=\nu-\gamma, \quad \frac{U_{l, l} h}{U_{l}}=\nu-\varphi, \quad \frac{U_{C, l} C}{U_{l}}=\nu \tag{70}
\end{equation*}
$$

where the last equality follows from the definition of $\nu$ and relations 67). Moreover, log-linearizing (1) yields

$$
\begin{equation*}
\widehat{C}_{t}=\frac{h_{m}}{h} \widehat{C}_{m, t}+\frac{h_{n}}{h} \widehat{C}_{n, t}, \quad \widehat{l}_{t}=-\frac{h_{m}}{1-h} \widehat{h}_{m, t}-\frac{h_{n}}{1-h} \widehat{h}_{n, t}, \quad \widehat{C}_{n, t}=\widehat{h}_{n, t} \tag{71}
\end{equation*}
$$

Relations 68 and (71) determine $\widehat{C}_{m, t}, \widehat{C}_{n, t}, \widehat{h}_{m, t}, \widehat{h}_{n, t}, \widehat{C}_{t}$ and $\widehat{l}_{t}$, given $\widehat{\lambda}_{t}$ and ${ }^{200} \widehat{W}_{t}$. Solving for $\widehat{C}_{m, t}$ and $\widehat{h}_{m, t}$ yields equations (28) in the main text and the associated coefficients (29). In addition, solving for $\widehat{C}_{t}$ and $\widehat{l}_{t}$, one can find

$$
\begin{equation*}
\left.\eta_{l, \lambda} \equiv \frac{\partial \widehat{l}_{t}}{\partial \widehat{\lambda}_{t}}\right|_{\widehat{W}_{t}}=\frac{\gamma}{\varphi(\nu-\gamma)+\nu \gamma} \frac{h}{l},\left.\quad \eta_{C, \lambda} \equiv \frac{\partial \widehat{C}_{t}}{\partial \widehat{\lambda}_{t}}\right|_{\widehat{W}_{t}}=\frac{\varphi}{\varphi(\nu-\gamma)+\nu \gamma} . \tag{72}
\end{equation*}
$$

We derive restrictions on parameters $\varphi, \gamma$ and $\nu$ such that $\eta_{l, \lambda}$ and $\eta_{C, \lambda}$ are both non-positive, so that consumption and leisure increase with wealth for any given price $W$, and they are thus non-inferior. To this purpose, we use equations 125 (72). The proof draws on Bilbiie [1]. Concavity of the utility function requires $U_{C C} \leq 0, U_{l l} \leq 0, U_{C C} U_{l l}-U_{C l}^{2} \geq 0$, which hold if and only if (i) $\gamma \geq \nu$, (ii) $\varphi \geq \nu$, (iii) $\gamma \varphi \geq \nu(\varphi+\gamma)$, respectively. By (72) and concavity requirements, $\eta_{l, \lambda}$ and $\eta_{C, \lambda}$ are both non-positive if and only if $\gamma \geq 0$ and $\varphi \geq 0$. In fact, if one of the two parameters is negative, either a non-inferiority condition is violated or the last condition for concavity is violated. But if $\gamma$ and $\varphi$ are positive, it is also the case that $\gamma \geq \gamma \varphi /(\gamma+\varphi)$ and $\varphi \geq \gamma \varphi /(\gamma+\varphi)$. Hence, the last condition for concavity, (iii), implies the first two, (i)-(ii). Therefore, $\gamma \geq 0$, $\varphi \geq 0$ and $\nu \leq \gamma \varphi /(\gamma+\varphi)$ are necessary and sufficient conditions for concavity of preferences and joint non-inferiority of consumption and leisure. Finally notice that, by equations (28) in the main text and the associated coefficients (29), market consumption is non-inferior if and only if total consumption is non-inferior.

Let $\eta_{c_{m}, \lambda}, \eta_{h_{m}, \lambda}, \eta_{c_{m}, w}$ and $\eta_{h_{m}, w}$ be the elasticities generated by the model for any given $h_{n}$ and $b_{1}$. Then, a model with $h_{n}=0$ can be calibrated such that values of parameters $\gamma, \varphi$ and $\nu$.

Proof For $h_{n}=0$ the Frisch elasticities reported in Section 3.1 of the paper state

$$
\begin{align*}
\eta_{c_{m}, \lambda} & =-\frac{\varphi}{\varphi(\nu-\gamma)+\nu \gamma}  \tag{73}\\
\eta_{h_{m}, \lambda} & =\eta_{c_{m}, \lambda} \frac{\gamma}{\varphi}  \tag{74}\\
\eta_{c_{m}, w} & =\frac{\nu}{\varphi(\nu-\gamma)+\nu \gamma}  \tag{75}\\
\eta_{h_{m}, w} & =\eta_{c_{m}, w}+\eta_{h_{m}, \lambda} \tag{76}
\end{align*}
$$

From (74) we get

$$
\begin{equation*}
\varphi=\gamma \frac{\eta_{c_{m}, \lambda}}{\eta_{h_{m}, \lambda}} \tag{77}
\end{equation*}
$$

which, combined with $\sqrt[73]{ }$ yields the following expression

$$
\begin{equation*}
\nu=\frac{\gamma \eta_{c_{m}, \lambda}-1}{\eta_{c_{m}, \lambda}+\eta_{h_{m}, \lambda}} \tag{78}
\end{equation*}
$$

Further combining (78) with (75) and 77) yields $\gamma$ as a function of Frisch elasticities, only

$$
\begin{equation*}
\gamma=\frac{\eta_{h_{m}, \lambda}}{\eta_{c_{m}, \lambda} \eta_{h_{m}, \lambda}+\eta_{c_{m}, \lambda} \eta_{c_{m}, w}+\eta_{h_{m}, \lambda} \eta_{c_{m}, w}} \geq 0 \tag{79}
\end{equation*}
$$

Equivalently, we can solve for $\nu$ and $\varphi$

$$
\begin{align*}
\nu & =\frac{\eta_{c_{m}, \lambda}}{\eta_{c_{m}, \lambda}+\eta_{h_{m}, \lambda}} \frac{\eta_{h_{m}, \lambda}}{\eta_{c_{m}, \lambda} \eta_{h_{m}, \lambda}+\eta_{c_{m}, \lambda} \eta_{c_{m}, w}}+\eta_{h_{m}, \lambda} \eta_{c_{m}, w} \\
\eta_{c_{m}, \lambda} & \frac{1}{\eta_{c_{m}, \lambda} \eta_{h_{m}, \lambda}}  \tag{81}\\
\varphi & =\frac{\eta_{c_{m}, \lambda}}{\eta_{c_{m}, \lambda} \eta_{h_{m}, \lambda}+\eta_{c_{m}, \lambda} \eta_{c_{m}, w}+\eta_{h_{m}, \lambda} \eta_{c_{m}, w}} \geq 0
\end{align*}
$$

Equality of the Frisch labor-supply elasticity between the model with and without the home sector follows from (76). Moreover, given $\gamma \geq 0$ and $\varphi \geq 0$, for non-inferiority of preferences we further need to show that $\nu \leq \frac{\gamma \varphi}{\gamma+\varphi}$, where

$$
\begin{equation*}
\frac{\gamma \varphi}{\gamma+\varphi}=\frac{\eta_{c_{m}, \lambda} \eta_{h_{m}, \lambda}}{\left(\eta_{c_{m}, \lambda}+\eta_{h_{m}, \lambda}\right)\left(\eta_{c_{m}, \lambda} \eta_{h_{m}, \lambda}+\eta_{c_{m}, \lambda} \eta_{c_{m}, w}+\eta_{h_{m}, \lambda} \eta_{c_{m}, w}\right)} \tag{82}
\end{equation*}
$$

The necessary condition for non-inferiority reduces to

$$
\begin{equation*}
-\frac{1}{\eta_{c_{m}, \lambda} \eta_{h_{m}, \lambda}}<0 \tag{83}
\end{equation*}
$$

which, given $\eta_{h_{m}, \lambda}>0$ and $\eta_{c_{m}, \lambda}>0$ always holds.
From the canonical form discussed in Section 3.2 in the main text, it follows $\eta_{h_{m}, w}$ also generate the same dynamics for all market variables.

Finally, consider the utility specification
$U\left(C_{t}, l_{t}, X_{t-1}\right)=\frac{\left[C_{t}-\psi\left(1-l_{t}\right)^{\bar{\nu}} C_{t}^{\bar{\gamma}} X_{t-1}^{1-\bar{\gamma}}\right]^{1-\bar{\sigma}}}{1-\bar{\sigma}}, \quad X_{t}=C_{t}^{\bar{\gamma}} X_{t-1}^{1-\bar{\gamma}}, \quad X_{-1}=1$,
and assume $h_{n, t}=0$, so that $C_{t}=C_{m, t}$ and $l_{t}=1-h_{m, t}$. The marginal utility of consumption is defined as
$\lambda_{t}=U_{C}\left(C_{t}, l_{t}, X_{t-1}\right) \Longrightarrow \lambda_{t}=\left[C_{t}-\psi h_{t}^{\bar{\nu}} C_{t}^{\bar{\gamma}} X_{t-1}^{1-\bar{\gamma}}\right]^{-\bar{\sigma}}\left(1-\psi \bar{\gamma} h_{t}^{\bar{\nu}} C_{t}^{\bar{\gamma}-1} X_{t-1}^{1-\bar{\gamma}}\right)$,
and the solution to the households' problem needs to satisfy

$$
\begin{equation*}
\lambda_{t} W_{t}=U_{l}\left(C_{t}, l_{t}, X_{t-1}\right) \quad \Longrightarrow W_{t}=\frac{\bar{\nu} \psi h_{t}^{\bar{\nu}-1} C_{t}^{\bar{\gamma}} X_{t-1}^{1-\bar{\gamma}}}{1-\psi \bar{\gamma} h_{t}^{\bar{\nu}} C_{t}^{\bar{\gamma}-1} X_{t-1}^{1-\bar{\gamma}}} \tag{85}
\end{equation*}
$$

so that equations (84) and (85) replace (3) and (5), respectively, while equation (66) vanishes because of the assumption $h_{n}=0$. As in the housework model analyzed above, we keep assumptions $K_{m} / Y=K_{n} / Y=G / Y=0$. Then, log-linear relations 68) collapse to

$$
\begin{gather*}
\widehat{\lambda}_{t}=(\nu-\gamma) \widehat{C}_{m, t}-\nu \widehat{h}_{m, t}+\frac{U_{C, X} X}{U_{C}} \widehat{X}_{t-1}  \tag{86}\\
\widehat{W}_{t}=\nu \widehat{C}_{m, t}-(\nu-\varphi) \widehat{h}_{m, t}-\widehat{\lambda}_{t}+\frac{U_{l, X} X}{U_{l}} \widehat{X}_{t-1}
\end{gather*}
$$

where $\varphi, \gamma$, and $\nu$, defined as in 69, map into utility parameters as follows,

$$
\varphi=\bar{\nu}+\bar{\gamma}-1, \quad \gamma=\frac{\bar{\gamma}}{\bar{\nu}}(\bar{\nu}+\bar{\gamma}-1), \quad \nu=\bar{\gamma}-\frac{\bar{\sigma} \bar{\nu}}{\bar{\nu}+\bar{\gamma}-1},
$$

and

$$
\frac{U_{C, X} X}{U_{C}}=\frac{\bar{\sigma}(1-\bar{\gamma})}{\bar{\nu}+\bar{\gamma}-1}-\frac{\bar{\gamma}(1-\bar{\gamma})}{\nu}, \quad \frac{U_{l, X} X}{U_{l}}=\frac{\bar{\sigma}(1-\bar{\gamma})}{\bar{\nu}+\bar{\gamma}-1}-(1-\bar{\gamma})
$$

Solving for $C_{m}$ and $h_{m}$ yields equation (31) in the text with coefficients

$$
\begin{equation*}
\eta_{C m, x}=-\eta_{C m, W} \frac{1-\bar{\gamma}}{\bar{\nu}}, \quad \eta_{h m, x}=-\left(\eta_{h m, W}+1\right) \frac{1-\bar{\gamma}}{\bar{\nu}}, \tag{87}
\end{equation*}
$$

that are zero if and only if $\bar{\gamma}=0$.
Given $\left(1-b_{1}\right)^{-1} \in[1,5]$, we choose parameters $\bar{\nu}, \bar{\sigma}$ and $\bar{\gamma}$ in the JR model in order to make the market consumption multiplier, the Frisch elasticity of labor supply and the intertemporal elasticity of substitution equal across the two models. Parameter $\psi$ is calibrated to match hours worked on the market. ${ }^{170}$ For $\left(1-b_{1}\right)^{-1} \in[1,5]$, figure 1 plots the following variables in both models after an exogenous increase in government expenditure normalized to one percentage point of steady-state GDP: fiscal multipliers, percentage deviations of leisure from the steady state, Frisch elasticities $\eta_{C m, W}$ and $\eta_{h m, \lambda}$, and parameter $\bar{\gamma}$. We find some interesting facts. First, matching the market consumption multiplier also implies the same responses of output and hours worked. In addition, for $\left(1-b_{1}\right)^{-1}$ between 1 and 2.25 , values close to the microeconomic estimates, complementarity and wealth effects are also identical across models, but they diverge for large substitutability. In this respect, evidence on time use seems to suggest values of $\bar{\gamma}$ closer to KPR rather than GHH preferences. Finally, irrespective of substitutability, in the model with home production, an increase in consumption expenditure is always associated with a fall in total consumption, as compared to the JR model.







Figure 1: Comparison of fiscal multipliers, the percentage deviations of leisure from the steady state, and the two Frisch elasticities, $\eta_{c m, w}$ and $\eta_{h m, \lambda}$, for different values of the elasticity of substitution between home and market goods, $\left(1-b_{1}\right)^{-1}$, for the housework model where $K_{m} / Y=K_{n} / Y=G / Y=0$, labeled as GHP, and a baseline model with Jaimovich-Rebelo
preferences (labeled as JR Preferences). Parameter $\gamma$ in the JR model is calibrated to match the consumption multiplier in the GHP model.

### 2.2. Inspecting the mechanism

Combining equations (28) in the main text immediately gives equation (32) and

$$
\begin{equation*}
\widehat{\lambda}_{t}=\frac{\eta_{C m, W}}{\eta_{C m, \lambda}} \widehat{W}_{t}-\frac{1}{\eta_{C m, \lambda}} \widehat{C}_{m, t} \tag{88}
\end{equation*}
$$

while a first-order approximation of equation (15) yields

$$
\begin{equation*}
\widehat{\lambda}_{t}=E_{t} \widehat{\lambda}_{t+1}-E_{t} \pi_{t+1}+r_{t}+\log (\beta) \tag{89}
\end{equation*}
$$

Equation (33) follows from combining 88) and 89. Using $\widehat{Y}_{t}=\widehat{C}_{m, t}+\widehat{g}_{t}$ into (28) and (33) to substitute for $\widehat{C}_{m, t}$ implies $^{2}$

$$
\begin{align*}
& \widehat{W}_{t}=\kappa \widehat{Y}_{t}-\kappa \frac{\eta_{h m, \lambda}}{\eta_{C m, \lambda}+\eta_{h m, \lambda}} \widehat{g}_{t},  \tag{90}\\
& \widehat{Y}_{t}=E_{t} \widehat{Y}_{t+1}-\eta_{C m, \lambda}\left(r_{t}-E_{t} \pi_{t+1}+\log (\beta)\right)-\eta_{C m, W}\left(E_{t} \widehat{W}_{t+1}-\widehat{W}_{t}\right) \\
& -\left(E_{t} \widehat{g}_{t+1}-\widehat{g}_{t}\right)
\end{align*}
$$

where $\kappa$ is defined as in equation (34), while from firms' optimality conditions, $\widehat{R M C}_{t}=\widehat{W}_{t}$. At the flexible-price equilibrium $\widehat{R M C}_{t}=0$ and the first equation in (90) imply the expression for $y_{t}^{n}$ displayed in the text, which, substituted for $\widehat{Y}_{t}$ in the second equation yields $r_{t}^{n}$. A conventional Phillips curve is obtained by log-linearizing firms' optimality condition 20 ,

$$
\pi_{t}=\beta E_{t} \pi_{t+1}+\frac{(1-\theta)(1-\theta \beta)}{\theta} \widehat{R M C}_{t}
$$

Using the fact $\widehat{R M C}_{t}=0$ at the flexible-price equilibrium, together with equations (90), the definitions of natural output, the natural interest rate and the output gap yields equations (34). We finally prove the statement in footnote 10. For simplicity define

$$
\bar{\kappa}=\kappa \frac{(1-\theta)(1-\theta \beta)}{\theta} .
$$

Let $r_{t}^{*}, \pi_{t}^{*}$ and $y_{t}^{*}$ satisfy both the IS and the Phillips curve (34). Then, the locally unique solution to the system formed by equations (34) and the interestrate rule $r_{t}=r_{t}^{*}+\phi_{\pi}\left(\pi_{t}-\pi_{t}^{*}\right)$, is $r_{t}^{*}, \pi_{t}^{*}$ and $y_{t}^{*}$ for any $\phi_{\pi}>1$. If indeed $r_{t}^{*}$,

[^1]$\pi_{t}^{*}$ and $y_{t}^{*}$ satisfy both the IS and the Phillips curve (34), the system formed by equations (34) and the interest-rate rule can be written as
\[

\left[$$
\begin{array}{c}
y_{t}-y_{t}^{*}  \tag{91}\\
\pi_{t}-\pi_{t}^{*}
\end{array}
$$\right]=\Omega \mathbf{A} E_{t}\left[$$
\begin{array}{l}
y_{t+1}-y_{t+1}^{*} \\
\pi_{t+1}-\pi_{t+1}^{*}
\end{array}
$$\right]
\]

where

$$
\begin{equation*}
\Omega=\left(\sigma+\bar{\kappa} \phi_{\pi}\right)^{-1} \tag{92}
\end{equation*}
$$

$$
\mathbf{A}=\left[\begin{array}{cc}
\sigma & 1-\phi_{\pi} \beta  \tag{93}\\
\sigma \bar{\kappa} & \bar{\kappa}+\beta \sigma
\end{array}\right]
$$

The system has a unique solution if and only if the eigenvalues of matrix $\mathbf{A}$ are both inside the unit circle. As Bullard and Mitra [4] show, this is the case if and only if $\phi_{\pi}>1$. Hence, $\phi_{\pi}>1$ guarantees that $y_{t}=y_{t}^{*}$ and $\pi_{t}=\pi_{t}^{*}$ is the unique solution. Set

$$
\begin{gathered}
\pi_{t}^{*}=\phi_{g} \widehat{g}_{t}, \\
y_{t}^{*}=\frac{\phi_{g}\left(1-\beta \rho_{g}\right) \theta}{\kappa(1-\theta)(1-\theta \beta)} \widehat{g}_{t} .
\end{gathered}
$$

and let $r_{t}^{*}$ be derived by substituting $\pi_{t}^{*}$ and $y_{t}^{*}$ in the IS equation. It must then be the case that the locally unique solution to equations (34) and $r_{t}=$ $r_{t}^{*}+\phi_{\pi}\left(\pi_{t}-\pi_{t}^{*}\right)$ is $r_{t}^{*}, \pi_{t}^{*}$ and $y_{t}^{*}$.

### 2.3. Welfare: consumption versus expenditure

In this section we study the role of houswork for the impact of government expenditure shocks on welfare. We follow the lines of [2] and use the nonlinear utility function together with the models' equilibrium conditions. Having households' preferences defined over consumption and leisure, where

$$
\begin{align*}
C_{t} & =\left[\alpha_{1} C_{m, t}^{b_{1}}+\left(1-\alpha_{1}\right) C_{n, t}^{b_{1}}\right]^{\frac{1}{b_{1}}}  \tag{94}\\
l_{t} & =1-h_{m, t}-h_{n, t}  \tag{95}\\
C_{n, t} & =h_{n, t} \tag{96}
\end{align*}
$$

the derivative of welfare with respect to $G_{t}$ states as

$$
\begin{aligned}
\frac{d U}{d G} & =\underbrace{U_{c} \alpha_{1}\left(\frac{C_{m, t}}{C_{t}}\right)^{b_{1}-1} \frac{d C_{m}}{d G}}_{\mathrm{A}}+\underbrace{U_{c}\left(1-\alpha_{1}\right)\left(\frac{C_{n, t}}{C_{t}}\right)^{b_{1}-1} \frac{d h_{n}}{d G}}_{\mathrm{B}} \\
& +\underbrace{U_{l} \frac{d l}{d h_{n}} \frac{d h_{n}}{d G}}_{\mathrm{C}}+\underbrace{U_{l} \frac{d l}{d h_{m}} \frac{d h_{m}}{d G}}_{\mathrm{D}}
\end{aligned}
$$

with $\frac{d l}{d h_{n}}=\frac{d l}{d h_{m}}=-1$. From households' optimality conditions we have

$$
\begin{align*}
\frac{U_{l}\left(C_{t}, l_{t}\right)}{U_{c}\left(C_{t}, l_{t}\right)} & =W_{t} \alpha_{1}\left(\frac{C_{m, t}}{C_{t}}\right)^{b_{1}-1}  \tag{97}\\
\frac{U_{l}\left(C_{t}, l_{t}\right)}{U_{c}\left(C_{t}, l_{t}\right)} & =\left(1-\alpha_{1}\right)\left(\frac{C_{n, t}}{C_{t}}\right)^{b_{1}-1} \tag{98}
\end{align*}
$$

such that terms B and C cancel out and we are left with terms A and D, only.

$$
\begin{align*}
\frac{d U}{d G} & =U_{c} \alpha_{1}\left(\frac{C_{m, t}}{C_{t}}\right)^{b_{1}-1} \frac{d C_{m}}{d G}-U_{l} \frac{d h_{m}}{d G} \\
& =U_{c} \alpha_{1}\left(\frac{C_{m, t}}{C_{t}}\right)^{b_{1}-1} \frac{d C_{m}}{d G}-W_{t} U_{c} \alpha_{1}\left(\frac{C_{m, t}}{C_{t}}\right)^{b_{1}-1} \frac{d h_{m}}{d G} \\
& =U_{c} \alpha_{1}\left(\frac{C_{m, t}}{C_{t}}\right)^{b_{1}-1}\left[\frac{d C_{m}}{d G}-W_{t} \frac{d h_{m}}{d G}\right] \tag{99}
\end{align*}
$$

The goods' market clearing condition and the resource constraint, respectively, read

$$
\begin{align*}
Y_{t} & =C_{m, t}+G_{t}  \tag{100}\\
Y_{t} & =h_{m, t} \Delta_{t} \tag{101}
\end{align*}
$$

Combining 100 and 101) then yields

$$
\begin{equation*}
h_{m, t}=\Delta_{t} C_{m, t}+\Delta_{t} G_{t} \tag{102}
\end{equation*}
$$

such that

$$
\begin{equation*}
\frac{d h_{m}}{d G}=\frac{d C_{m}}{d G} \Delta_{t}+\frac{d \Delta}{d G} C_{m, t}+\frac{d \Delta}{d G} G_{t}+\Delta_{t} \tag{103}
\end{equation*}
$$

Finally, combining (99) with 103 yields

$$
\begin{align*}
\frac{d U}{d G} & =U_{c} \alpha_{1}\left(\frac{C_{m, t}}{C_{t}}\right)^{b_{1}-1}\left[\frac{d C_{m}}{d G}-W_{t}\left(\frac{d C_{m}}{d G} \Delta_{t}+\frac{d \Delta}{d G} C_{m, t}+\frac{d \Delta}{d G} G_{t}+\Delta_{t}\right)\right] \\
& =U_{c} \alpha_{1}\left(\frac{C_{m, t}}{C_{t}}\right)^{b_{1}-1} W_{t} \Delta_{t}\left[\left(\frac{1}{W_{t} \Delta_{t}}-1\right) \frac{d C_{m}}{d G}-\frac{C_{m, t}}{\Delta_{t}} \frac{d \Delta}{d G}-\frac{G_{t}}{\Delta_{t}} \frac{d \Delta}{d G}-1\right] \\
& =\lambda_{t} W_{t} \Delta_{t}\left[\left(\frac{1}{W_{t} \Delta_{t}}-1\right) \frac{d C_{m}}{d G}-\frac{C_{m, t}}{\Delta_{t}} \frac{d \Delta}{d G}-\frac{G_{t}}{\Delta_{t}} \frac{d \Delta}{d G}-1\right] \tag{104}
\end{align*}
$$

## 3. Robustness: Modeling Assumptions

### 3.1. Distortionary Taxation and CES Production Functions

In this section, we show that our findings continue to hold in the case of distortionary taxation on capital and labor, and in the more general case of constant elasticity of substitution (CES) production functions both in the home and the market sector

$$
\begin{align*}
C_{n, t} & =\left[\alpha_{2}\left(K_{n, t}\right)^{b_{2}}+\left(1-\alpha_{2}\right)\left(h_{n, t}\right)^{b_{2}}\right]^{\frac{1}{b_{2}}}  \tag{105}\\
Y_{t} & =\left[\alpha_{3}\left(K_{m, t}\right)^{b_{3}}+\left(1-\alpha_{3}\right)\left(h_{m, t}\right)^{b_{3}}\right]^{\frac{1}{b_{3}}} \tag{106}
\end{align*}
$$

Assuming the presence of distortionary taxes on capital and labor, which we assume not to respond to the shock since we focus on deficit spending, the household's budget constraint becomes

$$
\begin{align*}
& E_{t}\left\{Q_{t, t+1} B_{t+1}\right\}+P_{t}\left(C_{m, t}+I_{t}\right) \\
& \quad \leq B_{t}+\left(1-\tau_{h}\right) W_{t} P_{t} h_{m, t}+\left(1-\tau_{k}\right) r_{t}^{k} P_{t} K_{m, t}+\delta \tau_{k} P_{t} K_{m, t}+T_{t} \tag{107}
\end{align*}
$$

Accordingly, the household's intratemporal conditions, 42 - 44, and the Euler equation for the optimal intertemporal allocation of the capital stock, 45, are replaced by

$$
\begin{align*}
\left(\frac{\alpha_{1}}{1-\alpha_{1}}\right)\left(\frac{C_{m, t}}{C_{n, t}}\right)^{b_{1}-1} & =\left(\frac{1-\alpha_{2}}{W_{t}\left(1-\tau_{h}\right)}\right)\left(\frac{C_{n, t}}{h_{n, t}}\right)^{1-b_{2}}  \tag{108}\\
\left(\frac{\alpha_{1}}{1-\alpha_{1}}\right)\left(\frac{C_{m, t}}{C_{n, t}}\right)^{b_{1}-1} & =\left(\frac{\alpha_{2}}{\left(1-\tau_{k}\right) r_{t}^{k}+\delta \tau_{k}}\right)\left(\frac{C_{n, t}}{K_{n, t}}\right)^{1-b_{2}}  \tag{109}\\
W_{t}\left(1-\tau_{h}\right)\left(1-h_{n, t}-h_{m, t}\right) & =\left(\frac{1-b}{b \alpha_{1}}\right) C_{m, t}^{1-b_{1}} C_{t}^{b_{1}} \tag{110}
\end{align*}
$$

$$
\begin{array}{r}
\beta E_{t}\left\{\frac{\lambda_{t+1}}{\lambda_{t}}\left[1+\frac{\xi}{K_{t}}\left(\frac{K_{t+1}}{K_{t}}-1\right)\right]^{-1}\right.  \tag{111}\\
\left.\left[1-\delta+\xi\left(\frac{K_{t+2}}{K_{t+1}}-1\right)\left(\frac{K_{t+2}}{K_{t+1}^{2}}\right)+\left(1-\tau_{k}\right) r_{t+1}^{k}+\delta \tau_{k}\right]\right\}=1
\end{array}
$$

The firms' optimality conditions, (53) and (54), become

$$
\begin{align*}
\alpha_{3} R M C_{t}\left(\frac{K_{m, t}(i)}{Y_{t}(i)}\right)^{b_{3}-1} & =r_{t}^{k}  \tag{112}\\
\left(1-\alpha_{3}\right) R M C_{t}\left(\frac{h_{m, t}(i)}{Y_{t}(i)}\right)^{b_{3}-1} & =W_{t} . \tag{113}
\end{align*}
$$

with (52) being replaced by

$$
\begin{equation*}
Y_{t}=\Delta_{t}^{-1}\left[\alpha_{3}\left(K_{m, t}\right)^{b_{3}}+\left(1-\alpha_{3}\right)\left(h_{m, t}\right)^{b_{3}}\right]^{\frac{1}{b_{3}}} \tag{114}
\end{equation*}
$$

For the definition of the equilibrium with distortionary taxation and CES production functions we further replace (39) by (105). All remaining equilibrium conditions, as defined in Appendix 1.4, remain unaffected. We set tax rates according to the base case in McGrattan, Rogerson and Wright [10, i.e., $\tau_{k}=0.55$ and $\tau_{h}=0.24$, and set $b_{2}=0.269$ and $b_{3}=0.054$ according to the estimates in McGrattan et al. [10]. Figure 2 shows that the relative performance of our 30 model, labeled as "GHP," compared to a counterfactual model, labeled as "No Home Sector," where hours worked and capital in the home sector are set to zero, is unaffected by the presence of distortionary taxes and by the assumption of CES production functions in both sectors.

### 3.2. External Habits in Consumption

In this section we analyze the implications of external habit formation. Households' period utility function now reads

$$
\begin{equation*}
U\left(\tilde{C}_{t}, h_{n, t}, h_{m, t}\right)=\frac{\left[\left(\tilde{C}_{t}\right)^{b}\left(1-h_{n, t}-h_{m, t}\right)^{1-b}\right]^{1-\sigma}-1}{1-\sigma} \tag{115}
\end{equation*}
$$

where

$$
\begin{equation*}
\tilde{C}_{t} \equiv C_{t}-h C_{t-1}^{A} \tag{116}
\end{equation*}
$$



Figure 2: Impulse responses with distortionary taxation, $\tau_{k}=0.55$ and $\tau_{h}=0.24$, and CES production functions both in the market $\left(b_{2}=0.269\right)$ and the home sector $\left(b_{3}=0.054\right)$. All remaining parameters are calibrated as in

Table 1 in the main text.
with $h>0$ being the habit persistence parameter, and $C_{t-1}^{A}$ representing past, aggregate consumption, which is taken as given by the household. The corresponding Lagrangian is now given by

$$
\begin{array}{r}
\mathcal{L}=E_{0} \sum_{t=0}^{\infty} \beta^{t} \frac{\left[\left[\left[\alpha_{1}\left(C_{m, t}\right)^{b_{1}}+\left(1-\alpha_{1}\right)\left(C_{n, t}\right)^{b_{1}}\right]^{\frac{1}{b_{1}}}-h C_{t-1}^{A}\right]^{b}\left(1-h_{n, t}-h_{m, t}\right)^{1-b}\right]^{1-\sigma}-1}{1-\sigma} \\
+\mu_{t}\left[\left(K_{n, t}\right)^{\alpha_{2}}\left(h_{n, t}\right)^{1-\alpha_{2}}-C_{n, t}\right] \\
+\gamma_{t}\left[K_{t}-K_{m, t}-K_{n, t}\right] \\
+\frac{\lambda_{t}}{P_{t}}\left\{B_{t}+W_{t} P_{t} h_{m, t}+r_{t}^{k} P_{t} K_{m, t}+T_{t}-E_{t}\left\{Q_{t, t+1} B_{t+1}\right\}-P_{t} C_{m, t}\right. \\
\left.-P_{t}\left[K_{t+1}+(1-\delta) K_{t}+\frac{\xi}{2}\left(\frac{K_{t+1}}{K_{t}}-1\right)^{2}\right]\right\}
\end{array}
$$

The corresponding first-order conditions are:

$$
\begin{gather*}
\left\{C_{m, t}\right\}: \quad \lambda_{t}=b \alpha_{1}\left(1-h_{n, t}-h_{m, t}\right)^{(1-b)(1-\sigma)} \tilde{C}_{t}^{b(1-\sigma)-1}\left(\frac{C_{m, t}}{C_{t}}\right)^{b_{1}-1}  \tag{117}\\
\left\{C_{n, t}\right\}: \quad \mu_{t}=b\left(1-\alpha_{1}\right)\left(1-h_{n, t}-h_{m, t}\right)^{(1-b)(1-\sigma)} \tilde{C}_{t}^{b(1-\sigma)-1}\left(\frac{C_{n, t}}{C_{t}}\right)^{b_{1}-1}  \tag{118}\\
\left\{h_{m, t}\right\}: \lambda_{t} W_{t}=(1-b) \tilde{C}_{t}^{b(1-\sigma)}\left(1-h_{n, t}-h_{m, t}\right)^{(1-b)(1-\sigma)-1}  \tag{119}\\
\left\{h_{n, t}\right\}: \mu_{t}\left(1-\alpha_{2}\right)\left(\frac{C_{n, t}}{h_{n, t}}\right)=(1-b) \tilde{C}_{t}^{b(1-\sigma)}\left(1-h_{n, t}-h_{m, t}\right)^{(1-b)(1-} \tag{q120}
\end{gather*}
$$

The remaining first-order conditions, (7) - 10), remain unaffected when assuming external habit persistence in households' utility function. Combining these new first-order conditions as in Appendix 1.1 yields the same equilibrium conditions as for our baseline model, except for the expression defining the marginal utility of wealth (47) and the optimality condition solving for the consumptionleisure tradeoff (44)

$$
\begin{equation*}
W_{t}\left(1-h_{n, t}-h_{m, t}\right)=\left(\frac{1-b}{b \alpha_{1}}\right) \tilde{C}_{t}\left(\frac{C_{m, t}}{C_{t}}\right)^{1-b_{1}} \tag{121}
\end{equation*}
$$

The equilibrium with external habit formation is thus defined as in Appendix 1.4 , except for 44 and (47), which are replaced by (121) and (117), respectively, and with $\tilde{C}_{t} \equiv C_{t}-h C_{t-1}$, given that in equilibrium $C_{t-1}=C_{t-1}^{A}$. In the vein of Appendix 1.5 the corresponding Frisch system states as:

$$
\begin{aligned}
f 1 & =b \alpha_{1}\left(1-h_{n, t}-h_{m, t}\right)^{(1-b)(1-\sigma)}\left(C_{t}-h C_{t-1}\right)^{b(1-\sigma)-1}\left(\frac{C_{m, t}}{C_{t}}\right)^{b_{1}-1}-\lambda_{t} \\
f 2 & =(1-b)\left(C_{t}-h C_{t-1}\right)^{b(1-\sigma)}\left(1-h_{n, t}-h_{m, t}\right)^{(1-b)(1-\sigma)-1}-\lambda_{t} W_{t} \\
f 3 & =\left(1-h_{n, t}-h_{m, t}\right) b\left(1-\alpha_{1}\right)\left(1-\alpha_{2}\right) \frac{C_{n, t}^{b_{1}}}{h_{n, t}}-(1-b) C_{t}^{b_{1}-1}\left(C_{t}-h C_{t-1}\right) \\
f 4 & =b\left(1-\alpha_{1}\right) \alpha_{2}\left(1-h_{n, t}-h_{m, t}\right)^{(1-b)(1-\sigma)}\left(C_{t}-h C_{t-1}\right)^{b(1-\sigma)-1} C_{t}^{1-b_{1}} \frac{C_{n, t}^{b_{1}}}{K_{n, t}}-r_{t}^{k} \lambda_{t} \\
f 5 & =\left[\alpha_{1}\left(C_{m, t}\right)^{b_{1}}+\left(1-\alpha_{1}\right)\left(C_{n, t}\right)^{b_{1}}\right]^{\frac{1}{b_{1}}}-C_{t} \\
f 6 & =\left(K_{n, t}\right)^{\alpha_{2}}\left(h_{n, t}\right)^{1-\alpha_{2}}-C_{n, t}
\end{aligned}
$$

where $y_{t}=\left\{C_{m, t}, C_{n, t}, C_{t}, h_{m, t}, h_{n, t}, K_{n, t}\right\}$ are the choice variables, whereas $x_{t}=\left\{\lambda_{t}, W_{t}, r_{t}^{K}, C_{t-1}\right\}$ are taken as given. As for the baseline housework model, we define $f=[f 1 ; f 2 ; f 3 ; f 4 ; f 5 ; f 6]$ and the matrix of unknown derivatives we

$$
Z_{y, x}=\left(\begin{array}{cccc}
\frac{\partial C_{m, t}}{\partial \lambda_{t}} & \frac{\partial C_{m, t}}{\partial W_{t}} & \frac{\partial C_{m, t}}{\partial r_{t}^{k}} & \frac{\partial C_{m, t}}{\partial C_{t-1}}  \tag{122}\\
\frac{\partial C_{n, t}}{\partial \lambda_{t}} & \frac{\partial C_{n, t}}{\partial W_{t}} & \frac{\partial C_{n, t}}{\partial r_{t}^{k}} & \frac{\partial C_{n, t}}{\partial C_{t-1}} \\
\frac{\partial C_{t}}{\partial \lambda_{t}} & \frac{\partial C_{t}}{\partial W_{t}} & \frac{\partial C_{t}}{\partial r_{t}^{k}} & \frac{\partial C_{t}}{\partial C_{t-1}} \\
\frac{\partial h_{m, t}}{\partial \lambda_{t}} & \frac{\partial h_{m, t}}{\partial W_{t}} & \frac{\partial h_{m, t}}{\partial r_{t}^{k}} & \frac{\partial h_{m, t}}{\partial C_{t-1}} \\
\frac{\partial h_{n, t}}{\partial \lambda_{t}} & \frac{\partial h_{n, t}}{\partial W_{t}} & \frac{\partial h_{n, t}}{\partial r_{t}^{k}} & \frac{\partial h_{n, t}}{\partial C_{t-1}} \\
\frac{\partial K_{n, t}}{\partial \lambda_{t}} & \frac{\partial K_{n, t}}{\partial W_{t}} & \frac{\partial K_{n, t}}{\partial r_{t}^{k}} & \frac{\partial K_{n, t}}{\partial C_{t-1}}
\end{array}\right)
$$

We then solve the following system for matrix $Z_{y, x}$

$$
\begin{equation*}
\underset{6 * 6}{J_{y}} \underset{6 * 3}{Z_{y, x}}+\underset{6 * 3}{J_{x}}=0 \tag{123}
\end{equation*}
$$

where $J_{y}$ is the Jacobian matrix of function $f$ with respect to the control variables, and $J_{x}$ is the Jacobian matrix of function $f$ with respect to the state variables.

### 3.3. Internal Habits in Consumption

While in Appendix 3.2 we analyze external habits, in this section we study the implications of internal habit formation. Households' period utility function is defined as

$$
\begin{equation*}
U\left(\overline{C_{t}}, h_{n, t}, h_{m, t}\right)=\frac{\left[\left(\overline{C_{t}}\right)^{b}\left(1-h_{n, t}-h_{m, t}\right)^{1-b}\right]^{1-\sigma}-1}{1-\sigma} \tag{124}
\end{equation*}
$$

where

$$
\begin{equation*}
\overline{C_{t}} \equiv C_{t}-h C_{t-1} \tag{125}
\end{equation*}
$$

with $h>0$ being the habit persistence parameter. Different from the case of external habits, the internal habit stock represents the household's own past consumption, $C_{t-1}$. The corresponding Lagrangian is now given by

$$
\begin{aligned}
& \mathcal{L}=E_{0} \sum_{t=0}^{\infty} \frac{\beta^{t}}{(1-\sigma)}\left\{\left[\left[\alpha_{1}\left(C_{m, t}\right)^{b_{1}}+\left(1-\alpha_{1}\right)\left(C_{n, t}\right)^{b_{1}}\right]^{\frac{1}{b_{1}}}-h\left[\alpha_{1}\left(C_{m, t-1}\right)^{b_{1}}+\left(1-\alpha_{1}\right)\left(C_{n, t-1}\right)^{b_{1}}\right]^{\frac{1}{b_{1}}}\right]^{b}\right. \\
&\left.\left(1-h_{n, t}-h_{m, t}\right)^{1-b}\right\}^{1-\sigma}-1 \\
&+\mu_{t}\left[\left(K_{n, t}\right)^{\alpha_{2}}\left(h_{n, t}\right)^{1-\alpha_{2}}-C_{n, t}\right] \\
&+\gamma_{t}\left[K_{t}-K_{m, t}-K_{n, t}\right]
\end{aligned} \begin{array}{r}
+\frac{\lambda_{t}}{P_{t}}\left\{B_{t}+W_{t} P_{t} h_{m, t}+r_{t}^{k} P_{t} K_{m, t}+T_{t}-E_{t}\left\{Q_{t, t+1} B_{t+1}\right\}-P_{t} C_{m, t}\right. \\
\left.-P_{t}\left[K_{t+1}+(1-\delta) K_{t}+\frac{\xi}{2}\left(\frac{K_{t+1}}{K_{t}}-1\right)^{2}\right]\right\}
\end{array}
$$

The only first-order conditions that are different compared to the external-habit model are the first-order conditions with respect to market and non-market consumption ( $C_{m, t}$ and $C_{n, t}$, respectively).

$$
\begin{array}{r}
\left\{C_{m, t}\right\}: \lambda_{t}=b \alpha_{1}\left(\frac{C_{m, t}}{C_{t}}\right)^{b_{1}-1}\left[l_{t}^{(1-b)(1-\sigma)}\left(\overline{C_{t}}\right)^{b(1-\sigma)-1}-\right.  \tag{126}\\
\left.\beta h l_{t+1}^{(1-b)(1-\sigma)}\left(\overline{C_{t+1}}\right)^{b(1-\sigma)-1}\right]
\end{array}
$$

where $l_{t}=1-h_{n, t}-h_{m, t}$. Different to the baseline model and the case of external habit formation, when the stock of habits is internal the marginal utility of wealth $\left(\lambda_{t}\right)$ is no longer equal to the marginal utility of market consumption $\left(U_{C_{m, t}}\right)$. Moreover, the marginal utility of wealth becomes a dynamic equation, 275 depending on expected marginal utility of market consumption $\left(U_{C_{m, t+1}}\right)$ and on expected consumption levels.

$$
\begin{align*}
&\left\{C_{n, t}\right\}: \mu_{t}=b\left(1-\alpha_{1}\right)\left(\frac{C_{n, t}}{C_{t}}\right)^{b_{1}-1} {\left[l_{t}^{(1-b)(1-\sigma)}\left(\overline{C_{t}}\right)^{b(1-\sigma)-1}-\right.}  \tag{127}\\
&\left.\beta h l_{t+1}^{(1-b)(1-\sigma)}\left(\overline{C_{t+1}}\right)^{b(1-\sigma)-1}\right]
\end{align*}
$$

We can then combine the first-order conditions as in our baseline model (see Appendix 1.1. The resulting equilibrium equations coincide with the ones for our baseline model (and with the ones for the model with external habits), except 280 for the expression for marginal utility of wealth and the optimality condition solving for the consumption-leisure tradeoff.

$$
\begin{align*}
& W_{t}\left(1-h_{n, t}-h_{m, t}\right)=\left(\frac{1-b}{b \alpha_{1}}\right) \overline{C_{t}}\left(\frac{C_{m, t}}{C_{t}}\right)^{1-b_{1}}\left[1-\beta h\left(\frac{l_{t+1}}{l_{t}}\right)^{(1-b)(1-\sigma)}\right. \\
&\left.\left(\frac{\overline{C_{t+1}}}{\overline{C_{t}}}\right)^{b(1-\sigma)-1}\right]^{-1} \tag{}
\end{align*}
$$

Equivalent to the case of external habits, the equilibrium with internal habit formation is thus defined as in Appendix 1.4, except for 44) and 47, which are replaced by 128 and 126 , respectively, and with $\overline{C_{t}} \equiv C_{t}-h C_{t-1}$. The
corresponding Frisch system states as:

$$
\begin{aligned}
f 1= & b \alpha_{1}\left(\frac{C_{m, t}}{C_{t}}\right)^{b_{1}-1}\left[l_{t}^{(1-b)(1-\sigma)}\left(C_{t}-h C_{t-1}\right)^{b(1-\sigma)-1}-\beta h l_{t+1}^{(1-b)(1-\sigma)}\left(C_{t+1}-h C_{t}\right)^{b(1-\sigma)-1}\right] \\
& -\lambda_{t} \\
f 2= & (1-b)\left(C_{t}-h C_{t-1}\right)^{b(1-\sigma)} l_{t}^{(1-b)(1-\sigma)-1}-\lambda_{t} W_{t} \\
f 3= & {\left[l_{t}^{(1-b)(1-\sigma)}\left(C_{t}-h C_{t-1}\right)^{b(1-\sigma)-1}-\beta h l_{t+1}^{(1-b)(1-\sigma)}\left(C_{t+1}-h C_{t}\right)^{b(1-\sigma)-1}\right] } \\
& b\left(1-\alpha_{1}\right)\left(1-\alpha_{2}\right) \frac{C_{n, t}^{b_{1}}}{h_{n, t}}-(1-b) C_{t}^{b_{1}-1}\left(C_{t}-h C_{t-1}\right)^{b(1-\sigma)} l_{t}^{(1-b)(1-\sigma)-1} \\
f 4= & {\left[l_{t}^{(1-b)(1-\sigma)}\left(C_{t}-h C_{t-1}\right)^{b(1-\sigma)-1}-\beta h l_{t+1}^{(1-b)(1-\sigma)}\left(C_{t+1}-h C_{t}\right)^{b(1-\sigma)-1}\right] } \\
& b\left(1-\alpha_{1}\right) \alpha_{2} \frac{C_{n, t}^{b_{1}}}{K_{n, t}} C_{t}^{1-b_{1}}-r_{t}^{k} \lambda_{t} \\
f 5= & {\left[\alpha_{1}\left(C_{m, t}\right)^{b_{1}}+\left(1-\alpha_{1}\right)\left(C_{n, t}\right)^{b_{1}}\right]^{\frac{1}{b_{1}}}-C_{t} } \\
f 6= & \left(K_{n, t}\right)^{\alpha_{2}}\left(h_{n, t}\right)^{1-\alpha_{2}}-C_{n, t} \\
f 7= & l_{t}-\left(1-h_{n, t}-h_{m, t}\right)
\end{aligned}
$$

where $y_{t}=\left\{C_{m, t}, C_{n, t}, C_{t}, h_{m, t}, h_{n, t}, K_{n, t}, l_{t}\right\}$ are the choice variables, whereas $x_{t}=\left\{\lambda_{t}, W_{t}, r_{t}^{K}, K_{t}, C_{t-1}\right\}$ are taken as given. As for the baseline housework model, we define $f=[f 1 ; f 2 ; f 3 ; f 4 ; f 5 ; f 6]$ and the matrix of unknown derivatives we are interested in

$$
Z_{y, x}=\left(\begin{array}{cccc}
\frac{\partial C_{m, t}}{\partial \lambda_{t}} & \frac{\partial C_{m, t}}{\partial W_{t}} & \frac{\partial C_{m, t}}{\partial r_{t}^{k}} & \frac{\partial C_{m, t}}{\partial C_{t-1}}  \tag{129}\\
\frac{\partial C_{n, t}}{\partial \lambda_{t}} & \frac{\partial C_{n, t}}{\partial W_{t}} & \frac{\partial C_{n, t}}{\partial r_{t}^{k}} & \frac{\partial C_{n, t}}{\partial C_{t-1}} \\
\frac{\partial C_{t}}{\partial \lambda_{t}} & \frac{\partial C_{t}}{\partial W_{t}} & \frac{\partial C_{t}}{\partial r_{t}^{k}} & \frac{\partial C_{t}}{\partial C_{t-1}} \\
\frac{\partial h_{m, t}}{\partial \lambda_{t}} & \frac{\partial h_{m, t}}{\partial W_{t}} & \frac{\partial h_{m, t}}{\partial r_{t}^{k}} & \frac{\partial h_{m, t}}{\partial C_{t-1}} \\
\frac{\partial h_{n, t}}{\partial \lambda_{t}} & \frac{\partial h_{n, t}}{\partial W_{t}} & \frac{\partial h_{n, t}}{\partial r_{t}^{k}} & \frac{\partial h_{n, t}}{\partial C_{t-1}} \\
\frac{\partial K_{n, t}}{\partial \lambda_{t}} & \frac{\partial K_{n, t}}{\partial W_{t}} & \frac{\partial K_{n, t}}{\partial r_{t}^{k}} & \frac{\partial K_{n, t}}{\partial C_{t-1}} \\
\frac{\partial l_{t}}{\partial \lambda_{t}} & \frac{\partial l_{t}}{\partial W_{t}} & \frac{\partial l_{t}}{\partial r_{t}^{k}} & \frac{\partial l_{t}}{\partial C_{t-1}}
\end{array}\right)
$$

We then solve the following system for matrix $Z_{y, x}$

$$
\begin{equation*}
\underset{6 * 6}{J_{y}} \underset{6 \times 3}{Z_{y, x}}+\underset{6 * 3}{J}=0 \tag{130}
\end{equation*}
$$

where $J_{y}$ is the Jacobian matrix of function $f$ with respect to the control variables, and $J_{x}$ is the Jacobian matrix of function $f$ with respect to the state
variables.

### 3.4. Real Wage Rigidity

 assume that real wages respond sluggishly to labor market conditions, as result of some unmodelled frictions in labor markets. Specifically, we assume$$
\begin{align*}
w_{t} & =\omega w_{t-1}+(1-\omega) m r s_{t} \\
w_{t} & =\omega w_{t-1}+(1-\omega) \ln \left[\left(\frac{1-b}{b \alpha_{1}}\right)\left(\frac{C_{m, t}^{1-b_{1}} C_{t}^{b_{1}}}{\left(1-h_{n, t}-h_{m, t}\right)}\right)\right] \tag{131}
\end{align*}
$$

where $w_{t}=\ln \left(W_{t}\right)$ represents the natural logarithm of the real wage and $m r s_{t}=-\frac{U_{h m, t}}{U_{c m, t}}$ the logarithm of the marginal rate of substitution between hours worked on the market and market consumption. The equilibrium with real wage rigidities is thus defined as in Appendix 1.4 except for 44 which is replaced by 131 .

## 4. Robustness: Monetary Policy Rules

In this section, we assess the robustness of our findings to two additional monetary policy rules for which we repeat the exercise presented in Section 4.4 of the paper, following Canova and Paustian [6]. In particular, we consider the following monetary policy rules.

- Taylor Rule with Output (in deviation from steady state) and


## Interest Rate Smoother (Rule 1):

$$
\begin{equation*}
\left(1+R_{t}\right)=\left(1+R_{t-1}\right)^{\rho_{m}}\left(\beta^{-1} \Pi_{t}^{\Phi_{\pi}}\left(\frac{Y_{t}}{Y}\right)^{\Phi_{Y}}\right)^{1-\rho_{m}} \tag{132}
\end{equation*}
$$

Among others, this rule has been considered by Del Negro and Schorfheide [7, Rabanal and Rubio-Ramírez [11], Del Negro, Schorfheide, Smets and Wouters [8], and Canova and Paustian [6].

- Simple Taylor Rule with Interest Rate Smoother (Rule 2):

$$
\begin{equation*}
\left(1+R_{t}\right)=\left(1+R_{t-1}\right)^{\rho_{m}}\left(\beta^{-1} \Pi_{t}^{\Phi_{\pi}}\right)^{1-\rho_{m}} \tag{133}
\end{equation*}
$$

We take 50,000 draws from uniform distributions of the following parameters, with their respective bounds: $\theta \in[0.2,0.9], \sigma \in[1,4], \xi \in[0,500], \rho_{m} \in[0,0.9]$, $\Phi_{\pi} \in[1.05,2.5], \rho_{g} \in[0,0.95]$ and $\Phi_{Y} \in[0,0.1]$ for Rule 1. All the other parameters are chosen as in Table 1 in the main text. As it becomes clear from figure 3, the two monetary policy rules do not differ significantly from our main specification.

Figure 3: Robustness analysis on the monetary policy rule. Median responses to a $G$ shock for 50,000 draws from uniform
distributions of the following parameters, with their respective bounds, as summarized in Table 1 in the paper: $\theta \in[0.2,0.9]$,
$\sigma \in[1,4], \xi \in[0,500], \rho_{m} \in[0,0.9], \Phi_{\pi} \in[1.05,2.5], \rho_{g} \in[0,0.95]$ and $\Phi_{Y} \in[0,0.1]$ for Rule 1 . All the other parameters are
chosen as in Table 1 in the main text.


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[^1]:    ${ }^{2}$ Recall that $G / Y=0, C / Y=1$ and $\widehat{g}_{t}=\frac{G_{t}}{Y}$.

