Financial Intermediation, House Prices, and the Welfare Effects of the U.S. Great Recession - **On-line Appendix**

Dominik Menno* and Tommaso Oliviero†

October 19, 2017

1 Mortgage rate spreads

Figure 1 shows the mortgage rate spread (nominal mortgage interest rate over Effective Fed Funds Rate) for different maturities. The picture looks very similar to the evidence presented in panel (d) of Figure 1 in the main text of the paper.

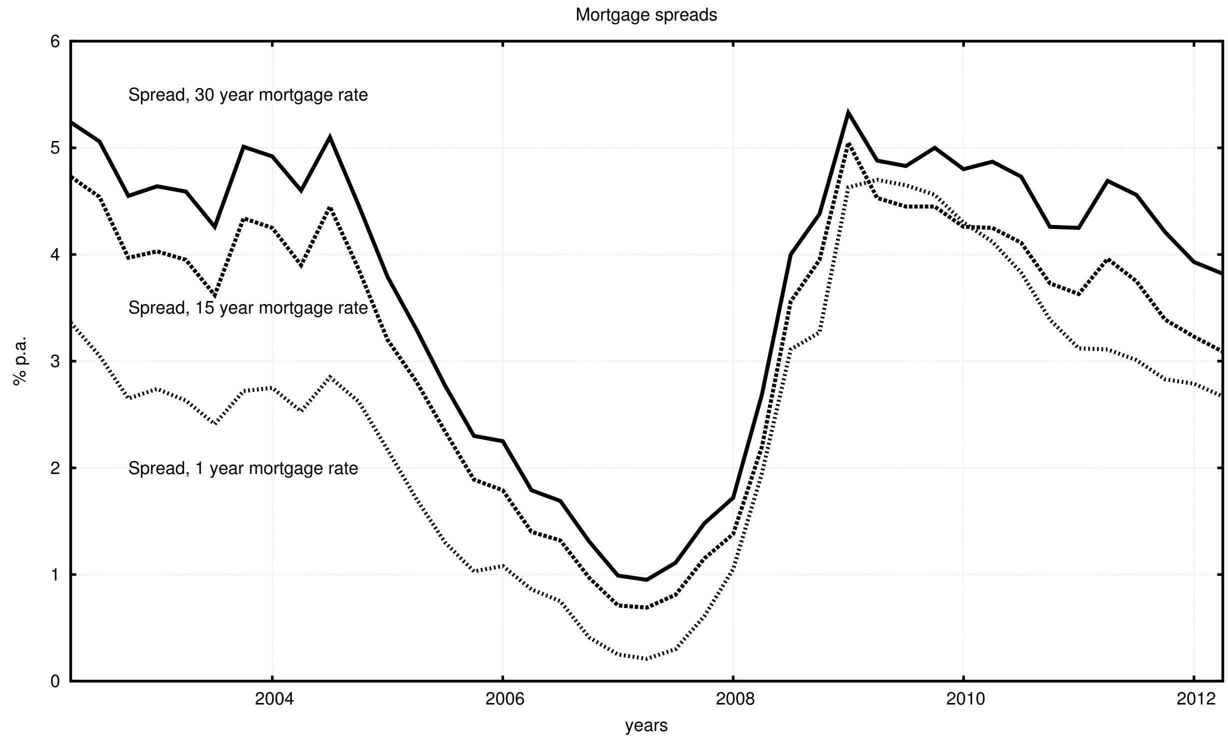
2 Regression analysis

In this section, we employ a simple regression analysis to show that in 2009, relative to 2007, borrowers' housing wealth falls by more than for savers and this difference is statistically and economically significant, also once controlling for ex-ante differences across households. The data comes from the SCF panel 2007- 2009. We use the change in the ratio between housing wealth and income between 2009 and 2007 as a dependent variable, and a dummy variable that indicates if the household is a borrower in 2007 (according to the definition in data appendix of the paper) as a main independent variable. We then uses three control variables: housing wealth, income and age at the household level in 2007. The choice of this set of controls is motivated by Landvoigt, Piazzesi, and Schneider (2015) who find that households' demand for housing is segmented according to differences

*Corresponding author. Affiliation: Aarhus University. E-mail: *dmenno@econ.au.dk*

†Affiliation: University of Naples Federico II and CSEF, Italy. E-mail: *tommaso.oliviero@unina.it*

Figure 1: Mortgage rate spreads for 1, 15, and 30 year mortgage rates



Notes: All spread series are relative to the effective Fed-Funds rate. Mortgage rate series are 30-Year Fixed Rate Mortgage Average in the United States, 15-Year Fixed Rate Mortgage Average in the United States, and 1-Year Adjustable Rate Mortgage Average in the United States. Data Source: FRED, Federal Reserve Economic Data, Federal Reserve Bank of St. Louis: Primary Mortgage Market Survey, Freddie Mac; Effective Federal Funds Rate, Board of Governors of the Federal Reserve System.

in age, income, wealth and access to the credit market. The results in table 1 document that the drop in housing wealth over income is about 70 to 100 percentage points bigger for borrowers than for savers in the considered specifications.

Table 1: Housing wealth drop between borrowers and savers

Dependent variable: Change in housing over income between 2009 and 2007	(1)	(2)	(3)	(4)	(5)
Borrower Dummy	-0.794***	-1.014***	-0.742***	-0.939***	-1.122***
	(0.143)	(0.163)	(0.144)	(0.147)	(0.167)
Age in 2007		-0.0201***			-0.0185***
		(0.00481)			(0.00474)
Income in 2007			0.836***		1.416***
			(0.135)		(0.248)
Housing wealth in 2007				-0.770***	-1.131***
				(0.215)	(0.239)
Constant	-0.120	1.031***	-0.233*	0.221	1.252***
	(0.121)	(0.306)	(0.125)	(0.142)	(0.323)
Observations	14551	14551	14551	14551	14551

Standard errors in parentheses

* $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$

The dependent variable is the difference of the ratio of housing wealth over income between 2007 and 2009. The borrower dummy is a dummy that takes a value equal to one if the household is a borrower. Age is measured by the number of years of the head of the households. Households' income and housing wealth are measured in millions of US dollars 2007. All regressions are weighted using sample weights provided by the SCF.

3 Discussion on the ex-ante vs ex post welfare gains and the role of the time discount factor

This section shows in a representative consumer framework how ex-post consumption equivalent welfare gains of a recession are computed and that ex-post welfare gains are decreasing in the discount factor whenever the shock is transitory, while the ex-ante welfare gain of fluctuations (i.e. the certainty equivalent of removing all cycles) is independent from the discount factor. Furthermore this appendix illustrates that the certainty equivalent of removing cycles is typically smaller as the ex-post welfare gain because the latter depends on the actual realization of the shock and the former depends on the volatility of the shock only. By definition, the volatility is a weighted average of the actual possible shocks, so it is by definition smaller than the size of the actual shocks.

Shock process. Income follows a Markov process with two possible realizations $y_H > y_L$ and symmetric transition matrix

$$\Pi = \begin{pmatrix} \pi & 1-\pi \\ 1-\pi & \pi \end{pmatrix} \qquad \text{with } \pi \in (0,1)$$

The parameter π governs the persistence of the Markov process. Notice that because the transition matrix is symmetric and $0 < \pi < 1$ the ergodic probability distribution is given by $(\tilde{\pi}, 1-\tilde{\pi}) = (0.5, 0.5)$. W.l.o.g. we assume $log(y_H) = \sigma$ and $log(y_L) = -\sigma$ for $\sigma > 0$ such that the unconditional mean is normalized to $E(log(y)) = 0$ and the variance is $Var(log(y)) = \sigma^2$. This implies that $E(y) = e^{\frac{1}{2}\sigma^2}$ where $y = e^{log(y)}$.

Consumer problem. The consumer maximizes expected life time utility s.t. the budget constraint $c_t = y_t \; \forall t > 0$ under the simplifying that there is no borrowing or lending. We focus on log-utility for simplicity but all the results carry over to the more general class of iso-elastic utility functions. The period by period utility function is given by

$$u(c) = log(c).$$

The value function of the representative consumer reads as

$$V(s) = \max_{c(s)} \left\{ u(c(s)) + \beta E[V(s')|s] \qquad s.t. \qquad c(s) \leq y(s) \right\} \qquad s = L, H$$

Since strictly concave utility function budget constraint will always bind and therefore the value function solves (in matrix notation)

$$V = u(y) + \beta \Pi V \qquad V = \begin{pmatrix} V(y_L) \\ V(y_H) \end{pmatrix} \qquad u(y) = \begin{pmatrix} u(y_L) \\ u(y_H) \end{pmatrix}$$

The solution is given by

$$V = (I_2 - \beta\Pi)^{-1} u(y)$$

where I_2 is the 2×2 identity matrix. Since there are two states only we can write this explicitly as

$$V = \frac{1}{(1-\beta)(1+\beta(1-2\pi)}\begin{pmatrix} 1-\beta\pi & \beta(1-\pi) \\ \beta(1-\pi) & 1-\beta\pi \end{pmatrix}\begin{pmatrix} u(y_L) \\ u(y_H) \end{pmatrix}$$

Welfare gains of a recession. The consumption equivalent welfare gain from moving from state H to L is defined as

$$V(y_L) = V(y_H, \lambda)$$

where λ is the compensation needed in percent of consumption in all periods to make the household indifferent between getting the low income and staying in the high income state

$$V(y_H, \lambda) = \frac{\beta(1-\pi)u(y_L(1+\lambda)) + (1-\beta\pi)u(y_H(1+\lambda))}{(1-\beta)(1+\beta(1-2\pi)}$$

Note that if $\lambda < 0$ then we call it a welfare loss of the recession because agents in the low state are worse off than in the high state and when $\lambda > 0$ then we call it a welfare gain of the recession. As we will show below given the specification here that generically $\lambda < 0$ whenever $y_L < y_H$ and $\gamma \geq 1$.

As we have assumed log utility, we can write $u(y(1+\lambda)) = \log(1+\lambda) + u(y)$. Using this, λ solves

$$V(y_L) = \frac{log(1+\lambda)(\beta(1-\pi) + (1-\beta\pi)) + \beta(1-\pi)log(y_L) + (1-\beta\pi)log(y_H)}{(1-\beta)(1+\beta(1-2\pi)}.$$

Rearranging gives

$$1 + \lambda = e^{(1-\beta)(V(y_L)-V(y_H))}$$

Notice that given the specification of $u(c)$, we have whenever $y_L < y_H$ we have that $|V(y_L)| < |V(y_H)|$ or $V(y_L) - V(y_H) < 0$. This means that in general $\lambda < 0$, so that the household is worse off in the recession. With other words we would need to take away consumption in the high state in order to make the household indifferent to the recession state.

The absolute value of ex post welfare gains of recession is decreasing in β while ex-ante welfare gains are not a function of the discount factor. It follows from discussion above that ex post welfare gains are a function of β and the higher β the smaller the absolute value of λ. With log utility, the ex-post welfare gain can be written explicitly as

$$log(1+\lambda) = \frac{1-\beta}{1+\beta(1-2\pi)}(-\Delta y)$$

where $\Delta y = log(y_H) - log(y_L) = 2\sigma$, so that up to first order approximation (using $log(1+\lambda) \simeq \lambda$) we have

$$\lambda \simeq -\frac{1-\beta}{1+\beta(1-2\pi)}2\sigma.$$

Therefore the ex-post welfare gain of a recession is always negative as long as $\beta < 1$. We can now show that the absolute value of the welfare gain is decreasing in β, as long as the shock process has no absorbing state, i.e. $0 < \pi < 1$:

$$\frac{\partial|\lambda|}{\partial\beta} = -\frac{2(1-\pi)}{(1+\beta(1-2\pi))^2}|2\sigma| < 0$$

Ex ante welfare gains of removing fluctuations (Certainty equivalent). Lucas (1987) defines (ex-ante) welfare gains of removing fluctuations in consumption as the certainty equivalent in consumption the household is to compensated with in a world with fluctuations to make her indifferent to a world without any fluctuations and just consuming the average consumption. Formally, we have

$$\tilde{\pi}V(y_H(1+\lambda^{CE})) + (1-\tilde{\pi})V(y_L(1+\lambda^{CE})) = V(E(y))$$

or

$$\frac{0.5}{1-\beta}\left[log(y_H) + log(1+\lambda^{CE})) + log(y_L) + log(1+\lambda^{CE}))\right] = \frac{1}{1-\beta}log(E(y))$$

where we have used the fact that $\tilde{\pi} = 0.5$ and $V(E(y)) = \frac{1}{1-\beta}u(E(y))$. Notice that given the assumptions on the stochastic process of income we have $E(log(y)) = 0$ and $log(E(y)) = \frac{1}{2}\sigma^2$. Therefore, λ^{CE} solves (up to first order approximation)

$$\lambda^{CE} = \frac{1}{2}\sigma^2$$

Comparing ex-ante and ex-post welfare costs. The above discussion illustrates that the consumption gain of removing all cycles permanently depends on the variance of the shock. Removing cycles permanently is increasing consumption permanently and therefore does not depend on discount factor of the household. It also depends only on the expected variation of the shock, and not the actual realizations of the shock. In contrast, the ex-post welfare gains of a recession depend on the actual size of the shock and the persistence of the shock. The ex-post welfare gains also depend on the discount factor because this determines the willingness of households to smooth the shock over time. Therefore, for the calibrated parameter values as used in this paper, the ex-post welfare gains are - depending on the actual value of the discount factor - one to two orders of magnitude larger than the ex-ante welfare gains of removing cycles (certainty equivalent).

Numerical illustration. This section shows how welfare gains expressed in consumption equivalents depend on the time discount factor. We do so because households with different time discount factors and facing temporary but persistent shocks will have different consumption equivalent welfare gains, even though the drop in their consumption is the same. With other words, the welfare gains expressed in consumption equivalents that we report in the main text are in general a function of β, the households' discount factor. Hence, the fact that borrowers and savers have different welfare gains in the recession, as we document in the main text, may then be just a reflection of the differences in their discount factor. In this section, we document how much of the difference in welfare gains is driven by differences in discount factors, given the parameter values in the benchmark calibration of the paper. To keep the analysis as simple as possible, we use a representative agent model and report welfare gains of a 5 percent drop in consumption for different time discount factors. We assume that exogenous processes and all other parameters are the same as in the benchmark calibration.

Note that in a representative agent framework with exogenous housing supply, the housing consumption is constant over time.

Table 2: Welfare gains of a 5% drop in consumption as a function of the discount factor (in percent)

Discount factor (β)	0.96	0.995	ratio (col 2/col 3)
Risk aversion			
$\gamma = 1$	-1.50	-0.25	6.11
$\gamma = 3$	-1.53	-0.25	6.05
$\gamma = 5$	-1.57	-0.26	6.00

Table 2 summarizes the results of this exercise. Column 2 reports the welfare losses of a 5 percent drop in consumption for households with a discount factor of $\beta = 0.96$ (equal to the calibrated discount factor for borrowers in the heterogeneous household model in the main text) and column 3 reports the welfare losses of a 5 percent consumption drop for households with a discount factor of $\beta = 0.995$ (equal to the calibrated discount factor for savers in the main text). Consumption in this simulation follows the same stochastic process as the process for income as in the calibration of the main text. Notice, however, that we assume that there is no housing consumption in the utility function. It is evident that the welfare losses of more impatient households are between 6 times larger than the welfare losses of more patient households. This analysis is useful to understand what difference in welfare gains we should expect just based on the difference in discount factors. Recall that in the benchmark simulations for the heterogeneous household model, borrowers lose more than 16 times more than savers. Comparing these numbers, we conclude that only a part of the difference in the consumption equivalent welfare gains as reported in the main text is driven by differences in discount factors.

Table 3: (Ex-ante) welfare gains of removing all consumption risk as a function of the discount factor (in percent)

Discount factor (β)	0.96	0.995	ratio (col 2/col 3)
Risk aversion			
$\gamma = 1$	-0.02	-0.02	1.00
$\gamma = 3$	-0.05	-0.05	1.00
$\gamma = 5$	-0.09	-0.09	1.00

Table 3 summarizes the consumption equivalent welfare gain of removing consumption risk permanently. Column 2 reports the certainty equivalent for households with a discount factor of

$\beta = 0.96$ (equal to the calibrated discount factor for borrowers in the heterogeneous household model in the main text) and column 3 reports for households with a discount factor of $\beta = 0.995$ (equal to the calibrated discount factor for savers in the main text). Consumption in this simulation follows the same stochastic process as the process for income as in the calibration of the main text. Again notice, that here we assume that there is no housing consumption in the utility function. Given the calibration of the income process in the main text, the standard deviation of income is equal to 1.8 percent. So from the formula derived above it follows that for $\gamma = 1$ (log utility) the certainty equivalent consumption gain of removing all risk should be approximately equal to $0.018^2/2$ or 0.0162 percent. This is very close to the exact computed value in row 1 of the table. It is evident that the certainty equivalent varies with the degree of risk aversion as shown by Lucas (1987) but it does not vary with the level of discount factors.

4 Numerical Details

This appendix describes the numerical solution of the equilibrium policy functions for the competitive equilibrium. The algorithm employed is an adoption of the time-iteration procedure with linear interpolation used in Grill and Brumm (2014). As we have only two agents, a fine grid for wealth is enough to deliver satisfactorily small Euler errors. For this reason, we do not adapt the grid around the points where the collateral constraint is binding, as proposed by Grill and Brumm (2014).

4.1 Equilibrium conditions

We want to describe the equilibrium in our economy in terms of policy functions that map the current state into current policies. Furthermore, we want to focus on recursive mappings - that is, time-invariant functions that satisfy the period-by-period first-order equilibrium conditions. In what follows, we characterize these equilibrium conditions in every detail. For each agent $i = b, s$, denote by $\nu_i(w, z)$ the Lagrange multiplier with respect to her budget constraint and by $\phi_i(w, z)$ the Kuhn- Tucker multiplier attached to her collateral constraint. In addition, we treat saving and debt as two separate assets: saving is an asset in which the agent can only take long positions, $s_i \geq 0$; debt (loans) is an asset with return R_L in which agents can only take short positions, $l_i \leq 0$.

Denote the Kuhn-Tucker multipliers attached to these inequalities as χ_i and μ_i, respectively. Then, for each tuple consisting of wealth and exogenous state today $\sigma = (w, z)$, the (time-invariant) policy and pricing functions have to satisfy the following system of equations (we will show below how to solve for these time-invariant functions):

- Agent's first order conditions

$$u_{i1}(c_i(\sigma), h_i(\sigma)) - \nu_i(\sigma) = 0$$
$$-\nu_i(\sigma) + \beta^i E[\nu_i(\sigma^+)|\sigma]R(\sigma) + \chi_i(\sigma) = 0, \qquad i = s, b$$
$$-\nu_i(\sigma) + \beta^i E[\nu_i(\sigma^+)|\sigma]R_L(\sigma) + \phi_i(\sigma) - \mu_i(\sigma) = 0$$
$$-\nu_i(\sigma)(q(\sigma) + \eta_i(h_i(\sigma) - \bar{h}_i)) + u_{i2}(c_i(\sigma), h_i(\sigma)) + \beta^i E[\nu_i(\sigma^+)q(\sigma^+)]|\sigma] + \phi_i(\sigma)mq(\sigma) = 0$$

- Agent's budget constraints

$$(1-\alpha)y(z) + \Upsilon_b(\sigma) + \frac{\omega \cdot q(\sigma)}{n_b} - l_b(\sigma) - s_b(\sigma) - q(\sigma)h_b(\sigma) - c_b(\sigma) - \frac{\eta_b}{2}(h_b(\sigma) - \bar{h}_b)^2 = 0$$
$$\left(1 - \alpha + \frac{\alpha}{n_s}\right) y(z) + \Upsilon_s(\sigma) + \frac{(1-\omega) \cdot q(\sigma)}{n_s} - l_s(\sigma) - s_s(\sigma) - q(\sigma)h_s(\sigma) - c_s(\sigma) - \frac{\eta_s}{2}(h_s(\sigma) - \bar{h}_s)^2 = 0$$

  NB: Here we have already used the definition for the borrower's wealth share and rewritten the budget constraints in these terms (see below the law of motion for wealth share as a reminder of how we defined the wealth share).

- Zero profits in the financial sector

$$\theta(z) \cdot R_L(\sigma) - R(\sigma) = 0$$

- Market clearing in housing and financial sector

$$n_s h_s(\sigma) + n_b h_b(\sigma) - 1 = 0$$
$$n_b l_b(\sigma) + n_s l_s(\sigma) + \theta(z) \cdot (n_b s_b(\sigma) + n_s s_s(\sigma)) = 0$$

- (Per-capita) Transfers

$$\Upsilon_s(\sigma) - \frac{(1-\theta(z))(n_b s_b(\sigma) + n_s s_s(\sigma))}{n_s} - \frac{\eta_s}{2}(h_s(\sigma) - \bar{h}_s)^2 = 0$$
$$\Upsilon_b(\sigma) - \frac{\eta_b}{2}(h_b(\sigma) - \bar{h}_b)^2 = 0$$

- Implicit "Law of motion" for borrower's wealth share

$$w^+(\sigma, z^+) \equiv n_b \frac{R_L(\sigma) l_b(\sigma) + R(\sigma) s_b(\sigma) + q(w^+(\sigma, z^+), z^+) h_b(\sigma)}{q(w^+(\sigma, z^+), z^+)}.$$

- Normalization of population shares

$$n_b + n_s = 1$$

- Complementary slackness conditions

$$\mu_i(\sigma) \geq 0, d_i(\sigma) \geq 0, \quad \mu_i(\sigma) \perp d_i(\sigma)$$
$$\chi_i(\sigma) \geq 0, s_i(\sigma) \geq 0, \quad \chi(\sigma) \perp s_i(\sigma), \qquad i = s, b$$
$$\phi_i(\sigma) \geq 0, CC_i(\sigma) \geq 0, \quad \phi_i(\sigma) \perp CC_i(\sigma)$$

where $CC_i(\cdot)$ is the collateral constraint of agent i, that is,

$$CC_i(\sigma) \equiv R_L(\sigma) l_i(\sigma) + mq(\sigma) h_i(\sigma) \geq 0$$

4.2 Algorithm

The structure of the above period-by-period equilibrium conditions can be summarized as follows: Given a guess for the policy and pricing functions in the next period - denoted by f^{prime} - we can compute the expectations in the agents' first order conditions. The functions that map current states to current policies - denoted by f - are then obtained by solving the static system of non-linear given in the previous subsection. More formally, the structure of the problem can be summarized as follows. For all tuples $\sigma = (w, z)$, we have

$$\psi(f^{prime})(\sigma, f(\sigma), \mu(\sigma)) = 0, \qquad \zeta(\sigma, f(\sigma)) \geq 0 \perp \mu(\sigma) \geq 0.$$

The system of equations $\psi[f^{prime}](\cdot)$ contains first order conditions of agents and the financial sector and market clearing conditions. The function $\zeta(\cdot)$ contains the sign restrictions and collateral constraints. $\mu(\cdot)$ denotes the respective Kuhn-Tucker multipliers. A recursive policy function f then solves $\psi[f](\sigma, f(\sigma)\mu(\sigma)) = 0$ such that the complementary slackness conditions are satisfied. The time iteration algorithm defined below finds the approximate recursive policy function iteratively.

In each iteration, taking as given a guess for f^{prime}, we obtain f by solving the above system of equations and then updating our guess by interpolating the obtained policy function on the implicitly defined next period wealth. The following box summarizes our algorithm in a form of Pseudo-code:

1. Select a grid $\mathcal{W}$, an initial guess f^{init} and an error tolerance ϵ. Set $f^{prime} = f^{init}$.
2. Make one time-iteration step:
   (a) For all $\sigma = (w, z)$, where $w \in \mathcal{W}$, find the function $f(\sigma)$ that solves
   $$\psi(f^{prime})(\sigma, f(\sigma), \mu(\sigma)) = 0, \qquad \zeta(\sigma, f(\sigma)) \geq 0 \perp \mu(\sigma) \geq 0.$$
   (b) Use the solution f and the guess f^{prime} to update wealth tomorrow and interpolate f on the obtained values for wealth tomorrow.
3. If $||f - f^{prime}|| < \epsilon$, go to step 4. Else set $f^{prime} = f$ and repeat step 2.
4. Set numerical solution $\tilde{f}$ equal to the solution of the infinite horizon problem, $\tilde{f} = f$.

In the reported simulations in the main text, we set ϵ so that the relative difference between f and f' is smaller than 1 percent. We also have experimented with smaller relative errors but it had no significant effects on the quantitative results of the paper.

4.3 Deterministic steady state

The deterministic steady state, when the lending constraint is binding ($\chi_s > 0$) and the collateral constraint is slack ($\psi_b = 0$). Assumption needed for this steady state to exist: $\beta_b < \theta\beta_s$, where $\theta = E(\theta_t)$. Note that this condition is satisfied in all of our calibrations.

$$
\begin{aligned}
R_L &= \frac{1}{\beta_b} \\
R &= \theta R_L \\
R &= \frac{1}{\beta_s}\left(1 + \frac{\chi_s}{\nu_s}\right) \\
s &= \bar{s} \\
l_b &= -\frac{n_s}{n_b}\bar{s} \\
c_b &= y(1-\alpha)\left(1 - (R_L - 1)\frac{n_b}{n_s}\frac{\bar{s}}{(1-\alpha)y}\right) \\
c_s &= y(1-\alpha+\alpha/n_s)\left(1 + (\theta R_L - 1)\frac{\bar{s}}{(1-\alpha+\alpha/n_s)y}\right) \\
\frac{qh_b}{(1-\alpha)y} &= \frac{1}{1-\beta_b}\frac{1-\phi_b}{\phi_b}\left(1 - (R_L - 1)\frac{n_b}{n_s}\frac{\bar{s}}{(1-\alpha)y}\right) \\
\frac{qh_s}{(1-\alpha+\alpha/ns)y} &= \frac{1}{1-\beta_s}\frac{1-\phi_s}{\phi_s}\left(1 + (\theta R_L - 1)\frac{\bar{s}}{(1-\alpha+\alpha/n_s)y}\right) \\
\nu_s &= u_1(c_s, h_s) \qquad n_s h_s + n_b h_b = 1
\end{aligned}
$$

The deterministic steady state when the lending constraint is slack $\chi_s = 0$ and the collateral constraint is binding ($\psi_b > 0$).

$$
\begin{aligned}
R_L &= \frac{1}{\beta_b}\left(1 + \frac{\psi_b}{\nu_b}\right) \\
R &= \theta R_L \\
R &= \frac{1}{\beta_s} \\
s &= \bar{s} \\
l_b &= -\frac{mqh_b}{R_L} \\
s_s &= -\frac{n_b}{n_s}\frac{1}{\theta}l_b \\
c_b &= y(1-\alpha)\left(1 - (R_L - 1)m\frac{qh_b}{y(1-\alpha)}\right) \\
c_s &= y(1-\alpha+\alpha/n_s)\left(1 + (R-1)\frac{s_s}{y(1-\alpha+\alpha/n_s)}\right) \\
\frac{qh_b}{(1-\alpha)y} &= \frac{1}{1-\beta_b}\frac{1-\phi_b}{\phi_b}\left(1 - (R_L - 1)m\frac{qh_b}{y(1-\alpha)}\right) \\
\frac{qh_s}{(1-\alpha+\alpha/ns)y} &= \frac{1}{1-\beta_s}\frac{1-\phi_s}{\phi_s}\left(1 + (R-1)\frac{s_s}{y(1-\alpha+\alpha/n_s)}\right) \\
\nu_s &= u_1(c_s, h_s) \qquad n_s h_s + n_b h_b = 1
\end{aligned}
$$

Calibration Targets used pre-2001

$$
\begin{aligned}
R_L &= 1.04 \\
\theta &= 0.98 \\
hwy_b &= \frac{qh_b}{(1-\alpha)y} = 2.5 \\
hwy_s &= \frac{qh_b}{(1-\alpha)y} = 2.5 \\
debtinc_b &= -\frac{R_L l_b}{(1-\alpha)y} = 1.2 \\
\alpha &= 0.27 \text{ (together with } n_b = 0.4 \text{ from SCF)}
\end{aligned}
$$

Calibration Targets used post-2001

$$R \simeq 1.005$$
$$LTV_b = \frac{R_L d_b}{q h_b} = 0.68 \qquad \text{(in 2007)}$$

Parameter values With the targets above can solve for following parameters. We use these as initial guesses for parameter values used in the calibration below:

$$y = 1 \qquad \text{(Normalization)}$$
$$\beta_s = 0.995 \qquad \text{(so that } R \text{ close to 1 if savings constr. not binding)}$$
$$\beta_b = 1/1.04$$
$$\bar{s} = \frac{n_b}{n_s}\frac{(1-\alpha)}{R_L}1.2$$
$$\phi_b = \left(1 + \frac{(1-\beta_b)hw_b}{1 - \frac{R_L - 1}{R_L} debtinc_b}\right)^{-1}$$
$$\phi_s = \left(1 + \frac{(1-\beta_s)hw_s}{\left(1 + (\theta R_L - 1)\frac{\bar{s}}{(1-\alpha+\alpha/n_s)}\right)}\right)^{-1}$$

4.4 Simulation of the model: transition towards unrestricted lending and Great Recession

For any parameter combination we proceed in the following steps to obtain the simulated time series 1998 - 2013 as shown in the figures of the paper.

Given a vector of parameters, we simulate N time series that have the length of $T = 24$ years (1990-2013). For each simulation i:

1. Simulate the economy with the lending constraint ($s_t \leq \bar{s}$) for a long time, and cut out the first 1000 periods so to make sure that initial conditions do not matter. The assumption here is that the U.S. economy previous to 2004 was in the steady state equilibrium of the lending restricted economy. Take the last 14 years of this simulated series to obtain a time series that

correspond to 1990 - 2003.

2. In period 0 (that corresponds to year 2004 in the data), simulate the model **without lending constraint on savers** by using the equilibrium law of motion for the lending unrestricted economy. With other words, the constraint $s_t \leq \bar{s}$ is removed after period 0. Or you could think of setting $\bar{s}$ to a very large number, so that the constraint does never bind in the whole state space. The initial condition for the borrowers' wealth share (the endogenous state variable in this model) is the wealth share in the last period of the restricted economy, obtained from the previous step.

3. At the same time feed into the model income and financial intermediation shocks as shown in figure 3 in the main text. The shock sequence from 2004 to 2013 is for income and financial intermediation, respectively: (y_H, θ_H) for $t = 2004 - 2008$, (y_L, θ_L) for $t = 2009$ and unrestricted in and after 2010. The latter means, in some simulations income/fin. intermediation will stay low also after the recession or return to the high state, according to the probabilities in the transition matrix. This makes sure that the variables return to their average when averaging over many simulations.

The time series and the moments of variables that we show in the figures and tables in the main text are then averages over N simulations, that is:

$$\bar{x}_t = \frac{1}{N} \sum_{i=1}^{N} \tilde{x}_{it} \qquad\qquad t = 1990, \ldots, 2013$$

where x_{it} is the value of variable x in period t, obtained from the simulation i. When we show ratios of the endogenous variables, then we report the ratio of the averages. For example the time series of the debt to income ratio for borrowers is computed as $\frac{\bar{R}_{Lt}\bar{d}_{bt}}{(1-\alpha)\bar{y}_t}$ where variables with a bar are averages over N simulations.

Robustness: does the averaging matter for the results? The reason why we average over many simulations is the fact that we have two point Markov chains as exogenous processes. Therefore, if we want to keep income or financial 'at the average' , we do not have an exogenous state for this, we just have high and low. In theory, this could be an issue for the welfare estimates in the paper but only to the extent that it affects the dynamics of borrowers' wealth share, the endogenous state variable in the model. Recall that the welfare estimates come from the change in the value function of the agents. The value function of agent i depends on the exogenous processes and the endogenous wealth share. In the paper we actually show the consumption equivalent welfare gains averaged over many simulations N, i.e. $\bar{\lambda}_t$. Notice we have compared this number with computing the welfare gains in the following way

$$\lambda_i = \begin{cases} \exp((1-\beta_i)(V_i(y_L,\theta_L,\bar{\omega}_{2009}) - V_i(y_H,\theta_H,\bar{\omega}_{2008}))) & \gamma = 1 \\ \left(\frac{V_i(y_L,\theta_L,\bar{\omega}_{2009})}{V_i(y_H,\theta_H,\bar{\omega}_{2008})}\right)^{\frac{1}{1-\gamma}} - 1 & \gamma \neq 1 \end{cases} \tag{1}$$

The results were not affected at all to whether averaging over the consumption equivalent welfare costs or evaluating the welfare costs using the average wealth share.

The second robustness check we did, was to simulate only one iteration, so that $N = 1$. In this simulation, we assumed that income and financial intermediation are in the high state forever, that is (y_H, θ_H), except during the Great recession (in period 2009) where financial intermediation and income drops for one period (y_L, θ_L). The resulting welfare estimates were very similar to the benchmark. Of course, the dynamics of the wealth share looks slightly different as the spread is low for a long time period before the recession, so importantly debt to income is slightly higher before the recession hits. This means that the implied dynamics of the wealth share are affected. Nevertheless, the difference are not large, therefore the consumption equivalent welfare costs are very similar to the benchmark in the paper. The reason is that the calibrated income process is asymmetric. The high state is much more likely than low states. Therefore the high income state is much closer to the average than the low income state. Therefore we should expect that this robustness check will not affect the wealth dynamics to a large extend. Also notice that what matters for the welfare estimates in 2009 are (i) the actual realizations of prices and quantities in 2009 and (ii) the expectation of households for future quantities and prices after 2009. Since both exogenous processes are persistent but mean reverting, households assume that the shocks will eventually die out and all variables will eventually return to their stationary levels. Therefore, the realizations of income and financial intermediation shocks and the implied prices and quantities after 2009 are irrelevant for the welfare estimates in 2009.

One way to get averages is to average over many simulations, hence we show then $E(x)$. In theory this can be important because the variables that we show are transform of the underlying variables, e.g. $f(x)$, and given our procedure what we are implicitly doing is to report $E(f(x))$. In practice however this is not an issue. In particular, we also have conducted one robustness check where we simulate the economy so that the path of income and financial intermediation

4.5 Calibration procedure.

Given the procedure to obtain the simulated time series as described in the previous subsection, we calibrate the parameters using the following steps:

1. Use the parametrization obtained from the deterministic steady state as an initial guess for β_i, ϕ_i, and $\bar{s}$, for $i = b, s$. Assume no housing adjustment costs: $\eta_i = 0$, $i = b, s$.

2. Simulate the economy as described by the steps in the previous subsection and compute the following moments averaging over the years 1990 - 2001: the borrowers' debt to income share ($R_L d_b / y_b$), housing wealth to income for borrowers ($q h_b / y_b$), housing wealth to income for savers ($q h_s / y_s$), and the mortgage interest rate (R_L). Then obtain the average real interest rate on savings R by averaging over 2002 - 2013.

3. For the parameters in the housing adjustment cost, we have to include one further step. For every parameter combination we first guess the average housing levels for each type in the lending restricted economy (i.e. $\bar{h}_b$, $\bar{h}_s$), solve the model, and compute the implied average housing levels for each type in the stationary distribution of the lending restricted economy. If this differs from the guess, we update the guess and compute again the implied housing levels in the stationary distribution (i.e. pre 2001). We iterate until the guessed levels are equal to the average housing levels in the stationary distribution.[1]

[1]This approach is very similar to a homotopy procedure. For every parameter combination, we first solve the model without adjustment costs, $\eta_b = \eta_s = 0$. The simulation of the ergodic distribution gives us an initial guess for $\bar{h}_b = \bar{h}_s$. Then we use this guess and subsequently increase the adjustment costs in small steps. For each housing

4. Compare model implied moments to the data moments. If they do not coincide update the guessed parameter values and repeat steps 1-4 until the moments coincide.

References

GRILL, M., AND J. BRUMM (2014): "Computing Equilibria in Dynamic Models with Occasionally Binding Constraints," *Journal of Economic Dynamics and Control*, 38, 142–160.

LANDVOIGT, T., M. PIAZZESI, AND M. SCHNEIDER (2015): "The housing market (s) of San Diego," *American Economic Review*, 105(4), 1371–1407.

LUCAS, R. E. (1987): *Models of business cycles*, vol. 26. Basil Blackwell Oxford.

adjustment costs, we keep updating the average housing levels. We prefer this procedure because we found that in particular the average house prices in the stochastic steady state are quiet different from the deterministic steady state value, and the level of house price affects the levels of other key variables in the model, in particular housing and debt levels.