Filling in Question 5: [Outline two tests of coefficient restrictions in OLS model].

5. A Cobb-Douglas production function $\log y = \beta_0 + \beta_1 \log L + \beta_2 \log K + \varepsilon$ is estimated by OLS (the numbers to follow are approximately what is found from Table F6.1 in Greene–they are rounded so the calculations are easier).The coefficients are estimated as: 1.17, .60, and .37 respectively and

$$s^2(X'X)^{-1} = \begin{matrix} 0.095 & -0.018 & 0.001 \\ -0.018 & 0.014 & -0.009 \\ 0.001 & -0.009 & 0.006 \end{matrix}$$

   (a) Form a 95% confidence interval for $\beta_1 + \beta_2$.

   (b) Form a 95% confidence interval for $\beta_1 - \beta_2$.

   (c) Test the hypothesis that $\beta_1 + \beta_2 = 1$.

   (d) Test the hypothesis that $\beta_1 = \beta_2$.

   (e) Outline at least one other method by which these hypotheses can be tested.

Filling in question 13: [Combining two estimators to get a more efficient estimator]

13. Suppose θ_0, the true value of a parameter is 0 (this makes no difference in the result to be obtained) and we have two estimators of θ, $\hat{\theta}$ and $\tilde{\theta}$ which are consistent and $\hat{\theta}$ is efficient. It is suggested to combine the two estimators to obtain a super estimator θ^* by taking a weighted average of the two estimators: $\theta^* = \alpha\,\hat{\theta} + (1-\alpha)\,\tilde{\theta}$.

    (a) Demonstrate that $V(\theta^*) = \alpha^2 V(\hat{\theta}) + 2\alpha(1-\alpha)E(\hat{\theta}\,\tilde{\theta}) + (1-\alpha)^2 V(\tilde{\theta})$. Then write $\tilde{\theta} = \hat{\theta} + \varepsilon$ so $E(\hat{\theta}\,\tilde{\theta}) = V(\hat{\theta}) + E(\varepsilon\hat{\theta})$.

    (b) Show that the optimal choice of α is 1 if and only if $E(\varepsilon\hat{\theta}) = 0$ and conclude that an efficient estimator is uncorrelated with an inefficient estimator.

Two new questions on GMM.

27. The central χ^2 distribution with ν 'degrees of freedom' has a density function given by

$$f(x) = \frac{e^{-x/2} x^{(\nu/2)-1}}{2^{(\nu/2)} \Gamma(\frac{\nu}{2})}, \qquad x > 0,$$

where $\Gamma(\cdot)$ is the complete gamma function. (Just regard $\Gamma(\cdot)$ as a 'black box' for this question.) Although the simplest genesis of the χ^2 distribution is that it is the sum of the squares of ν independent $N(0,1)$ variables, it is not required that ν be an integer but simply that $\nu > 0$ (and $x > 0$). So ν can be thought of as unknown parameter that we would like to estimate from $i.i.d.$ data $x_1, x_2, ...x_n$. The $\chi^2(\nu)$ distribution has mean ν and variance 2ν.

(a) Show how to estimate ν by the method of maximum likelihood. (It is OK to state a solution is terms of $\Gamma(\cdot)$ and its derivative(s).)

(b) Explain how to estimate ν by GMM using two moments of the sample data.

(c) Which estimate will be more efficient?

(d) How would the answers change if we wanted to estimate ν for the $N(v, 2\nu)$ distribution?

28. We write the structural equation

$$y_1 = \alpha + \beta_1 y_2 + \sum_{k=1}^{K} \gamma_k X_k + \varepsilon$$

to indicate that y_2 is endogenous and that there are K variates that are exogenous.

(a) What does it mean for y_2 to be 'endogenous'?

Suppose that there are J variates Z_j, $j = 1, ..., J$; $(J > 1)$ such that Z_j is uncorrelated with ε.

(b) What moment conditions are implied by the foregoing?

(c) Show how to exploit these moment conditions to estimate α, β, and γ by GMM.

(d) Characterize precisely the $(K + 2)$ moment conditions that can be exploited to 'efficiently' estimate α, β, and γ, assuming homoscedasticity wherever that simplifies the exposition.