Review Questions.

1. Write and explain the basic linear regression model in matrix notation, and explain the assumptions that are usually made in connection with it. Which of these assumptions are required for the coefficients to have a 'causal' interpretation? If the assumptions required for a 'causal' interpretation fail, what, if any, interpretation remains?

2. 'The linear nature of the linear regression makes it useless for anything that is not linear. And nearly everything is "not linear"'. Discuss.

3. Derive and explain the formula $b_{OLS} = (X'X)^{-1}X'y$. State assumptions under which it is possible to derive the distribution of b_{OLS}, and do the derivation.

4. Let y be a random variable such that $E(y|X) = \mu(X)$. Show that $\mu(X)$ minimizes $E([y - \mu(X)]^2)$. What if any significance does this have?

5. [Outline two tests of coefficient restrictions in OLS model].

6. State, demonstrate and interpret the Gauss–Markov Theorem, 'OLS is BLUE'.

7. Suggest three mechanisms that cause error terms to be correlated with explanatory or 'right hand side' variables. Why does this matter? Discuss the use of instrumental variables for each case.

8. What is the method of instrumental variables? Derive and interpret the formula $b_{IV} = (Z'X)^{-1}Z'y$. Derive the asymptotic distribution of b_{IV} under suitable assumptions.

9. 'While the classical measurement error model gives rise to a correlation between the disturbances and the explanatory variable that is measured with error, it is not true that every instance of a variable's observable value deviating from its 'true' value has serious consequences.' Discuss.

10. What is a 'proxy variable' and under what conditions is the use of a proxy variable (i) useful (ii) valid?

11. What is the method of 'multiple indicators' and what problem is it designed to solve?

12. What is the method of 'two stage least squares'? Is 2SLS an IV estimator? What is its asymptotic variance?

13. [Combining two estimators to get a more efficient estimator]

14. 'The assumption of exogeneity cannot be tested.' Discuss.

15. Write a simple supply and demand model in structural form where both supply and demand depend on price, and demand, but not supply, is affected by an exogenous variable.

    (a) Give an example of such a variable.

    (b) Derive the corresponding reduced form.

    (c) From knowledge or an estimate of the reduced form, in this example can any of the structural parameters be identified?

    (d) Discuss identification in the context of this simple model.

16. Explain the genesis of the multiple equation structural model

$$y_t'\Gamma + x_t'B = \varepsilon_t' \tag{1}$$

Derive its corresponding reduced form supposing that ε_t is normal and with contemporaneous covariance Σ for each x; state the 'supposition' more carefully at the outset of your derivation. What is the conditional distribution of y_t given x_t? Discuss the relation between the structural and reduced forms, with particular attention to the issues of identification.

17. (Double credit.) Let $f(y|\theta)$ be the density of y for parameter θ and suppose we have $i = 1, ..., n$ independently sampled y_i's drawn from $f(y|\theta)$.

    (a) Write the joint density of the data.

    (b) Write the likelihood and log–likelihood functions for θ.

    (c) What is the ML estimate of θ?

    (d) Under what conditions is the MLE consistent and asymptotically normal?

    (e) State and derive the 'likelihood equation'.

    (f) State and derive the 'information matrix equality'.

    (g) Define consistency for an estimator. Demonstrate that the MLE is consistent.

    (h) Demonstrate that the MLE is asymptotically normal and derive its variance.

    (i) Explain how to estimate the variance of the MLE.

18. The random variable y takes the value 1 with probability

$$p(y = 1|X_i) = \frac{e^{X_i\beta}}{1 + e^{X_i\beta}}$$

and has value 0 otherwise. This is called 'logit'.

(a) Write the likelihood and log likelihood functions when y is independently sampled from this model.

(b) Derive the MLE of β as the solution to the likelihood equation.

(c) Derive two or more estimates of the variance of $\hat{\beta}_{MLE}$, and compare.

19. Define and interpret the Kullback Leibler Information Criterion (KLIC). Define quasi maximum likelihood estimation (QMLE). What role does the KLIC play in QMLE? Briefly discuss the properties of QMLE estimators.

20. Demonstrate that 'OLS is the MLE of the CLRM with normality'. Write the likelihood and log–likelihood functions and the likelihood equation. Derive two ways to estimate the variance of the ML estimators of β and σ^2.

21. Define and discuss the likelihood ratio test. Contrast it with the Wald test. How are nonlinear hypotheses handled in these two procedures?

22. Define the τ^{th} quantile of the scalar random variable y. Define the check function. Show that the τ^{th} sample quantile can be found by solving an optimization problem using the check function.

23. (a) Explain how the estimate $med(y|X)$ as an optimization problem. Be sure to show the 'first order conditions' or 'estimating equations'.

    (b) Briefly, how can your answer be applied to other quantiles?

    (c) 'There is no point to estimating conditional medians or other conditional quantiles. The Gauss Markov theorem tells us OLS is the best estimator of the linear model.' Discuss.

24. Interpret the estimated conditional quantiles of daily maximum temperatures in Canberra (or was it Auckland?)

25. Consider the conditional quantile model:

$$Q_y(\tau|x) = \beta(\tau)x.$$

    (a) Explain how an estimate of $\beta(\tau)$ can be obtained for any τ

(b) Suppose the assumptions of the classical linear regression model are met, excluding normality. What properties will $\beta(\tau)$ have for different values of τ? Illustrate for the case of scalar x.

(c) Suppose we have the CLRM except that the disturbances are heteroscedastic. Illustrate two possibilities for scalar x: the variance of the disturbances (i) grows with x (ii) declines with x.

(d) Can 'true quantile lines' ever cross? Can estimated quantile lines ever cross? Explain and interpret.

26. In what sense are conditional quantile estimates robust?

27. How might quantile methods be used to investigate changes in income stratification over time?

28. GMM questions 1-3 to follow.