# International Economics EUI 2010

## Optimal policy in New Keynesian open-economy models

### Monetary policy issues in open economy

- do exchange rate movements have desirable stabilization and allocative properties? Or, on the contrary, should policymakers curb exchange rate volatility and be concerned with, and attempt to correct, currency misalignments?

- Should monetary policy respond to international variables such as terms of trade, international cycle, or global imbalances, on top and beyond of their influence on the domestic output gap and inflation?

- Are there large gains the international community could reap by strengthening cross-border policy cooperation?

We now extend the NK framework to this questions — New Open Economy Macroeconomics (see Obstfeld and Rogoff 1995).

Difference from closed-economy analysis. Need to account explicitly for different forms of heterogeneity that naturally arise in an international context,

- product specialization, national differences in technology, preferences, financial market developments and asset holdings (instances of ex-ante heterogeneity across countries),

- asymmetric nature of shocks, as well as endogenous fluctuations in the wealth distribution in response to shocks (ex-post heterogeneity).

- monetary policy problems are addressed using as many policy instruments as there are monetary authorities in the model economy. Along this dimension as well, however, there could be heterogeneity in objectives and policy conducts.

Model encompassing 6 specifications: Complete markets vs. Financial Autarky vs. Bond Economy; Export prices are sticky in the currency of either the producer, or the market of destination.

*A workhorse open economy model (Obstfeld Rogoff)*

In this workhorse model, nominal rigidities interact with three other sources of distortions: the first is the monopoly power in production, familiar from the (closed-economy) new-Keynesian literature; the other two are specific to international analysis.

- imperfections in international financial markets,

- incentives to deviate from globally optimal policies stemming from the assumption that countries have monopoly power on their terms of trade.

I. *Preferences and technology*

$$V^j = E_0 \left\{ \sum_{t=0}^{\infty} \beta^t \left[ U\left(C_t^j, \zeta_{C,t}\right) + L\left(\frac{M_{t+1}^j}{P_t}, \zeta_{M,t}\right) - \frac{1}{n}\int_0^n V\left(y_t(h), \zeta_{Y,t}\right) dh \right] \right\}. \quad (1)$$

Households obtain utility from consumption and the liquidity services of holding money, while they receive disutility from contributing to the production of all domestic goods $y_t(h)$ with a separable disutility. $\zeta_{C,t}$, $\zeta_{M,t}$, $\zeta_{Y,t}$ denote country specific shocks to preferences towards consumption, real money balances and production, respectively. Risk is pooled internally to the extent that agents participate in the production of all goods and receive an equal share of production revenue.

As in BB and CGG:

$$
\begin{aligned}
U\left(C_t^j, \zeta_{C,t}\right) &= \zeta_{C,t}\frac{C_t^{j\,1-\sigma} - 1}{1-\sigma} \\
L\left(\frac{M_{t+1}^j}{P_t}, \zeta_{M,t}\right) &= \lambda\zeta_{M,t}\frac{\left(\frac{M_{t+1}^j}{P_t}\right)^{1-\sigma} - 1}{1-\sigma} \\
V\left(y_t(h), \zeta_{Y,t}\right) &= \frac{\zeta_{Y,t}^{-\eta} y_t(h)^{1+\eta}}{1+\eta}
\end{aligned} \tag{2}
$$

Demand by household j of H and F good bundles

$$C_{\mathsf{H},t}(j) \equiv \left[\left(\frac{1}{n}\right)^{1/\theta} \int_0^n C_t(h,j)^{\frac{\theta-1}{\theta}} dh\right]^{\frac{\theta}{\theta-1}}, \tag{3}$$

$$C_{\mathsf{F},t}(j) \equiv \left[\left(\frac{1}{1-n}\right)^{1/\theta} \int_n^1 C_t(f,j)^{\frac{\theta-1}{\theta}} df\right]^{\frac{\theta-1}{\theta}}$$

The full consumption basket, $C_t$,

$$C = \left[a_{\mathsf{H}}^{1/\phi} C_{\mathsf{H}}^{\frac{\phi-1}{\phi}} + a_{\mathsf{F}}^{1/\phi} C_{\mathsf{F}}^{\frac{\phi-1}{\phi}}\right]^{\frac{\phi}{\phi-1}}, \qquad \phi > 0. \tag{4}$$

The CPI is

$$P_t = \left[a_{\mathsf{H}} {P_{\mathsf{H},t}}^{1-\phi} + (1 - a_{\mathsf{H}})\, {P_{\mathsf{F},t}}^{1-\phi}\right]^{\frac{1}{1-\phi}}, \tag{5}$$

where $P_{\mathsf{H},t}$ is the price sub-index for home-produced goods and $P_{\mathsf{F},t}$ is the price sub-index for foreign produced goods, both expressed in the domestic currency:

$$P_{\mathsf{H},t} \equiv \left[\frac{1}{n}\int_0^n P_t(h)^{1-\theta} dh\right]^{\frac{1}{1-\theta}}, \qquad P_{\mathsf{F},t} \equiv \left[\frac{1}{1-n}\int_0^n P_t(f)^{1-\theta} df\right]^{\frac{1}{1-\theta}} \tag{6}$$

Foreign country analog:

$$P_t^* = \left[(1 - a_{\mathsf{F}}^*)\, {P_{\mathsf{H},t}^*}^{1-\phi} + a_{\mathsf{F}}^* {P_{\mathsf{F},t}^*}^{1-\phi}\right]^{\frac{1}{1-\phi}}. \tag{7}$$

Let $\Theta_t$ denote the real exchange rate, and $\mathcal{T}_t$ is the terms of trade:

$$\Theta_t = \frac{\mathcal{E}_t P_t^*}{P_t} \qquad \mathcal{T}_t = \frac{P_{\mathsf{F},t}}{\mathcal{E}_t P_{\mathsf{H},t}^*}$$

Demand for the variety $h$ and $f$ by household $j$ :

$$\begin{aligned} C_t(h,j) &= a_{\mathsf{H}} \left(\frac{P_t(h)}{P_{\mathsf{H},t}}\right)^{-\theta} \left(\frac{P_{\mathsf{H},t}}{P_t}\right)^{-\phi} C_t^j, \qquad (8) \\ C_t(f,j) &= (1-a_{\mathsf{H}}) \left(\frac{P_t(f)}{P_{\mathsf{F},t}}\right)^{-\theta} \left(\frac{P_{\mathsf{F},t}}{P_t}\right)^{-\phi} C_t^j; \end{aligned}$$

Assuming the law of one price holds (i.e. $P_t(h) = \mathcal{E}_t P_t^*(h)$ and $P_t(f) = \mathcal{E}_t P_t^*(f)$), total demand of good $h$ and $f$ can then been written as:

$$y_t^d(h) = \left(\frac{P_t(h)}{P_{\mathsf{H},t}}\right)^{-\theta} \left[\left(\frac{P_{\mathsf{H},t}}{P_t}\right)^{-\phi} \left(a_{\mathsf{H}} C_t + a_{\mathsf{H}}^* \frac{1-n}{n} \Theta_t^{\phi} C_t^*\right)\right] \qquad (9)$$

$$y_t^d(f) = \left(\frac{P_t(f)}{P_{\mathsf{F},t}}\right)^{-\theta}\left[\left(\frac{P_{\mathsf{F},t}}{P_t}\right)^{-\phi}\left((1-a_{\mathsf{H}})\frac{n}{1-n}C_t + \Theta_t^{\phi}(1-a_{\mathsf{H}}^*)C_t^*\right)\right], \tag{10}$$

The individual flow budget constraint for the representative agent in the Home country can be generically written as:

$$M_t + B_{\mathsf{H},t+1} + \int q_{\mathsf{H},t+1}(s_{t+1})\mathcal{B}_{\mathsf{H},t+1}(s_{t+1})\,ds_{t+1} \leq M_{t-1} + (1+i_t)B_{\mathsf{H},t} + \mathcal{B}_{\mathsf{H},t} + (1-\tau_t)\frac{\int P_t(h)y_t(h)dh}{n} - P_{\mathsf{H},t}T_t - P_{\mathsf{H},t}C_{\mathsf{H},t} - P_{\mathsf{F},t}C_{\mathsf{F},t},$$

whereas the Euler equations determining the intertemporal profile of consumption and savings are

$$\frac{U_C\left(C_t, \zeta_{C,t}\right)}{P_t} = (1+i_t)\,E_t\left[\beta\frac{U_C\left(C_{t+1}, \zeta_{C,t+1}\right)}{P_{t+1}}\right], \tag{11}$$

$$\frac{U_C\left(C_t^*, \zeta_{C,t}^*\right)}{P_t^*} = (1 + i_t^*)\, E_t\left[\beta \frac{U_C\left(C_{t+1}^*, \zeta_{C,t+1}^*\right)}{P_{t+1}^*}\right]. \tag{12}$$

For the Government

$$\tau_t \int P_t(h) y_t(h) dh = P_{\mathsf{H},t} \int T_t^j + \int \left(M_t^j - M_{t-1}\right), \tag{13}$$

$$\tau_t^* \int P_t^*(f) y_t^*(f) df = P_{\mathsf{F},t}^* \int T_t^{j*} + \int \left(M_t^j - M_{t-1}\right). \tag{14}$$

## II. *Calvo-Yun pricing in Open Economy*

Let $\alpha \in [0; 1)$ denote the fraction of randomly chosen producers not allowed to change the nominal price of the goods they produce. Let $\mathcal{P}_t(h)$ denote the price optimally chosen by the firm $h$ for the domestic market at time $t$. What about the price in the foreign market?

Conceptually, there are at least two ways to model nominal rigidities in the export market.

- PCP: Seminal contributions in the literature (after OR 1995) posit that prices are rigid in currency of the producers: firms set export prices in domestic currency, letting the foreign currency price of their product vary with the exchange rate. This hypothesis is dubbed ‘producer currency pricing’ or PCP.

- LCP: The PCP assumption is questioned by an important strand of the literature (pioneered by Betts and Devereux 2000), subscribing the alternative view that firms preset prices in domestic currency for the domestic market, and in foreign currency for the market of destination. This hypothesis is dubbed ‘local currency prices’ or LCP.

**Pricing under PCP** Firms resetting prices choose $\mathcal{P}_t(h)$ and the price in domestic currency to charge abroad, for convenience '$\mathcal{E}_t\mathcal{P}_t^*(h)$' as to maximize:

$$
\begin{aligned}
& Max_{p_t(h),\mathcal{E}_t p_t^*(h)} \; E_t \sum_{s=0}^{\infty} \{(\alpha\beta)^s \\
& \left\{ \frac{U_{C,t+s}}{P_{t+s}} (1-\tau_{t+s}) \left[ p_t(h) \left( \frac{p_t(h)}{P_{\mathsf{H},t+s}} \right)^{-\theta} \left( \frac{P_{\mathsf{H},t+s}}{P_{t+s}} \right)^{-\phi} (a_{\mathsf{H}} C_{t+s}) \right.\right. \\
& \left. + \mathcal{E}_t p_t^*(h) \left( \frac{\mathcal{E}_t p_t^*(h)}{\mathcal{E}_{t+s} P^*_{\mathsf{H},t+s}} \right)^{-\theta} \left( \frac{P^*_{\mathsf{H},t+s}}{P^*_{t+s}} \right)^{-\phi} \left( a^*_{\mathsf{H}} \frac{1-n}{n} C^*_{t+s} \right) \right] \\
& \left. - V\left( y_{t+s}(h), \zeta_{Y,t+s} \right) \right\}
\end{aligned}
\qquad (15)
$$

where revenues and costs are measured in utils and an asterisk denotes prices in Foreign currency. Let $y^d_{t+s}(h)$ be the total demand of the good at time $t+s$ under the circumstances that the prices $\mathcal{P}_t(h)$ and $\mathcal{E}_t\mathcal{P}_t^*(h)$ still apply at $t+s$, and let $\mu_t = \frac{\theta}{(1-\tau_{t+s})(\theta-1)}$.The FOCs are

$$E_t \sum_{s=0}^{\infty} (\alpha\beta)^s \left\{ \left[ \frac{U_{C,t+s}}{P_{t+s}} \mathcal{P}_t(h) - \mu_t V_y \left( y^d_{t+s}(h), \zeta_{Y,t+s} \right) \right] \left[ \left( \frac{\mathcal{P}_t(h)}{P_{\mathsf{H},t+s}} \right)^{-\theta} \left( \frac{P_{\mathsf{H},t+s}}{P_{t+s}} \right)^{-\phi} (a_{\mathsf{H}} C_t) \right] \right\} = 0$$

$$E_t \sum_{s=0}^{\infty} (\alpha\beta)^s \left\{ \left[ \frac{U_{C,t+s}}{P_{t+s}} \mathcal{E}_t \mathcal{P}^*_t(h) - \mu_t V_y \left( y^d_{t+s}(h), \zeta_{Y,t+s} \right) \right] \left[ \left( \frac{\mathcal{E}_t \mathcal{P}^*_t(h)}{\mathcal{E}_{t+s} P^*_{\mathsf{H},t+s}} \right)^{-\theta} \left( \frac{P^*_{\mathsf{H},t+s}}{P^*_{t+s}} \right)^{-\phi} \left( a^*_{\mathsf{H}} \frac{1-n}{n} C^*_t \right) \right] \right\} = 0$$

The last term on the left hand side of each condition is the demand for the good $h$ in the Home and Foreign market, respectively, at the price chosen at time $t$

— these two terms indeed sum up to $y^d(h)$. The firm's problem is solved by

$$E_t \sum_{s=0}^{\infty} (\alpha\beta)^s \left[ \frac{U_{C,t+s}}{P_{t,t+s}} p_t(h) - \frac{\theta}{(1-\tau_{t+s})(\theta-1)} V_y\left(y^d_{t+s}(h), \zeta_{Y,t+s}\right) \right] y^d_{t+s} = 0 \tag{16}$$

$$\mathcal{E}_t \mathcal{P}^*_t(h) = \mathcal{P}_t(h) \qquad for\ all\ h$$

Note: demand elasticities are constant and symmetric across borders => firms optimally choose identical prices in both market (the law of one price holds) independently of barriers to good markets integration. The above solution hence implies

$$\mathcal{E}_t P^*_{H,t} = P_{H,t} \text{ and } \quad P_{F,t} = \mathcal{E}_t P^*_{F,t}$$

The terms of trade move one-to-one with the exchange rate, as well as with the domestic relative price of imports faced by consumers: $\mathcal{T}_t = P_{\mathsf{F},t}/\mathcal{E}_t P^*_{\mathsf{H},t} = \mathcal{E}_t P^*_{\mathsf{F},t}/P_{\mathsf{H},t} = P_{\mathsf{F},t}/P_{\mathsf{H},t}$.

Since all the producers that can choose their price set it to the same value, we obtain the following equations

$$P_{\mathsf{H},t}^{1-\theta} = \alpha P_{\mathsf{H},t-1}^{1-\theta} + (1-\alpha)\,\mathcal{P}_t(h)^{1-\theta}, \tag{17}$$

$$P_{\mathsf{F},t}^{*1-\theta} = \alpha P_{\mathsf{F},t-1}^{*1-\theta} + (1-\alpha)\,\mathcal{P}_t^*(f)^{1-\theta}.$$

describing the dynamic evolution of $P_{\mathsf{H},t}$ and $P_{\mathsf{F},t}$.

**Price setting under LCP** The PCP assumption is questioned by an important strand of the literature (pioneered by Betts and Devereux 2000), subscribing the alternative view that firms preset prices in domestic currency for the domestic market, and in foreign currency for the market of destination. This hypothesis is dubbed 'local currency prices' or LCP. Under this hypothesis, firms choose $\mathcal{P}_t^*(h)$ instead of $\mathcal{E}_t\mathcal{P}_t^*(h)$ and the first order condition for this price is

$$E_t \sum_{s=0}^{\infty} (\alpha\beta)^s \left\{ \left[ \frac{U_{C,t+s}}{P_{t+s}} \mathcal{E}_{t+s} \mathcal{P}_t^*(h) - \frac{\theta}{(1-\tau_{t+s})(\theta-1)} V_y \left( y_{t+s}^d(h), \zeta_{Y,t+s} \right) \right] \left[ \left( \frac{\mathcal{P}_t^*(h)}{P_{\mathsf{H},t+s}^*} \right)^{-\theta} \left( \frac{P_{\mathsf{H},t+s}^*}{P_{t+s}^*} \right)^{-\phi} \left( a_{\mathsf{H}}^* \frac{1-n}{n} C_t^* \right) \right] \right\} = 0$$

- Note: when a firm can re-optimize, it can do so both in the domestic and export markest. With LCP, for a firm not re-optimizing its price, exchange rate pass-through is zero.

Let $\Delta_t$ denote deviations from the law of one price (LOOP): for the Home country, we can write

$$\Delta_{\mathsf{H},t} = \mathcal{E}_t P^*_{\mathsf{H},t} / P_{\mathsf{H},t}$$

As $P^*_{\mathsf{H},t}$ and $P_{\mathsf{H},t}$ are sticky, the law of one price is violated with any movement in the currency value. Specifically, nominal depreciation tends to increase the Home firms' receipts in Home currency from selling goods abroad, relative to the Home market: nominal depreciation raises $\Delta_{\mathsf{H},t}$. Because of deviations from the LOOP, the Home terms of trade $\mathcal{T}_t = P_{\mathsf{F},t}/\mathcal{E}_t P^*_{\mathsf{H},t}$ will generally be different from the domestic price of imported goods, $P_{\mathsf{F},t}/P_{\mathsf{H},t}$.

*III. International asset markets and exchange rate determination*

The market equilibrium crucially differ depending on the asset market structure.

**Complete markets**

$$\beta \frac{U_C\left(C_{t+1}, \zeta_{C,t+1}\right)}{U_C\left(C_t, \zeta_{C,t}\right)} \frac{P_{t+1}}{P_t} = \beta \frac{U_C\left(C_{t+1}^*, \zeta_{C,t+1}^*\right)}{U_C\left(C_t^*, \zeta_{C,t}^*\right)} \frac{\mathcal{E}_{t+1} P_{t+1}^*}{\mathcal{E}_t P_t^*}. \tag{18}$$

With zero net foreign assets initially:

$$\frac{C_t^{-\sigma} \zeta_{C,t}}{P_t} = \frac{(C_t^*)^{-\sigma} \zeta_{C,t}^*}{\mathcal{E}_t P_t^*} \tag{19}$$

Given Home and Foreign monetary policy, this equation fully determines the exchange rate in both nominal and real terms.

**Incomplete-market economy: financial autarky**

The individual flow budget constraint for the representative agent $j$ in the Home country is:

$$M_t \leq M_{t-1} - P_{\mathsf{H},t} T_t + (1 - \tau_t) \frac{\int P_t(h) y_t(h) dh}{n} - P_{\mathsf{H},t} C_{\mathsf{H},t} - P_{\mathsf{F},t} C_{\mathsf{F},t}. \quad (20)$$

Barring international trade in asset, the value of domestic production has to be equal to the level of public and private consumption in nominal terms. Aggregating private and public budget constraints, we have:

$$P_t C_t = \int P_t(h) y_t(h) dh. \quad (21)$$

The value of imports should equal the value of exports:

$$n P_{\mathsf{F},t} C_{\mathsf{F},t} = (1 - n) \mathcal{E}_t P^*_{\mathsf{H},t} C^*_{\mathsf{H},t}. \quad (22)$$

Using the definitions of terms of trade $\mathcal{T}_t$ and real exchange rate $\Theta_t$, we can rewrite the trade balance condition in terms of aggregate consumption:

$$n\,(1 - a_{\mathsf{H}})\,\mathcal{T}_t^{1-\phi} C_t = (1 - n)\,a_{\mathsf{H}}^{*} \Theta_t^{\phi} C_t^{*}. \tag{23}$$

For given monetary policy in the two countries, it is this equation — balanced trade — that determines exchange rates.

**Incomplete-market economy: trade in assets**

Between the two polar cases, Home and Foreign agents hold an international bond, $B_{\mathsf{H}}$, which pays in units of Home currency and is zero in net supply; in addition they may hold other securities in the amounts $\alpha_{it}$, yielding ex-post returns $R_{it}$. The individual flow budget constraint is:

$$M_t + B_{\mathsf{H},t+1} + \sum_i \alpha_{i,t+1} \leq M_{t-1} + (1 + i_t) B_{\mathsf{H},t} + \sum_i \alpha_{i,t} R_{i,t}$$
$$+ (1 - \tau_t) \frac{\int P_t(h) y_t(h) dh}{n} - P_{\mathsf{H},t} T_t - P_{\mathsf{H},t} C_{\mathsf{H},t} - P_{\mathsf{F},t} C_{\mathsf{F},t}. \qquad (24)$$

The equilibrium condition is

$$E_t \left[ \beta \frac{U_C \left( C_{t+1}, \zeta_{C,t+1} \right)}{U_C \left( C_t, \zeta_{C,t} \right)} \frac{P_{t+1}}{P_t} R_{i,t+1} \right] = E_t \left[ \beta \frac{U_C \left( C^*_{t+1}, \zeta^*_{C,t+1} \right)}{U_C \left( C^*_t, \zeta^*_{C,t} \right)} \frac{\mathcal{E}_t P^*_t}{\mathcal{E}_{t+1} P^*_{t+1}} R_{i,t+1} \right]. \qquad (25)$$

which holds for each individual asset (or portfolio of assets). The case of international trade in one bond is easily obtained from the above imposing $\alpha_{it} = 0$.

**Differences between the complete-market and the incomplete-market economy**

- First, while exchange rates reflect only shocks to fundamentals (thus acting as 'shock absorber') in both economies, when markets are incomplete their equilibrium value will differ from the efficient one, irrespective of nominal rigidities, due to this form of asset-market frictions.

- Incomplete international risk sharing will generally be imperfect, resulting in inefficient fluctuations in aggregate demand across countries, as shocks open a wedge between national wealth. Let $D_t$ denote the welfare-relevant cross-country demand imbalance, defined as the following PPP-adjusted measure of cross-country demand differential:

$$\mathcal{D}_t = \left(\frac{C_t}{C_t^*}\right)^{\sigma} \left(\frac{1}{\Theta_t}\frac{\zeta^*_{C,t}}{\zeta_{C,t}}\right) \tag{26}$$

  Under complete markets $\mathcal{D}_t = 1$ always. With incomplete markets, $\mathcal{D}_t$ will fluctuate inefficiently contingent on shocks.

## 0.1 Natural and efficient allocations

With flexible prices, equilibrium is determined by the following first set of conditions, plus exchange rate determination, defined later:

$$
\begin{aligned}
U_C\left(C_t, \zeta_{C,t}\right)\frac{P_{\mathsf{H},t}}{P_t} &= \mu_t V_y\left(y_t, \zeta_{Y,t}\right) \\
\zeta_{C,t} C_t^{-\sigma}\frac{P_{\mathsf{H},t}}{P_t} &= \mu_t \left( \frac{\left(\frac{P_{\mathsf{H},t}}{P_t}\right)^{-\phi}\left(a_{\mathsf{H}} C_t + a_{\mathsf{H}}^{*}\frac{1-n}{n}\Theta_t^{\phi} C_t^{*}\right)}{\zeta_{Y,t}} \right)^{\eta} \qquad (27)
\end{aligned}
$$

$$U_C\left(C_t^*,\zeta_{C,t}^*\right)\frac{P_{\mathsf{F},t}^*}{P_t^*} = \mu_t V_y\left(y_t,\zeta_{Y,t}^*\right)$$

$$\zeta_{C,t}^* C_t^{*-\sigma}\frac{P_{\mathsf{F},t}^*}{P_t^*} = \mu_t\left(\frac{\left(\frac{P_{\mathsf{F},t}^*}{P_t^*}\right)^{-\phi}\left((1-a_{\mathsf{H}})\frac{n}{1-n}\Theta_t^{-\phi}C_t+\left(1-a_{\mathsf{H}}^*\right)C_t^*\right)}{\zeta_{Y,t}^*}\right)^{\eta} \quad (28)$$

whereas, holding the law of one price, the terms of trade and the real exchange rate can be written as follows :

$$\mathcal{T}_t = \frac{P_{\mathsf{F},t}}{P_{\mathsf{H},t}}$$

$$\Theta_t^{1-\phi} = \frac{a_{\mathsf{H}}^* {P_{\mathsf{H},t}}^{1-\phi}+\left(1-a_{\mathsf{H}}^*\right){P_{\mathsf{F},t}}^{1-\phi}}{a_{\mathsf{H}}{P_{\mathsf{H},t}}^{1-\phi}+(1-a_{\mathsf{H}})\,{P_{\mathsf{F},t}}^{1-\phi}} = \frac{a_{\mathsf{H}}^*+\left(1-a_{\mathsf{H}}^*\right)\mathcal{T}_t^{1-\phi}}{a_{\mathsf{H}}+(1-a_{\mathsf{H}})\,\mathcal{T}_t^{1-\phi}};$$

In log-deviations from steady state — assuming that in steady state the net foreign asset position is zero. Denoting with an upper-bar steady-state values, $\widehat{x}_t = \ln x_t/\bar{x}$ will represent deviations under sticky prices, while $\widetilde{x}_t = \ln x_t/\bar{x}$ will represent deviations under flexible prices. Letting $\mu$ denote the equilibrium markup $(\mu_t = \theta/\left((\theta - 1)(1 - \tau_t)\right))$, a log-linear approximation around the steady state of the above equations will yield:

$$\widetilde{\Theta}_t = (a^* + a - 1)\,\widetilde{\mathcal{T}}_t \tag{29}$$

$$\widehat{\zeta}_{C,t} - \sigma \widetilde{C}_t - (1-a)\,\widetilde{\mathcal{T}}_t =$$
$$= \eta \left[ \begin{array}{c} \left(\widehat{G}_t - \left(\widehat{\zeta}_{Y,t} - \frac{\widehat{\mu}_t}{\eta}\right)\right) + \phi\,(1-a)\,\widetilde{\mathcal{T}}_t + \\ \left(a_{\mathsf{H}} + (1-a_{\mathsf{H}})\,\bar{\mathcal{T}}^{1-\phi}\right)^{\frac{\phi}{1-\phi}} \left(a_{\mathsf{H}}\frac{\bar{C}}{\bar{Y}} n \widetilde{C}_t + a^*_{\mathsf{H}} \frac{\bar{C}^* \bar{\Theta}^{\phi}}{\bar{Y}} \frac{1-n}{n}\left(\widetilde{C}^*_t + \phi \widetilde{\Theta}_t\right)\right) \end{array} \right]$$

$$\widehat{\zeta}^*_{C,t} - \sigma \widetilde{C}^*_t + (1-a^*)\,\widetilde{\mathcal{T}}_t =$$
$$= \eta \left[ \begin{array}{c} \left(\widehat{G}^*_t - \left(\widehat{\zeta}^*_{Y,t} - \frac{\widehat{\mu}^*_t}{\eta}\right)\right) - \phi\,(1-a^*)\,\widetilde{\mathcal{T}}_t + \\ \left(a^*_{\mathsf{H}}\bar{\mathcal{T}}^{\phi-1} + \left(1-a^*_{\mathsf{H}}\right)\right)^{\frac{\phi}{1-\phi}} \left( \begin{array}{c} (1-a_{\mathsf{H}})\,\frac{\bar{C}\bar{\Theta}^{-\phi}}{\bar{Y}^*}\frac{n}{1-n}\left(\widetilde{C}_t - \phi\widetilde{\Theta}_t\right) + \\ \left(1-a^*_{\mathsf{H}}\right)\frac{\bar{C}^*}{\bar{Y}^*}(1-n)\,\widetilde{C}^*_t \end{array} \right) \end{array} \right]$$

where $a$, $a^*$,$\bar{Y}$, $\bar{Y}^*$ are:

$$1 - a^* = \frac{a_{\mathsf{H}}^*}{a_{\mathsf{H}}^* + \left(1 - a_{\mathsf{H}}^*\right) \bar{\mathcal{T}}^{1-\phi}}, \qquad 1 - a = \frac{(1 - a_{\mathsf{H}}) \bar{\mathcal{T}}^{1-\phi}}{a_{\mathsf{H}} + (1 - a_{\mathsf{H}}) \bar{\mathcal{T}}^{1-\phi}}$$

$$\bar{Y} = \left[a_{\mathsf{H}} + (1 - a_{\mathsf{H}}) \bar{\mathcal{T}}^{1-\phi}\right]^{\frac{\phi}{1-\phi}} \left[\left(a_{\mathsf{H}} \bar{C} + \frac{1-n}{n} a_{\mathsf{H}}^* \bar{C}^* \bar{\Theta}^{\phi}\right)\right]$$

$$\bar{Y}^* = \left[a_{\mathsf{H}}^* \bar{\mathcal{T}}^{\phi-1} + (1 - a_{\mathsf{H}}^*)\right]^{\frac{\phi}{1-\phi}} \left[\left(\frac{n}{1-n} (1 - a_{\mathsf{H}}) \bar{\Theta}^{-\phi} \bar{C} + (1 - a_{\mathsf{H}}^*) \bar{C}^*\right)\right].$$

Exchange rate determination depends on the structure of international financial markets. With complete market:

$$\widetilde{\Theta}_t = \left(\widehat{\zeta}^*_{C,t} - \widehat{\zeta}_{C,t}\right) + \sigma\left(\widetilde{C}_t - \widetilde{C}^*_t\right) \tag{30}$$

With financial autarky, balance trade implies

$$\widetilde{\Theta}_t = \frac{a^* + a - 1}{\phi\left(a^* + a\right) - 1}\left(\widetilde{C}_t - \widetilde{C}^*_t\right) \tag{31}$$

The system of equations (29) and either (30) or (31) map all the shocks in the four endogenous variables $\left(\widetilde{\Theta}_t, \widetilde{C}_t, \widetilde{C}^*_t \text{ and } \widetilde{\mathcal{T}}_t\right)$, characterizing the natural rate global allocation under either complete asset markets, or international financial autarky.

By the first welfare theorem, the *efficient* allocation is equivalent to the decentralized equilibrium with flexible prices *and* complete markets above, in which markups levels and fluctuations are neutralized with appropriate subsidies ($\mu_t = 0$), so that $U_C(\cdot)\frac{P_{\mathsf{H},t}}{P_t} = V_y(\cdot)$ and $U_C^*(\cdot)\frac{P_{\mathsf{F},t}^*}{P_t^*} = V_y^*(\cdot)$. In what follows, we will denote the efficient allocation (corresponding to (a) complete markets, (b) flexible prices and (c) production subsidies such that $\mu_t = 0$ with a superscript 'fb'.

**Interdependence**

In general, the international transmission of shocks can be expected to be shaped by a large set of structural characteristics of the economy, ranging from financial market development and integration, to vertical interactions between producers and retailers, which are not accounted for by our workhorse model. One advantage of the workhorse model specified in this section is that, with complete markets and flexible prices, it yields an admittedly special yet intuitive and parsimonious benchmark characterization of the international transmission, stressing output linkages.

Imposing symmetry ($n = 1 - n$ and $a_{\mathsf{H}} = 1 - a^*_{\mathsf{H}}$):

$$
\begin{aligned}
(\eta+\sigma)\,\widetilde{Y}^{fb}_{H,t} &= \left[2a_{\mathsf{H}}\left(1-a_{\mathsf{H}}\right)\left(\sigma\phi-1\right)\right]\left(\widetilde{\mathcal{T}}^{fb}_t\right)-\left(1-a_{\mathsf{H}}\right)\left(\widehat{\zeta}_{C,t}-\widehat{\zeta}^*_{C,t}\right)+\widehat{\zeta}_{C,t}+\eta\widehat{\zeta}_{Y,t} \quad (32) \\
(\eta+\sigma)\,\widetilde{Y}^{fb}_{F,t} &= \left[2a_{\mathsf{H}}\left(1-a_{\mathsf{H}}\right)\left(\sigma\phi-1\right)\right]\left(-\widetilde{\mathcal{T}}^{fb}_t\right)+\left(1-a_{\mathsf{H}}\right)\left(\widehat{\zeta}_{C,t}-\widehat{\zeta}^*_{C,t}\right)+\widehat{\zeta}^*_{C,t}+\eta\widehat{\zeta}^*_{Y,
\end{aligned}
$$

$$
\left[4\left(1-a_{\mathsf{H}}\right)a_{\mathsf{H}}\phi+\frac{\left(2a_{\mathsf{H}}-1\right)^2}{\sigma}\right]\widetilde{\mathcal{T}}^{fb}_t=\left(\widetilde{Y}^{fb}_{H,t}-\widetilde{Y}^{fb}_{F,t}\right)-\frac{2a_{\mathsf{H}}-1}{\sigma}\left(\widehat{\zeta}_{C,t}-\widehat{\zeta}^*_{C,t}\right) \tag{33}
$$

- the terms of trade channel of transmission: foreign shocks, such as gains in productivity $\widehat{\zeta}^*_{Y,t}$, affect the level of activity in the Home country, $\widetilde{Y}_{H,t}$ via movements in $\widetilde{\mathcal{T}}^{fb}_t$: Home and Foreign output will move either in the same or in the opposite direction depending on whether $\sigma\phi < 1$, or $\sigma\phi > 1$ — goods are complement or substitute in the Pareto-Edgeworth sense.

- However, note that the value of $\sigma\phi$ alone does not fully characterize cross-border output spillovers. National outputs responds to preference shocks abroad independently of the terms-of-trade channel.

- In turn, the terms of trade now change one-to-one with output differential, but also move proportionally to the differential in preference shocks independently of output movements:

$$\sigma = \phi = 1 \qquad => \qquad \widetilde{\mathcal{T}}_t^{fb} = \left(\widetilde{Y}_{H,t}^{fb} - \widetilde{Y}_{F,t}^{fb}\right) - \frac{2a_{\mathsf{H}} - 1}{\sigma}\left(\widehat{\zeta}_{C,t} - \widehat{\zeta}_{C,t}^{*}\right)$$

## 0.2 The Open-Economy Phillips Curve

To characterize the allocation with nominal rigidities, derive the counterparts to the New-Keynesian Phillips Curve (NKPC) in our open-economy model. Log-linearizing the price setting decisions (either (16) or (**??**) and the evolution for the price indexes (17).

The NKPC writes Home inflation of the domestically produced good as a function of expected inflation and current marginal costs:

$$\pi_{H,t} = \beta E_t \pi_{H,t+1} + \frac{(1-\alpha\beta)(1-\alpha)}{\alpha(1+\theta\eta)} \left[\sigma \widehat{C}_t - \widehat{\zeta}_{C,t} + \eta\left(\widehat{Y}_{H,t} - \widehat{\zeta}_{Y,t}\right) + \widehat{\mu}_t + (1-a_{\mathsf{H}})\left(\widehat{\mathcal{T}_t} + \widehat{\Delta}_{\mathsf{H},t}\right)\right]$$

Using the aggregate demand for domestic output

$$\widehat{C}_t = \widehat{Y}_{H,t} - (1 - a_{\mathsf{H}}) \left[ \phi \left( \widehat{\mathcal{T}_t} + \widehat{\Theta}_t \right) - \left( \widehat{C}_t - \widehat{C}_t^* \right) \right]$$

together with the definition of $\mathcal{D}_t$ in (26) to substitute out the consumption differential:

$$\sigma\widehat{C}_t = \sigma\widehat{Y}_{H,t} - (1 - a_{\mathsf{H}}) \left[ \sigma\phi\widehat{\mathcal{T}_t} + (\sigma\phi - 1)\,\widehat{\Theta}_t - \widehat{\mathcal{D}}_t - \left( \widehat{\zeta}_{C,t} - \widehat{\zeta}^*_{C,t} \right) \right],$$

and equation (32) for $\widetilde{Y}^{fb}_{H,t}$, the open-economy NK Phillips Curve takes the general form:

$$\pi_{H,t} = \beta E_t \pi_{H,t+1} + \frac{(1 - \alpha\beta)(1 - \alpha)}{\alpha(1 + \theta\eta)} \cdot \left\{ \begin{array}{c} (\eta + \sigma) \left( \widehat{Y}_{H,t} - \widetilde{Y}^{fb}_{H,t} \right) + \widehat{\mu}_t + \\ -(1 - a_{\mathsf{H}}) \cdot \left[ (\sigma\phi - 1) \left( \widehat{\mathcal{T}_t} - \widetilde{\mathcal{T}}^{fb}_t + \widehat{\Theta}_t - \widetilde{\Theta}^{fb}_t \right) - \widehat{\Delta}_{\mathsf{H},t} - \widehat{\mathcal{D}}_t \right] \end{array} \right\} \quad (34)$$

- In the closed-economy counterpart of our model ($a_{\mathsf{H}} = 1$), baseline New-Keynesian specification with only one sector.

- In open economies ($a_{\mathsf{H}} < 1$), however, inflation responds to additional factors:

  - cross-country misalignments in international relative prices of goods ($\widehat{\mathcal{T}_t} + \widehat{\Delta}_{\mathsf{H},t}$) and in the relative price of consumption, $\widehat{\Theta}_t$, both measured with respect to their efficient levels $\widetilde{\mathcal{T}}_t^{fb}$ and $\widetilde{\Theta}_t^{fb}$. For future reference, note that the relative price terms drop out from the NKPC in the particular case in which $\sigma\phi = 1$.

  - the welfare-relevant measure of cross-country demand $\widehat{\mathcal{D}}_t$. which is $\widehat{\mathcal{D}}_t = 0$ in the efficient allocation with perfect risk sharing.

Compare to PC in a closed-economy model with two sectors, in which the parameter $a_{\mathsf{H}}$ would index the weight of the two goods in consumption. With a representative agent, the Phillips Curve for sectoral inflation (see e.g. Woodford 2003, chapter 3) is also a function of the efficient gap of the relative price between the two goods, ($\widehat{\mathcal{T}}_t - \widetilde{\mathcal{T}}_t^{fb}$ in our notation). BUT in closed economy one representative agent supplies labor inputs to the two sectors, while in an open-economy setting, there are multiple agents with generally different preferences, supplying good-specific labor inputs. In closed-economy analyses the output gap is usually referred to aggregate output. The coefficient multiplying relative prices is a function of labor elasticity, that is, $\eta\phi + 1$, instead of $1 - \sigma\phi$. Furthermore, price discrimination and deviations from the law of one price $\widehat{\Delta}_{\mathsf{H},t}$ are only conceivable in a heterogenous-agent economy. In comparing the two settings, a final important issue refers to the possibility of aggregating multiple agents into a world representative agent — as discussed below, this will require either the assumption of complete markets within and across borders, or some restrictions on preferences and shocks.