

International Economics

EUI 2010

Properties of the New Keynesian Model

- Transmission of monetary policy
- Instrument (money and interest rates) rules
- The effects of monetary and productivity shocks: impulse responses
- Is the NK PC consistent with the data?

The NK model is described by:

$$\text{Dynamic IS} : \hat{y}_t = -\frac{1}{\sigma} \left(\hat{i}_t - E_t \hat{\pi}_{t+1} \right) + E_t \hat{y}_{t+1}$$

$$\text{NKPC} : \hat{\pi}_t = \frac{(1 - \gamma\beta)(1 - \gamma)}{\gamma} \widehat{mc}_t + \beta E_t \hat{\pi}_{t+1}$$

$$\text{(mc)} : \widehat{mc}_t = \left(\sigma + d \frac{n^{ss}}{1 - n^{ss}} \right) \hat{y}_t - \left(1 + d \frac{n^{ss}}{1 - n^{ss}} \right) z_t$$

$$\text{(LM)} : \widehat{m}_{t+1} - \hat{p}_t = \frac{\sigma}{b} \hat{y}_t - \frac{1}{b} \hat{i}_t$$

plus $z_t = \gamma_z z_{t-1} + \varepsilon_{zt}$, with $\varepsilon_{zt} \sim \text{nid}(0, V_z)$, and a description of monetary policy (to be specified below).

- Solve the “Dynamic IS” forwards:

$$\hat{y}_t = \hat{c}_t = -\frac{1}{\sigma} \sum_{s=0}^{\infty} E_t \left(\hat{i}_{t+s} - E_t \hat{\pi}_{t+1+s} \right)$$

output=demand is determined by a geometric average of short term real interest rates over the indefinite future: the yield of a very-long term zero coupon bond

- Therefore **monetary policy can affect output only through current and future expected real interest rates**
- The reason is simple: From the Euler equation it follows that the path of output is determined by the real interest rate - a lower real interest rate increases current output relative to future expected output because it makes current consumption cheap relative to future consumption
- Hence, the real interest rate is key for understanding the transmission mechanism: A positive monetary expansion that lowers the real interest rate raises demand, which in turn raises output

- Why does output rise? Some firms cannot change their prices. Hence, they meet the higher demand at given prices
- However, by the first order condition in the labor market, higher activity translates into higher nominal wages
- Implications: with the monetary expansion, the mark-up of non-adjusting firms fall below its desired level. **Hence, countercyclical movements in mark-up are key**
- Recall: shocks to marginal costs (productivity) can affect current output only insofar as they affect real rates

The transmission of monetary policy

- We need to close the model by introducing a description of monetary policy
- First, let's look at the effects of monetary policy assuming an *exogenous money growth rule*:

$$\hat{g}_{mt} = \rho_m \hat{g}_{mt-1} + e_t^m$$

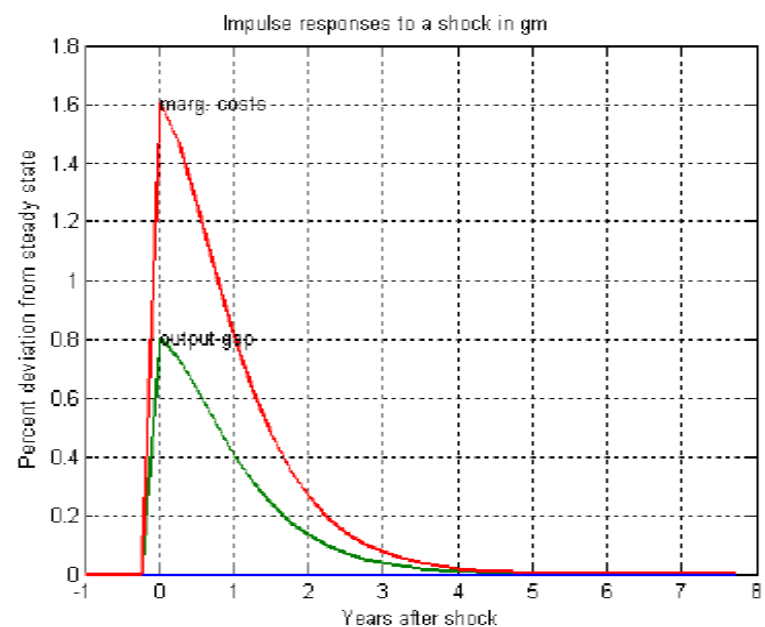
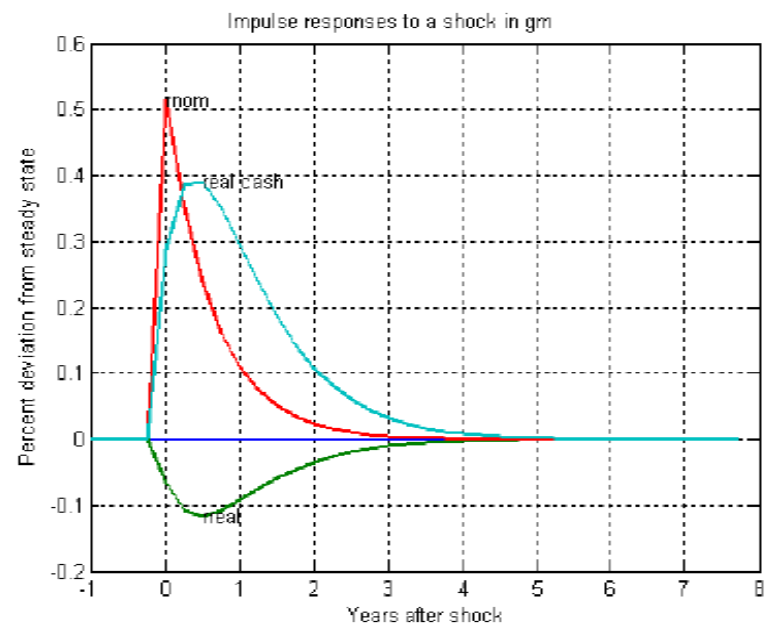
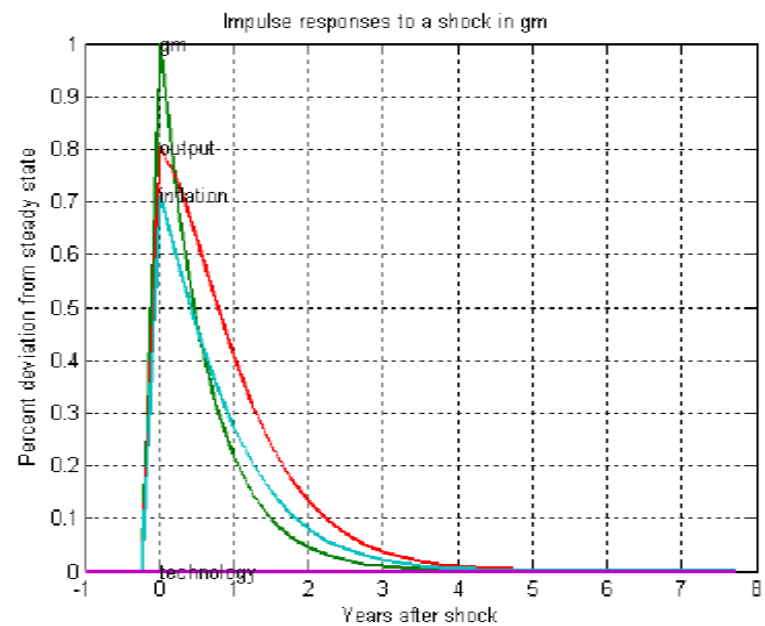
we can write:

$$\begin{aligned} m_{t+1} &= g_{mt} m_t \Rightarrow \\ m_{t+1}/p_t &= (m_t/p_{t-1}) g_{mt} (p_{t-1}/p_t) \Rightarrow \\ \widehat{m}_{t+1} - \widehat{p}_t &= \widehat{m}_t - \widehat{p}_{t-1} - \widehat{\pi}_t + \widehat{g}_{mt} \end{aligned}$$

- Recall: substituting the Euler equation into the money demand equation yields:

$$2\hat{y}_t = \frac{1}{\sigma} E_t \hat{\pi}_{t+1} + \frac{b}{\sigma} (\hat{m}_{t+1} - \hat{p}_t) + E_t \hat{y}_{t+1}$$

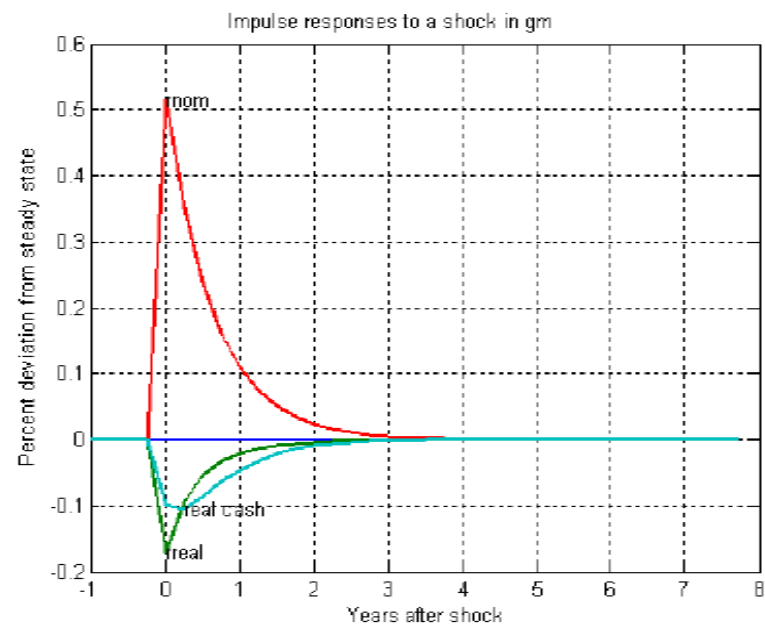
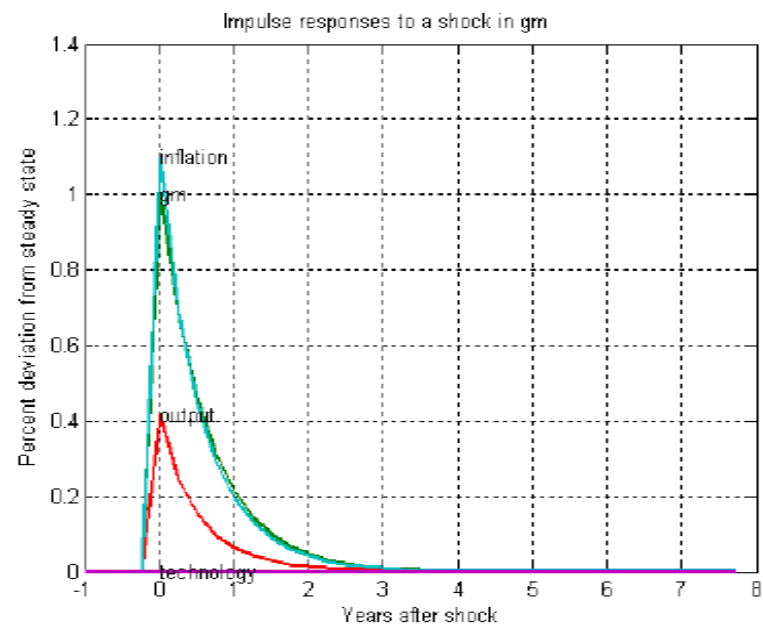
- To investigate the properties of the model, set $\beta = 0.99$, $b = d = \sigma = 1$, $\rho_m = 0.68$
- Results when $\gamma = 3/4$ so that prices stay fixed for one year (4 quarters = $1/(1 - \gamma)$ on average)
- Plot: (g_m , output, inflation, technology) (nominal and real interest rates, cash balances) (marginal costs and output gap)



- We get large and persistent output effects
- Also large and persistent effect on inflation and a drop in the real interest rate
- In these dimensions, the impulse-responses from the model appears qualitatively in line with empirical evidence.
- But there are some problems:
 1. **Lack of inflation persistence:** Inflation increases most upon impact - in the VAR literature, inflation rises persistently following a bell-shaped pattern (inflation persistence)

2. The empirical evidence also seems to indicate a **hump-shaped response of output**

3. **How much price stickiness is needed?** The above diagrams assume that prices remain fixed for a year on average (4 quarters) - this sounds unlikely. Recent studies indicate that 3-6 months is more realistic. Below we show the results for $\gamma = 1/2$ which is in the upper end of these more recent estimates.



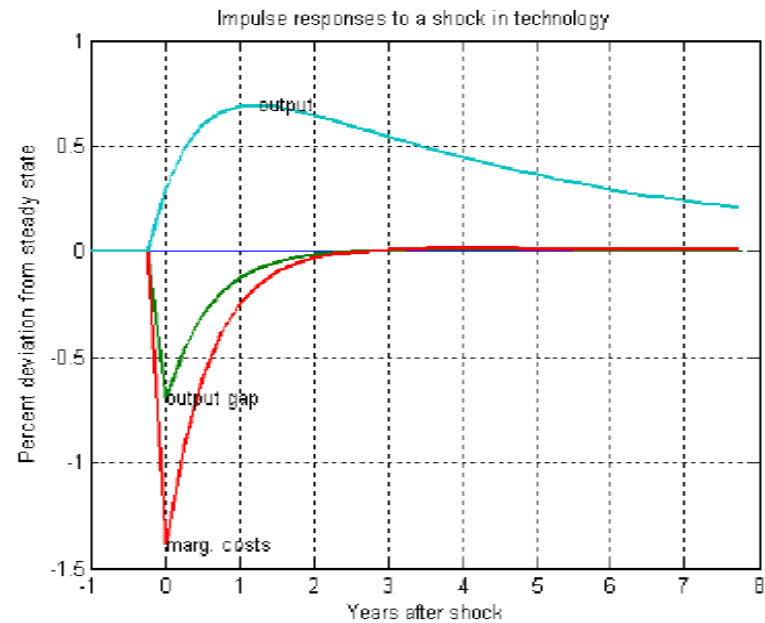
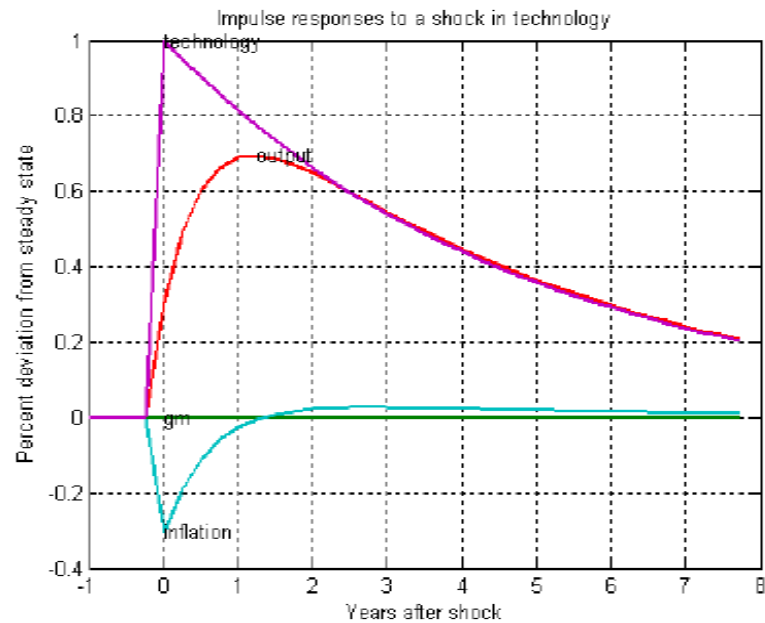
- Much smaller output effects and much larger inflation effects when prices are more flexible
4. **Distribution of profits may be weird:** Suppose there is a positive money supply shock. This increases nominal marginal costs but some firms cannot

change prices. If $\gamma = 3/4$, after 4 quarters there is still a large number of firms that have not adjusted prices:

$$\vartheta = \gamma^4 = 81/256 = 31.6\%$$

If the money supply shock is large enough, these firms will be making large negative profits.

5. Response to technology shocks: In the sticky price model, technology shocks increase output but the elasticity of output is smaller than unity - some firms cannot change prices. (For an exogenously given monetary policy) when the elasticity of output is less than one, technology shocks will lead to a decline in hours worked.



- Notice that output is below its efficient level (the output gap is negative) due to price stickiness
- Whether hours worked respond positively or negatively to technology shocks is one of the most hotly debated topics in current research. This implication of sticky price models may / may not be consistent with the empirical evidence.

Monetary policy (interest rate) rules

- Above we analyzed the effects of monetary policy when the money supply rule is entirely exogenous.
- There is no endogenous policy responses to macro variables: this is not what we observe, as much of monetary policy is arguably endogenous — Central Banks respond to shocks
- Let's now introduce systematic policy rules, first arbitrary ones, then we will derive them as optimal rule from the model itself.
- To start with we will focus on instrument rule, in the form of interest rate rules (rather than money growth rules).

- We will see two important things:
 - (a) Not all monetary policy rules lead to equilibrium determinacy
 - (b) The policy rule fundamentally affects the response of the economy to shocks
- The importance of the policy rule simply follows from the fact that, as we have seen before, the real interest rate is key for understanding the effects of monetary policy — therefore the rule followed by monetary authorities in setting the nominal interest rate affects the economy through the implied path for real interest rates

- Consider an interest rate rule with interest rate ‘smoothing’ in the form:

$$\begin{aligned}\hat{i}_t &= \rho_i \hat{i}_{t-1} + e_t^i \\ \rho_i &< 1\end{aligned}$$

where e_t^i denotes stochastic monetary policy shocks. Note that this is exogenous as was the monetary rule above

- Implicitly here, the Central Bank adjusts the money supply growth rate according to its interest rate rule
- In this case, the NK model is given by the equations:

$$\begin{aligned}\hat{y}_t &= -\frac{1}{\sigma} (\hat{i}_t - E_t \hat{\pi}_{t+1}) + E_t \hat{y}_{t+1} \\ \hat{\pi}_t &= \beta E_t \hat{\pi}_{t+1} + \kappa \hat{y}_t - \lambda z_t \\ \hat{i}_t &= \rho_i \hat{i}_{t-1} + e_t^i\end{aligned}$$

since the money demand equation in the model is redundant — it simply determines real cash balances

- Rewrite the equations above as:

$$\begin{bmatrix} 1 & 0 & 0 \\ -\kappa & 1 & 0 \\ 0 & 0 & \rho_i \end{bmatrix} \begin{bmatrix} \hat{y}_t \\ \hat{\pi}_t \\ \hat{i}_{t-1} \end{bmatrix} = \begin{bmatrix} 1 & \frac{1}{\sigma} & -\frac{1}{\sigma} \\ 0 & \beta & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} E_t \hat{y}_{t+1} \\ E_t \hat{\pi}_{t+1} \\ \hat{i}_t \end{bmatrix} + \begin{bmatrix} 0 \\ -\lambda z_t \\ -e_t^i \end{bmatrix}$$

- Pre-multiplying with the inverse of the matrix on the left hand side we get:

$$\begin{bmatrix} \hat{y}_t \\ \hat{\pi}_t \\ \hat{i}_{t-1} \end{bmatrix} = \begin{bmatrix} 1 & \frac{1}{\sigma} & -\frac{1}{\sigma} \\ \kappa & \beta + \frac{\kappa}{\sigma} & -\frac{\kappa}{\sigma} \\ 0 & 0 & \frac{1}{\rho_i} \end{bmatrix} \begin{bmatrix} E_t \hat{y}_{t+1} \\ E_t \hat{\pi}_{t+1} \\ \hat{i}_t \end{bmatrix} + \begin{bmatrix} 0 \\ -\lambda z_t \\ -\frac{1}{\rho_i} e_t^i \end{bmatrix}$$

- This is a system of linear expectational difference equations with 2 forward-looking variables: Saddle path stability therefore requires that two of the roots

of the matrix on the right hand side are inside the unit circle

- Clearly the root of the interest rate $\left(\frac{1}{\rho_i}\right)$ is greater than unity. Thus the roots of:

$$\begin{bmatrix} 1 & \frac{1}{\sigma} \\ \kappa & \beta + \frac{\kappa}{\sigma} \end{bmatrix}$$

must both be inside the unit circle

- We find the roots, ξ_i , from solving:

$$\begin{vmatrix} 1 - \xi & \frac{1}{\sigma} \\ \kappa & \beta + \frac{\kappa}{\sigma} - \xi \end{vmatrix} = 0$$

The roots are given by:

$$\xi_i = \frac{1 + \beta + \frac{\kappa}{\sigma} \pm \sqrt{D}}{2}$$
$$D = \left(1 + \beta + \frac{\kappa}{\sigma}\right)^2 + 4\frac{\kappa}{\sigma}$$

Note that $\sqrt{D} > \left(1 + \beta + \frac{\kappa}{\sigma}\right)$ and therefore $1 + \beta + \frac{\kappa}{\sigma} + \sqrt{D} > 2\left(1 + \beta + \frac{\kappa}{\sigma}\right) > 2$. Thus, the largest of these roots is also outside the unit circle

- This implies that **the equilibrium is locally indeterminate** (many solutions in the neighborhood of $(\tilde{y}_t, 0)$ that fulfill the equilibrium conditions) and there can be self-fulfilling expectational equilibria
- Intuition: Consider a rise in expected inflation which (according to the “IS curve”) increases output since the nominal interest rate is not affected due to the interest rate rule (i.e. the real interest rate decreases)

- The increase in output, according to the Phillips curve then increases actual inflation: In other words, the expected higher inflation is self-fulfilling
- This indicates that, **to get stability, we must have an interest rate rule that increases the nominal interest rate in response to inflation**

- Consider now a **Taylor-type rules** (without interest rate smoothing):

$$\hat{i}_t = \alpha_\pi \hat{\pi}_t + \alpha_y y_t^{gap} + e_t^i$$

Here y_t^{gap} is the percentage deviation of output from its flexible price level (see last lecture):

$$\begin{aligned} y_t^{gap} &= \hat{y}_t - \tilde{y}_t \\ \tilde{y}_t &= \frac{\left(1 + d \frac{n^{ss}}{1 - n^{ss}}\right)}{\left(\sigma + d \frac{n^{ss}}{1 - n^{ss}}\right)} z_t \end{aligned}$$

- The Taylor rule attempts to stabilize inflation (with weight α_π) and to target the flexible price level of output (with weight α_y). It thus describes a monetary policy that systematically and endogenously responds to the conditions of the economy
- The arguments in the previous slides suggests that, in order to get determinacy, α_π must be high relative to α_y

- Were $\alpha_y = 0$, determinacy simply requires that $\alpha_\pi > 1$ — in this case, an increase in expected inflation increases the real interest rate which lowers output and therefore eliminates self-fulfilling expectations
- Sometimes the literature refers to (α_π, α_y) as the **preferences of the Central Banker** — a soft-one would be $(\alpha_\pi/\alpha_y$ small), a tough-nosed $(\alpha_\pi/\alpha_y$ large)
- Now look at the response of the economy to shocks for a slightly generalized version of the Taylor rule, allowing for “interest rate smoothing” through ρ_i :

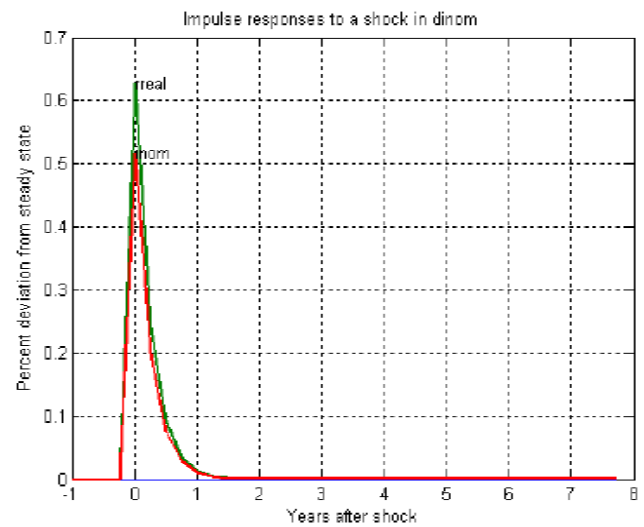
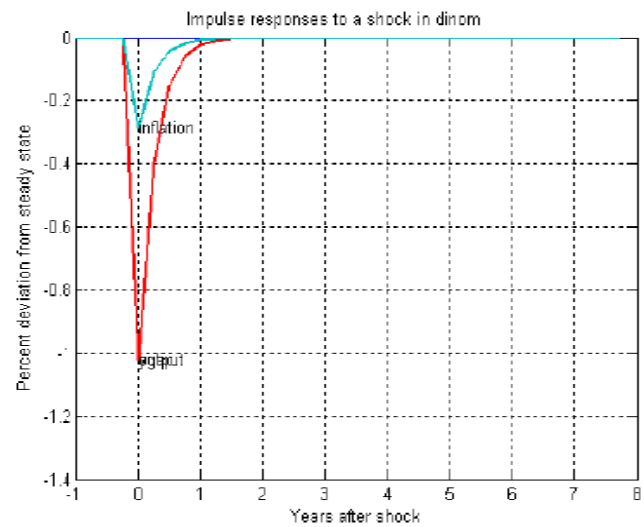
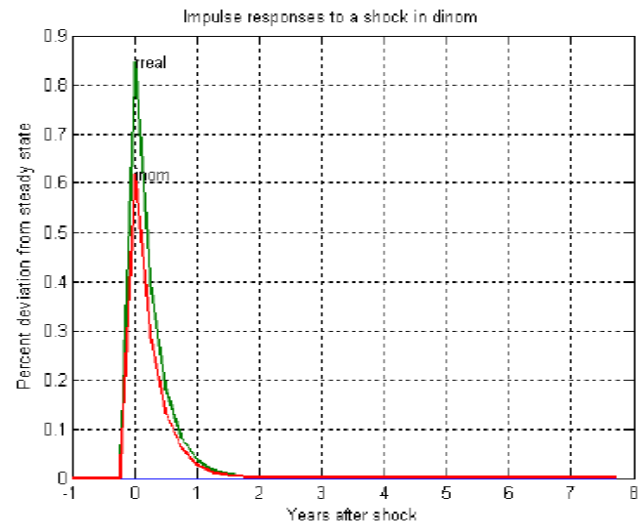
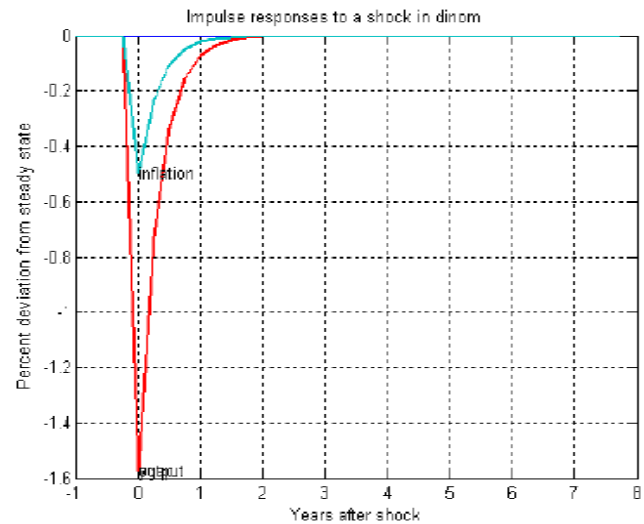
$$\hat{i}_t = \rho_i \hat{i}_{t-1} + (1 - \rho_i) (\alpha_\pi \hat{\pi}_t + \alpha_y y_t^{gap}) + e_t^i$$

- Top row is for a softie, $(\alpha_\pi, \alpha_y) = (1.5, 0.5)$
- Bottom row is for a tough nosed central banker $(\alpha_\pi, \alpha_y) = (5, 0.5)$

- In both cases, I set $\rho_i = 0.75$

1. The Effects of Monetary Policy Shocks

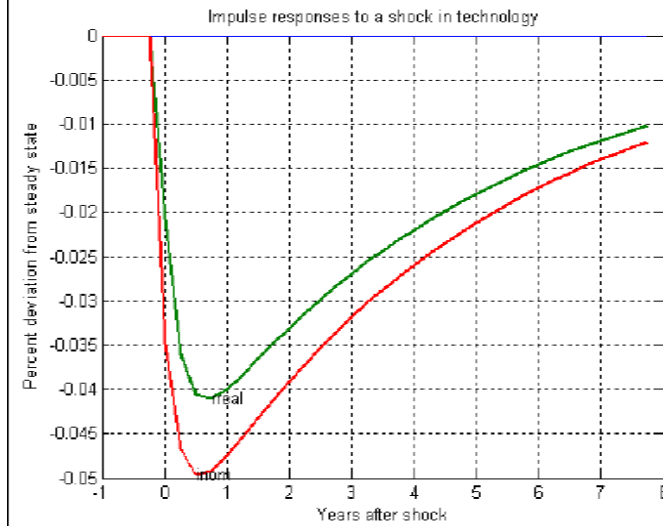
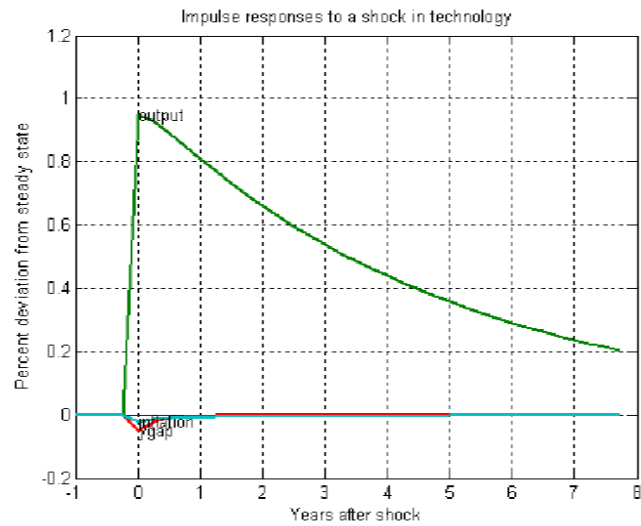
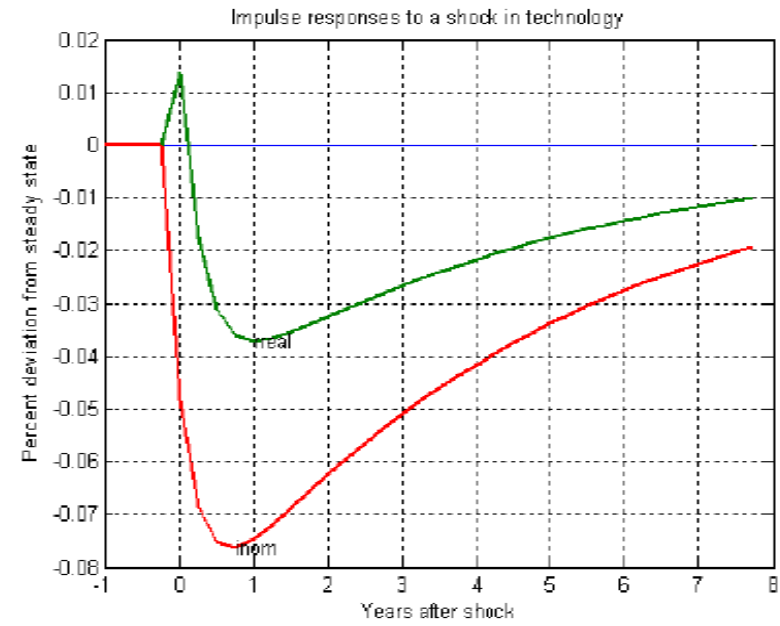
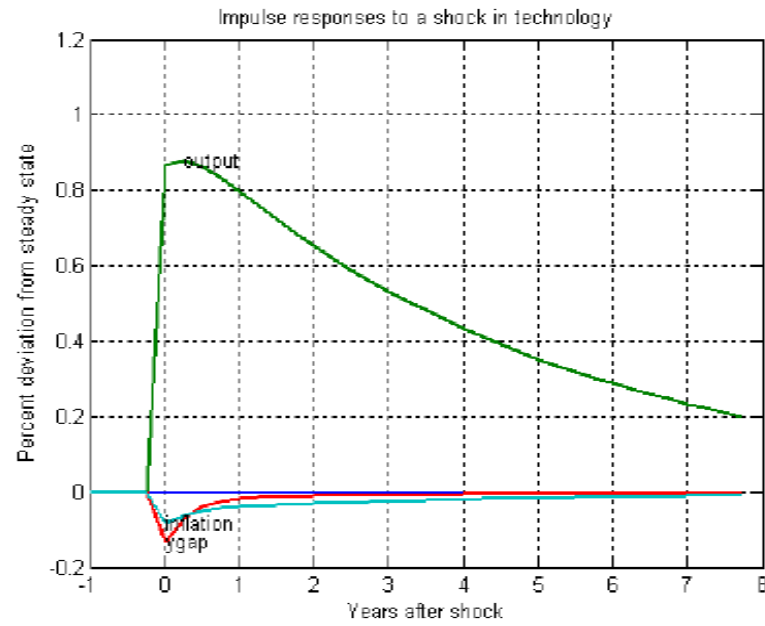
- plot: (inflation, output gap ygap) (nominal and real interest rates)



- The weaker CB is more “destabilizing” - monetary policy shocks have a larger impact on real interest rates

2. The Effects of Technology Shocks

- plot: (output, inflation, output gap) (nominal and real interest rates)



- Two conclusions:

1. The dynamics of the economy in response to the technology shock depends on the policy rule.
2. Although the tough nosed central banker cares less about the output gap, she succeeds better in stabilizing the economy in the sense of eliminating the output gap. More about this in future lectures.

Is the NK Phillips curve “consistent” with the data?

- Galí and Gertler, Journal of Monetary Economics, 1999, estimate the NK Phillips curve and generalize it slightly by allowing for backward looking price setters as well.
- Specifically, the NK Phillips curve is given by:

$$\hat{\pi}_t = \frac{(1 - \gamma\beta)(1 - \gamma)}{\gamma} \widehat{mc}_t + \beta E_t \hat{\pi}_{t+1}$$

- Galí and Gertler assume a Cobb-Douglas production function:

$$y_t = \exp(z_t) k_t^{\alpha_k} n_t^{\alpha_n}$$

In this case, real marginal costs are given as:

$$\begin{aligned} mc_t &= \frac{W_t/P_t}{\partial y_t / \partial n_t} = \frac{W_t}{P_t \alpha_n y_t / n_t} \\ &= \frac{W_t n_t}{\alpha_n P_t Y_t} = \frac{s_t^n}{\alpha_n} \\ s_t^n &= \frac{W_t n_t}{P_t Y_t} \end{aligned}$$

i.e. real marginal costs depend only on the labor share of income

- Galí and Gertler allow for a ‘hybrid’ NK curve by assuming that a share $1 - \omega$ of producers are forward looking, while another share ω are backward looking and set prices simply adjusting past prices for realized (lagged) inflation. This

implies a 'hybrid' NK curve:

$$\hat{\pi}_t = (1 - \omega) \frac{(1 - \gamma\beta)(1 - \gamma)}{\gamma} \widehat{mc}_t + \varsigma_f E_t \hat{\pi}_{t+1} + \varsigma_b \hat{\pi}_{t-1}$$

$$\varsigma_f = \frac{\beta\gamma}{\gamma + \omega(1 - \gamma(1 - \beta))}$$

$$\varsigma_b = \frac{\omega}{\gamma + \omega(1 - \gamma(1 - \beta))}$$

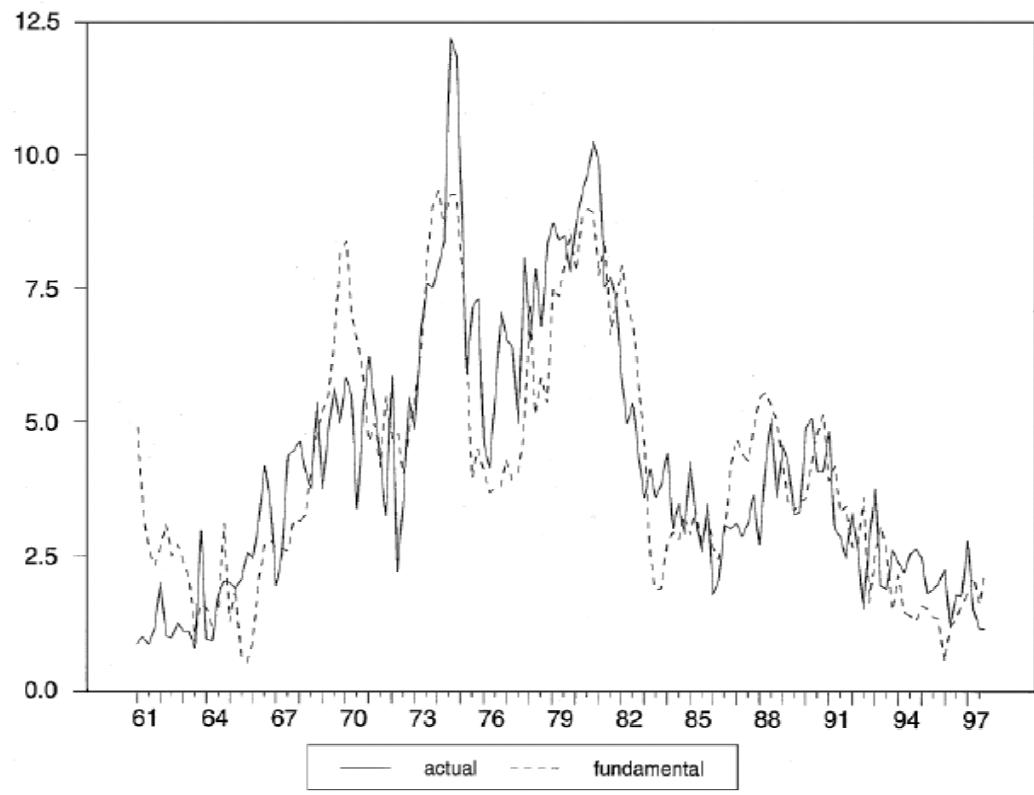
where we get the standard NK Phillips curve when $\omega = 0$

- Questions: (a) Does it fit the data, (b) how sticky are prices, (c) is it important to allow for 'rule-of-thumb' producers (the backward looking ones). Estimate relationship using US quarterly data for the period 1960-1997
- Notation: θ corresponds to our γ and (γ_f, γ_b) correspond to our $(\varsigma_f, \varsigma_b)$

Table 2
Estimates of the new hybrid Phillips curve

	ω	θ	β	γ_b	γ_r	λ
GDP deflator						
(1)	0.265 (0.031)	0.808 (0.015)	0.885 (0.030)	0.252 (0.023)	0.682 (0.020)	0.037 (0.007)
(2)	0.486 (0.040)	0.834 (0.020)	0.909 (0.031)	0.378 (0.020)	0.591 (0.016)	0.015 (0.004)
Restricted β						
(1)	0.244 (0.030)	0.803 (0.017)	1.000	0.233 (0.023)	0.766 (0.015)	0.027 (0.005)
(2)	0.522 (0.043)	0.838 (0.027)	1.000	0.383 (0.020)	0.616 (0.016)	0.009 (0.003)
NFB deflator						
(1)	0.077 (0.030)	0.830 (0.016)	0.949 (0.019)	0.085 (0.031)	0.871 (0.018)	0.036 (0.008)
(2)	0.239 (0.043)	0.866 (0.025)	0.957 (0.021)	0.218 (0.031)	0.755 (0.016)	0.015 (0.006)

- Estimate of γ (θ in their notation) implies that prices remain fixed for an average of about 5 quarters - very high!
- In most regressions, there is a significant share of producers (ω) that appear to use rule-of-thumb pricing — this is a concern
- Galí and Gertler, however, argue that the rule-of-thumb producers are not very important, because their share is much smaller than the share of forward looking producers
- How well does the model do at forecasting inflation? Gali and Gertler construct an inflation forecast on the basis of the parameter estimates above and compare it to actual inflation:



- The model does pretty well and tracks particularly well the deflation in the 1990's
- In sum: The NK Phillips curve works OK but there appears to be a need for including a backward looking term in there — introducing this through rule-of-thumb producers is ad-hoc

Appendix: An extension of the model Indexation

- The NK model assumes that non-adjusters cannot change their price at all. This is perhaps a bit too strong an assumption — after all, it might be easy to write down price contracts/pricing rules including automatic indexation (so the price rises with inflation unless it's reoptimized)
- Hence, assume that with probability $(1 - \gamma)$ the firm can reset its price to its optimal level, with probability γ the firm cannot reset to the optimal level but its preset price nonetheless adjust with the **realized** past inflation rate
- When the firm gets the chance to reset its price, it faces the optimization

problem:

$$\begin{aligned} & \max_{P_{it}^*(s)} H_{it}(p_{it}^*) \frac{1}{p_t} + E_t \sum_{\tau=t+1}^{\infty} \gamma^{\tau-t} R_{t,\tau} \left(H_{i\tau} \left(p_{it}^* X_{t,\tau} \right) \right) \frac{1}{p_\tau} \\ & : X_{t,\tau} = 1 \text{ for } \tau = t \\ & : X_{t,\tau} = \pi_t \times \pi_{t+1} \times \dots \times \pi_{t+\tau-1} \end{aligned}$$

where $H_{it}(p_{it}^*)$ denotes the nominal profits and π_t denotes the inflation rate between period $t - 1$ and period t

- Imagine, for example, that the firm adjusts in period t but not in period $t + 1$. Then indexation means that the price of its good in period $t + 1$ is $p_{it}^* \pi_t$
- Going through the same algebra as in the last lecture, we get

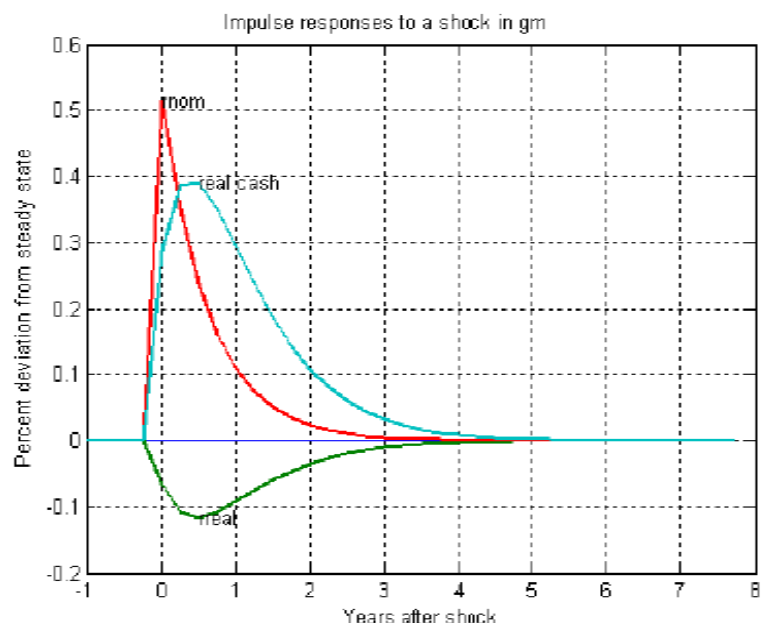
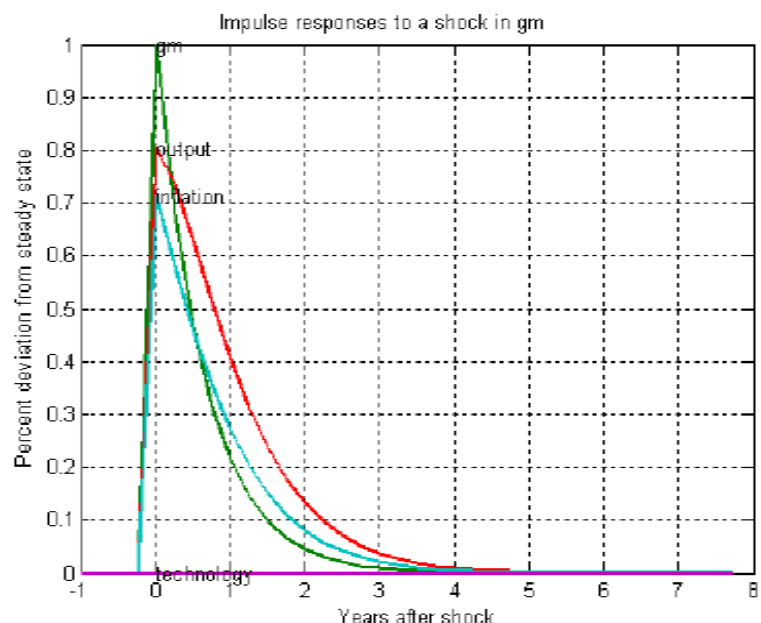
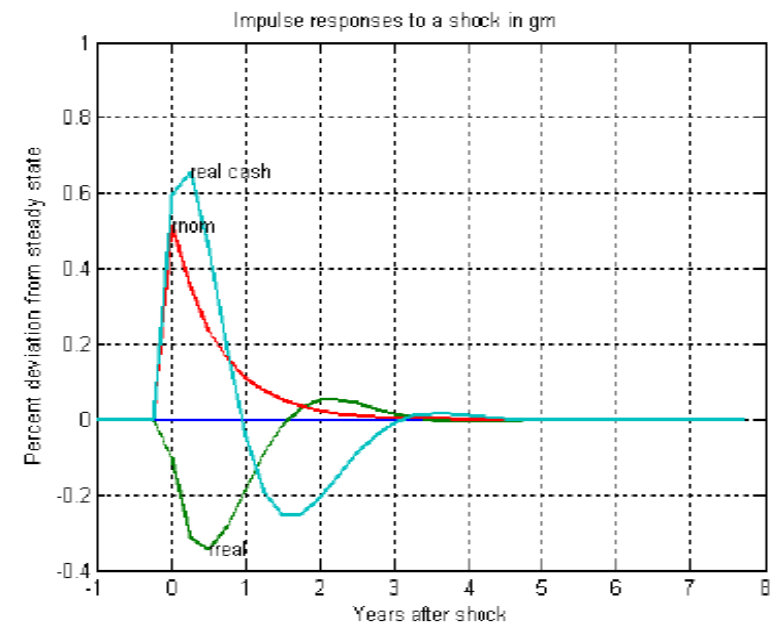
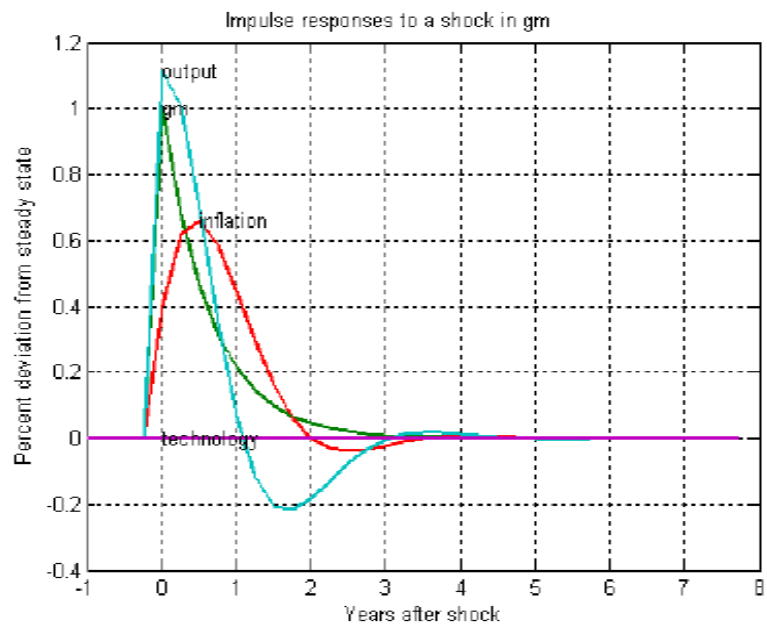
$$\hat{\pi}_t - \hat{\pi}_{t-1} = \beta (E_t \hat{\pi}_{t+1} - \hat{\pi}_t) + \frac{(1 - \beta\gamma)(1 - \gamma)}{\gamma} \widehat{mc}_t$$

- This looks like the standard NK Phillips curve but there is one very important change: It relates the change in inflation to marginal costs and the expected future change in inflation - therefore, we will get inflation persistence since:

$$\hat{\pi}_t = \frac{\beta}{1 + \beta} E_t \hat{\pi}_{t+1} + \frac{1}{1 + \beta} \hat{\pi}_{t-1} + \frac{(1 - \beta\gamma)(1 - \gamma)}{\gamma(1 + \beta)} \widehat{mc}_t$$

i.e. the Phillips curve relates current inflation to past and future inflation

- Below we compare the indexation model (top row) with the non-indexation (bottom row) for $\gamma = 3/4$



- We see that:
 1. Non-indexation gives rise to more persistent monetary policy shocks and to “hump-shaped” response of inflation due to the induced inflation persistence.
 2. The effects on output, however, are less persistent under indexation — the reason is that the fall in the real interest rate is less persistent
- Thus, while indexation gives rise to longer lasting effects on inflation and to a delayed peak response, it moderates the output effects.
- Eichenbaum and Fisher (2005) re-estimate the Phillips curve introducing indexation. They also add a few other features that (a) make marginal costs increasing, and (b) imply countercyclical mark-ups under flexible prices. With these modifications they find that:

Table 3: Frequency of Re-optimization: Full Sample Results

Deflator	ϵ	Rental Market for Capital		Firm-Specific Capital			
		θ	$\frac{1}{1-\theta}$	θ	$\frac{1}{1-\theta}$	θ	$\frac{1}{1-\theta}$
GDP	0	0.88	8.3	0.83	5.9	0.72	3.6
		[0.78, 0.98]	[4.5, 50.0]	[0.65, 0.97]	[2.9, 33.3]	[0.53, 0.95]	[2.1, 20.0]
	10	0.83	5.9	0.79	4.8	0.70	3.3
		[0.70, 0.96]	[3.3, 25.0]	[0.60, 0.96]	[2.5, 25.0]	[0.51, 0.94]	[2.0, 16.7]
	33	0.76	4.2	0.72	3.6	0.66	2.9
		[0.60, 0.95]	[2.5, 20.0]	[0.52, 0.94]	[2.1, 16.7]	[0.46, 0.93]	[1.9, 14.3]
PCE	0	0.86	7.1	0.80	5.0	0.69	3.2
		[0.77, 0.96]	[4.3, 25.0]	[0.62, 0.95]	[2.6, 20.0]	[0.51, 0.91]	[2.0, 11.0]
	10	0.81	5.3	0.76	4.2	0.67	3.0
		[0.68, 0.94]	[3.1, 16.7]	[0.57, 0.93]	[2.3, 14.3]	[0.49, 0.90]	[2.0, 10.0]
	33	0.73	3.7	0.69	3.2	0.63	2.7
		[0.57, 0.91]	[2.3, 11.1]	[0.49, 0.91]	[2.0, 11.1]	[0.44, 0.88]	[1.8, 8.3]

Note: Estimates based on labor's share equal to 2/3, a 10% markup and a 2.5% quarterly depreciation rate.

- Note: θ in their notation denotes γ in our notation. The last row reports the estimates when there are increasing marginal costs and countercyclical optimal mark-ups
- The estimates of the Calvo parameter are now more realistic implying that firms can re-adjust their prices every 3 quarters.

Determinacy with forward-looking interest rules

Posit that central bank responds to expected inflation (not to current inflation):

$$i_t = r_t^{nr} + \phi_\pi E_t \{\pi_{t+1}\} + \phi_y E_t \{\tilde{y}_{t+1}\}$$

Verify that, to ensure determinacy, ϕ_π needs to be large (as before), but not too large. Intuitively: start with expectations of positive inflation (which is the percentage change in prices). If the central bank reacts very strongly, the contraction in current demand is deflationary: the price level actually falls, validating the initial inflation expectations.

Reacting to 'current inflation' rules out this possibility: the fall in current inflation would bring the interest rate down.