International Economics

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NOMINAL RIGIDITIES and the foundations of the New Keynesian Model

- 1. Sticky prices and imperfect competion

  2. The Baseline NK model

     – Calvo-Yun pricing

     – General equilibrium

  3. The gaps with the ‘natural rate’ allocation

Sticky Prices and Imperfect Competition

- Sticky price models: Firms must stand ready to satisfy any demand that they face at the (possibly predetermined) price that they quote

- This is incompatible with competitive behavior on the part of firms => Thus, we need to introduce some sort of imperfect competition in goods markets.

- Many different ways of doing this: monopoly, oligopoly etc.

- Most popular in macroeconomics is to assume monopolistic competition: Many firms, each has some price setting power but is small (atomistic) and takes the behavior of all other firms for given - i.e. no strategic interaction

- In a monopolistic competition framework, an individual firm faces the profit maximization problem:

$$\begin{aligned} \max_{p_i} \Pi &= d\left(p_i/P\right) p_i - h(d\left(p_i/P\right)) \\ d' &< 0,\ h' > 0,\ h'' \geq 0 \end{aligned}$$

where p_i is the price of the firm's output, and P is the general price index which the firm takes for given; d is real demand for the firm's output, h are *nominal* costs.

- The first order condition is:

$$\begin{aligned} d'\left(p_i/P\right) p_i/P + d\left(p_i/P\right) &= h'(d\left(p_i/P\right))d'\left(p_i/P\right) 1/P \\ &\Rightarrow \\ p_i\left(1 + \frac{d\left(p_i/P\right)}{d'\left(p_i/P\right)}\frac{P}{p_i}\right) &= h'(d\left(p_i/P\right)) \end{aligned}$$

$$\Rightarrow$$

$$\begin{aligned}
\underbrace{p_i}_{\text{nominal price}} &= \underbrace{\mu\left(p_i/P\right)}_{\text{markup}} \quad \underbrace{h'\left(d\left(p_i/P\right)\right)}_{\text{nominal marginal costs}} \\
\mu\left(p_i/P\right) &= \frac{\theta\left(p_i/P\right)}{\theta\left(p_i/P\right)-1} \\
\theta\left(p_i/P\right) &= -\frac{d'\left(p_i/P\right)}{d\left(p_i/P\right)}\frac{p_i}{P}
\end{aligned}$$

- i.e. the price is determined as a mark-up $(\mu \geq 1)$ over the marginal costs

- the mark-up depends on the price-elasticity of demand (θ) – the more elastic is demand, the lower will be the mark-up

- **Important feature 1**: Higher mark-up increases the wedge between prices and marginal costs and depresses output - level and dynamics of mark-ups are key in imperfect competition models

- **Important feature 2**: If the price elasticity of demand is constant, changes in prices occur due to changes in marginal costs

- As equilibrium prices are above marginal costs, model with imperfect competition are consistent with the idea that, *within limits,* a firm can meet actual demand fluctuations without changing its (sticky) price and still get non-negative profits.

- Finally, because h are **nominal** costs, we can also express the first-order condition as:

$$\frac{p_i}{P} = \mu\left(p_i/P\right) mc_i\left(d\left(p_i/P\right)\right)$$

  where mc_i are real marginal costs

- The mark-up will be constant when the demand function displays constant price elasticity:

$$d\left(p_i/P\right) = \left(p_i/P\right)^{-\eta} D$$

and marginal costs will be independent of the scale of production when the production function displays constant returns to scale.

- Under these conditions, the pricing rule becomes:

$$\begin{aligned} \frac{p_i}{P} &= \mu m c_i \\ \mu &= \frac{\eta}{\eta - 1} > 1 \end{aligned}$$

- These conditions are imposed in much of the New Keynesian literature that has adopted sticky prices

Why might prices be "sticky"?

1. Behavioral reasons (customer relations, market share concerns etc.)

2. Informational reasons (firms not aware that optimally they should change prices)

3. Costly to change prices - if the extra profit that arise from changing prices is small enough, better to keep prices stable

4. Contractual reasons: Prices set in advance for some period

- Concentrate on 3&4 but lots of current interest in 1&2 as well

The 'baseline' New Keynesian Model with Calvo Staggered Contracting

- Preferences are given as:

$$\begin{aligned} V_0 &= E_0 \sum_{t=0}^{\infty} \beta^t u\left(c_t, m_{t+1}/p_t, l_t\right) \\ l_t &= 1 - n_t \\ u_t &= \frac{c^{1-\sigma}}{1-\sigma} + \frac{\vartheta}{1-b}\left(\frac{m_{t+1}}{p_t}\right)^{1-b} + \phi \frac{l_t^{1-d}}{1-d} \end{aligned}$$

  Note the separability between consumption and real money balances. This implies that money is neutral and superneutral if prices are flexible. Now, because of nominal stickiness, these propositions will hold only in the long run

- The budget constraint is

$$\begin{aligned} & c_t + \frac{b_{t+1}}{p_t} + \frac{m_{t+1}}{p_t} \\ = \; & w_t n_t + \tau_t + \frac{(1 + i_{t-1})\, b_t}{p_t} + \frac{m_t}{p_t} + \Pi_t \end{aligned}$$

  where profits from ownership of firms are rebated to the households (note: it abstracts from capital accumulation)

- Households rent out labor to the firms at the real price w_t, which they take as given. Different from the product market, *labor market is competitive*

- The first order conditions for the households problem are:

$$
\begin{aligned}
c_t^{-\sigma} &= \beta E_t c_{t+1}^{-\sigma}(1+r_t) \\
\frac{\phi(1-n_t)^{-d}}{c_t^{-\sigma}} &= w_t \\
\frac{\vartheta(m_{t+1}/p_t)^{-b}}{c_t^{-\sigma}} &= \frac{i_t}{1+i_t} \\
1+i_t &= (1+r_t)\,p_{t+1}/p_t
\end{aligned}
$$

Note that wages are flexible.

Final Goods Firms

- It is useful to assume that there are two sectors in the economy: A final goods sector and an intermediate goods sector.

- The final goods sector is competitive and firms produce a single homogeneous sector. This good is produced using inputs of a continuum of differentiated intermediate goods.

- The final goods sector technology is given as a CES aggregator with elasticity of substitution $1/\rho > 1$:

$$y_t = \left(\int_0^1 y_{it}^{1-\rho} di \right)^{1/(1-\rho)}$$

where y_{it} denotes the input of intermediate goods of variety i. These goods are purchased at price p_{it}. Minimizing costs then gives us the demand for each variety:

$$
\begin{aligned}
y_{it}^d &= \left(\frac{p_{it}}{p_t}\right)^{-1/\rho} y_t \\
p_t &= \left(\int_0^1 p_{it}^{1-1/\rho} di\right)^{1/(1-1/\rho)}
\end{aligned}
$$

- p_t is the general price level that enters households' budget constraint

- Note that the price elasticity of demand is constant - therefore, from the vantage point of an imperfectly competitive supplier, the **optimal** mark-up is constant and equal to $1/(1-\rho) > 1$

Intermediate Goods Firms

- Intermediate goods firms produce output using labor.

- The production function is given as:

$$y_{it} = \exp(z_t)\, n_{it}$$

where z_t is a stochastic productivity shock:

$$\begin{aligned} z_t &= \gamma_z z_{t-1} + \varepsilon_{zt} \\ & \quad \varepsilon_{zt} \tilde{\ } nid(0, V_z) \end{aligned}$$

- We will assume that firms are unable to optimize prices in each period: some rule or contract will make this possible only at certain time. Hence pricing will be 'time dependent'

- Two standard ways of looking at **time-dependent pricing** models in macroeconomics are Taylor staggered contracting and Calvo price setting

- In the **Taylor set-up**: A large number of firms, each firm sets a price that is predetermined (by contract?) for N periods, but firms do so in staggered fashion — in period t, a share $1/N$ of firms set prices that are predetermined between periods t and $t+N$; in period $t+1$, another share $1/N$ of firms set prices that are predetermined between $t+1$ and $t+N+1$, etc.

- Therefore at any point in time, only $1/N$ of the firms can actively adjust their predetermined price

- The **Calvo set-up** assumes a probabilistic set-up in which at the beginning of every period firms receive a signal (phone call), with a given probability, telling them that they can re-optimize their price (otherwise, they have to charge the predetermined price)

- Let γ be the probability that an individual firm **does not** get the opportunity to adjust its price and let the arrival rate of the signal/phone-calls be distributed as a Poisson process (so that the probability of getting a phone-call this morning is independent of when I last got such a phone-call).

- If there are many firms (a continuum), γ is not only the probability that an individual firm does not receive the phone-call but also the proportion of firms that do not get the phone call

- Therefore, a share $1-\gamma$ of firms adjust their prices every period. In expectations, the time between price adjustment (the 'expected life' of a price) is $(1-\gamma)\sum_{i=0}^{\infty} i\gamma^{i-1} = 1/(1-\gamma)$.

- How will firms set prices in a Calvo setting? They need to maximize the expected present value of profits taking into account the likelihood with which they can reset prices in the future

- In expectations, the present value of the firm's profits is:

$$W_{it} = \Pi_{it}\frac{1}{p_t} + E_t \sum_{s=1}^{\infty} \gamma^s R_{t,t+s} \Pi_{is} \frac{1}{p_s}$$

where $R_{t,t+s}$ is the discount factor of the owners of the firm (which coincides with the households), for consumption between date t and date $t+s$. Note that in each period nominal profits are converted into real by dividing for the general price level.

- Were prices fully flexible ($\gamma = 0$), the firms' problem is equivalent to a sequence of static problems since it could simply re-optimize every period

- The more sticky are prices, the more forward looking the firm needs to be

- Consider a firm that gets the chance to re-optimize it's price at date t. In expectations, we can write the present value of its profits as

$$\begin{aligned}
&\Pi_{it}\left(p_{it}^{*}\right)\frac{1}{p_{t}}+\\
&+\left(1-\gamma\right)E_{t}R_{t,t+1}\Pi_{i,t+1}\left(p_{i,t+1}^{*}\right)\frac{1}{p_{t+1}}+\\
&+\gamma E_{t}R_{t,t+1}\Pi_{i,t+1}\left(p_{i,t}^{*}\right)\frac{1}{p_{t+1}}+\\
&+\left(1-\gamma\right)\left(1-\gamma\right)E_{t}R_{t,t+2}\Pi_{i,t+2}\left(p_{i,t+2}^{*}\right)\frac{1}{p_{t+2}}+\\
&+\left(1-\gamma\right)\gamma E_{t}R_{t,t+2}\Pi_{i,t+2}\left(p_{i,t+2}^{*}\right)\frac{1}{p_{t+2}}+\\
&+\gamma\left(1-\gamma\right)E_{t}R_{t,t+2}\Pi_{i,t+2}\left(p_{i,t+1}^{*}\right)\frac{1}{p_{t+2}}+\\
&+\gamma^{2}E_{t}R_{t,t+2}\Pi_{i,t+2}\left(p_{i,t}^{*}\right)\frac{1}{p_{t+2}}+....
\end{aligned}$$

- Note that, when setting the price at time t, the only terms that matters for the firm's problem are the ones including $p^*_{i,t}$, which determine the cash flow and profits from sales as long as the firm cannot re-optimize.

- Hence we can rewrite the firm's problem as

$$\max_{P^*_{it}(s)} \Pi_{it}\left(p^*_{it}\right)\frac{1}{p_t} + E_t \sum_{\tau=t+1}^{\infty} \gamma^{\tau-t} R_{t,\tau}\left(\Pi_{i\tau}\left(p^*_{it}\right)\right)\frac{1}{p_\tau} + ...$$

where the future terms involve the price that we set **today**, p^*_{it}, relative to the **future** general price level p_τ

- The first order condition for this problem is:

$$\underbrace{\Pi'_{it}\left(p^*_{it}\right)\frac{1}{p_t}}_{\text{effect on current profit}} + \underbrace{E_t \sum_{\tau=t+1}^{\infty} \gamma^{\tau-t} R_{t,\tau}\left(\Pi'_{i\tau}\left(p^*_{it}\right)\right)\frac{1}{p_\tau}}_{\text{expected present value of effects on future profits}} = 0$$

- Now write:

$$
\begin{aligned}
\Pi_{it} &= p_{it} y_{it}^{d} - h\left(y_{it}^{d}\right) \\
&= p_{it}\left(\frac{p_{it}}{p_t}\right)^{-1/\rho} y_t - p_t mc_t \left(\frac{p_{it}}{p_t}\right)^{-1/\rho} y_t - \varphi
\end{aligned}
$$

where φ is a fixed cost and mc_t denotes marginal costs, which are independent of the scale of production and identical across firms due to the assumption of constant returns to scale and identical productivity shocks

- The derivative of nominal profits is then given as:

$$
\begin{aligned}
\Pi_{it}' &= \left(\frac{p_{it}}{p_t}\right)^{-1/\rho} y_t - \frac{1}{\rho}\frac{p_{it}}{p_t}\left(\frac{p_{it}}{p_t}\right)^{-1/\rho-1} y_t + \frac{1}{\rho} mc_t \left(\frac{p_{it}}{p_t}\right)^{-1/\rho-1} y_t \\
&= \left[\left(1-\frac{1}{\rho}\right) p_t^{1/\rho-1} p_{it} + \frac{1}{\rho} mc_t p_t^{1/\rho}\right] p_t p_{it}^{-1/\rho-1} y_t
\end{aligned}
$$

- Therefore, we can write the intertemporal first-order condition as (work out details):

$$(p_{it}^*)^{-1/\rho-1}\, E_t \sum_{\tau=t}^{\infty} \gamma^{\tau-t} R_{t,\tau} \left\{ \left[\left(1-\frac{1}{\rho}\right) p_\tau^{1/\rho-1} p_{it}^* + \frac{1}{\rho} mc_\tau p_\tau^{1/\rho} \right] y_\tau \right\} = 0$$

$$\Rightarrow$$

$$p_{it}^* = \frac{1/\rho}{1/\rho-1} \frac{E_t \sum_{\tau=t}^{\infty} \gamma^{\tau-t} R_{t,\tau} mc_\tau p_\tau^{1/\rho} y_\tau}{E_t \sum_{\tau=t}^{\infty} \gamma^{\tau-t} R_{t,\tau} p_\tau^{1/\rho-1} y_\tau}$$

- Were price fully flexible ($\gamma = 0$), the price would be equal to the mark-up over marginal costs

- When prices are not fully flexible, however, the firm needs to be forward looking and prices depend on a weighted average of future expected marginal costs.

- Rearranging the above, the optimal price equates in expectation the present discounted value of the revenue from charging p^*_{it}, to the present discounted value of marginal costs, augmented by the equilibrium markup:

$$p^*_{it} E_t \sum_{\tau=t}^{\infty} \gamma^{\tau-t} R_{t,\tau} p_\tau^{1/\rho-1} y_\tau = \frac{1/\rho}{1/\rho - 1} E_t \sum_{\tau=t}^{\infty} \gamma^{\tau-t} R_{t,\tau} mc_\tau p_\tau^{1/\rho} y_\tau$$

- Implication 1: There is long-run neutrality - an permanent increase in marginal costs, increases prices one-to-one in the long-run

- Implication 2: There is short-run non-neutrality - since only a fraction of the firms can adjust prices, an increase in the money-supply raising demand will not be neutral in the short run

General Equilibrium Implications - the New Keynesian Model

- We will now analyze the properties of the model in general equilibrium: Hence we take the conditions above and impose general equilibrium conditions

- We also impose symmetry – all firms that are given the chance to adjust their prices choose the same price

- The model is now summarized by the following equations:

$$
\begin{aligned}
y_t^{-\sigma} &= \beta E_t y_{t+1}^{-\sigma} (1 + i_t) \, p_t / p_{t+1} \\
\frac{\phi (1 - n_t)^{-d}}{y_t^{-\sigma}} &= w_t \\
\frac{\vartheta (m_{t+1}/p_t)^{-b}}{y_t^{-\sigma}} &= \frac{i_t}{1 + i_t} \\
y_{it} &= \exp(z_t) \, n_{it} \\
mc_{it} &= w_t / \exp(z_t) \\
p_t &= \left(\int_0^1 p_{it}^{1-1/\rho} di \right)^{1/(1-1/\rho)} \\
p_t^* &= \frac{1/\rho}{1/\rho - 1} E_t \frac{\sum_{\tau=t}^{\infty} (\gamma\beta)^{\tau - t} \, mc_\tau p_\tau^{1/\rho} y_\tau^{1-\sigma}}{\sum_{\tau=t}^{\infty} (\gamma\beta)^{\tau - t} \, p_\tau^{1/\rho - 1} y_\tau^{1-\sigma}}
\end{aligned}
$$

plus a money-supply rule, to be specified below. Note that we have used that $R_{t,\tau} = \beta^{\tau - t} E_t c_{t+1}^{-\sigma} / c_t^{-\sigma}$ and the goods market clearing condition, $c_t = y_t$

and $y_t = \left(\int_0^1 y_{it}^{1-\rho} di\right)^{1/(1-\rho)}$

- To analyze the properties of the model, we will log-linearize the above conditions around the steady-state of the economy.

- We will assume that there is no inflation in the steady-state – in the absence of technological progress, this implies that the net steady-state money growth rate g_m is equal to unity (our money supply rule)

- The steady-state conditions are:

$$
\begin{aligned}
1+i^{ss} &= 1+r^{ss}=1/\beta \\
\frac{\phi\left(1-n^{ss}\right)^{-d}}{\left(y^{ss}\right)^{-\sigma}} &= w^{ss} \\
\frac{\vartheta\left(g_m m^{ss}/p^{ss}\right)^{-b}}{\left(y^{ss}\right)^{-\sigma}} &= \frac{i^{ss}}{1+i^{ss}} \\
y^{ss} &= c^{ss}=n^{ss} \\
mc^{ss} &= w^{ss} \\
p_t &= \left(\int_0^1 p_{it}^{1-1/\rho}di\right)^{1/(1-1/\rho)}=p^{ss} \\
p^{ss} &= \frac{1/\rho}{1/\rho-1}mc^{ss}=1
\end{aligned}
$$

- Denoting with a $\widehat{}$ on a variable percentage deviations from steady state values, this is what you get:

$$
\begin{aligned}
\widehat{y}_t &= -\frac{1}{\sigma}\left(\widehat{i}_t - E_t\widehat{\pi}_{t+1}\right) + E_t\widehat{y}_{t+1} \\
\widehat{w}_t &= \sigma\widehat{y}_t + \phi\frac{n^{ss}}{1-n^{ss}}\widehat{n}_t \\
\widehat{y}_t &= \widehat{n}_t + z_t \\
\widehat{mc}_t &= \widehat{w}_t - z_t \\
\widehat{p}_t &= \gamma\widehat{p}_{t-1} + (1-\gamma)\,\widehat{p_t^*} \\
\widehat{p_t^*} &= (1-\gamma\beta)\left(\widehat{p}_t + \widehat{mc}_t\right) + \gamma\beta E_t\widehat{p_{t+1}^*} \\
\widehat{m}_{t+1} - \widehat{p}_t &= \frac{\sigma}{b}\widehat{y}_t - \frac{1}{b}\widehat{i}_t
\end{aligned}
$$

- Interpretation

1. is the linearized Euler equation: output is constant unless the real interest rate changes

2. is the linearized labor supply equation: real wages depend on output and hours worked

3. is the linearized production function

4. defines marginal cost (linearized), which are increasing in nominal wages and decreasing in the productivity shock

5. defines the price index, which is a weighted average of the past price (with weight γ, the probability of not updating the price) and the current re-set price (with weight $1 - \gamma$)

6. is the linearized money demand equation: real cash balances depend on output and nominal interest rates

- Using the definition of the price index above

$$\widehat{p_t^*} = \frac{1}{1-\gamma}\left(\widehat{p}_t - \gamma \widehat{p}_{t-1}\right)$$

combined with the condition for the optimal price we get the **New-Keynesian Phillips Curve NKPC**

$$\pi_t = \frac{(1-\gamma\beta)(1-\gamma)}{\gamma}\widehat{mc}_t + \beta E_t \pi_{t+1}$$

Current inflation is determined by (a) current marginal cost, and (b) **expected future inflation**. The second term enters because future inflation affects optimal price setting today

- Iterating the NKPC forward we have

$$\pi_t = \xi E_t \sum_{\tau=t}^{\infty} \beta^{\tau-t} \widehat{mc}_\tau$$
$$\xi = \frac{(1-\gamma\beta)(1-\gamma)}{\gamma}$$

According to the NKPC, current inflation is determined **solely by current and future expected marginal costs.** So while prices are sticky, inflation is not, in the sense that current inflation does not depend on past inflation

- Combining the labor market condition, the production function and the definition of marginal costs, the model can be summarized as follows:

$$
\begin{aligned}
\widehat{y}_t &= -\frac{1}{\sigma}\left(\widehat{i}_t - E_t\widehat{\pi}_{t+1}\right) + E_t\widehat{y}_{t+1} \\
\pi_t &= \frac{(1-\gamma\beta)(1-\gamma)}{\gamma}\widehat{mc}_t + \beta E_t\pi_{t+1} \\
\widehat{mc}_t &= \left(\sigma + \phi\frac{n^{ss}}{1-n^{ss}}\right)\widehat{y}_t - \left(1 + \phi\frac{n^{ss}}{1-n^{ss}}\right)z_t \\
\widehat{m}_{t+1} - \widehat{p}_t &= \frac{\sigma}{b}\widehat{y}_t - \frac{1}{b}\widehat{i}_t
\end{aligned}
$$

plus $z_t = \gamma_z z_{t-1} + \varepsilon_{zt}$, with $\varepsilon_{zt}\tilde{\ } nid\,(0, V_z)$, and a description of how monetary policy is conducted (see below).

- The first of these equations is referred to as the **forward-looking IS curve** in the NK literature

- The second equation is the forward looking **NK Phillips Curve**

- The third equation shows that marginal costs falls with productivity gains (z) and increase with output, i.e. with a demand-determined rise in output which raises the demand for labor hence wages.

- The fourth equation is the money demand equation (which then combined with a money supply eqation is sometimes referred to as an **LM curve**)

Important Remarks

1. The steady-state is inefficient - this is not due to sticky prices but due to monopolistic competition. This inefficiency, however, can be eliminated with an employment subsidy so that prices equal marginal costs (and the marginal rate of substitution between consumption and labor equal the marginal rate of transformation). The requires that the subsidy is equals the mark-up (the firms labor cost equal the real wage divided by the subsidy - hence prices will equal marginal costs)

2. To close the model we need to specify monetary policy. When prices are sticky, money is non-neutral: monetary policy will be key for understanding the business cycle properties of the model.

3. The Phillips curve can also be formulated replacing marginal costs with the 'output gap', defined as the difference between current output and output, other things equal, would obtain if all prices were flexible, the 'natural rate.' To do this, note that when prices are flexible, the firms' first-order condition implies that:

$$\frac{p_{it}}{p_t} = 1 = \frac{1/\sigma}{1/\sigma - 1} mc_t$$

Therefore, **in the symmetric flexible price equilibrium, marginal costs are constant,** hence $w_t = z_t$**:** nominal wages move one-to-one with productivity

- **Natural rate allocation:** Denote with $\widetilde{x}$ the log-deviation of x from the steady-state in the *flexible price allocation*. From the above it follows that:

$$
\begin{aligned}
\sigma\widetilde{y}_t + \phi\frac{n^{ss}}{1-n^{ss}}\widetilde{n}_t &= w_t \Rightarrow \\
\sigma\widetilde{y}_t + \phi\frac{n^{ss}}{1-n^{ss}}\left(\widetilde{y} - z_t\right) &= z_t \Rightarrow \\
\left(1 + \phi\frac{n^{ss}}{1-n^{ss}}\right) z_t &= \left(\sigma + \phi\frac{n^{ss}}{1-n^{ss}}\right)\widetilde{y}_t
\end{aligned}
$$

there is a linear relation between z_t and $\widetilde{y}_t$. As money is neutral, the only reasons why output can deviate from the steady state are shocks to technology (expressively modelled in the simple specification above), preferences and markup (not explicitly modelled above).

- Upon subtitution, we can then rewrite marginal costs as:

$$\begin{aligned}\widehat{mc}_t &= \left(\sigma + \phi \frac{n^{ss}}{1-n^{ss}}\right)\widehat{y}_t - \left(1 + \phi \frac{n^{ss}}{1-n^{ss}}\right) z_t \\ &= \left(\sigma + \phi \frac{n^{ss}}{1-n^{ss}}\right)(\widehat{y}_t - \widetilde{y}_t) \\ &= \left(\sigma + \phi \frac{n^{ss}}{1-n^{ss}}\right) y_t^{gap}\end{aligned}$$

where y_t^{gap} is the (log-deviation) of output from its flexible price level.

- Thus, the Phillips curve can be expressed as:

$$\pi_t = \frac{(1-\gamma\beta)(1-\gamma)}{\gamma}\left(\sigma + \phi \frac{n^{ss}}{1-n^{ss}}\right) y_t^{gap} + \beta E_t \pi_{t+1}$$

- This looks a bit like an "old style Phillips curves:"

$$\begin{aligned} \pi_t &= a\pi_{t-1} + b y_t^{gap} \\ \pi_t &= a E_{t-1} \pi_t + b y_t^{gap} \end{aligned}$$

but there are two very important differences:

(a) The NK Phillips curve is forward looking in inflation - current inflation depends on the expected path of current and future marginal costs. The old style Phillips curve is instead either backward looking, or is forward looking in the price level.

(b) Output gap is measured by the deviation from the flexible price level of output and derives from marginal cost variations. In old style analysis, output gap often proxied by deviations from a trend, corresponding to a traditional notion of potential output which does not necessarily coincides with the flex-price equilibrium.

Appendix

1. Derivation of First-Order Condition for the Firm's Profit Maximization Problem

- The intertemporal first-order condition is:

$$E_t \sum_{\tau=t}^{\infty} \gamma^{\tau-t} R_{t,\tau} \left(\pi'_{i\tau} \left(p^*_{it} \right) \right) \frac{1}{p_\tau} \quad = \quad 0$$

$$\Rightarrow$$

$$\left(p^*_{it} \right)^{-1/\rho-1} E_t \sum_{\tau=t}^{\infty} \gamma^{\tau-t} R_{t,\tau} \left\{ \left[\left(1 - \frac{1}{\rho} \right) p_\tau^{1/\rho-1} p^*_{it} + \frac{1}{\rho} mc_\tau p_\tau^{1/\rho} \right] y_\tau \right\} = 0$$

note that the first term on the left hand side is a constant that therefore drops out.

- Therefore the above condtions is:

$$E_t \sum_{\tau=t}^{\infty} \gamma^{\tau-t} R_{t,\tau} \left\{ \left[\left(1 - \frac{1}{\rho}\right) p_\tau^{1/\rho-1} p_{it}^* + \frac{1}{\rho} mc_\tau p_\tau^{1/\rho} \right] y_\tau \right\} = 0$$

- Re-arranging now gives us:

$$p_{it}^* = \frac{1/\rho}{1/\rho - 1} \frac{E_t \sum_{\tau=t}^{\infty} \gamma^{\tau-t} R_{t,\tau} mc_\tau p_\tau^{1/\rho} y_\tau}{E_t \sum_{\tau=t}^{\infty} \gamma^{\tau-t} R_{t,\tau} p_\tau^{1/\rho-1} y_\tau}$$

2. Derivation of the Phillips Curve

We have seen that:

$$\begin{aligned} \widehat{p_t^*} &= \frac{1}{1-\gamma} \left(\widehat{p}_t - \gamma \widehat{p}_{t-1}\right) \\ \widehat{p_t^*} &= (1 - \gamma\beta) \left[\widehat{p}_t + \widehat{mc}_t\right] + (1 - \gamma\beta) E_t \widehat{p_{t+1}^*} \end{aligned}$$

Combining them

$$\frac{1}{1-\gamma}\left(\widehat{p}_t-\gamma\widehat{p}_{t-1}\right)=\left(1-\gamma\beta\right)\left(\widehat{p}_t+\widehat{mc}_t\right)+\gamma\beta E_t\frac{1}{1-\gamma}\left(\widehat{p}_{t+1}-\gamma\widehat{p}_t\right)$$

which can be re-arranged as to yield

$$\pi_t=\frac{\left(1-\gamma\beta\right)\left(1-\gamma\right)}{\gamma}\widehat{mc}_t+\beta E_t\pi_{t+1}$$