International Macroeconomic Fluctuations

Lecture 2 - Intratemporal Trade Models - Multigood Models

So far pure intertemporal trade which ignores:

- Transmission through trade in goods
- Importance of variations in relative prices
- Distortions in trade that works through relative prices

Types of multi-good models

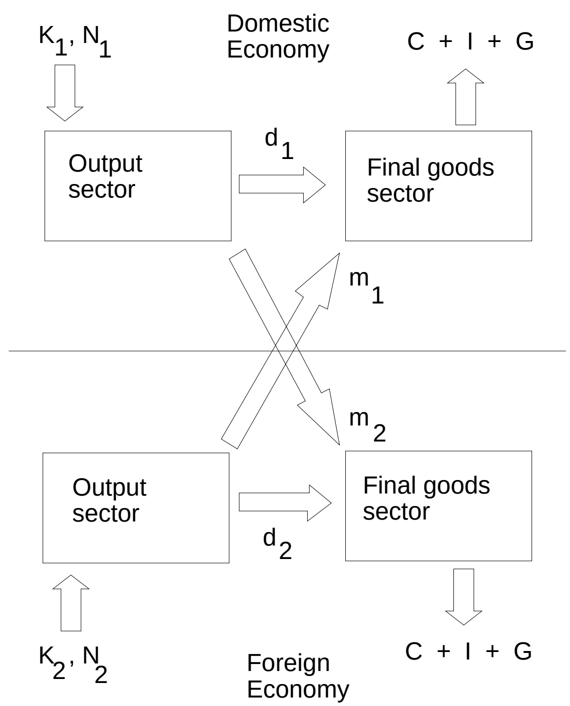
- Multiple traded goods: Ricardian / H0 / Intra-industry trade models
- Traded - Non-traded goods models

Business cycle literature has focused on:

- Specialization (Ricardian)
- Traded - Non-Traded
- very little on Intra-industry (but beginning literature) and HO settings

"Workhorse Model"

- 2 countries and 2 traded goods



- complete specialization; domestic and foreign goods imperfect substitutes

- competitive firms (imperfect competition can be allowed for)

- complete markets

- Armington aggregator set-up: elasticity of substitution between domestic and foreign goods identical for all final uses

- Model summarized by:

$$
\begin{aligned}
V_{i0} &= \max \sum_{t=0}^{\infty} \beta^t u\left(d_{it}^c c_{it}, l_{it}\right) \\
h_{it} &= c_{it} + g_{it} + i_{it} \\
k_{it+1} &= (1-\delta)\, k_{it} + i_{it} \\
h_{it} &= \left(\omega_1 \left(s_{it}^i\right)^{(\rho-1)/\rho} + \omega_2 \left(s_{it}^j\right)^{(\rho-1)/\rho}\right)^{\rho/(\rho-1)} \\
y_{it} &= s_{it}^i + s_{jt}^i \\
y_{it} &= A_{it} \left(k_{it}^{1-\alpha}\right)\left(n_{it}^{\alpha}\right) \\
T &= l_{it} + n_{it}
\end{aligned}
$$

- Final goods: CES aggregate of foreign and domestic "intermediate goods", $1/\rho$ is the elasticity of substitution

- (ω_1, ω_2) : taste or share parameters, taste for home-goods

- note that model can be rewritten so that there's trade in final goods by assuming that each final good is an identical CES function of domestic and foreign goods

The planner's problem can be written as:

$$
\begin{aligned}
& V(z_t) \\
= \; & \max\left(\Omega_1 u\left(d^c_{1t} c_{1t}, (T - n_{1t})\right) + \Omega_2 u\left(d^c_{2t} c_{2t}, (T - n_{2t})\right)\right) \\
& + \beta E_t V(z_{t+1}) \\
z_t = \; & (A_{1t}, A_{2t}, g_{1t}, g_{2t}, d^c_{1t}, d^c_{2t}, k_{1t}, k_{2t})
\end{aligned}
$$

subject to resource and technology constraints

First-order necessary conditions:

$$
\begin{aligned}
\Omega_i d^c_{it} u_c\left(d^c_{it} c_{1t}, (T - n_{it})\right) &= \lambda_{it} \\
\Omega_i d^c_{it} u_c\left(d^c_{it} c_{it}, (T - n_{it})\right) &= \mu_{it} \alpha A_{it} \left(k_{it}^{1-\alpha}\right) \left(n_{it}^{\alpha-1}\right)
\end{aligned}
$$

$$
\begin{aligned}
\beta E_t \left[\lambda_{it+1} (1-\delta) + \mu_{it+1} (1-\alpha) A_{it+1} \left(k_{it+1}^{1-\alpha} \right) \left(n_{it+1}^{\alpha-1} \right) \right] &= \lambda_{it} \\
\lambda_{it} \omega_1 \left(s_{it}^{i} \right)^{-1/\rho} \left(\omega_1 \left(s_{it}^{i} \right)^{(\rho-1)/\rho} + \omega_2 \left(s_{it}^{j} \right)^{(\rho-1)/\rho} \right)^{1/(\rho-1)} &= \mu_{it} \\
\lambda_{it} \omega_2 \left(s_{it}^{j} \right)^{-1/\rho} \left(\omega_1 \left(s_{it}^{i} \right)^{(\rho-1)/\rho} + \omega_2 \left(s_{it}^{j} \right)^{(\rho-1)/\rho} \right)^{1/(\rho-1)} &= \mu_{jt}
\end{aligned}
$$

where λ_{it} is the multiplier on $h_{it} = c_{it} + g_{it} + i_{it}$ and μ_{it} is the multiplier on $s_{it}^{i} + s_{jt}^{i} = \left(\omega_1 \left(s_{it}^{i} \right)^{(\rho-1)/\rho} + \omega_2 \left(s_{it}^{j} \right)^{(\rho-1)/\rho} \right)^{\rho/(\rho-1)}$

Some definitions

1. The terms of trade: The terms of trade are defined as the price of imports relative to the price of exports

- The multipliers μ can be interpreted as the shadow prices of the goods

- Hence, country $i's$ terms of trade are given as:

$$p_{it} = \frac{\mu_{jt}}{\mu_{it}}$$

2. The real exchange rate: The real exchange rate is defined as the price of the foreign consumption basket relative to the price of the domestic consumption basket translated into the same currency. Here, given that we have a real model, the real exchange rate is simply the ratio of the price of the foreign basket to the price of the domestic basket.

- Although the law of one price holds, real exchange does not identically equal 1

- Reason is that consumption baskets differ across countries

- Since λ is the multiplier on the consumption good resource constraints, the real exchange can be defined as

$$q_{it} = \frac{\lambda_{jt}}{\lambda_{it}}$$

Implications / Interpretations of the first order necessary conditions

- **Risk sharing**: Combining the conditions for c_{it} and c_{jt} :

$$\Omega_1 d_{1t}^c u_c \left(d_{1t}^c c_{1t}, (T - n_{1t})\right) = \frac{\lambda_{it}}{\lambda_{jt}} \Omega_2 d_{2t}^c u_c \left(d_{2t}^c c_{2t}, (T - n_{2t})\right)$$

- this includes multipliers so looks different from intertemporal trade model

- however, from above, we know that the ratio of the consumption good multipliers can be interpreted as the real exchange rate:

$$\Omega_1 d_{1t}^c u_c \left(d_{1t}^c c_{1t}, (T - n_{1t})\right) = \frac{1}{q_{it}} \Omega_2 d_{1t}^c u_c \left(d_{1t}^c c_{1t}, (T - n_{1t})\right)$$

- the real exchange rate now introduces a wedge between domestic and foreign marginal utility of consumption

- a decrease in q_{it} implies that the planner ceteris paribus reallocates consumption from country 1 to country 2

- the wedge introduced by the real exchange rate might potentially help addressing the problem of (too) high cross-country consumption correlations

- however this will depend on the covariance between real exchange rate and relative marginal utility

- **The terms of trade**: combining the conditions for s_i^i and for s_i^j :

$$p_{it} = \frac{\mu_{jt}}{\mu_{it}} = \frac{\omega_2 \left(s_{it}^j\right)^{-1/\rho}}{\omega_1 \left(s_{it}^i\right)^{-1/\rho}}$$

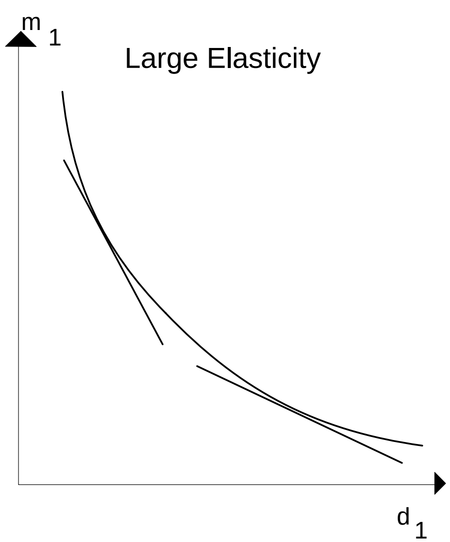
- ie. the terms of trade, in equilibrium, equal the marginal rate of transformation between domestic and foreign goods (the slope of the isoquants)

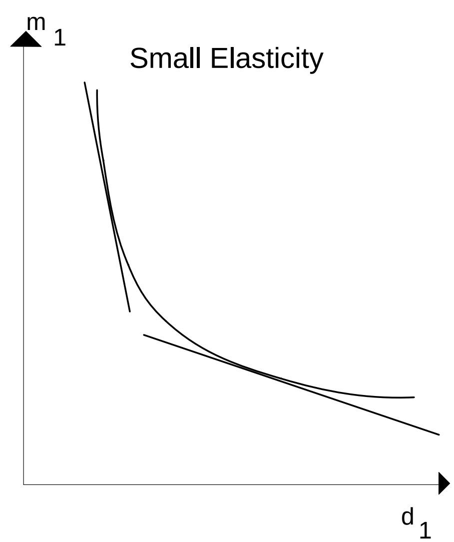
- important note: The elasticity of the terms of trade to quantities depend crucially on the elasticity of substitution, ρ

- **When goods are highly substitutable**, the elasticity of p_{it} to quantities is small - or said otherwise, quantities react very elastically to prices

- **When goods are poor substitutes**, the elasticity of p_{it} to quantities is large - or, quantities react inelastically to prices

- this price-quantity relationship turns out to be a problem ...





Small price changes	Big price changes
Big quantity changes	Small quantity changes

- **The real exchange rate:** can be written as a function of the terms of trade:

$$q_{it} = \frac{\left(1 + \left(\frac{\omega_2}{\omega_1}\right)^{\rho} p_{it}^{1-\rho}\right)^{1/(\rho-1)}}{\left(p_{it}^{1-\rho} + \left(\frac{\omega_2}{\omega_1}\right)^{\rho}\right)^{1/(\rho-1)}}$$

- Although the LOP holds, real exchange rate only equal to one identically when $\omega_1 = \omega_2$: In this case consumption baskets are identical across countries and therefore, given LOP, absolute PPP will hold

- this means that shocks that affect terms of trade will affect real exchange rate

- the elasticity of the real exchange rate to the terms of trade is:

$$\frac{\partial q_{it}}{\partial p_{it}} \frac{p_{it}}{q_{it}} = \frac{1 - \left(\frac{\omega_2}{\omega_1}\right)^{\rho}}{\left(1 + \left(\frac{\omega_2}{\omega_1}\right)^{\rho}\right)}$$
$$= 1 - 2s^m$$

- where s^m is the steady-state import share, $s^m = ps_i^j / y_i$

1. the elasticity of the real exchange rate to the terms of trade is smaller than 1 in absolute value

2. the elasticity of the real exchange rate to the terms of trade is positive (negative) if there is taste for homegoods (taste for foreign goods), ie. if $\omega_1 > \omega_2$ $(\omega_1 > \omega_2)$

- also implications for the division of work and capital across countries

- from the conditions for n_{it} and n_{jt} :

$$\frac{\Omega_i d_{it}^l u_l \left(d_{it}^c c_{it}, (T - n_{it})\right)}{\Omega_j d_{jt}^l u_l \left(d_{jt}^c c_{jt}, \left(T - n_{jt}\right)\right)} = \frac{1}{p_{it}} \frac{\alpha A_{it} \left(k_{it}^{1-\alpha}\right) \left(n_{it}^{\alpha-1}\right)}{\alpha A_{jt} \left(k_{jt}^{1-\alpha}\right) \left(n_{jt}^{\alpha-1}\right)}$$

- depending upon the covariance between terms of trade and the shocks to the economy, this may lead to a weaker or stronger tendency for negative cross-country hours and output correlations than in a single-good economy

- from the conditions for k_{it+1} and k_{jt+1} :

$$\frac{\beta E_t \left[\lambda_{it+1} (1 - \delta) + \mu_{it+1} (1 - \alpha) A_{it+1} \left(k_{it+1}^{1-\alpha}\right) \left(n_{it+1}^{\alpha-1}\right)\right]}{\beta E_t \left[\lambda_{jt+1} (1 - \delta) + \mu_{jt+1} (1 - \alpha) A_{jt+1} \left(k_{jt+1}^{1-\alpha}\right) \left(n_{jt+1}^{\alpha-1}\right)\right]} = \frac{1}{q_{it}}$$

- hence, relative prices introduce a wedge between “own currency” expected marginal products of capital

Questions

1. Does this model with specialization lead to different implications for cross-country comovements?

- reasonable to expect so:

(a) Increases in domestic final use of goods ($c_{it},\ i_{it},\ g_{it}$) needs inputs of both foreign and domestic intermediate goods - thus some transmission through trade

(b) Real exchange rate introduces wedge between marginal utilities of consumption in the risk sharing condition - hence, one may expect that this model predicts lower cross-country consumption correlation

(c) Because domestic and foreign goods are imperfect substitutes, less tendency for strong negative cross-country correlation of investment

2. How does the model perform in accounting for trade flows?

3. The behavior of relative prices

Backus, Kehoe and Kydland, American Economic Review, 1994

- only technology shocks, 2 countries

- calibration: US vs RoW (apart from technology shocks which are calibrated to US vs. Europe)

- standard parameters: $n/T = 0.30$, $CRRA = 1/IES = 2$, $\beta = 0.99$, $\alpha = 0.64$, $\delta = 0.025$, technology shock process as in BKK, 1992

- new parameters:

1. Import share: 15% - this matches US but not Europe. This calibrates the "taste for home goods"

2. Elasticity of substitution between domestic and foreign goods: 1.5 - this is an important variable as discussed earlier

- show results for calibration with slightly higher import share (22% - the OECD average) intratemporal trade IS important:

- some international transmission through international trade - domestic productivity shock increases both domestic and foreign output

however: the effects are moderate:

- risk sharing still implies strong cross-country consumption correlation

- still strong tendency for negative cross-country investment correlations

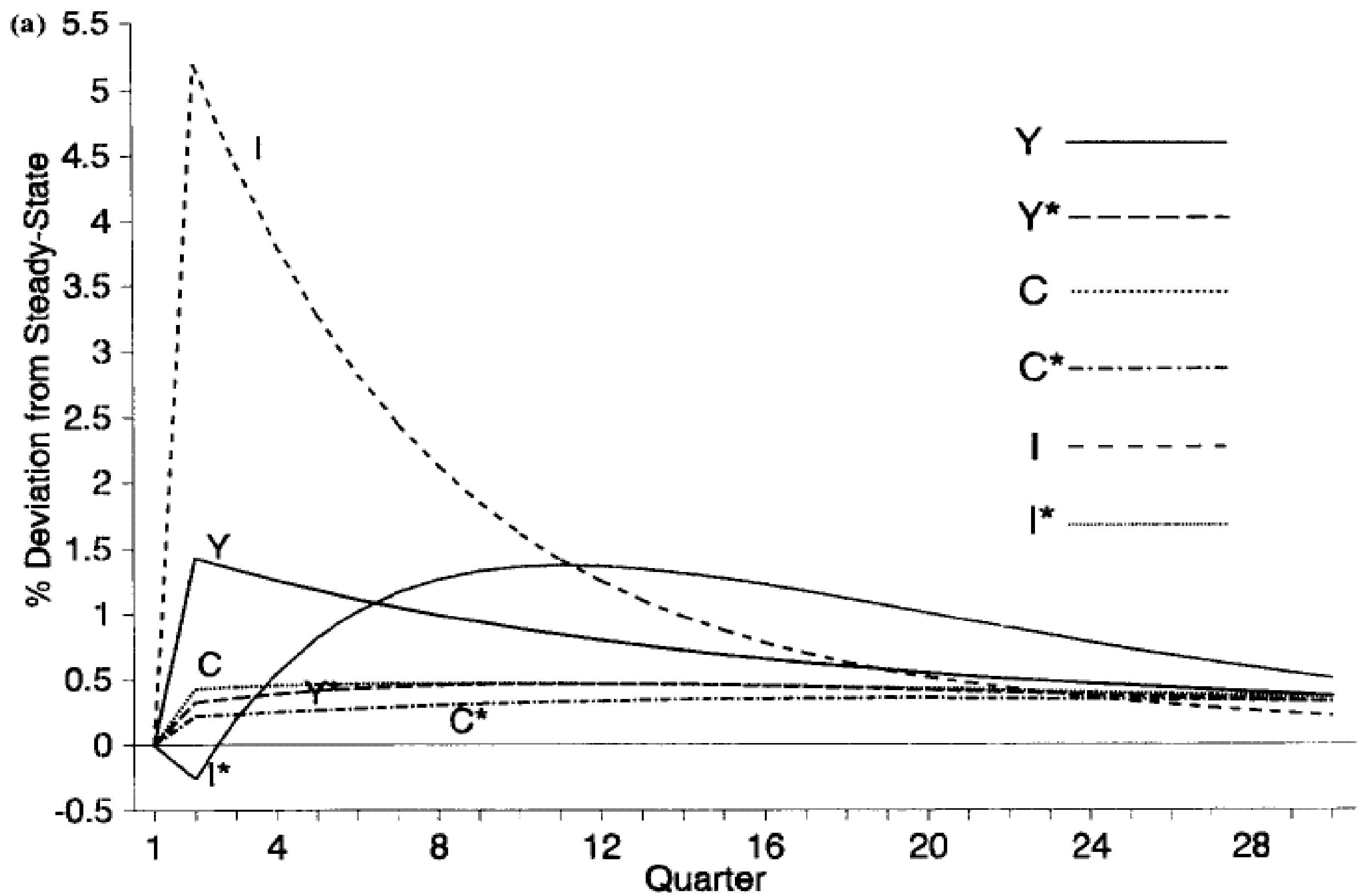


Figure 1:

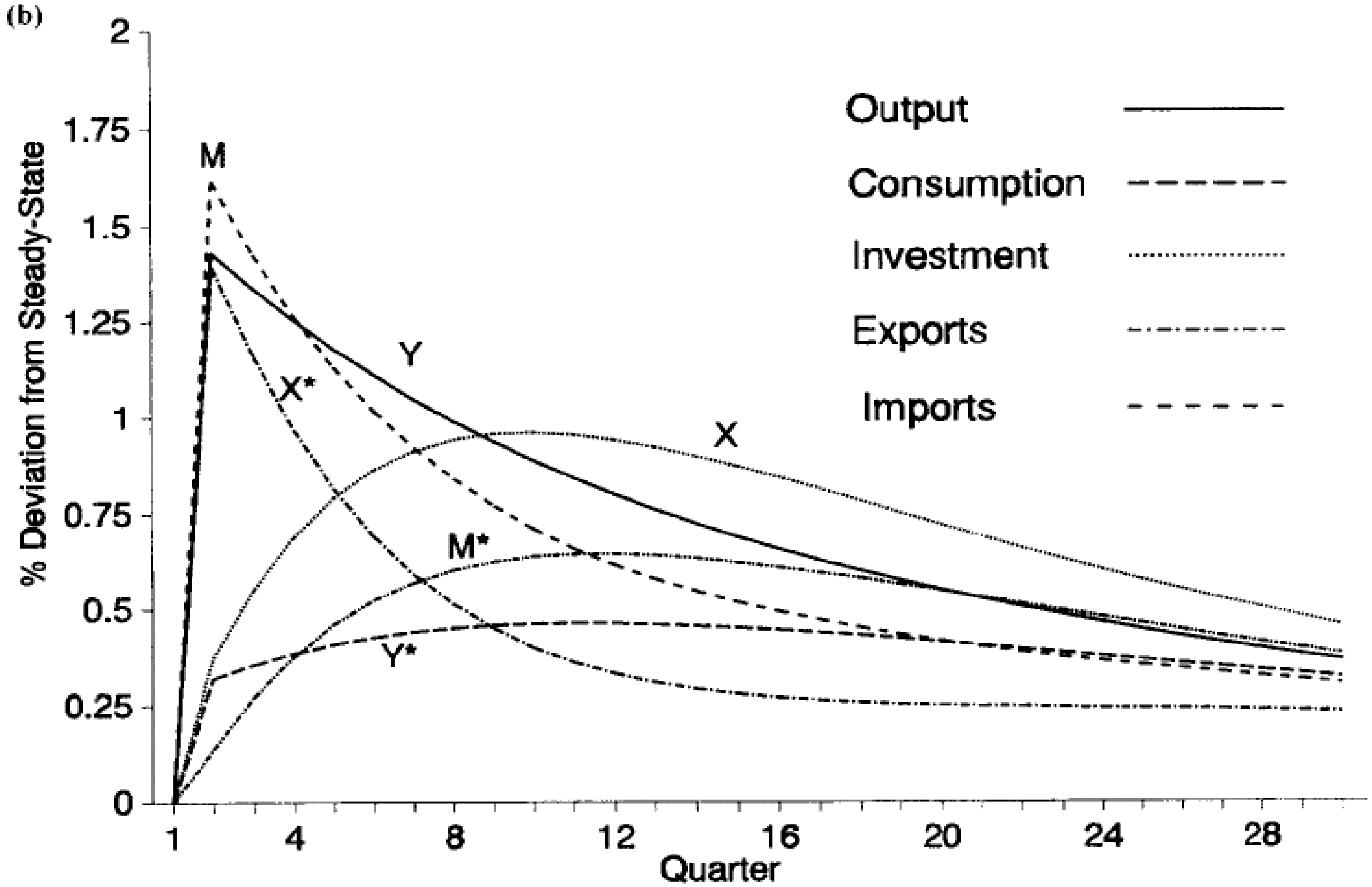


FIGURE 1. Impulse response. (a) Output components. (b) Trade variables.

Figure 2:

Robustness checks:

- large spill-overs - higher direct transmission through shocks

- low elasticity of substitution between domestic and foreign goods: should make trade link more important

- high elasticity of substitution between domestic and foreign goods: should make trade link less important

- government spending shocks: should lead to stronger output co-movements

TABLE 4. Simulation results: international comovements

Moment	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
COR (y,y*)	0.175	0.157	0.328	0.091	0.182	0.194	0.509	0.379	0.018	0.027
	(0.167)	(0.158)	(0.153)	(0.172)	(0.167)	(0.166)	(0.147)	(0.160)	(0.175)	(0.174)
COR (c,c*)	0.714	0.759	0.596	0.766	0.732	0.748	0.730	0.672	0.865	0.874
	(0.119)	(0.117)	(0.140)	(0.108)	(0.110)	(0.101)	(0.112)	(0.126)	(0.055)	(0.053)
COR (i,i*)	−0.330	−0.508	−0.184	−0. 479	−0.321	−0.310	0.278	0.026	−0.337	−0.265
	(0.148)	(0.118)	(0.161)	(0.127)	(0.149)	(0.152)	(0.172)	(0.174)	(0.146)	(0.153)
COR (g,g*)	—	—	—	—	0	1	—	—	—	—
	—	—	—	—	—	—	—	—	—	—
COR (z,z*)	0.281	0.280	0.281	0.281	0.281	0.281	0.513	0.395	0.281	0.281
	(0.156)	(0.148)	(0.156)	(0.156)	(0.156)	(0.156)	(0.143)	(0.155)	(0.156)	(0.156)
COR (exp, exp*)	0.116	−0.101	0.279	−0.248	−0.068	0.136	0.671	0.417	—	—
	(0.200)	(0.224)	(0.138)	(0.183)	(0.195)	(0.166)	(0.122)	(0.159)	—	—
COR (imp, imp*)	−0.086	−0.219	−0.393	−0.227	−0.131	−0.066	0.444	0.202	—	—
	(0.170)	(0.161)	(0.159)	(0.171)	(0.169)	(0.171)	(0.151)	(0.169)	—	—
COR (n,n*)	−0.059	−0.140	0.437	−0.271	0.050	0.182	0.555	0.385	−0.281	−0.406
	(0.166)	(0.155)	(0.138)	(0.157)	(0.167)	(0.166)	(0.134)	(0.155)	(0.136)	(0.142)
S(nx/y)	0.441	0.526	0.507	0.582	0.497	0.441	0.287	0.371	0.716	0.790
	(0.060)	(0.073)	(0.065)	(0.080)	(0.067)	(0.060)	(0.037)	(0.038)	(0.124)	(0.143)

(1) Standard model, baseline parameterization.
(2) Standard model, big spill-over in technology.
(3) Standard model, low elasticity of substitution.
(4) Standard model, high elasticity of substitution.
(5) Standard model with uncoordinated government spending
(6) Standard model with perfectly coordinated government spending.
(7) Model with VAR(2) shocks, US–Canada specification.
(8) Model with VAR(2) shocks, US–Germany specification.
(9) Single good model with asymmetric countries, share of country 1 = 20%.
(10) Single good model with asymmetric countries, share of country 1 = 5%.
See Table 6 for parameter values used in the simulations. Moments relate to the averages over 1000 simulations. Numbers in parentheses are standard deviations.

Figure 3:

Table 1
Business Cycle moments of OECD data

Variable	Standard deviations (%)	1st order autocorrelation	Correlation with output	Correlation with terms of trade	Cross-country correlations
Output	1.41 [0.91;2.41]	0.80 [0.33;0.94]	–	–	0.33 [0.15;0.41]
Consumption	1.34 [0.57;2.01]	0.78 [0.55;0.94]	0.69 [0.36;0.87]	–	0.22 [0.04;0.34]
Gov. spending	1.17 [0.70;2.88]	0.66 [0.00;0.92]	0.02 [−0.39;0.58]	–	–
Investment	4.23 [2.95;6.62]	0.83 [0.48;0.96]	0.74 [0.55;0.94]	–	0.30 [0.13;0.44]
Exports	3.27 [2.13;4.41]	0.67 [0.34;0.91]	0.44 [0.06;0.78]	–	–
Imports	4.29 [2.62;6.51]	0.79 [0.51;0.94]	0.67 [0.63;0.88]	–	–
Net exports	0.96 [0.45;1.64]	0.73 [0.42;0.93]	−0.27 [−0.61;0.06]	0.035 [−0.30;0.51]	–
Employment	1.21 [0.58;2.48]	0.73 [0.20;0.96]	0.48 [−0.01;0.86]	–	0.20 [−0.09;0.35]
Terms of trade	3.19 [1.17;5.30]	0.88 [0.70;0.94]	−0.08 [−0.36;0.20]	–	–
Real exch. rate	4.03 [1.48;7.93]	0.80 [0.75;0.88]	0.07 [−0.55;0.44]	0.39 [−0.07;0.75]	–

Notes: All data were taken from the OECD national accounts. The sample period is 1970.1–2000.4 and the data are quarterly. The terms of trade are computed as the implicit imports deflator divided by the implicit exports deflator. Net exports are computed as exports minus imports divided by output, all in current prices. The real exchange rate is measured by the real effective exchange rate. Output components are employed are measured in per capita terms. All data, except net-exports, are in logs and are detrended using the Hodrick–Prescott filter using a value of 1600 for the smoothing parameter. The ratio of net exports to output was also Hodrick–Prescott filtered.
Numbers in brackets are the minimum and the maximum of the moments across the 14 countries in the sample.

Figure 4:

Conclusions: although does better than intertemporal trade model, the effects are very moderate and 'quantity puzzle' remains

How does it do on relative prices?

- terms of trade are volatile and persistent

- real exchange rates are volatile - and more so than terms of trade - and persistent

- positive but imperfect correlation between terms of trade and real exchange rate (real exchange rate here defined as foreign price level divided by domestic price level)

- model implies persistent terms of trade (and real exchange rate movements) just like in the data in response to persistent technology shocks

- but implies terms of trade volatility much too low - standard deviation 6-8 times lower than in the data

TABLE 3—PROPERTIES OF NET EXPORTS, REAL OUTPUT, AND THE TERMS OF TRADE IN THEORETICAL ECONOMIES

	Standard deviation (percent)			Autocorrelation			Correlation		
Economy	nx	y	p	nx	y	p	(nx, y)	(nx, p)	(y, p)
Benchmark	0.30	1.38	0.48	0.61	0.63	0.83	−0.64	−0.41	0.49
	(0.02)	(0.18)	(0.06)	(0.07)	(0.10)	(0.05)	(0.07)	(0.08)	(0.14)
Large elasticity	0.33	1.41	0.35	0.63	0.64	0.88	−0.57	−0.05	0.43
	(0.03)	(0.18)	(0.05)	(0.07)	(0.18)	(0.03)	(0.08)	(0.09)	(0.14)
Small elasticity	0.37	1.33	0.76	0.61	0.63	0.77	−0.66	−0.80	0.51
	(0.03)	(0.18)	(0.07)	(0.07)	(0.10)	(0.05)	(0.07)	(0.09)	(0.16)
Two shocks	0.33	1.33	0.57	0.62	0.65	0.78	−0.57	−0.05	0.39
	(0.03)	(0.15)	(0.07)	(0.08)	(0.08)	(0.06)	(0.15)	(0.17)	(0.17)
Time to build	0.28	1.34	0.51	0.60	0.63	0.52	−0.61	−0.40	0.50
	(0.02)	(0.17)	(0.06)	(0.17)	(0.10)	(0.16)	(0.07)	(0.08)	(0.12)
Time to ship	0.24	1.35	0.48	0.65	0.66	0.66	−0.56	−0.51	0.61
	(0.02)	(0.18)	(0.05)	(0.07)	(0.08)	(0.09)	(0.08)	(0.09)	(0.11)
No capital	0.18	1.14	1.29	0.71	0.61	0.64	0.66	0.99	0.68
	(0.01)	(0.15)	(0.09)	(0.06)	(0.11)	(0.07)	(0.06)	(0.00)	(0.06)
Government shocks	0.16	0.17	0.30	0.67	0.67	0.67	−0.55	1.00	−0.55
	(0.03)	(0.02)	(0.05)	(0.11)	(0.08)	(0.11)	(0.13)	(0.00)	(0.13)
Perfect substitutes	16.90	2.22	—	−0.10	0.76	—	0.10	—	—
	(1.14)	(0.29)		(0.18)	(0.05)		(0.04)		

Figure 5:

- since import share around 15-20 percent, standard deviation of real exchange rate is (1-2*0.2)=60% lower than terms of standard deviation - an order of magnitude lower than in the data

- lower elasticity of substitution implies higher relative price volatility

- does sufficiently low elasticity "solve" this problem (of low terms of trade volatility)?

- potentially yes, but creates another problem

- low elasticity implies little variation in quantities: import ratio (imported goods to domestic goods) becomes constant

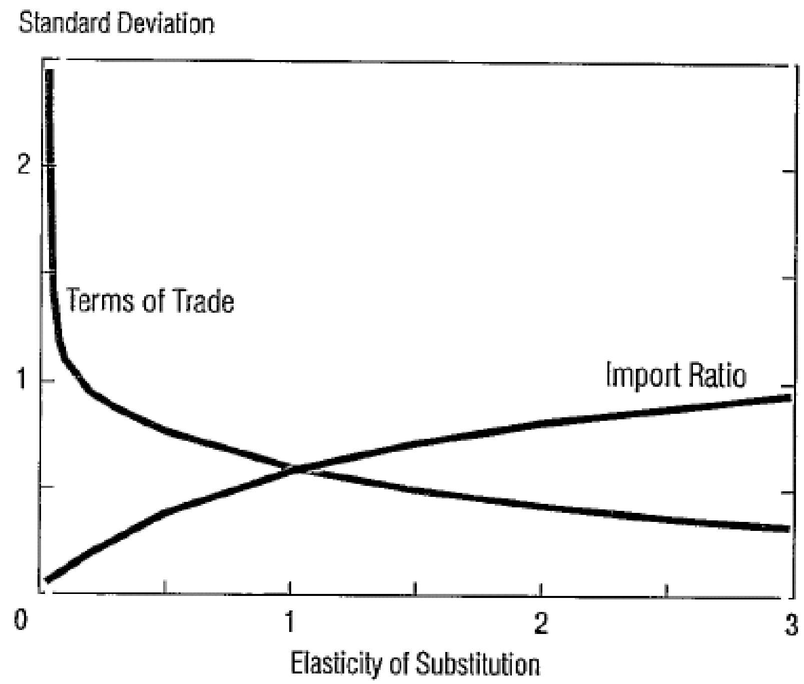

Standard Deviation
2
1
0
Terms of Trade
Import Ratio
1
2
3
Elasticity of Substitution

Figure 6:

Incomplete Markets

- Single good model: Incomplete markets matter little unless shocks are very persistent - wealth effects are not very big unless shocks are very persistent

- What about multiple goods models?

- Incomplete markets may matter even less - at least if there are only technology shocks

- The basic reason: Relative price of good declines in response to increase in relative productivity

- One can even establish an equivalence result

- Cole & Obstfeld, 1991

- Consider a model with no financial trade - ie. current account needs to be identically equal to zero

- Recall: single good model - no financial trade = no goods trade

- With multiple traded goods instead - no financial trade = balanced foreign trade

- They show that - as a special case - when elasticity of substitution between domestic and foreign goods = 1: equivalence between incomplete markets and complete market allocation in response to endowment shocks

- Suppose endowment economy - no investment, no government expenditure, no taste shocks, no leisure

$$
\begin{aligned}
u\left(C_t^h\right) &= \frac{1}{1-\sigma}\left[\left(c_t^h\right)^{\theta}\left(d_t^h\right)^{1-\theta}\right]^{1-\sigma} \\
u\left(C_t^f\right) &= \frac{1}{1-\sigma}\left[\left(c_t^f\right)^{\theta}\left(d_t^f\right)^{1-\theta}\right]^{1-\sigma}
\end{aligned}
$$

- ie. unit elasticity of substitution between the goods

- note also that expenditure shares assumed identical

- endowments stochastic

$$
\begin{aligned}
c_t^h + c_t^f &= C_t \\
d_t^h + d_t^f &= D_t
\end{aligned}
$$

- Consider two cases

1. Complete markets

2. Financial autarky

- **Complete markets** - social planner maximizes weighted sum of domestic and foreign agents' utilities subject to resource constraints

- Because of complete markets, this can be broken into separate problems for any state/date pair

- Social planner's problem

$$\max \Omega \frac{1}{1-\sigma}\left[\left(c^h\right)^\theta \left(d^h\right)^{1-\theta}\right]^{1-\sigma} + (1-\Omega)\frac{1}{1-\sigma}\left[\left(c^f\right)^\theta \left(d^f\right)^{1-\theta}\right]^{1-\sigma}$$

$$st$$

$$: \quad c^h + c^f = C$$

$$: \quad d^h + d^f = D$$

- Because welfare weights are constant, intuitively, the optimal allocation must look like (see the appendix for details)

$$\begin{aligned} c^h &= \chi C \\ c^f &= (1-\chi) C \\ d^h &= \chi D \\ d^f &= (1-\chi) D \end{aligned}$$

- and the complete markets terms of trade are given as:

$$p^{CM} = \frac{1-\theta}{\theta}\frac{C}{D}$$

- Moreover, χ can be found as:

$$\chi = \frac{1}{1 + \left(\frac{1-\Omega}{\Omega}\right)^{1/\sigma}} \in (0, 1)$$

- **Financial autarky** - each agent maximizes her utility subject to budget constraint, and foreign trade needs to be balanced

- Because of balanced trade and because goods are non-storable, agents will face static maximization problems:

$$\begin{aligned}
dom &: \max \frac{1}{1-\sigma}\left[\left(c^h\right)^{\theta}\left(d^h\right)^{1-\theta}\right]^{1-\sigma} \\
st &: c^h + pd^h = C \\
for &: \max \frac{1}{1-\sigma}\left[\left(c^f\right)^{\theta}\left(d^f\right)^{1-\theta}\right]^{1-\sigma} \\
st &: c^f + pd^f = pD
\end{aligned}$$

- solving this (and clearing the markets) gives:

$$\begin{aligned}
c^h &= \theta C \\
c^f &= (1-\theta) C \\
d^h &= \theta D \\
d^f &= (1-\theta) D
\end{aligned}$$

- and:

$$p^{FA} = \frac{1-\theta}{\theta}\frac{C}{D}$$

- The two allocations are thus identical if:

$$\begin{aligned} \chi &= \theta \Rightarrow \\ \theta &= \frac{1}{1+\left(\frac{1-\Omega}{\Omega}\right)^{1/\sigma}} \end{aligned}$$

- Note: This is a counterexample - sometimes, incomplete markets may not matter at all - this is not generally the case

- However - indicates that terms of trade have a tendency to provide automatic insurance against particular types of shocks

Incomplete Markets, Non-Tradables and Distribution

Corsetti, Dedola, and Leduc (ReStud, 2008) extend the BKK model to:

- Tradables and non-tradables: Consumers derive utility from both tradable and non-tradable goods

- Bond economy

- Distrubution: In order to supply tradable goods to consumers, retailers must purchase tradables and combine them with non-tradables. This breaks the link between tradables goods prices across countries

- They show that this model can deliver

- High real exchange rate volatility - and high terms of trade volatility

- low cross-country consumption correlations but high output, investment and employment cross-correlations

- This, however, relies on low elasticity of substitution between domestic and foreign goods and the model implies low trade volatility (low volatility of the import ratio)

- Giancarlo will cover this paper in details

Endogenous Incomplete Markets: Kehoe and Perri, Econometrica

- so far considered three versions of capital markets:

1. Complete markets with full set of contingent claims, fully observable, fully enforceable contracts

2. Financial autarky

3. Trade in limited number of assets (most often specified as non-contingent bonds only)

- 1 is clearly quite extreme and perhaps also unrealistic. But useful as a starting case.

- 2 is not much less unrealistic. But perhaps useful as a starting case as well.
- 3 slightly more general - but perhaps still unrealistic
- However both 2 and 3 limit the set of claims that can be traded and introduce “exogenous” constraints on financial markets
- Here: friction in international capital markets: international loans not fully enforceable
- gives rise to “endogenous” borrowing constraints

Model:

- 2 countries
- single good model
- firms perfectly competitive
- preferences:

$$V_i\left(s^0\right) = \sum_{t=0}^{\infty} \beta^t \sum_{s^t} \pi\left(s^t\right) U\left(c_i\left(s^t\right), n_i\left(s^t\right)\right)$$

- technology:

$$y_i\left(s^t\right) = F\left(k_i\left(s^{t-1}\right), A_i\left(s^t\right) n_i\left(s^t\right)\right)$$

- world resource constraint

$$\sum_{i=1}^{2}\left(c_i\left(s^t\right) + k_i\left(s^t\right)\right) \leq \sum_{i=1}^{2}\left(F\left(k_i\left(s^{t-1}\right), A_i\left(s^t\right) n_i\left(s^t\right)\right) + (1-\delta) k_i\left(s^{t-1}\right.\right.$$

- **Complete markets:**

$$\max \Omega_1 \sum_{t=0}^{\infty} \beta^t \sum_{s^t} \pi\left(s^t\right) U\left(c_1\left(s^t\right), n_1\left(s^t\right)\right)$$
$$+\Omega_2 \sum_{t=0}^{\infty} \beta^t \sum_{s^t} \pi\left(s^t\right) U\left(c_2\left(s^t\right), n_2\left(s^t\right)\right)$$

- subject to:

$$\sum_{i=1}^{2}\left(c_i\left(s^t\right)+k_i\left(s^t\right)\right)\leq\sum_{i=1}^{2}\left(F\left(k_i\left(s^{t-1}\right),A_i\left(s^t\right)n_i\left(s^t\right)\right)+(1-\delta)\,k_i\left(s^{t-1}\right.\right.$$

- **Limited enforcement:** The countries cannot be forced to pay - pay only when they want to

- so what makes them want to pay?

- either: the stick - punish them if they don't

- or: the carrot - reward them if they pay

- here: if a country reneges, it will be barred from any future access to international capital markets: it must revert to autarky

- the enforcement constraint is then that:

$$\begin{array}{rcl} \sum_{r=t}^{\infty} \beta^{r-t} \sum_{s^t} \pi\left(s^r | s^t\right) U\left(c_i\left(s^r\right), n_i\left(s^r\right)\right) & \geq & V_i\left(k_i\left(s^{t-1}\right), s^t\right) \forall t \\ \shortparallel & & \geq\shortparallel \\ \text{value of contract} & \geq & \text{value of autarky} \end{array}$$

- where:

$$\begin{aligned} &: \quad V_i\left(k_i\left(s^{t-1}\right), s^t\right) = \max \sum_{r=t}^{\infty} \beta^{r-t} \sum_{s^t} \pi\left(s^r | s^t\right) U\left(c_i\left(s^r\right), n_i\left(s^r\right)\right) \\ st &: \quad \left(c_i\left(s^r\right) + k_i\left(s^r\right)\right) \leq \left(F\left(k_i\left(s^{r-1}\right), A_i\left(s^r\right) n_i\left(s^r\right)\right) + (1-\delta) k_i\left(s^{r-1}\right)\right) \\ & \quad\quad k_i\left(s^{t-1}\right) \text{ given} \end{aligned}$$

- how does one solve this model?

- K&P use a pseudo-planning setup

- planner now maximizes:

$$\max \Omega_1 \sum_{t=0}^{\infty} \beta^t \sum_{s^t} \pi\left(s^t\right) U\left(c_1\left(s^t\right), n_1\left(s^t\right)\right)$$
$$+\Omega_2 \sum_{t=0}^{\infty} \beta^t \sum_{s^t} \pi\left(s^t\right) U\left(c_2\left(s^t\right), n_2\left(s^t\right)\right)$$

- subject to:

$$
\begin{aligned}
\beta^t \pi\left(s^t\right) \lambda_i\left(s^t\right) \quad &: \quad \sum_{i=1}^{2}\left(c_i\left(s^t\right)+k_i\left(s^t\right)\right) \leq \\
& \quad \sum_{i=1}^{2}\left(F\left(k_i\left(s^{t-1}\right), A_i\left(s^t\right) n_i\left(s^t\right)\right)+(1-\delta) k_i\left(s^{t-1}\right)\right) \\
\beta^t \pi\left(s^t\right) \mu_i\left(s^t\right) \quad &: \quad \sum_{r=t}^{\infty} \beta^{r-t} \sum_{s^t} \pi\left(s^r \mid s^t\right) U\left(c_i\left(s^r\right), n_i\left(s^r\right)\right) \geq \\
& \quad V_i\left(k_i\left(s^{t-1}\right), s^t\right) \forall t
\end{aligned}
$$

- Resource constraint will bind with equality because of locally non-satiated preferences

- But enforcement constraint will sometimes be binding and other times not

- Moreover, enforcement constraint involves future values - the problem is non-recursive

- Reformulate the problem as a recursive one using recursive contract setup - apply partial summation formulae

- the terms in the Lagrangian that involve the enforcement constraints are given by:

$$\beta^t \pi\left(s^t\right) \mu_i\left(s^t\right)\left[\sum_{r=t}^{\infty} \beta^{r-t} \sum_{s^t} \pi\left(s^r | s^t\right) U\left(c_i\left(s^r\right), n_i\left(s^r\right)\right) - V_i\left(k_i\left(s^{t-1}\right), s^t\right)\right]$$

- now use that $\pi\left(s^r\right) = \pi\left(s^r | s^t\right) \pi\left(s^t\right)$

- The original Lagrangian can be written as:

$$\sum_{i=1}^{2}\sum_{t=0}^{\infty}\sum_{s^t}\beta^t\pi\left(s^t\right)(\Omega_i U\left(c_i\left(s^t\right),n_i\left(s^t\right)\right)$$
$$+\mu_i\left(s^t\right)\left[\sum_{r=t}^{\infty}\sum_{s^t}\beta^{r-t}\pi\left(s^r|s^t\right)U\left(c_i\left(s^r\right),n_i\left(s^r\right)\right)-V_i\left(k_i\left(s^{t-1}\right),s^t\right)\right])$$
$$+\sum_{t=0}^{\infty}\sum_{s^t}\beta^t\pi\left(s^t\right)\lambda\left(s^t\right)$$
$$\sum_{i=1}^{2}[\left(F\left(k_i\left(s^{t-1}\right),A_i\left(s^t\right)n_i\left(s^t\right)\right)+(1-\delta)k_i\left(s^{t-1}\right)\right)$$
$$-\left(c_i\left(s^t\right)+k_i\left(s^t\right)\right)]$$

- now use the formula for the conditional probability:

$$\sum_{i=1}^{2}\sum_{t=0}^{\infty}\sum_{s^t}\beta^t\pi\left(s^t\right)[(M_i\left(s^{t-1}\right)U\left(c_i\left(s^t\right),n_i\left(s^t\right)\right)$$
$$+\mu_i\left(s^t\right)(U\left(c_i\left(s^t\right),n_i\left(s^t\right)\right)-V_i\left(k_i\left(s^{t-1}\right),s^t\right))]$$
$$+\sum_{t=0}^{\infty}\sum_{s^t}\beta^t\pi\left(s^t\right)\lambda\left(s^t\right)$$
$$\sum_{i=1}^{2}[\left(F\left(k_i\left(s^{t-1}\right),A_i\left(s^t\right)n_i\left(s^t\right)\right)+(1-\delta)k_i\left(s^{t-1}\right)\right)$$
$$-\left(c_i\left(s^t\right)+k_i\left(s^t\right)\right)]$$

- where

$$\begin{aligned} M_i\left(s^t\right) &= M_i\left(s^{t-1}\right)+\mu_i\left(s^t\right) \\ M_i\left(s^{-1}\right) &= \Omega_1 \end{aligned}$$

- observations: This is now recursive - still involves inequality constraint, however

- notice how this leads to a problem with an effectively state-and-time dependent welfare weight

- The first-order conditions (for c, n, k) are:

$$M_i\left(s^{t-1}\right)U_c\left(c_i\left(s^t\right), n_i\left(s^t\right)\right)+\mu_i\left(s^t\right)U_c\left(c_i\left(s^t\right), n_i\left(s^t\right)\right) = \lambda\left(s^t\right)$$

$$\begin{aligned} & M_i\left(s^{t-1}\right)U_n\left(c_i\left(s^t\right), n_i\left(s^t\right)\right) + \mu_i\left(s^t\right)U_n\left(c_i\left(s^t\right), n_i\left(s^t\right)\right) \\ = & -F_n\left(k_i\left(s^{t-1}\right), A_i\left(s^t\right)n_i\left(s^t\right)\right) \end{aligned}$$

$$\lambda\left(s^t\right) = \beta \sum_{s^{t+1}} \pi\left(s^{t+1}|s^t\right)\lambda\left(s^{t+1}\right)$$
$$\left(F_k\left(k_i\left(s^t\right), A_i\left(s^{t+1}\right) n_i\left(s^{t+1}\right)\right) + (1-\delta)\right)$$
$$-\beta \sum_{s^{t+1}} \pi\left(s^{t+1}|s^t\right)\mu_i\left(s^{t+1}\right) V_{ik}\left(k_i\left(s^t\right), s^{t+1}\right)$$

- and the complementary slackness condition:

$$\mu_i\left(s^t\right)\left(U\left(c_i\left(s^t\right), n_i\left(s^t\right)\right) - V_i\left(k_i\left(s^{t-1}\right), s^t\right)\right) = 0$$

- or after rearraging (and combining)

$$\frac{U_c\left(c_1\left(s^t\right), n_1\left(s^t\right)\right)}{U_c\left(c_2\left(s^t\right), n_2\left(s^t\right)\right)} = \frac{M_2\left(s^{t-1}\right) + \mu_2\left(s^t\right)}{M_1\left(s^{t-1}\right) + \mu_1\left(s^t\right)}$$
$$-\frac{U_n\left(c_i\left(s^t\right), n_i\left(s^t\right)\right)}{U_c\left(c_i\left(s^t\right), n_i\left(s^t\right)\right)} = F_n\left(k_i\left(s^{t-1}\right), A_i\left(s^t\right) n_i\left(s^t\right)\right)$$

$$
\begin{aligned}
U_c\left(c_i\left(s^t\right), n_i\left(s^t\right)\right) \;=\;& \beta \sum_{s^{t+1}} \pi\left(s^{t+1}|s^t\right) \frac{M_i\left(s^{t+1}\right)}{M_i\left(s^t\right)} \\
& \left(F_k\left(k_i\left(s^t\right), A_i\left(s^{t+1}\right) n_i\left(s^{t+1}\right)\right) + (1-\delta)\right) \\
& -\beta \sum_{s^{t+1}} \pi\left(s^{t+1}|s^t\right) \frac{\mu_i\left(s^{t+1}\right)}{M_i\left(s^t\right)} V_{ik}\left(k_i\left(s^t\right), s^{t+1}\right)
\end{aligned}
$$

- note the impact on the risk sharing condition: When a country "hits" the enforcement constraint, it will be rewarded with a higher welfare weight

- When will this happen? when a country has high productivity - it suffers less from autarky and will thus have to be rewarded

- hence, domestic consumption will rise more in response to domestic positive productivity shock than under complete markets

- moreover, if shocks are persistent, these effects will be persistent

- also effects on employment investment and output

1. Since domestic consumption will rise more than under complete markets, smaller increase (decrease) in domestic (foreign) employment

2. smaller tendency for negative investment correlations

3. and therefore also smaller tendency for negative output comovements

- calibration and results

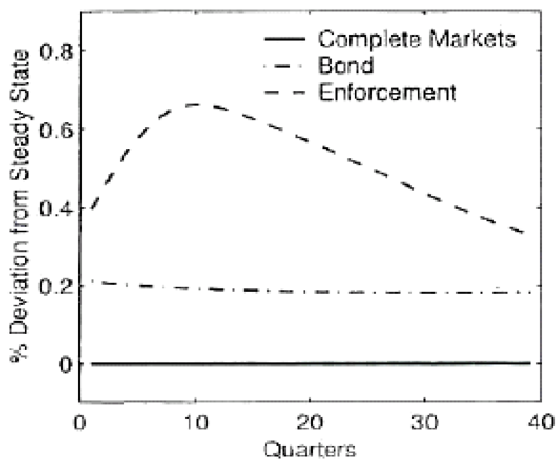
- labor share $= 0.64$, depreciation rate 2.5% per quarter
- Cobb-Douglas preferences, CRRA $= 2$, $\beta = 0.99$
- persistence of technology shocks $= 0.95$, no spill-overs, correlation of shocks $= 0.25$
- US-Europe on size

TABLE 2. Business cycle statistics: Baseline parameters

Statistic	Data	Economy with: No Adjustment Costs: Complete Markets	Bond	Enforcement	Adjustment Costs: Complete Markets	Bond
Volatility						
% Standard deviations						
GDP	1.72 (.20)	2.01	1.94	1.33	1.37	1.34
Net Exports/GDP	0.15 (.01)	13.04	12.42	0.06	0.36	0.33
% Standard deviations relative to GDP						
Consumption	0.79 (.05)	0.19	0.21	0.28	0.27	0.29
Investment	3.24 (.17)	25.23	25.06	3.04	3.42	3.24
Employment	0.63 (.04)	0.56	0.54	0.50	0.52	0.49
Domestic Comovement						
Correlations with GDP						
Consumption	0.87 (.03)	0.90	0.93	0.93	0.90	0.94
Investment	0.93 (.02)	0.07	0.08	0.99	0.95	0.95
Employment	0.86 (.03)	0.99	0.99	0.99	0.99	0.99
Net Exports/GDP	−0.36 (.09)	0.06	0.06	0.27	−0.02	−0.05
International Correlations						
Home and Foreign GDP	0.51 (.13)	−0.46	−0.43	0.25	0.09	0.12
Home and Foreign Consumption	0.32 (.17)	0.28	0.13	0.29	0.77	0.62
Home and Foreign Investment	0.29 (.17)	−0.99	−0.99	0.33	−0.17	−0.09
Home and Foreign Employment	0.43 (.11)	−0.58	−0.53	0.23	−0.15	−0.04

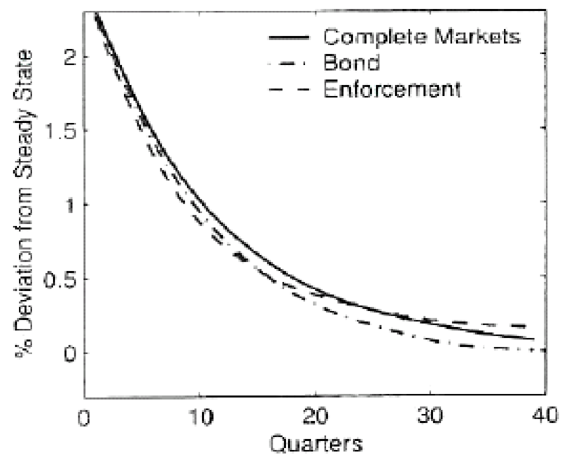
Figure 7:

2. **Foreign/Home Ratio of Marginal Utilities**



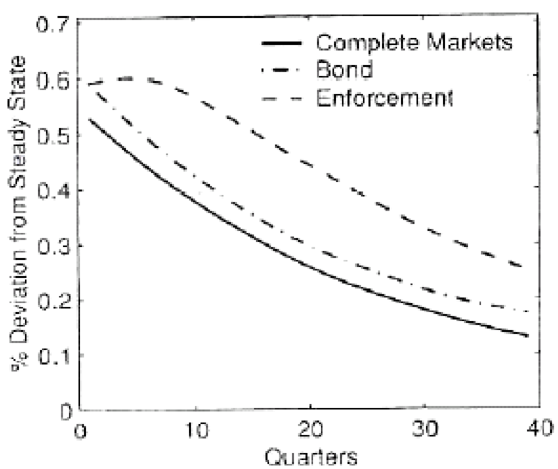
3. **Output**

3a. Home Country

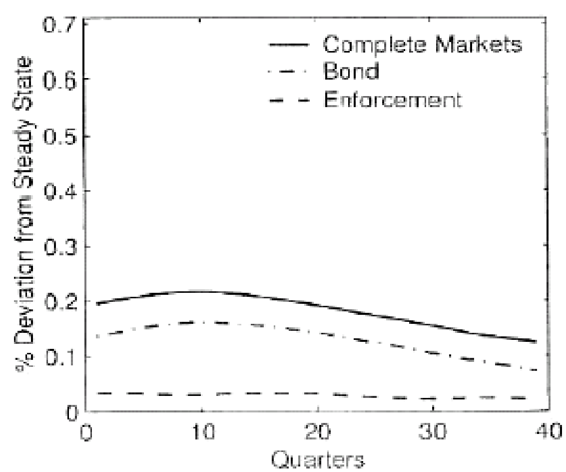


4. **Consumption**

4a. Home Country

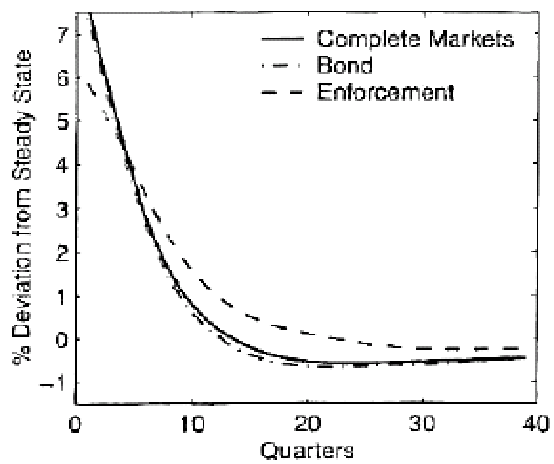


4b. Foreign Country

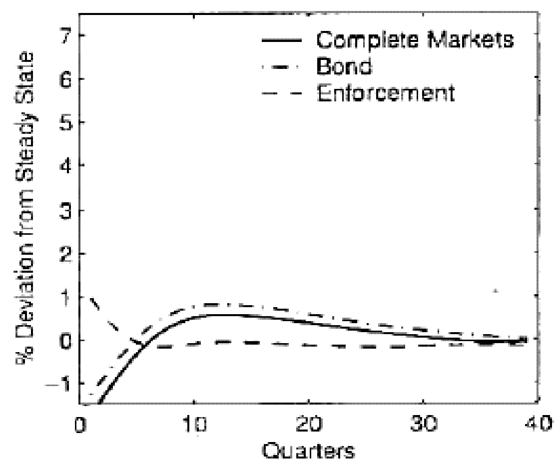


5. **Investment**

5a. Home Country

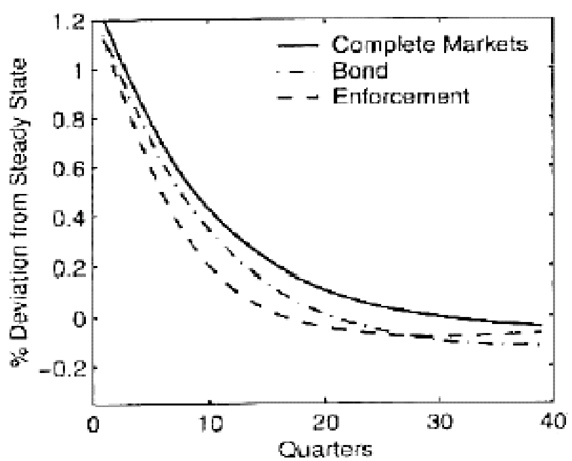


5b. Foreign Country

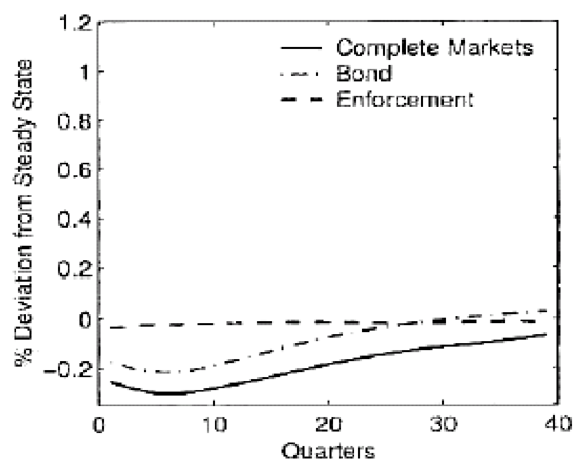


6. **Employment**

6a. Home Country



6b. Foreign Country



7. **Home Country Net Exports**

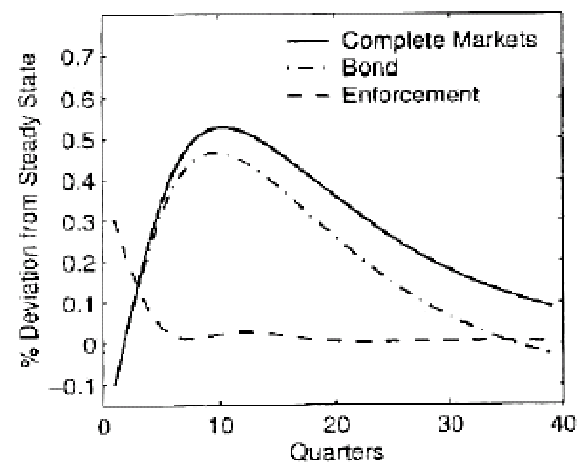


Figure 8:

- seems to work quite well (with exception of procyclical net-exports)

- however:

1. One good model: financial autarky = trade autarky - very severe punishment. Does this matter? Yes, in multiple good setting, we have seen that terms of trade movements neutralize wealth effects of technology shocks.

2. We have also seen in single good model that model with / without trade behaves very similar - so a bit puzzling why there's a big effect

3. Intuitively, one would think that enforcement constraints matter for countries with bad shocks - here it goes the other way