

# Open-economy macroeconomic models for policy evaluation: An analytical survey

*Giancarlo Corsetti*

May 2008

# Contents

<b>1</b>	<b>Five benchmark results</b>	<b>7</b>
<b>2</b>	<b>One framework, 4 models</b>	<b>13</b>
2.1	The setup . . . . .	14
2.2	Macroeconomic synthesis . . . . .	47
<b>3</b>	<b>International transmission</b>	<b>51</b>

<b>4</b>	<b>Optimal monetary policy and international coordination (with commitment)</b>	<b>62</b>
4.1	Flex-price model benchmark (model 1) . . . . .	65
4.2	Optimal policies in the PCP model (model 2) . . . . .	66
4.2.1	Terms of trade manipulation under PCP . . . . .	72
4.3	Optimal policy in the LCP model (model 3) . . . . .	79
4.3.1	Implications for exchange rate regimes . . . . .	83
4.3.2	Numerical Appraisal of LCP models . . . . .	86
4.4	Optimal policy in the dollar pricing model (model 4) . . . . .	106

<b>5</b>	<b>Pass-through as a choice variable</b>	<b>110</b>
<b>6</b>	<b>Discretion vs. commitment</b>	<b>118</b>

## *Introduction*

In this lecture we use a workhorse model to shed light on main issues in open economy macro monetary economics. The model leads to a set of benchmark results establishing basic principles or raising fundamental questions around the following classical theme:

- What is the international transmission mechanism of monetary shock? What are the allocative properties of the exchange rate?
- Should monetary policy have an ‘international dimension’, i.e. respond to international variables such as terms of trade or international cycle beyond its optimal response to domestic output gap and inflation?

- Is international policy cooperation desirable?
- Should governments curb exchange rate volatility?

These questions have been addressed by the so-called New Open-Economy Macroeconomics (after Obstfeld and Rogoff JPE 1995). The core of this literature has delivered five results which are not general, but provide important conceptual benchmarks, against which richer models which performs well quantitatively can measure up.

# 1 Five benchmark results

1. Monetary policy rules supporting the flexible price allocation are optimal: no rule welfare-dominates complete marginal cost and output gap stabilization. This result can hold under different assumptions regarding nominal rigidities, including staggered prices setting and partial adjustment (see e.g. Clarida, Gertler and Galí 2002). Optimal monetary rules are completely ‘inward-looking’: welfare-maximizing central banks stabilize the GDP deflator, while letting the CPI fluctuate with movements in the relative price of imports. There is no need for monetary policies to react to international variables.

The result does not hold in general. In the presence of multiple distortions monetary authorities are generally able to exploit nominal rigidities and improve

welfare relative to such allocation (see Benigno and Benigno 2003, Corsetti and Dedola 2005, De Paoli 2006, Sutherland 2004 among others). Yet, holding PCP, it is unclear whether and under which conditions deviating from full domestic stabilization could yield significant welfare gains.

2. The New-Keynesian theory has emphasized welfare costs from relative price dispersion when private pricing decisions are not synchronized (see e.g. Galí and Monacelli 2003). Early NOEM contributions have instead pioneered the analysis of the effect of uncertainty on the level of prices and economic activity (see e.g. Corsetti and Pesenti 2005, Kollmann 2002 and Sutherland 2005 for a quantitative assessment).

Similar effects, with potentially stronger welfare implications, are caused by a noisy conduct of monetary policy and exchange rate variability (Obstfeld and

Rogoff 1998). Notably, Broda (2006) provides evidence consistent with the (NOEM) prediction that incomplete stabilization and monetary/exchange rate noise transpire into higher price levels and real appreciation.

3. A third result is a clear-cut argument in favour of policies which are not 'inward looking.' To the extent that exporters' revenues and markups are exposed to exchange rate uncertainty, firms' optimal pricing strategies internalize the monetary policy of the importing country. When foreign firms' profits are exposed to exchange rate uncertainty, optimal monetary rules are no longer focusing on stabilizing domestic marginal costs. The importance of Foreign shocks in the conduct of monetary policy depends on the degree of openness of the economy, measured by the overall share of imports in the CPI (see Corsetti and Pesenti 2005a and Sutherland 2005, for a discussion of intermediate degrees of pass-through, and Smets and Wouters 2002 and Monacelli 2005 for models with staggered price setting).

4. With local currency pricing, the absence of expenditure switching effects (thus negative spillovers) from exchange rate movements is an argument in favour of containing terms of trade movements (although not an argument in favor of fixed exchange rates).

In the model of this lecture specified with symmetric preferences, the optimal policy rules actually prevent any short-run fluctuations of the exchange rate, a point stressed by Devereux and Engel (2003). But this exact result only holds when the weights of Home and Foreign goods in final expenditure are assumed to be identical across countries: Home and Foreign monetary authorities de facto stabilize the same weighted average of marginal costs. The presence of non-traded goods or some Home bias in consumption would obviously imply asymmetries in the optimal monetary stances, which would be incompatible

with a fixed exchange rate (Duarte and Obstfeld 2004, Corsetti 2006). Even if, with LCP, exchange rate variability does not perform any role in adjusting international prices, a fixed rate regime would impose unwarranted constraints on the efficient conduct of monetary policy.

5. The gains from international policy coordination may actually be quite small. This lecture provides examples with either PCP or LCP behavior, where optimal monetary rules are identical whether national policymakers act independently or cooperatively (maximizing an equally weighted sum of national welfare functions). When this exact result breaks down (depending on the elasticity of substitution between Home and Foreign tradables, and/or sector-specific shocks in the presence of nontradables), gains from coordination usually remain small (see e.g. Pappa 2004, Benigno and Benigno 2006).

The lesson from the NOEM literature stressed by Obstfeld and Rogoff (2002), is a new welfare-based argument against coordination: once policymakers independently pursue efficient stabilization policies in their own country (i.e. they 'keep their house in order'), the room for improving welfare through cooperation is quite limited (see Canzoneri et al. 2005 for a discussion). Recall that in absolute value, the gains from stabilizing the business cycle in these models are minuscule.

## 2 One framework, 4 models

The benchmark results will be derived using a two-country New Open Economy Macroeconomic (NOEM) model building on ‘Cole and Obstfeld economies,’ (e.g. Corsetti and Pesenti (QJE 2001, JME 2005), Devereux and Engel (REStud 2003), Obstfeld and Rogoff (JIE 2000, QJE 2002)). Specifying preferences as log+Cobb-Douglas aggregator of domestic and foreign consumption, one can simplify the analysis immensely, making it possible to solve seemingly complex model in closed form, and inspect a number of questions through tractable analytical expressions.

This section on the model should be familiar from previous lectures.

## 2.1 The setup

Two symmetric countries, Home and Foreign. In each country there are households, firms, and a government. Home households and firms are defined over a continuum of unit mass, with indexes  $j \in [0, 1]$  and  $h \in [0, 1]$ . Foreign households and firms are also defined over a continuum of unit mass, with indexes  $j^* \in [0, 1]$  and  $f \in [0, 1]$ .

Households are immobile across countries and they own national firms. Firms in each country specialize in the production of a *country-specific good*. Each firm produces a *variety (brand)* of the national good which is an imperfect substitute to all other varieties under conditions of *monopolistic competition*. Labor market is competitive. Markets are complete.

## *Home households*

The one-period utility of household  $j$  is:

$$U_t(j) = \ln C_t(j) - \kappa \ell_t(j) + \chi \ln \frac{\mathcal{M}_t(j)}{P_t}$$

where  $C_t(j)$  is now a Cobb-Douglas basket (that is, a CES basket with unit elasticity) of the Home and Foreign goods with equal weights (1/2, 1/2):

$$C_t(j) = C_{H,t}(j)^{1/2} C_{F,t}(j)^{1/2}$$

and  $C_{H,t}(j)$  and  $C_{F,t}(j)$  are CES baskets of, respectively, Home and Foreign varieties (for identical elasticity  $\theta > 1 =$  substitution between domestic goods and imports)

$$C_{H,t}(j) = \left( \int_0^1 C_t(h, j)^{1-\frac{1}{\theta}} dh \right)^{\frac{\theta}{\theta-1}} \quad C_{F,t}(j) = \left( \int_0^1 C_t(f, j)^{1-\frac{1}{\theta}} df \right)^{\frac{\theta}{\theta-1}}$$

*Foreign households are analogously characterized*

The one-period utility of household  $j^*$  is:

$$U_t^*(j^*) = \ln C_t^*(j^*) - \kappa \ell_t^*(j^*) + \chi \ln \frac{M_t^*(j^*)}{P_t^*}$$

where  $C_t^*(j^*)$  is a Cobb-Douglas basket:

$$C_t^*(j^*) = C_{H,t}^*(j^*)^{1/2} C_{F,t}^*(j^*)^{1/2}$$

and  $C_{H,t}^*(j^*)$ ,  $C_{F,t}^*(j^*)$  are CES baskets of, respectively, Home and Foreign varieties:

$$C_{H,t}^*(j^*) = \left( \int_0^1 C_t^*(h, j^*)^{1-\frac{1}{\theta}} dh \right)^{\frac{\theta}{\theta-1}} \quad C_{F,t}^*(j^*) = \left( \int_0^1 C_t^*(f, j^*)^{1-\frac{1}{\theta}} df \right)^{\frac{\theta}{\theta-1}}$$

## *Price indexes*

For given Home-currency prices of the varieties,  $p_t(h)$  and  $p_t(f)$ , the utility-based CPI,  $P_t$ , is defined as:

$$P_t = 2P_{H,t}^{1/2}P_{F,t}^{1/2}$$

where:

$$P_{H,t} = \left( \int_0^1 p_t(h)^{1-\theta} dh \right)^{\frac{1}{1-\theta}} \quad P_{F,t} = \left( \int_0^1 p_t(f)^{1-\theta} df \right)^{\frac{1}{1-\theta}} .$$

$P_t$  is the minimum expenditure associated with consumption of one unit of the index  $C_t$ .

## *Demand*

The Home-country individual demand curves for varieties  $h$  and  $f$  are, respectively:

$$C_t(h, j) = \left( \frac{p_t(h)}{P_{H,t}} \right)^{-\theta} \frac{1}{2} \left( \frac{P_{H,t}}{P_t} \right)^{-1} C_t(j)$$
$$C_t(f, j) = \left( \frac{p_t(f)}{P_{F,t}} \right)^{-\theta} \frac{1}{2} \left( \frac{P_{F,t}}{P_t} \right)^{-1} C_t(j)$$

and the optimal composition of nominal spending is:

$$P_{H,t}C_{H,t}(j) = P_{F,t}C_{F,t}(j) = \frac{1}{2}P_tC_t(j)$$

*Similar expressions hold in the Foreign country*

$$P_t^* = 2P_{H,t}^{*1/2} P_{F,t}^{*1/2} \quad P_{H,t}^* = \left( \int_0^1 p_t^*(h)^{1-\theta} dh \right)^{\frac{1}{1-\theta}} \quad P_{F,t}^* = \left( \int_0^1 p_t^*(f)^{1-\theta} df \right)^{\frac{1}{1-\theta}}$$

$$C_t(h, j^*) = \left( \frac{p_t^*(h)}{P_{H,t}^*} \right)^{-\theta} \frac{1}{2} \left( \frac{P_{H,t}^*}{P_t^*} \right)^{-1} C_t^*(j^*)$$

$$C_t(f, j^*) = \left( \frac{p_t^*(f)}{P_{F,t}^*} \right)^{-\theta} \frac{1}{2} \left( \frac{P_{F,t}^*}{P_t^*} \right)^{-1} C_t^*(j^*)$$

$$P_{H,t}^* C_{H,t}^*(j^*) = P_{F,t}^* C_{F,t}^*(j^*) = \frac{1}{2} P_t^* C_t^*(j^*)$$

## *Budgets (1)*

Home households own the portfolio of Home firms, hold the Home currency,  $\mathcal{M}$ , receive wages and profits from the firms and pay non-distortionary (lump-sum) net taxes  $TX$ , denominated in Home currency.

*Assume complete markets* — households have access to a full set of Arrow-Debreu securities. Using a sequential formulation (see e.g. Ljungqvist and Sargent (2000)), let  $Q(s_{t+1} | s_t)$  denote the price of one unit of Home currency delivered in period  $t + 1$  contingent on the state of nature at  $t + 1$  being  $s_{t+1}$ . With complete markets,  $Q(s_{t+1} | s_t)$  is the same for all individuals. Let  $B_t(s_{t+1}, j)$  denote the claim to  $B_t(s_{t+1}, j)$  units of Home currency at time  $t + 1$  in the state of nature  $s_{t+1}$ , that household  $j$  buys at time  $t$  and brings into time  $t + 1$ .  $B_t^*(s_t, j)$  and  $Q^*(s_{t+1} | s_t)$  are similarly defined in terms of units of Foreign currency.

## *Budgets (2)*

The individual flow budget constraint for household  $j$  in the Home country is:

$$\mathcal{M}_t(j) + \sum_{s_{t+1}} B_t(s_{t+1}, j)Q(s_{t+1} | s_t) + \mathcal{E}_t \sum_{s_{t+1}} B_t^*(s_{t+1}, j)Q^*(s_{t+1} | s_t) \leq$$
$$\mathcal{M}_{t-1}(j) + B_{t-1}(s_t, j) + \mathcal{E}_t B_{t-1}^*(s_t, j) + W_t \ell_t(j) + \mathcal{P}_t(j) - TX_t(j) - P_t C_t(j)$$

Recall:  $\mathcal{E}_t$  denotes the nominal exchange rate (defined as Home currency per unit of Foreign currency).

The utility function and the budget constraint of the Foreign representative household are similarly defined.

$$F.O.C.S (C_t(j), M_t(j), \ell_t(j)) (1)$$

Home household  $j$  maximizes expected utility subject to budget constraint. F.O.C.S with respect to  $C_t(j)$ ,  $M_t(j)$  and  $\ell_t(j)$  are familiar:

$$\frac{1}{C_t(j)} - \Lambda_t(j)P_t = 0$$

$$\frac{\chi}{M_t(j)} - \Lambda_t(j) + \beta E_t \Lambda_{t+1}(j) = 0$$

$$-\kappa + W_t \Lambda_t(j) = 0$$

where  $\Lambda_t(j)$  is the Lagrangian multiplier associated with the flow budget constraint at time  $t$ .

*F.O.C.S (C<sub>t</sub>(j), M<sub>t</sub>(j), ℓ<sub>t</sub>(j)) (2)*

Define the nominal discount rate between time  $t$  and  $t + \tau$  for the household  $j$  as

$$D_{t,t+\tau}(j) = \beta^\tau \frac{\Lambda_{t+\tau}(j)}{\Lambda_t(j)} = \beta^\tau \frac{C_t(j)}{C_{t+\tau}(j)} \frac{P_t}{P_{t+\tau}}$$

Workers equate the marginal rate of substitution between consumption and leisure,  $\kappa C_t(j)$ , to the real wage in consumption units,  $W_t/P_t$ .

Note that the previous expressions imply equalization of consumption across agents, or:

$$C_t(j) = C_t, \quad \Lambda_t(j) = \Lambda_t, \quad D_{t,t+\tau}(j) = D_{t,t+\tau}.$$

### *F.O.C.S (securities) (1)*

Without loss of generality, we focus only on Home-currency securities. The first order conditions with respect to each Arrow-Debreu security yield:

$$Q(s_{t+1} | s_t) = \beta \cdot \Pr(s_{t+1} | s_t) \frac{\partial U_{t+1} / \partial C_{t+1}}{\partial U_t / \partial C_t} \frac{P_t}{P_{t+1}}$$

where  $\Pr(s_{t+1} | s_t)$  denotes the probability of state  $s_{t+1}$  at time  $t+1$  conditional on the realization of state  $s_t$  at  $t$ . Similar results hold for the representative Foreign household. Namely, the first order conditions with respect to the Arrow-Debreu securities yield:

$$\frac{\mathcal{E}_{t+1}}{\mathcal{E}_t} Q(s_{t+1} | s_t) = \beta \cdot \Pr(s_{t+1} | s_t) \frac{\partial U_{t+1}^* / \partial C_{t+1}^*}{\partial U_t^* / \partial C_t^*} \frac{P_t^*}{P_{t+1}^*}$$

### *F.O.C.S (securities) (2)*

By combining the two expressions above we obtain:

$$\frac{\partial U_{t+1}/\partial C_{t+1}}{\partial U_t/\partial C_t} = \frac{\partial U_{t+1}^*/\partial C_{t+1}^*}{\partial U_t^*/\partial C_t^*} \frac{\mathcal{E}_t P_t^*/P_t}{\mathcal{E}_{t+1} P_{t+1}^*/P_{t+1}}$$

and therefore:

$$\frac{P_t C_t}{P_{t+1} C_{t+1}} = \frac{\mathcal{E}_t P_t^* C_t^*}{\mathcal{E}_{t+1} P_{t+1}^* C_{t+1}^*}$$

The rate of growth of marginal utility is equal to the rate of real depreciation (the rate of growth of the real exchange rate). Defining  $\mu_t = P_t C_t$  and  $\mu_t^* = P_t^* C_t^*$  we can write

$$\frac{\mu_t}{\mu_{t+1}} = \frac{\mathcal{E}_t \mu_t^*}{\mathcal{E}_{t+1} \mu_{t+1}^*}$$

### *F.O.C.S (securities) (3)*

Iterating the above expression we can rewrite the above with respect to some initial date 0:

$$\mu_t = \left( \frac{\mu_0}{\mathcal{E}_0 \mu_0^*} \right) \mathcal{E}_t \mu_t^* = \text{constant} \cdot \mathcal{E}_t \mu_t^*$$

In a symmetric world, Home and Foreign consumption are ex ante identical, hence the constant in the above expression is equal to one (recall risk-sharing wedge  $\Phi$  above).

The equilibrium exchange rate is therefore equal to the ratio of Home to Foreign monetary stance:  $\mathcal{E}_t = \mu_t / \mu_t^*$  and

$$P_t C_t = \mathcal{E}_t P_t^* C_t^*.$$

## *Pricing bonds*

Let's price one period nominal bonds that are traded internationally. In the case of bonds denominated in domestic currency, yielding the nominal interest rate  $i$ , we have the following *Euler equation*:

$$-\Lambda_t + \beta (1 + i_t) E_t \Lambda_{t+1} = 0$$

In the case of bonds denominated in Foreign currency, and yielding  $i^*$  we have

$$-\Lambda_t \mathcal{E}_t + \beta (1 + i_t^*) E_t \Lambda_{t+1} \mathcal{E}_{t+1} = 0$$

which can also be written as:

$$\frac{1}{C_t} = \beta (1 + i_t^*) E_t \left( \frac{1}{C_{t+1}} \frac{1 + \mathcal{D}_{t+1}}{1 + \pi_{t+1}} \right)$$

## *Firms*

The production functions in the two countries are linear in labor:

$$Y_t(h) = Z_t \ell_t(h) \qquad Y_t^*(f) = Z_t^* \ell_t^*(f)$$

where  $Z_t$  and  $Z_t^*$  are two country-specific productivity processes. Note that the resource constraint for Home variety  $h$  is now:

$$Y_t(h) = \int_0^1 C_t(h, j) dj + \int_0^1 C_t^*(h, j^*) dj^*$$

and similarly for Foreign variety  $f$ :

$$Y_t^*(f) = \int_0^1 C_t(f, j) dj + \int_0^1 C_t^*(f, j^*) dj^*$$

## *Demand schedules*

Aggregating across  $j$ -agents we obtain total Home demand for variety  $h$ :

$$\int_0^1 C_t(h, j) dj = \left( \frac{p_t(h)}{P_{H,t}} \right)^{-\theta} \int_0^1 C_{H,t}(j) dj = \left( \frac{p_t(h)}{P_{H,t}} \right)^{-\theta} C_{H,t}$$

Similarly, total Foreign demand for variety  $h$  is obtained by aggregating over  $j^*$ -agents:

$$\int_0^1 C_t^*(h, j^*) dj^* = \left( \frac{p_t^*(h)}{P_{H,t}^*} \right)^{-\theta} \int_0^1 C_{H,t}^*(j^*) dj^* = \left( \frac{p_t^*(h)}{P_{H,t}^*} \right)^{-\theta} C_{H,t}^*$$

so that Home firm  $h$  faces the following demand schedule for its product:

$$Y_t(h) = \left( \frac{p_t(h)}{P_{H,t}} \right)^{-\theta} \left( \frac{P_{H,t}}{P_t} \right)^{-1} \frac{1}{2} C_t + \left( \frac{p_t^*(h)}{P_{H,t}^*} \right)^{-\theta} \left( \frac{P_{H,t}^*}{P_t^*} \right)^{-1} \frac{1}{2} C_t^*$$

### *Home marginal cost*

As above: Home firm  $h$  minimizes costs  $W_t \ell_t(h)$  subject to the above technology. The Lagrangian multiplier associated with this problem is the nominal marginal cost  $MC_t(h)$ , equal to:

$$MC_t(h) = MC_t = \frac{W_t}{Z_t}$$

or using the F.O.C. with respect to  $\ell$ :

$$MC_t = \frac{\kappa \mu_t}{Z_t}$$

Firms operating under conditions of monopolistic competition take into account the downward-sloping demand for their product and set prices to maximize their value. Firms are small, in the sense that they ignore the impact of their pricing and production decisions on aggregate variables and price indexes.

### *Home firm $h$ 's nominal profits*

Can be written as:

$$\begin{aligned}\mathcal{P}_t(h) &= p_t(h) \int_0^1 C_t(h, j) dj + \mathcal{E}_t p_t^*(h) \int_0^1 C_t(h, j^*) dj^* - W_t \ell_t(h) \\ &= p_t(h) \int_0^1 C_t(h, j) dj + \mathcal{E}_t p_t^*(h) \int_0^1 C_t(h, j^*) dj^* \\ &\quad - \frac{W_t}{Z_t} \left( \int_0^1 C_t(h, j) dj + \int_0^1 C_t^*(h, j^*) dj^* \right) \\ &= (p_t(h) - MC_t) \int_0^1 C_t(h, j) dj + (\mathcal{E}_t p_t^*(h) - MC_t) \int_0^1 C_t(h, j^*) dj^* \\ &= (p_t(h) - MC_t) \left( \frac{p_t(h)}{P_{H,t}} \right)^{-\theta} C_{H,t} + (\mathcal{E}_t p_t^*(h) - MC_t) \left( \frac{p_t^*(h)}{P_{H,t}^*} \right)^{-\theta} C_{H,t}^*\end{aligned}$$

## MODEL 1: FLEXIBLE PRICE

Home firms set prices to maximize  $\mathcal{P}_t(h)$  with respect to  $p_t(h)$  and  $p_t^*(h)$ . This implies:

$$p_t(h) = \mathcal{E}_t p_t^*(h) = \frac{\theta}{\theta - 1} MC_t$$

Both prices are equal to the marginal cost augmented by a constant markup  $\theta / (\theta - 1)$ . With identical elasticity  $\theta$  across countries, the law of one price holds: the same good  $h$  sells at the same price in both markets when expressed in terms of the same currency (even if firms could segment these markets).

### 3 MODELS WITH NOMINAL RIGIDITIES: *same Home prices*

At time  $t - 1$ , firms preset the price(s) at which they sell their good in the Home and Foreign countries at time  $t$  (only for one period). They do so by maximizing the value of the firm, i.e. expected discounted profits  $E_{t-1} \left( D_{t-1,t} \mathcal{P}_t(h) \right)$ . The first order condition with respect to the *domestic-currency price*  $p(h)$  is:

$$E_{t-1} \left\{ D_{t-1,t} \left( \frac{p_t(h)}{P_{H,t}} \right)^{-\theta} C_{H,t} \right\} = \theta E_{t-1} \left\{ D_{t-1,t} \frac{p_t(h) - MC_t}{p_t(h)} \left( \frac{p_t(h)}{P_{H,t}} \right)^{-\theta} C_{H,t} \right\}$$

Recalling that  $D_{t-1,t} = \beta P_{t-1} C_{t-1} / P_t C_t$ ,  $C_{H,t} = P_t C_t / 2 P_{H,t}$ , and observing that all prices  $p_t(h)$  are symmetric, thus  $p_t(h) = P_{H,t}$ :

$$p_t(h) = P_{H,t} = \frac{\theta}{\theta - 1} E_{t-1} (MC_t)$$

Recall results in closed-economy model.

Logically, *the Foreign-currency price of exports*  $p_t^*(h)$  can be set in two different ways, depending on the specific currency in which Home exports are priced.

**MODEL 2: PCP** *A Model with producer currency pricing (PCP):*

Exports are priced and invoiced in domestic (producer's) currency, firm  $h$  maximizes  $E_{t-1} \left( D_{t-1,t} \mathcal{P}_t(h) \right)$  with respect to  $\mathcal{E}_t p_t^*(h)$ , setting the price of variety  $h$  according to:

$$E_{t-1} \left\{ D_{t-1,t} \left( \frac{\mathcal{E}_t p_t^*(h)}{\mathcal{E}_t P_{H,t}^*} \right)^{-\theta} C_{H,t}^* \right\} =$$

$$\theta E_{t-1} \left\{ D_{t-1,t} \frac{\mathcal{E}_t p_t^*(h) - MC_t}{\mathcal{E}_t p_t^*(h)} \left( \frac{\mathcal{E}_t p_t^*(h)}{\mathcal{E}_t P_{H,t}^*} \right)^{-\theta} C_{H,t}^* \right\}$$

## MODEL 2: PCP

Rearranging:

$$\mathcal{E}_t p_t^*(h) = \frac{\theta}{\theta - 1} \frac{E_{t-1} \left( D_{t-1,t} MC_t \left( \frac{\mathcal{E}_t p_t^*(h)}{\mathcal{E}_t P_{H,t}^*} \right)^{-\theta} C_{H,t}^* \right)}{E_{t-1} \left( D_{t-1,t} \left( \frac{\mathcal{E}_t p_t^*(h)}{\mathcal{E}_t P_{H,t}^*} \right)^{-\theta} C_{H,t}^* \right)}$$

Recalling that  $D_{t-1,t} = \beta P_{t-1} C_{t-1} / P_t C_t$ ,  $C_{H,t}^* = \varepsilon_t P_t^* C_t^* / 2 (\varepsilon_t P_{H,t}^*)$ , and observing that all prices  $\varepsilon_t p_t^*(h)$  are symmetric, thus  $\varepsilon_t p_t^*(h) = \varepsilon_t P_{H,t}^*$ , we obtain:

$$\begin{aligned} \varepsilon_t p_t^*(h) = \varepsilon_t P_{H,t}^* &= \frac{\theta}{\theta - 1} \frac{E_{t-1} \left( MC_t \frac{\varepsilon_t P_t^* C_t^*}{P_t C_t} \frac{1}{\varepsilon_t P_{H,t}^*} \right)}{E_{t-1} \left( \frac{\varepsilon_t P_t^* C_t^*}{P_t C_t} \frac{1}{\varepsilon_t P_{H,t}^*} \right)} \\ &= \frac{\theta}{\theta - 1} \frac{E_{t-1} \left( MC_t \frac{\varepsilon_t P_t^* C_t^*}{P_t C_t} \right)}{E_{t-1} \left( \frac{\varepsilon_t P_t^* C_t^*}{P_t C_t} \right)} = \frac{\theta}{\theta - 1} E_{t-1} (MC_t) \end{aligned}$$

## MODEL 2: PCP

$$\mathcal{E}_t P_{H,t}^* = \frac{\theta}{\theta - 1} E_{t-1} (MC_t) = P_{H,t}$$

the equilibrium price is equilibrium markup times expected marginal costs.

- Foreign-currency prices  $P_{H,t}^*$  move one-to-one with the nominal exchange rate, leaving the export price  $\mathcal{E}_t P_{H,t}^*$  unchanged when expressed in Home currency
  - In other words, there is *full exchange rate pass-through*.
- The *law of one price holds* independently of market segmentation. With identical demand elasticities, firms charge the same price (in domestic currency) everywhere.

### MODEL 3: LCP: *A model with local currency pricing*

The export price is preset in Foreign currency, firm  $h$  maximizes expected discounted profits  $E_{t-1} \left( D_{t-1,t} \mathcal{P}_t(h) \right)$  with respect to  $p_t^*(h)$ . The first order condition is:

$$\begin{aligned} E_{t-1} \left( D_{t-1,t} \mathcal{E}_t \left( \frac{p_t^*(h)}{P_{H,t}^*} \right)^{-\theta} C_{H,t}^* \right) &= \\ = \theta E_{t-1} \left( D_{t-1,t} \frac{\mathcal{E}_t p_t^*(h) - MC_t}{p_t^*(h)} \left( \frac{p_t^*(h)}{P_{H,t}^*} \right)^{-\theta} C_{H,t}^* \right) \end{aligned}$$

### MODEL 3: LCP

The optimal price in foreign currency is the equilibrium markup times expected marginal costs *expressed in foreign currency*

$$\begin{aligned} p_t^*(h) = P_{H,t}^* &= \frac{\theta}{\theta - 1} \frac{E_{t-1} \left( MC_t D_{t-1,t} \left( \frac{1}{P_{H,t}^*} \right)^{-\theta} C_{H,t}^* \right)}{E \left( D_{t-1,t} \varepsilon_t \left( \frac{1}{P_{H,t}^*} \right)^{-\theta} C_{H,t}^* \right)} \\ &= \frac{\theta}{\theta - 1} \frac{E_{t-1} \left( \frac{MC_t}{\varepsilon_t} \frac{P_t^* C_t^* \varepsilon_t}{P_t C_t} \right)}{E \left( \frac{\varepsilon_t P_t^* C_t^*}{P_t C_t} \right)} = \frac{\theta}{\theta - 1} E_{t-1} \left( \frac{MC_t}{\varepsilon_t} \right) \end{aligned}$$

### MODEL 3: LCP

$$P_{H,t}^* = \frac{\theta}{\theta - 1} E_{t-1} \left( \frac{MC_t}{\mathcal{E}_t} \right) \text{ in general different from } \frac{P_{H,t}}{\mathcal{E}_t}$$

- Home export prices expressed in Foreign currency do not move when the exchange rate changes
  - *Zero exchange rate pass-through*
- Exchange rate movements will induce *deviations from the law of one price*.
  - Implicit assumption: international arbitrage is not possible — markets are segmented.

## **MODEL 4: DOLLAR PRICING”**

A last model is obtained by combining PCP and LCP in an asymmetric world.

All exports prices are preset in one currency only.

- Overall then we have 4 models differing in pricing assumptions: flexible, PCP, LCP and Dollar pricing.

We now look at the remaining variables in these 4 models.

**IN ALL THESE FOUR MODELS**, the *resource constraint* for the Home output is:

$$\begin{aligned} Z_t \ell_t = C_{H,t} + C_{H,t}^* &= \frac{1}{2} \left( \frac{P_t C_t}{P_{H,t}} + \frac{P_t^* C_t^*}{P_{H,t}^*} \right) \\ &= \frac{1}{2} \left( \frac{P_t C_t}{P_{H,t}} + \frac{P_t C_t}{\varepsilon_t P_{H,t}^*} \right) = \frac{P_t}{2} \left( \frac{1}{P_{H,t}} + \frac{1}{\varepsilon_t P_{H,t}^*} \right) C_t \end{aligned}$$

which can be synthetically be written

$$Z_t \ell_t = \frac{1}{\tau_t} C_t$$

the variable  $\tau$  is the *consumption-output terms of trade* (how many units of output households need to buy one unit of consumption).

*Similarly, for the Foreign economy*

$$C_t^* = Z_t^* l_t^* \tau_t^* \quad \frac{1}{\tau_t^*} \equiv \frac{P_t^*}{2} \left( \frac{1}{P_{F,t}^*} + \frac{\mathcal{E}_t}{P_{F,t}} \right)$$

As we will see below, the variables  $\tau_t^*$  and  $\tau_t$  are indexes of international spillovers, reflecting the macroeconomic impact of fluctuations of relative prices and terms of trade on the Home and Foreign economy.

*The natural rate of employment:* using the resource constraint together with optimal prices it can be shown that:

- Absent nominal rigidities, (full) employment in both economies is constant regardless of the shocks:

$$l_t = l_t^* = \frac{\theta - 1}{\theta \kappa} = \bar{l}$$

- In the presence of nominal rigidities, instead, the flex-price employment level holds in expectations:

$$E_{t-1}(l_t) = E_{t-1}(l_t^*) = \bar{l}$$

regardless of export pricing.

## *Monetary policy and the government budget constraint (1)*

Observe that optimal money holdings can be written as:

$$\frac{\mathcal{M}_t(j)}{\chi} = P_t C_t \frac{1 + i_t}{i_t} = \frac{\mu_t}{1 - \beta E_t (\mu_t / \mu_{t+1})}$$

We note here that two nominal aggregate demand variables  $\mu_t = P_t C_t$ ;  $\mu_t^* = P_t^* C_t^*$  can also be interpreted as indicators of Home and Foreign monetary stance.

As there is no public spending, the government uses seigniorage revenues and taxes to finance transfers. The public budget constraint is simply:

$$\mathcal{M}_t - \mathcal{M}_{t-1} + \int_0^1 T X_t(j) dj = 0$$

and in equilibrium money supply equals demand, or  $\mathcal{M}_t = \int_0^1 \mathcal{M}_t(j) dj$ .

*A digression on an equivalent specification of our model (1)*

As we assume unit elasticity of substitution between domestic and foreign goods, and log preferences, one can show that the equilibrium allocation would be identical if, instead of assuming complete markets, one assumes that only one period nominal bonds are traded internationally, and net foreign wealth is initially zero. See Corsetti and Pesenti 2001 and 2005a,b.

Note  $\mu$  here is nominal spending (or monetary stance), not distribution margin!

## 2.2 Macroeconomic synthesis

To summarize: given the exogenous variables  $Z_t, Z_t^*, \mu_t, \mu_t^*$  and the prices  $P_{H,t}, P_{F,t}, P_{H,t}^*, P_{F,t}^*$ , the macroeconomics of the two-country model is described by the system of 13 equations in 13 endogenous variables  $\mathcal{E}_t, P_t, P_t^*, C_t, C_t^*, \tau_t, \tau_t^*, l_t, l_t^*, C_{H,t}, C_{F,t}, C_{H,t}^*,$  and  $C_{F,t}^*$ :

*The model at a glance: common equations to the 4 models...*

$$\mathcal{E}_t = \mu_t / \mu_t^*$$

$$\mu_t = P_t C_t$$

$$P_t = 2P_{H,t}^{1/2} P_{F,t}^{1/2}$$

$$C_t = Z_t \ell_t \tau_t$$

$$\frac{1}{\tau_t} \equiv \frac{P_t}{2} \left( \frac{1}{P_{H,t}} + \frac{1}{\mathcal{E} P_{H,t}^*} \right)$$

$$P_{H,t} C_{H,t} = \frac{1}{2} P_t C_t$$

$$P_{H,t}^* C_{H,t}^* = \frac{1}{2} P_t^* C_t^*$$

$$\mu_t^* = P_t^* C_t^*$$

$$P_t^* = 2P_{H,t}^{*1/2} P_{F,t}^{*1/2}$$

$$C_t^* = Z_t^* \ell_t^* \tau_t^*$$

$$\frac{1}{\tau_t^*} \equiv \frac{P_t^*}{2} \left( \frac{1}{P_{F,t}^*} + \frac{\mathcal{E}_t}{P_{F,t}} \right)$$

$$P_{F,t} C_{F,t} = \frac{1}{2} P_t C_t$$

$$P_{F,t}^* C_{F,t}^* = \frac{1}{2} P_t^* C_t^*$$

...plus optimal prices for model 1, model 2...

1st: flexible prices:

$$\begin{aligned}
 P_{H,t} &= \frac{\theta \kappa}{\theta - 1} \frac{\mu_t}{Z_t} & P_{F,t} &= \varepsilon_t P_{F,t}^* = \varepsilon_t \frac{\theta \kappa}{\theta - 1} \frac{\mu_t^*}{Z_t^*} \\
 P_{H,t}^* &= \frac{P_{H,t}}{\varepsilon_t} = \frac{1}{\varepsilon_t} \frac{\theta \kappa}{\theta - 1} \frac{\mu_t}{Z_t} & P_{F,t}^* &= \frac{\theta \kappa}{\theta - 1} \frac{\mu_t^*}{Z_t^*}
 \end{aligned}$$

2nd: Nominal rigidities with PCP: export prices set in the producer's currency:

$$\begin{aligned}
 P_{H,t} &= \frac{\theta \kappa}{\theta - 1} E_{t-1} \left( \frac{\mu_t}{Z_t} \right) & P_{F,t} &= \varepsilon_t P_{F,t}^* = \varepsilon_t \frac{\theta \kappa}{\theta - 1} E_{t-1} \left( \frac{\mu_t^*}{Z_t^*} \right) \\
 P_{H,t}^* &= \frac{P_{H,t}}{\varepsilon_t} = \frac{1}{\varepsilon_t} \frac{\theta \kappa}{\theta - 1} E_{t-1} \left( \frac{\mu_t}{Z_t} \right) & P_{F,t}^* &= \frac{\theta \kappa}{\theta - 1} E_{t-1} \left( \frac{\mu_t^*}{Z_t^*} \right)
 \end{aligned}$$

...model 3 and 4

3rd: *LCP*: export prices are set in the consumer's currency:

$$\begin{aligned} P_{H,t} &= \frac{\theta\kappa}{\theta-1} E_{t-1} \left( \frac{\mu_t}{Z_t} \right) & P_{F,t} &= \frac{\theta\kappa}{\theta-1} E_{t-1} \left( \frac{\mu_t}{Z_t^*} \right) \\ P_{H,t}^* &= \frac{\theta\kappa}{\theta-1} E_{t-1} \left( \frac{\mu_t^*}{Z_t} \right) & P_{F,t}^* &= \frac{\theta\kappa}{\theta-1} E_{t-1} \left( \frac{\mu_t^*}{Z_t^*} \right) \end{aligned}$$

4th: *Dollar Pricing*: export prices set in only one currency (say, Home):

$$\begin{aligned} P_{H,t} &= \frac{\theta\kappa}{\theta-1} E_{t-1} \left( \frac{\mu_t}{Z_t} \right) & P_{F,t} &= \frac{\theta\kappa}{\theta-1} E_{t-1} \left( \frac{\mu_t}{Z_t^*} \right) \\ P_{H,t}^* &= \frac{P_{H,t}}{\varepsilon_t} = \frac{1}{\varepsilon_t} \frac{\theta\kappa}{\theta-1} E_{t-1} \left( \frac{\mu_t}{Z_t} \right) & P_{F,t}^* &= \frac{\theta\kappa}{\theta-1} E_{t-1} \left( \frac{\mu_t^*}{Z_t^*} \right) \end{aligned}$$

### 3 International transmission

*International transmission under flexible prices*

$$\begin{aligned} \ell &= \bar{\ell} \Rightarrow Y_H = Z\bar{\ell} \\ C &= Z\bar{\ell}\tau = Z\bar{\ell} \left( \frac{P_H}{P_F} \right)^{1/2} = Z\bar{\ell} \left( \frac{Z^*}{Z} \right)^{1/2} = Z^{1/2} (Z^*)^{1/2} \bar{\ell} = C^* \end{aligned}$$

- Transmission of productivity shocks is 'positive.' As Home country is better off because of higher productivity, Foreign also benefits via an improvements of their terms of trade.
- $\mu$  does not matter: an increase in nominal spending is matched by an adjustment in nominal prices.

*International transmission under PCP*

$$\begin{aligned}
 \ell_t &= \frac{\mu_t/Z_t}{E_{t-1}(\mu/Z)} \bar{\ell}, & \ell_t^* &= \frac{\mu_t^*/Z_t^*}{E_{t-1}(\mu_t^*/Z_t^*)} \bar{\ell} \\
 \tau_t &= \frac{1}{2} \left( \frac{E_{t-1}(\mu_t/Z_t)}{E_{t-1}(\mu_t^*/Z_t^*)} \frac{1}{\mathcal{E}_t} \right)^{1/2}, & \tau_t^* &= \frac{1}{2} \left( \frac{E_{t-1}(\mu_t^*/Z_t^*)}{E_{t-1}(\mu_t/Z_t)} \mathcal{E}_t \right)^{1/2} \\
 C_t &= \bar{\ell} \frac{\mu_t^{1/2} \mu_t^{*1/2}}{2 [E_{t-1}(\mu_t/Z_t)]^{1/2} [E_{t-1}(\mu_t^*/Z_t^*)]^{1/2}} = C_t^*
 \end{aligned}$$

- Home productivity shocks only affect Home employment (labor ‘gap’).
- Demand shocks have *spillovers on consumption, but not on output abroad*.

- A monetary expansion depreciates  $\mathcal{E}_t$  and deteriorates the Home terms of trade: *monetary transmission is positive*, in the sense that raises consumption abroad for any level of labor effort.
- With nominal rigidities, anticipation of future Home productivity growth, given interest rates, also raises current demand and Home consumption. The terms of trade deterioration raises Foreign consumption in line with domestic consumption.
  - Recall from previous lectures that anticipation of future growth does not have any effect on current demand in Financial Autarky, but may lead to appreciation in a bond economy.

### *Expenditure switching effects with PCP*

Under PCP, the terms of trade  $P_F/\mathcal{E}P_H^*$  are equal to  $P_F^*\mathcal{E}/P_H$ . Since  $P_H$  and  $P_F^*$  are preset, the Home terms of trade worsens with a nominal depreciation of the Home currency (i.e. a higher  $\mathcal{E}$ ). When the Home currency weakens, Home goods are cheaper relative to Foreign goods in both the Home and the Foreign country.

As demand shifts in favor of the goods with the lowest relative price, world consumption of Home goods increases relative to consumption of Foreign goods: '*expenditure switching effects*' of exchange rate movements.

*International transmission with LCP (1)*

$$l_t = \frac{1}{2} \left( \frac{\mu_t/Z_t}{E_{t-1}(\mu_t/Z_t)} + \frac{\mu_t^*/Z_t}{E_{t-1}(\mu_t^*/Z_t)} \right) \bar{l}$$
$$l_t^* = \frac{1}{2} \left( \frac{\mu_t^*/Z_t^*}{E_{t-1}(\mu_t^*/Z_t^*)} + \frac{\mu_t/Z_t^*}{E_{t-1}(\mu_t/Z_t^*)} \right) \bar{l}$$

- Current productivity only affect domestic employment.
- *Demand shocks and monetary policies have spillovers on output and employment overseas.*

*International transmission with LCP (2)*

$$\tau_t = \frac{\left( \frac{E_{t-1}(\mu_t/Z_t)}{E_{t-1}(\mu_t/Z_t^*)} \right)^{1/2}}{1 + \frac{E_{t-1}(\mu_t/Z_t)}{E_{t-1}(\mu_t^*/Z_t)} \frac{1}{\mathcal{E}_t}}$$
$$\tau_t^* = \frac{\left( \frac{E_{t-1}(\mu_t^*/Z_t^*)}{E_{t-1}(\mu_t^*/Z_t)} \right)^{1/2}}{1 + \frac{E_{t-1}(\mu_t^*/Z_t^*)}{E_{t-1}(\mu_t/Z_t^*)} \mathcal{E}_t}$$

Since prices are preset in local currency, a *depreciation of  $\mathcal{E}_t$  improves the Home terms of trade  $P_F/\mathcal{E}P_H^*$* : it increases Home exporters' sales revenue and reduces Foreign exporters' sales revenue, without effects on consumer prices. Thus, a depreciation of  $\mathcal{E}_t$  has now a positive impact on  $\tau_t$  and negative on  $\tau_t^*$ — the opposite of the PCP case.

### *International transmission with LCP (3)*

There are no monetary spillovers on consumption. A home monetary shock raise  $C$  at Home and  $\ell^*$  abroad: '*beggar-thy-neighbor*' transmission of monetary policy.

$$C_t = \bar{\ell} \frac{\mu_t}{2 [E_{t-1} (\mu_t / Z_t)]^{1/2} [E_{t-1} (\mu_t / Z_t^*)]^{1/2}}$$
$$C^* = \bar{\ell}^* \frac{\mu_t^*}{2 [E_{t-1} (\mu_t^* / Z_t^*)]^{1/2} [E_{t-1} (\mu_t^* / Z_t)]^{1/2}}$$

*No expenditure switching effects with LCP*

With prices preset in local currency, exchange rate fluctuations do not affect the relative price faced by importers and consumers.

There is no 'expenditure switching effect' of exchange rate movements.

*International transmission with Dollar Pricing (1)*

$$l_t = \frac{\mu_t/Z_t}{E_{t-1}(\mu_t/Z_t)} \bar{l}$$

$$l_t^* = \frac{1}{2} \left( \frac{\mu_t^*/Z_t^*}{E_{t-1}(\mu_t^*/Z_t^*)} + \frac{\mu_t/Z_t^*}{E_{t-1}(\mu_t/Z_t^*)} \right) \bar{l}^*$$

$$\tau_t = \frac{[E_{t-1}(\mu_t/Z_t)]^{1/2}}{2 [E_{t-1}(\mu_t/Z_t^*)]^{1/2}} \quad \tau_t^* = \frac{\left( \varepsilon_t \frac{E_{t-1}[\mu_t^*/Z_t^*]}{E_{t-1}[\mu_t/Z_t]} \right)^{1/2}}{1 + \frac{E_{t-1}[\mu_t^*/Z_t^*]}{E_{t-1}[\mu_t/Z_t^*]} \varepsilon_t}$$

*International transmission with Dollar Pricing (2)*

$$C_t = \bar{\ell} \frac{\mu_t}{2 [E_{t-1} (\mu_t/Z)]^{1/2} [E_{t-1} (\mu_t/Z_t^*)]^{1/2}}$$
$$C_t^* = \bar{\ell}^* \frac{\mu_t^{1/2} \mu_t^{*1/2}}{2 [E_{t-1} (\mu_t^*/Z_t^*)]^{1/2} [E_{t-1} (\mu_t/Z_t)]^{1/2}}$$

- Home depreciation has no macroeconomic effects in the Home country: *output, consumption, and terms of trade are all insulated* from external shocks.

### *International transmission with Dollar Pricing (3)*

- A Home depreciation has two different effect on the Foreign country: (a) it lowers import prices in the Foreign country, thus improving  $\tau^*$ ; (b) it reduces sales revenue of Foreign exporters, thus lowering  $\tau^*$ . Which effect prevails depends on the sign of  $\mathcal{E}_t^{-1/2} - \mathcal{E}_t^{1/2} E_{t-1}(\mu_t^*/Z_t^*) / E_{t-1}(\mu_t/Z_t)$ .
  - Note that, when evaluated around a non-stochastic equilibrium, the previous expression is zero: a Home depreciation has no first-order effects on  $\tau^*$ .
- Home monetary policy has spillovers for both Foreign output and consumption: if labor increases by, say,  $\Delta\ell^*$ , consumption increases by  $Z^*\Delta\ell^*$ .

## 4 Optimal monetary policy and international coordination (with commitment)

### *Non-cooperative optimal policy*

Assume monetary authorities have perfect commitment. Define  $\mathcal{W} = E(U)$  and  $\mathcal{W}^* = E(U^*)$ . In the absence of international coordination, Home policymakers determine their optimal monetary stance by maximizing  $\mathcal{W}$  with respect to  $\mu$  while taking  $\mu^*$  as given. Foreign authorities behave in the same way. The two monetary stances define the following Nash equilibrium:

$$\begin{aligned}\mu_{Non-Coop} &= \arg \max_{\mu} \mathcal{W} \\ \mu_{Non-Coop}^* &= \arg \max_{\mu^*} \mathcal{W}^*\end{aligned}$$

## *Cooperation*

To characterize cooperative policymaking, instead, we posit that policymakers jointly maximize an equally weighted average of Home and Foreign welfare:

$$\{\mu_{Coop}, \mu_{Coop}^*\} = \arg \max_{\mu, \mu^*} \left[ \frac{1}{2} \mathcal{W} + \frac{1}{2} \mathcal{W}^* \right]$$

whereas the weights coincide with the size of each country.

- In all our models, recall that  $E_{t-1}(\ell_t) = \bar{\ell}$ , so that the second term in utility is independent of monetary policy and we need focus on consumption only. In fact, welfare can be written as:

$$\mathcal{W} = E_{t-1} \ln C_t + \text{constant (independent of } \mu)$$

## *Distortions in the economy*

- Monopoly power in production.
- Monopoly power on the terms of trade: atomistic firms disregards the effect of their supply/pricing decisions on the aggregate terms of trade of the country.
- Nominal rigidities.

## 4.1 Flex-price model benchmark (model 1)

$$\begin{aligned}P_{H,t} &= \frac{\theta\kappa}{\theta-1}\Gamma_t \\ \bar{\ell} &= \frac{\theta-1}{\theta\kappa} \Rightarrow Y_H = Z\bar{\ell} \\ C &= Z^{1/2}(Z^*)^{1/2}\bar{\ell} = C^*\end{aligned}$$

where  $\Gamma$  is the nominal scale of the economy.

- Note that there two of the three distortions listed above are present in this economy: monopoly power in production and the terms of trade.

## 4.2 Optimal policies in the PCP model (model 2)

$$\begin{aligned} \max_{\mu_t} E_{t-1} \ln C_t &= E_{t-1} \ln \frac{\theta - 1}{\theta \kappa} \frac{\mu_t^{1/2} \mu_t^{*1/2}}{2 [E_{t-1} (\mu_t / Z_t)]^{1/2} [E_{t-1} (\mu_t^* / Z_t^*)]^{1/2}} \\ &= \frac{1}{2} E_{t-1} \ln \mu_t + \frac{1}{2} E_{t-1} \ln \mu_t^* - \frac{1}{2} \ln E_{t-1} (\mu_t / Z_t) - \frac{1}{2} \ln E_{t-1} (\mu_t^* / Z_t^*) \end{aligned}$$

$$F.O.C. : \quad \frac{1}{2} \frac{1}{\mu_t} - \frac{1}{2} \frac{1/Z_t}{E_{t-1} (\mu_t / Z_t)} = 0$$

This is precisely the same expression one would obtain for a closed economy. Home monetary policy responds one-to one to real shocks, stabilizing Home firms' marginal costs.

*With PCP, optimal inward-looking policies support a flex-price allocation*

The F.O.C. above is solved by

$$\mu = \Gamma Z$$

Substituting into optimal pricing

$$P_{H,t} = \frac{\theta\kappa}{\theta - 1} E_{t-1} \left( \frac{\mu_t}{Z_t} \right) = \frac{\theta\kappa}{\theta - 1} \Gamma_t$$

The optimal policy consists in a commitment to provide a nominal anchor for the economy,  $\Gamma$  (nominal trend) and deviate from such stance only when (a) current productivity shocks in the economy threaten to destabilize marginal costs and (b) anticipated shocks threaten to raise demand for given current productivity, both (a) and (b) moving employment and output from their potential levels. Optimal policy is *'inward looking'*.

### *Friedman view of the exchange rate*

Friedman (1953) suggested that, in a world with nominal price rigidities, exchange rate movements facilitate efficient adjustment of international relative prices. With flexible prices, the relative price of Home goods falls in response to a positive productivity shock. With sticky prices, adjustment is achieved via an exchange rate depreciation (corresponding to Home monetary expansion relative to Foreign), that lowers the international price of the Home goods relative to Foreign goods.

- Note however that in the model efficient exchange rate adjustment in response to productivity shock only occurs if monetary policy is optimally conducted. Exchange rates does not provide any 'automatic stabilization' independent of policy.

*How large are the gains from international policy cooperation? (1)*

In the model with PCP above, national objective function for the Foreign policy makers is identical to the Home objective function  $\mathcal{W}=\mathcal{W}^*$ . Maximizing an average of  $\mathcal{W}$  and  $\mathcal{W}^*$  yields exactly the the same optimal policy prescriptions as the Nash optimal rule. The non-cooperative rules remain the best policy rules.

- The implication of this result is straightforward: international policy cooperation is redundant: by *'keeping one's house in order'*, policymakers are already able to achieve economic efficiency.

### *Important implications of suboptimal stabilization*

While the gains from going from Nash optimal policy to coordinated policy may be small, how large are the gains from stabilization? Suppose monetary authorities deviate from optimal rules, say positing  $\mu = Z^\zeta$ . By Jensen's inequality

$$P_{H,t} = \frac{\theta\kappa}{\theta - 1} E_{t-1} \left( \frac{\mu_t}{Z_t} \right) = \frac{\theta\kappa}{\theta - 1} \Gamma E_{t-1} Z_t^{\zeta-1} \begin{matrix} \leq \\ \geq \end{matrix} \frac{\theta\kappa}{\theta - 1} \Gamma$$

depending on whether  $\zeta \begin{matrix} \leq \\ \geq \end{matrix} 1$ . With  $\zeta < 1$  — insufficient stabilization of marginal costs —, firms will set higher prices than in a fully stabilized economy.

Expected employment remains at  $\bar{\ell}$ . But per effect of uncertainty average output will be lower (it falls too much when productivity is low, raises too little when  $Z$  is high). While this improves the terms of trade of the country, consumption is lower than in a fully stabilized economy. Welfare overall worsens.

### *Assessing the gains from stabilization?*

- Assume lognormality in the model above and posit  $\zeta = 0$ , the case of no stabilization. Welfare costs are:

$$\mathcal{W}|_{\mu=Z} - \mathcal{W}|_{\mu=1} = \frac{1}{2} \left( E_{t-1} \ln Z_t + \ln E_{t-1} \frac{1}{Z_t} \right) \approx \frac{1}{2} \text{Var}_{t-1} \ln Z_t$$

- In more general specification of the utility function (power instead of log), you can get larger losses as a consequence of monetary noise (irrelevant with log utility). (compare results from DSGE models with second order approximations).
- Note that consumption fluctuates with productivity in the efficient equilibrium, but is constant in the inefficient one.

### 4.2.1 Terms of trade manipulation under PCP

An important question is why, in the above model specification, policymakers target a flex-price allocation without attempting to improve or manipulate the terms of trade of the country in their favor. We have seen above they could do so, by changing the intensity of the monetary response to shocks  $\zeta$ .

An intuitive answer builds on the fact that, in the Cobb-Douglas case, output and prices move proportionally, leaving relative wealth unaffected: terms of trade manipulation is ineffective. This is not true in more general model specifications, such as:

$$U(.) = \frac{C^{1-\sigma}}{1-\sigma} = \frac{\left( [a_H^{1-\rho} C_H^\rho + a_F^{1-\rho} C_F^\rho]^{\frac{1}{\rho}} \right)^{1-\sigma}}{1-\sigma}$$

## *Optimal deviations from flex-price allocation*

Moving away from the Cobb-Douglas aggregator, it turns out that, under symmetry, policymakers will still optimally target the flex-price allocation under cooperation, but not under Nash, except when  $\sigma + \rho = 1$  — see e.g. the propositions in Benigno and Benigno Restud 2003. Why?

Note that the marginal utility of consuming  $C_H$

$$\frac{\partial U}{\partial C_H} = \left( [a_H^{1-\rho} C_H^\rho + a_F^{1-\rho} C_F^\rho] \right)^{\frac{1-\sigma-\rho}{\rho}} a_H^{1-\rho} C_H^{\rho-1}$$

is increasing in  $C_F$  if  $\sigma + \rho < 1$ , decreasing otherwise. In the first case, the two goods are complements in the Pareto-Edgeworth sense, substitutes otherwise (see e.g. Obstfeld and Rogoff book). They are neither complements, nor substitutes for  $\sigma + \rho = 1$ .

*Goods are substitutes.*

Starting from an allocation with full stabilization, the Home policymakers have a clear incentive to improve the Home terms of trade in their favor: as imports are good substitute for domestic goods, a higher price for domestic output raises the C-utility by raising its content in terms of imports. They therefore choose incomplete stabilization  $\zeta < 1$ .

However, in a symmetric Nash equilibrium, policymakers in both countries will attempt to do so simultaneously. Their attempt will be self-defeating, and the world equilibrium will end up being welfare-dominated by the flex-price allocation. This is why, with cooperation, there are some gains relative to Nash. Cooperation simply restore the flex-price allocation.

*Goods are complements.*

In this case, the marginal utility from domestic goods is increasing in the consumption of foreign goods (this is the definition of complements in the P-E sense). In general, less (more) consumption of Home goods naturally lowers (increases) the demand for imports. Why? Consider the instance of a positive productivity shock at Home. By overstabilizing the shock (i.e. producing more than in the flex-price allocation —  $\zeta > 1$ ), policymakers can use the extra output to buy more units of foreign goods, raising the marginal utility of domestically produced goods (i.e. containing the fall of marginal utility from the now more abundant domestic goods). With overstabilization, policymakers worsen the Home terms of trade, and let the economy produce higher average output. HW: what about the case of a negative shock?

This a-symmetric Nash strategy is inefficient — with a positive Home shocks, foreign households consume too much Home goods, and produce above the natural rate of output. Cooperation can improve welfare, moving the economy back to full stabilization of marginal costs.

When  $\sigma + \rho = 1$ , there is no incentive to deviate from price (=marginal cost) stability.

*Asset markets:* An open issue is the importance of asset markets as regards the incentive of policymakers to exploit terms of trade. Sutherland 2004 unpublished paper stresses that the difference between Nash and cooperation is small with *incomplete markets* , but may become very large with *complete markets*. Suppose goods are substitutes: the incentive to understabilize and improve the terms of trade is greater under perfect insurance, since any adverse relative wealth effect from producing less is undone by financial contracts.

Note that optimal policies in the cases above only respond to domestic shocks, and under- or over-stabilize the domestic output gap, ignoring foreign shocks. Strictly speaking, however, they are not ‘inward-looking’ as they are driven by terms of trade considerations.

The literature also analyzes the implications of including non-traded goods, showing that one obtain richer results than in the benchmark even without

moving from the Cobb-Douglas to a CES aggregator of consumption goods (see Obstfeld and Rogoff 2002 and Canzoneri et al. JIE 2005).

### 4.3 Optimal policy in the LCP model (model 3)

Back to our baseline (Cobb-Douglas) model:

$$\begin{aligned} \max_{\mu_t} E_{t-1} \ln C_t &= E_{t-1} \ln \frac{\theta - 1}{\theta \kappa} \frac{\mu_t}{2 [E_{t-1} (\mu_t / Z_t)]^{1/2} [E_{t-1} (\mu_t / Z_t^*)]^{1/2}} \\ &= E_{t-1} \ln \mu_t - \frac{1}{2} \ln E_{t-1} (\mu_t / Z_t) - \frac{1}{2} \ln E_{t-1} (\mu_t / Z_t^*) \end{aligned}$$

$$F.O.C. : \quad \frac{1}{\mu_t} - \frac{1}{2} \frac{1/Z_t}{E_{t-1} (\mu_t / Z_t)} - \frac{1}{2} \frac{1/Z_t^*}{E_{t-1} (\mu_t / Z_t^*)} = 0$$

In the Foreign country, the optimal policy will solve:

$$F.O.C. : \quad \frac{1}{\mu_t^*} - \frac{1}{2} \frac{1/Z_t^*}{E_{t-1} (\mu_t^* / Z_t^*)} - \frac{1}{2} \frac{1/Z_t}{E_{t-1} (\mu_t^* / Z_t)} = 0$$

## *The case for an 'international dimension' in monetary policy*

Home monetary policy responds to both Home and Foreign shocks, with weights given by the relative importance of domestic and foreign goods in consumption.

- Suppose that Home monetary authorities ignore the influence of their decisions on the price of Home imports. For the reason discussed above, import prices will tend to be inefficiently high. On the other hand, if Home monetary authorities want to stabilize Foreign firms' marginal costs, they can only do so at the cost of raising costs and markup uncertainty for Home producers, resulting in higher Home good prices. It follows that, to maximize Home welfare, Home policymakers should optimally trade-off the stabilization of marginal costs of all producers (domestic and foreign) selling in the Home markets.

*With LCP:*

*The equilibrium is inefficient:* since marginal costs will not be completely stabilized, prices (purchasing power) will be on average higher (lower) than with PCP.

*Optimal policy is not 'inward looking.'* Because of the effect of marginal costs uncertainty on pricing, policy makers face a trade-off between stabilizing domestic and foreign prices. Trade-off larger, the more open the economy.

De facto, however, the optimal policy prescribe to move towards targeting CPI, instead of the GDP deflator as in PCP.

## *No gains from monetary coordination with LCP?*

With LCP, Home Monetary policy shocks have large spillovers on employment and output abroad. In spite of this, LCP does not necessarily imply monetary interdependence: in our specification, actually, there are *no gains from international policy coordination* (special to Cobb-Douglas parameterization).

The reason is that there are no spillovers on Foreign consumption: recall that  $C^* = \mu^*/P^*$  does not move with  $\mu$  since with LCP  $P^*$  is predetermined. Because of the 'natural rate property' of the model

$$E_{t-1}l = \bar{l}$$

coordinating monetary policy is useless in addressing the inefficiency due to labor and output spillovers.

### 4.3.1 Implications for exchange rate regimes

A flexible exchange rate regime is optimal in all the models we study in this lecture. However, note that the system of the first order conditions of the policy problem in the LCP case is solved by a common policy  $\mu_t = \mu_t^*$  — responding to the same average of Home and Foreign shocks. As a result, the nominal and real exchange rate is not contingent on productivity shocks.

Some authors conclude from this that *LCP can be seen as an argument in favor of fixed exchange rates*. This argument is wrong. For instance, country may still prefer different long run inflation rates  $\Gamma$  and  $\Gamma^*$  (leading to trends for the nominal exchange rate). Second, most important, LCP really provides an argument that optimal CPI stability implies curbing the variability of the terms of trade, NOT of real (and nominal) exchange rates.

## *Home Bias and optimal monetary policy (1)*

To see this, introduce Home Bias in preferences (Corsetti 2006):

$$C_t = C_H^{a_H} C_F^{1-a_H} \quad C_t^* = C_H^{1-a_H} C_F^{a_H}$$

The solution of the model is almost identical except for the price indexes, implying that in a flex price or PCP equilibrium efficient risk sharing requires:

$$\frac{C}{C^*} = \frac{\varepsilon P^*}{P} = (TOT)^{2a_H-1}$$

the real exchange rate is no longer constant, but fluctuates with the terms of trade. Our analysis above is a special case for  $a_H = 1/2$ .

## *Home Bias and optimal monetary policy (2)*

With LCP, the first order condition of the policy problems are now

$$\begin{aligned}\frac{1}{\mu_t} - a_H \frac{1/Z_t}{E_{t-1}(\mu_t/Z_t)} - (1 - a_H) \frac{1/Z_t^*}{E_{t-1}(\mu_t/Z_t^*)} &= 0 \\ \frac{1}{\mu_t^*} - (1 - a_H) \frac{1/Z_t}{E_{t-1}(\mu_t^*/Z_t)} - a_H \frac{1/Z_t^*}{E_{t-1}(\mu_t^*/Z_t^*)} &= 0\end{aligned}$$

which is not solved by  $\mu = \mu^*$  unless shocks are symmetric or  $a_H = 1/2$ . Optimal policy still stabilizes a weighted average of domestic and foreign marginal costs. By doing so, central bankers implicitly curb terms of trade volatility. It is easy to see, however, that a fixed-exchange rate regime will never be efficient in this environment, as this will translate into a binding constraint on the conduct of optimal policies.

### 4.3.2 Numerical Appraisal of LCP models

We now revisit the key results on LCP using the new-Keynesian model introduced at the end of the lecture on local currency price stability of imports (Calvo pricing, LCP, upstream and downstream producers) — Corsetti Dedola Leduc 2007. We focus on cooperation under complete markets. The exercise is run in ‘dynare++’.

- The planner chooses the growth rates of money in the Home and Foreign economies, to maximize an equally weighted average of Home and Foreign welfare, subject to the first-order conditions for households and firms and the economy-wide resource constraints (utility from real balances disregarded).

- The optimal policy that has been in place for a long enough time that initial conditions do not matter.
- Fiscal subsidies, financed via lump-sum taxation, ensure that steady state with zero inflation is first best.
- Results shown are averages of 500 simulations for 100 periods each.

### *Calibration*

- Standard separable utility function with risk aversion set to 2;

- The markup of downstream firms in steady state is 15 percent;
- The trade elasticity is 1.5, imports are 10 percent of aggregate output in steady state;
- Upstream and downstream firms update their prices with a 0.5 probability;
- Upstream and downstream technology shocks have autocorrelation set to 0.95, standard deviation of innovations to 0.7%.
- The price of traded goods accounts for 50 percent of final price

Recall *key specification feature* (useful from didactical point of view):

- Upstream firms employ labor to produce intermediate goods.
- Downstream firms manipulate upstream goods with stochastic productivity **WITHOUT** using labor.

Hence a monetary expansion which raising nominal demand and (via competitive labor markets) nominal wages, raises marginal costs for upstream firms; it affects marginal costs for downstream firms only to the extent it causes upstream firms to change their prices.

## *Methodology and road map*

Close-up inspection of the model to understand optimal policy

- in response to upstream shock and downstream shock in isolation
- close economy vs. open economy

Methodology: analyze the relative weight attached by monetary authorities to different components of CPI looking at the effects of stabilization on the relative variance of markups, prices and quantities (output and consumption).

## *CLOSED ECONOMY – UPSTREAM SHOCKS*

This is the new-Keynesian benchmark: complete stabilization of upstream marginal costs and prices is possible and desirable: there is no inflation, no price dispersion.

How? match any change in upstream marginal costs driven by productivity, with a change in the monetary stance in the opposite direction, which ultimately moves nominal wages in tandem with productivity.

Key: in the model specification, fluctuations in nominal wages are not consequential for downstream firms, by virtue of our assumption that these firms employ no labor resources in producing final goods.

## *CLOSED ECONOMY – DOWNSTREAM SHOCKS*

This deviates from the new-Keynesian benchmark: it is optimal to expand in response to positive productivity shocks at retail level (raising demand in line with productivity). But this cannot guarantee price stability. the reason is that more demand raises nominal wages, thus marginal costs of upstream firms. This leads to price dispersion upstream, reflected in price dispersion downstream. The optimal policy faces a trade-off between inflation and output.

Complete price stability is neither feasible, nor optimal.

## *OPEN ECONOMY – UPSTREAM SHOCKS*

This is the Calvo-pricing equivalent of the LCP case studied above. Complete price stabilization is not optimal in an open economy setting. The numerical results in the table show:

- the variability of the CPI is close to, but not zero; domestic and imported goods prices are actually much more variable than the CPI.
- prices and markups in both countries fluctuate much less for domestic goods than for imported goods: monetary policymakers concentrate their efforts to reduce the volatility of markups of domestic producers selling in the domestic markets.

*Table 1: shocks to upstream firms*

Standard deviation of:	Flex prices Home bias	Optimal policy Home bias	Optimal policy No Home Bias
<b>Inflation rates:</b>	0	.01	0
domestic goods	.19	.11	.25
Imported goods	.74	.43	.25
Home producers	.19	.19	.50
Import prices: dock	.74	.87	.50
Export Prices: dock	.74	.87	.50
<b>International prices</b>			
<i>RER</i>	1.38	1.62	0
<i>TOT</i>	2.30	2.10	2.06

*Table 1: shocks to upstream firms*

Standard deviation of	flex prices <i>HB</i>	optimal policy <i>HB</i>	Optimal policy NO Home Bias
<b>Deviations from the LOP</b>			
producer level	0	.57	
consumer level	0	0.39	
<b>Markups Home producers</b>			
upstream selling at Home	0	.06	.17
upstream selling abroad	0	.26	.17
downstream: domestic	0	.24	.65
downstream: imports	0	1.12	.65
<b>Quantities</b>			
Consumption	0.87	0.9	.81
Home hours	.41	.43	.47

By mirroring the logic of the preset price model, assume the policymakers pursue an allocation where there is no price dispersion in domestic market for domestically produced goods: domestic goods prices remain constant and identical to each other, but monetary reactions to domestic shocks would affect exchange rates creating both

- misalignment of the relative price between domestic and foreign goods (recall that the law of one price does not hold as a function of differences in domestic inflation);
- import price dispersion at consumer level (since a constant fraction of Foreign producers would react to, e.g., Home depreciation, by inefficiently raising the price they charge to Home downstream firms).

## *TOT and RER*

With LCP, endogenous changes in monetary stance across countries tend to be positively correlated. Optimal national monetary policies thus curb the *volatility of the terms of trade*, relative to the case of flexible prices (see table above). The more so, the lower the Home bias in consumption.

- This is because, with LCP, nominal exchange rate movements do not help correct international relative prices. They are actually counterproductive.

RER volatility depends on other considerations. For instance, without home bias, national monetary policies react to the same weighted average of shocks, and are perfectly correlated: volatility of the real exchange rate is 0.

## *OPEN ECONOMY – DOWNSTREAM SHOCKS*

Monetary authorities would never be able to achieve complete stability of final prices, not even in a closed-economy environment.

Complete price stability at consumer level requires monetary policy to respond to technology shocks downstream. Since the resulting fluctuations in wages induce (inefficient) price dispersion among upstream firms, it follows that final producers will face different costs of their intermediate input, depending on which industry they operate in.

Model embeds **Friedman's** analysis of cost-push inflation (see Nelson [2007]).

- Positive productivity shock downstream  $\Rightarrow$  monetary policy optimally expands
  - $\Rightarrow$  raise consumers' demand and shift labor supply  $\Rightarrow$  income effects drive up nominal wages  $\Rightarrow$  upstream producers charge higher prices.
  - $\Rightarrow$  downstream firms raise their prices because of 'increasing costs of intermediate goods'.

Changes in prices which appear to be motivated by costs consideration, are actually the result of a demand stimulus, working its way up through the vertical links between downstream and upstream producers, and ultimately raising the price of scarce production inputs supplied in competitive markets.

*Nontradability of consumer goods.* Even when consumer expenditure is not biased towards domestic goods, consumption baskets would still be effectively different across countries.

- When the expenditure weights  $a_H$  and  $a_F$  are identical — a case of no home bias in terms of upstream products — monetary authorities would efficiently provide the same degree of stabilization across all categories of domestic and imported goods.
- Yet, in contrast to the case of upstream disturbances only, the optimal monetary stance will be sufficiently different across countries as to induce nominal and real exchange rate fluctuations in response to country-specific shocks at downstream level. (see e.g. Duarte and Obstfeld [2007]).

Table 2: shocks to downstream firms

Standard deviation of:	flex prices Home bias	optimal policy Home bias	Optimal policy No Home Bias
<b>Inflation rates:</b>	0	.12	.13
domestic goods	.02	.13	.12
Imported goods	.09	.11	.12
Home producers	.69	.38	.39
Import prices: dock	.77	.44	.39
Export Prices: dock	.77	.44	.39
<b>International prices</b>			
<i>RER</i>	2.62	2.91	2.75
<i>TOT</i>	.27	0.72	.78

Table 2: shocks to downstream firms

Standard deviation of:	flex prices Home bias	optimal policy Home bias	Optima policy No Home Bias
<b>Deviations from the LOP</b>			
producer level	0	.27	.29
consumer level	2.46	2.68	2.66
<b>Markups Home producers</b>			
upstream selling at Home	0	.58	.57
upstream selling abroad	0	.56	.57
downstream: domestic	0	.04	.04
downstream: imports	0	.11	.04
<b>Quantities</b>			
Consumption	0.99	1.08	1.05
Home hours	.51	.47	.42

## *Optimal stabilization and RER vs TOT*

- In Table 2, at an optimum the variability of CPI inflation is not zero, but remains remarkably stable for different degrees of home bias in consumption.
  - Because of vertical interactions, complete stabilization of prices is not feasible (marginal costs of downstream firms not symmetric). Yet it is optimal to get close to CPI stability.
- What instead varies considerably with the degree of home bias is the variability of markups across sectors, since home bias shifts the weight of monetary stabilization away from imported goods.

- Yet, because of nontradability, the real exchange rate is now much more volatile than the terms of trade, even in the flexible price allocation.
  - This is because of the combined effects of nominal rigidities, and the presence of nontradable components in final goods.
- Under the optimal policy, the real exchange rate is more volatile, but the terms of trade less volatile, than under flexible prices.
  - a caution against strong policy prescriptions on the need to curb the volatility of the real exchange rate (see e.g. Devereux and Engel [2007]).
- Recall that in the model the elasticity of the upstream producer's demand curve falls with the rate of change in final goods prices. Inflation differentials

motivate optimal price discrimination across border, causing deviations from the law of one price

- this suggests a novel (relative to the literature) argument for policy emphasis on final price stabilization, as a way to contain monopolistic distortions and inefficient deviations from the law of one price.

## 4.4 Optimal policy in the dollar pricing model (model 4)

Back to the baseline model one more time. When world exports are priced in Home currency, Home optimal monetary policy is still described by Nash LCP. Instead, in the Foreign country welfare is:

$$\begin{aligned}
 E_{t-1} \ln C_t^* &= E_{t-1} \ln \frac{\theta - 1}{\theta \kappa} \frac{\mu_t^{1/2} \mu_t^{*1/2}}{2 [E_{t-1} (\mu_t^*/Z_t^*)]^{1/2} [E_{t-1} (\mu_t/Z_t)]^{1/2}} \\
 &= \frac{1}{2} E_{t-1} \ln \mu_t + \frac{1}{2} E_{t-1} \ln \mu_t^* - \frac{1}{2} \ln E_{t-1} (\mu_t^*/Z_t^*) - \frac{1}{2} \ln E_{t-1} (\mu_t/Z_t)
 \end{aligned}$$

and optimal policy rules satisfy:

$$\frac{1}{2} \frac{1}{\mu_t^*} - \frac{1}{2} \frac{1/Z_t^*}{E_{t-1} (\mu_t^*/Z_t^*)} = 0$$

## *Optimal monetary rule with dollar pricing (2)*

In a Nash equilibrium, the country that issues the vehicle currency (Home) optimally responds to shocks hitting the global economy.

The country that uses the vehicle currency (Foreign) only needs to stabilize domestic prices and markups.

### *Incentive for cooperation and dollar pricing*

The interest in this case mainly concerns its implication for the desirability of international policy cooperation. World welfare indeed increases when monetary policy rules are designed in a cooperative way (by maximizing an equally weighted average of the two national welfare functions).

However, the cooperative and noncooperative optimal policy rules coincide for the Foreign country, but not for the Home country.

The 'contribution' to cooperation is therefore unilateral: only the Home country is expected to modify its rules. This raises an interesting issue, as of whether there is any incentive for this country to enter any binding cooperative agreement as regards stabilization policy.

*A final note and a caveat about different way to interpret optimal policies*

- PCP: target GDP deflator (let exchange rate movements correct relative prices)
- LCP: target CPI (minimize consumption prices overall).

Caveat: there are many equivalent ways to rearrange endogenous variables (prices, output gaps, etc.) as policy targets.

What about cost-push shocks? Rigidities in both product and labor markets?...

## 5 Pass-through as a choice variable

*An extension of the baseline model*

Suppose that firms preset export prices in foreign currency but are able to modify them after observing exchange rate changes. In this new setup, the extent to which the Foreign-currency prices of Home exports move with the exchange rate is a choice variable, predetermined by Home firms at time  $t - 1$ . In other words, the elasticity of exchange rate pass-through can endogenously be zero (as in the LCP case), one (as in the PCP case), or any intermediate number.

### *A useful specification*

Formally, by definition of pass-through elasticity  $\eta_t^* \equiv \partial \ln p_t^*(h) / \partial \ln (1/\mathcal{E}_t)$ , Foreign-currency prices of Home brands are:

$$p_t^*(h) \equiv \frac{\tilde{p}_t(h)}{\mathcal{E}_t^{\eta_t^*}} \quad 0 \leq \eta_t^* \leq 1$$

where  $\tilde{p}_t(h)$  is the predetermined component of the Foreign-currency price of good  $h$  that is not adjusted to variations of the exchange rate during period  $t$ . Home firms choose  $\tilde{p}_t(h)$  and  $\eta_t^*$  one period in advance at time  $t - 1$  in order to maximize discounted profits.

### *First order conditions*

with respect to  $p_t^*(h)$  and  $\eta^*$  yield:

$$p_t^*(h) = \frac{\theta}{\theta - 1} \frac{1}{\mathcal{E}_t^{\eta^*}} \frac{E_{t-1} \left( Q_{t-1,t} p_t^*(h)^{-\theta} P_{H,t}^{*\theta} C_{H,t}^* MC_t \right)}{E_{t-1} \left( Q_{t-1,t} p_t^*(h)^{-\theta} P_{H,t}^{*\theta} C_{H,t}^* \mathcal{E}_t^{1-\eta^*} \right)}$$

$$Cov_{t-1} \left[ MC_t / \mathcal{E}_t^{1-\eta_t^*}, \ln \mathcal{E}_t \right] = 0$$

The latter is a critical condition. At an optimum, the (reciprocal of the) markup in the export market must be uncorrelated with the (log of the) exchange rate. Trivially, if  $\mathcal{E}$  is constant or fully anticipated, *any* degree of pass-through is consistent with the previous expression. But if  $\mathcal{E}$  is not perfectly predictable, the optimal degree of pass-through will depend on monetary policy and the structure of the shocks.

*Why?* In equilibrium, Home *ex-post* real profits in the Foreign market are proportional to

$$\tilde{p}_t(h) - MC_t / \mathcal{E}_t^{1-\eta^*}$$

a concave function of  $\mathcal{E}$  for  $\eta^* < 1$ . Keeping everything else constant, exchange rate shocks reduce expected profits from exports. In general, however, to assess the overall exposure of profits to exchange rate uncertainty it is crucial to know whether the underlying shocks make unit export revenue and marginal costs covary positively. In fact, if  $MC$  is high in those states of nature in which  $\tilde{p}\mathcal{E}^{1-\eta^*}$  is high as well, exchange rate uncertainty has little or no effect on profits.

When the firm chooses the degree of pass-through, it equates the ‘marginal costs’ of imperfect pass-through, in terms of reduced expected revenue, with the ‘marginal benefits’ of imperfect pass through, in terms of increased covariance between revenue and marginal costs.

## *Pass-through and monetary policy (1)*

Suppose that there are no productivity shocks and the only source of uncertainty is exogenous monetary volatility.

- $\mu$  is the only source of uncertainty:

$$Cov_{t-1} \left[ \mu_t^{\eta_t^*}, \ln \mu_t \right] = 0$$

which is solved by  $\eta_t^* = 0$ . Home monetary shocks affect both Home marginal costs and the exchange rate:  $\mathcal{E}$  depreciates in those states of nature in which marginal costs increase. By setting  $\eta_t^* = 0$  Home firms insure that export sales revenue and marginal costs move in parallel, leaving the export markup unaffected.

## *Pass-through and monetary policy (2)*

- $\mu^*$  is the only source of uncertainty:

$$Cov_{t-1} \left[ (\mu_t^*)^{\eta_t^* - 1}, -\ln \mu_t^* \right] = 0$$

which is solved by  $\eta_t^* = 1$ . Home marginal cost are uncorrelated with the exchange rate. By choosing full pass-through and letting export prices absorb exchange rate changes, Home firms can insulate their export sales revenue from currency fluctuations and avoid any uncertainty of markup and profitability in the Foreign market.

Note that these examples shed light on the reason why countries with high and unpredictable monetary volatility should also exhibit a high degree of pass-through, and vice versa — a view expressed by Taylor [2000].

### *Monetary and real uncertainty.*

In this case, patterns of endogenous intermediate pass-through can emerge, as the following example illustrates. If the Home monetary authority adopted the policy  $\mu_t = \left(Z_t^{-2} \mu_t^*\right)^{-1}$ , then it would be optimal for Home firms to choose  $\eta_t^* = 0.5$ . Abroad, we would need  $MC_t^* \mathcal{E}_t^{1-\eta_t}$  to be uncorrelated with the exchange rate. This would be the case, for instance, if  $\mu_t^* = Z_t^{-4} / (Z_t^*)^{-5}$  and  $\eta_t = 0.6$ .

But what is the equilibrium if monetary authorities maximize agents' expected utility, taken into account the optimality conditions of the firm? The following slide provides the answer.

## *Pass-through and optimal monetary rules*

The following allocation is an equilibrium supporting an (constrained-) efficient allocation:

$$\begin{aligned} MC_t &= E_{t-1}(MC_t), \quad MC_t^* = E_{t-1}(MC_t^*), \\ \eta_t &= \eta_t^* = 1 \end{aligned}$$

Monetary policies fully stabilize the national economies by closing output and employment gaps. Exchange rates are volatile, their conditional variance being proportional to the volatility of  $Z_t/Z_t^*$ . Purchasing power parity holds and there is full pass-through of exchange rate changes into prices. This equilibrium characterizes an *optimal float*.

## 6 Discretion vs. commitment

*the PCP case*

Under discretion, the Home policymakers maximize agents' current utility with respect to  $\mu$  after observing the shocks  $Z$  and  $Z^*$ , taking firms' prices and Foreign policy as given. The Foreign policymaker solves a similar problem. Focus on the PCP model. The first order conditions are

$$\frac{\mu}{Z} = \frac{1}{2\theta - 1} E \left( \frac{\mu}{Z} \right); \quad \frac{\mu^*}{Z^*} = \frac{1}{2\theta - 1} E \left( \frac{\mu^*}{Z^*} \right)$$

Note that (abstracting from reputation mechanisms) the above condition cannot be part of a rational expectations equilibrium unless

$$\frac{1}{2\theta - 1} = 1 \quad \Rightarrow \quad \theta = 2 = \frac{1}{1 - \text{import share}}$$

### *Inflationary or deflationary bias in the PCP case*

In general, once prices are set, there is always an incentive for the policy makers to expand/contract monetary policy relative to private expectations, depending on the sign of the inequality:

$$1 - \text{import share} = \frac{1}{2} \gtrless \left[ \frac{\theta - 1}{\theta} \right]$$

Intuitively, inflationary bias prevails if the monopolistic distortions in production are sufficiently important relative to terms of trade distortions, whose magnitude depends — among other things — on the degree of openness of the economy (in our specification, the import share in consumption is 1/2). Since the exchange rate affects the price of a relatively small share of consumption goods, policymakers are less concerned with adverse import price movements than with the distortions associated with monopoly power in production.

*Monopoly in production vs. monopoly power on terms of trade*

- When  $\theta$  is sufficiently small relative to openness —  $\theta < 2$  and markups are above 100 percent — a benevolent discretionary policymaker will have an incentive to raise output above market equilibrium.
- When  $\theta$  is sufficiently large, the reverse is true. A monetary expansion, while raising output and employment, also increases the price of a substantial proportion of consumption goods. The terms of trade movement becomes the dominant concern in discretionary policy making, leading to a deflationary bias. Reducing the degree of pass-through blunts the adverse terms of trade effects of monetary expansion.

## *Open economy vs. closed economy trade-offs*

Clearly, in a closed economy import share of demand is zero. Discretionary policy satisfies

$$\frac{\mu}{Z} = \frac{\theta}{\theta - 1} E \left( \frac{\mu}{Z} \right)$$

which unambiguously implies inflationary bias. Monopolistic distortions in production create an incentive for the policymakers to expand demand and bring output to its Pareto-efficient level  $Z/k$ . This is general not true in an open economy.

### *Discretion vs. commitment in the LCP case*

Reducing the degree of pass-through would clearly blunt the terms of trade effects of monetary policy. The solutions to the policy problems in the LCP model under discretion are:

$$\frac{\mu}{Z} = \frac{2\theta}{\theta - 1} E \left( \frac{\mu}{Z} \right); \quad \frac{\mu^*}{Z^*} = \frac{2\theta}{\theta - 1} E \left( \frac{\mu^*}{Z^*} \right)$$

In this case, discretionary policy is unambiguously biased towards surprise monetary expansions.

### *Addressing supply distortions with fiscal policy...*

Suppose now that governments can use a fiscal instrument to correct average domestic monopolistic distortions. Suppose that it sets a subsidy to production at the rate  $(1 - \zeta)^{-1}$  such that the average markup is driven to zero:

$$\frac{1}{2} \left( \frac{\theta}{\theta - 1} \right) (1 - \zeta) = 1$$

Observe that the subsidy rate is smaller than in the closed-economy case (in our specification, it is half the size than in closed economy).

- It is easy to verify that, in the PCP model with the above subsidy in place, the first order conditions of the policy problems under discretion coincide with the first order conditions under commitment. They both imply  $\mu = \alpha Z$ .

*...does not ensure equivalence between discretion and commitment*

But the equivalence between discretionary policy and optimal policy under commitment does not hold in general — as firms' profits may still be exposed to exchange rate variability.

In fact, discretionary policy in the LCP model accounting for production subsidies is:

$$\frac{\mu}{Z} = \frac{2\theta}{\theta - 1} (1 - \zeta) E \left( \frac{\mu}{Z} \right)$$

but under commitment we have:

$$\frac{1}{\mu_t} - \frac{1}{2} \frac{1/Z_t}{E_{t-1}(\mu_t/Z_t)} - \frac{1}{2} \frac{1/Z_t^*}{E_{t-1}(\mu_t/Z_t^*)} = 0$$

Even if the subsidy eliminates the inflationary bias, a policy  $\mu = Z$  is optimal under discretion but not under commitment.

## *Why?*

The reason is that, under discretion, national policymakers take goods' prices as given, and therefore find it optimal to respond to domestic productivity shocks while ignoring the effects of domestic monetary policy on the markup of producers abroad. In that case, however, Foreign exporters would react to an increase in the variability of their markups by raising average prices in the Home country.

Under commitment, instead, Home policymakers take these effects into account and respond to both Home and Foreign shocks. They contain exchange rate and terms of trade movements so as to reduce their effects on the income of Foreign producers, trading off complete stabilization of Home producers' profits with lower import prices.