MACROECONOMIC DYNAMICS WITH PERFECT RISK SHARING

PORTFOLIO DIVERSIFICATION WITH HOME BIAS

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Outline

Work out a model with full diversification of macro risk with a limited menu of assets — this is a generalization of Lucas and C&O in a common framework building on Heatcote and Perri 2004, Corsetti and Bergin 2005 and Arespa 2007.

- Analysis of the link between home bias in portfolio and home bias in consumption in economies with high risk sharing.

- Consequences of adding frictions: nominal rigidities.

Introduction

Important goal pursued by open macro: integration of macro dynamics (current account investment and saving) with portfolio analysis in general equilibrium.

- Classical literature on diversification is developed in partial equilibrium (Kouri 1976, Branson and Henderson 1985; Adler and Dumas 1983).

Stylized facts after liberalization plus deregulation of financial markets

- Explosion of cross-border asset holdings

- Emergence of large current account imbalances (see graph below from Lane and Milesi-Ferretti jie 2007).

What are the open issues?

- Degree of risk sharing made possible by financial market integration (recall that the international transmission of shocks depends on it).

- Asset pricing: volatility risk and micro-structure of markets.

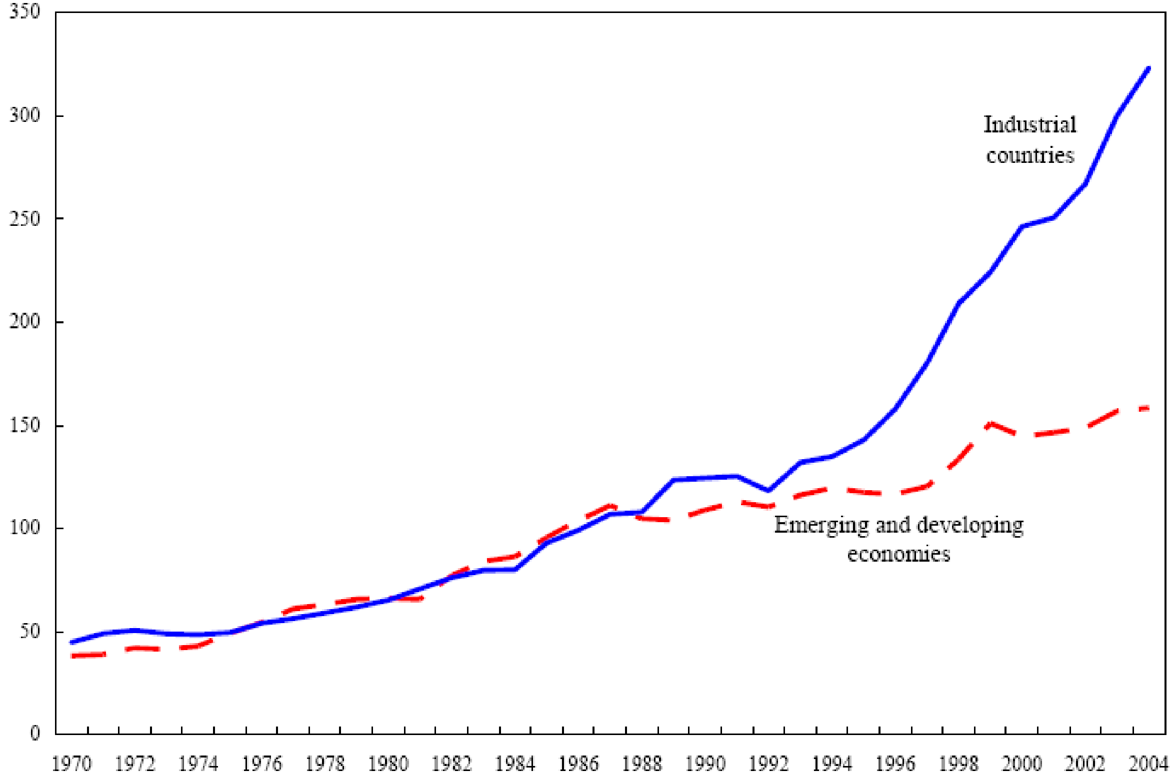
- Portfolio composition (French and Poterba Home bias).

- Financing investment (the Feldstein-Horioka puzzle)..

Introduction

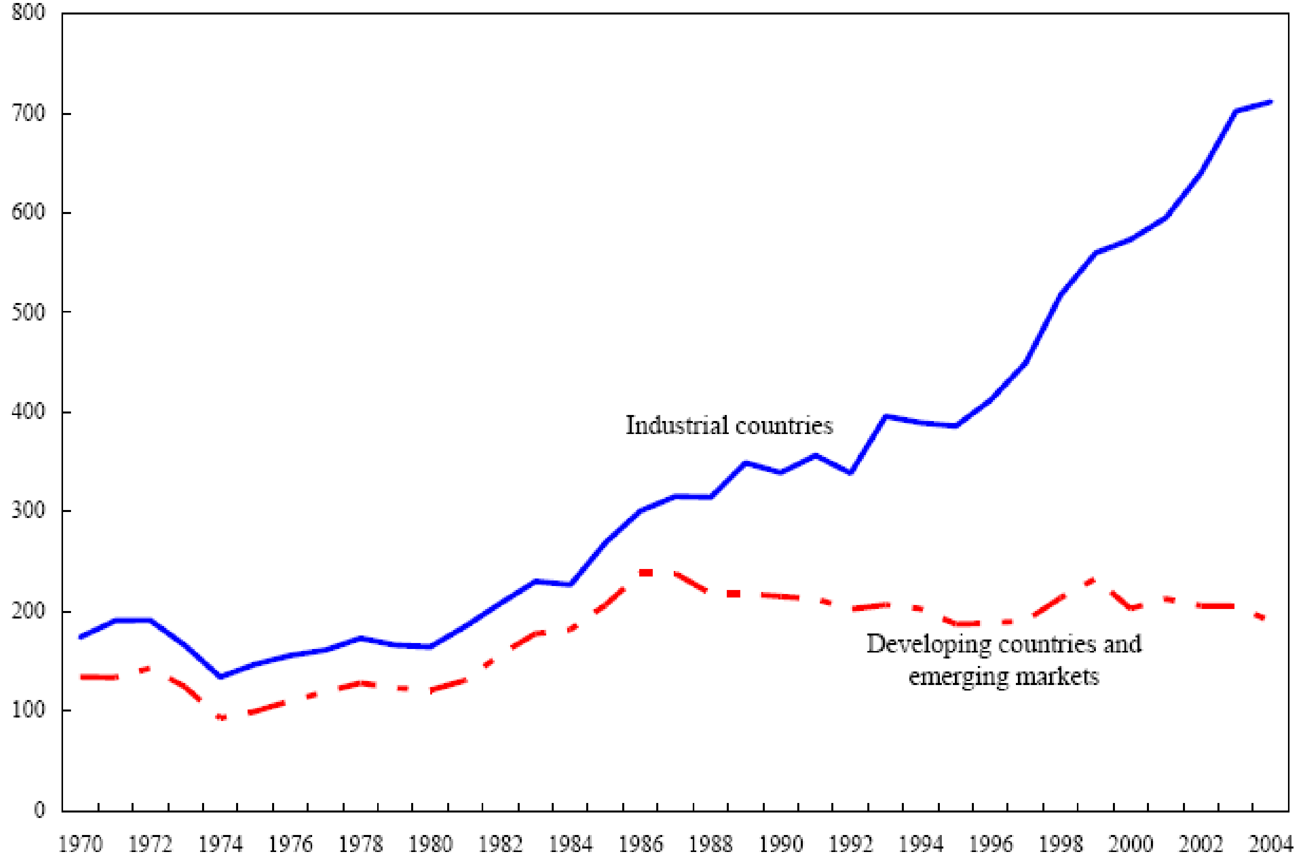
- The literature on portfolio analysis in open economy has so far developed along three research directions:
  - Posit a limited set of assets and perfect risk sharing: derive optimal holdings (e.g. Lucas 82; Heatcote Perri 2007; Kollmann 2005; Courdacier Kollmann and Martin 2007).
  - Posit a limited set of assets and small noise: derive optimal portfolio holdings (e.g. Devereux and Sutherland 2006).
  - Endogenize returns and assets traded in equilibrium (e.g. Broner and Ventura 2007).

Figure 3. International Financial Integration:
Industrial Group and Emerging Markets/Developing Countries Group, 1970-2004



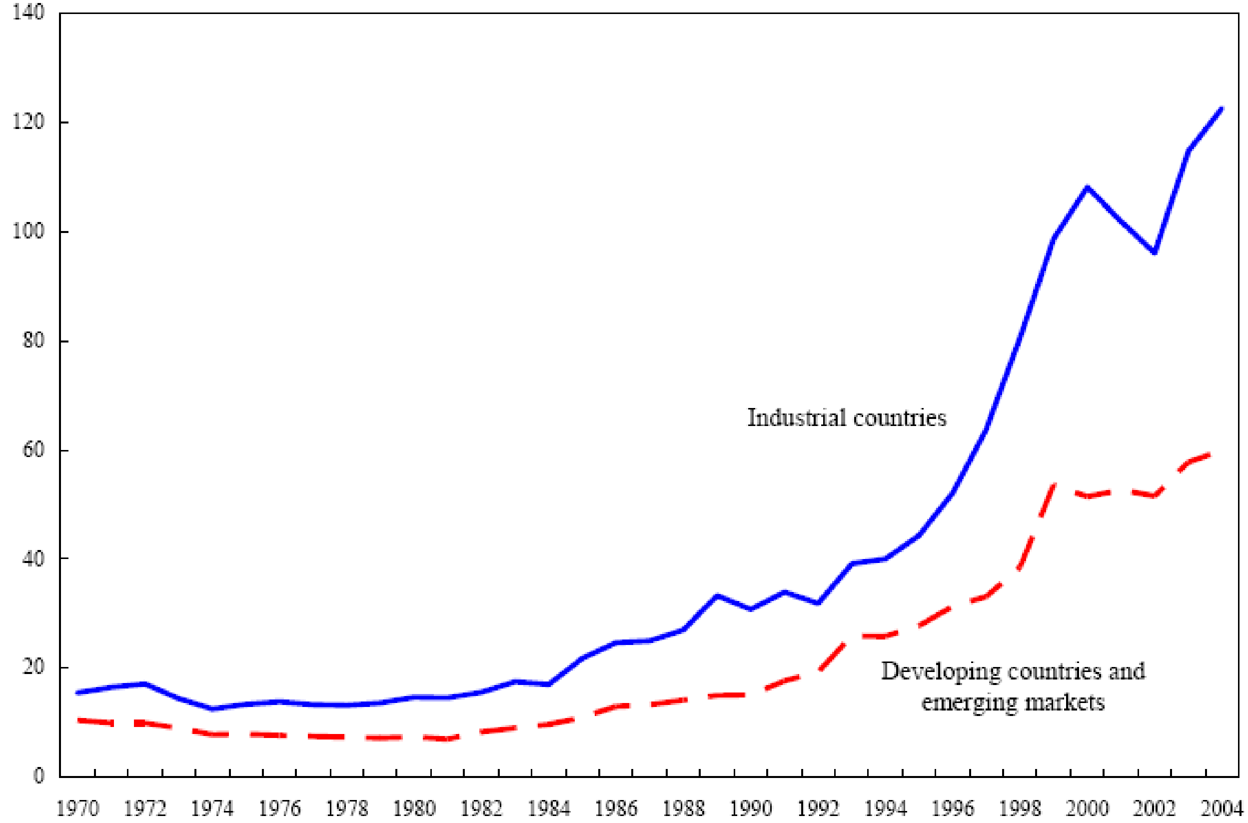
Note: Ratio of sum of foreign assets and liabilities to GDP, 1970-2004.

Figure 4. Financial Integration versus Trade Integration:
Industrial Group and Emerging Markets/Developing Countries Group, 1970-2004



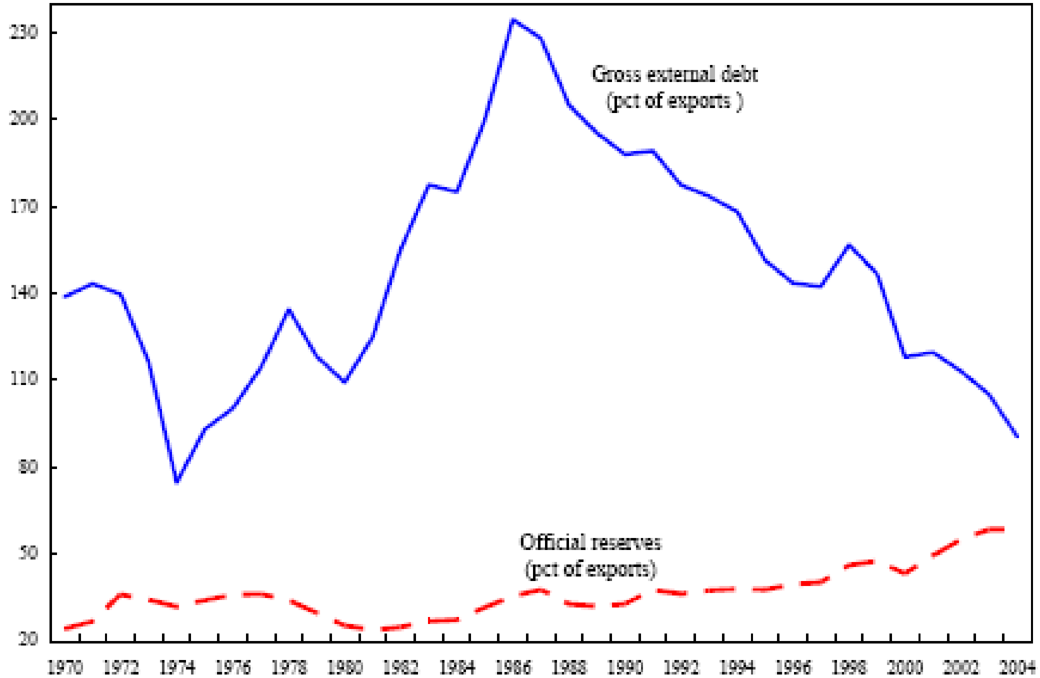
Note: Sum of external assets and liabilities in percent of sum of exports and imports.

Figure 5. International Equity Integration: Industrial Country Group and Emerging Markets and Developing Country Group, 1970-2004



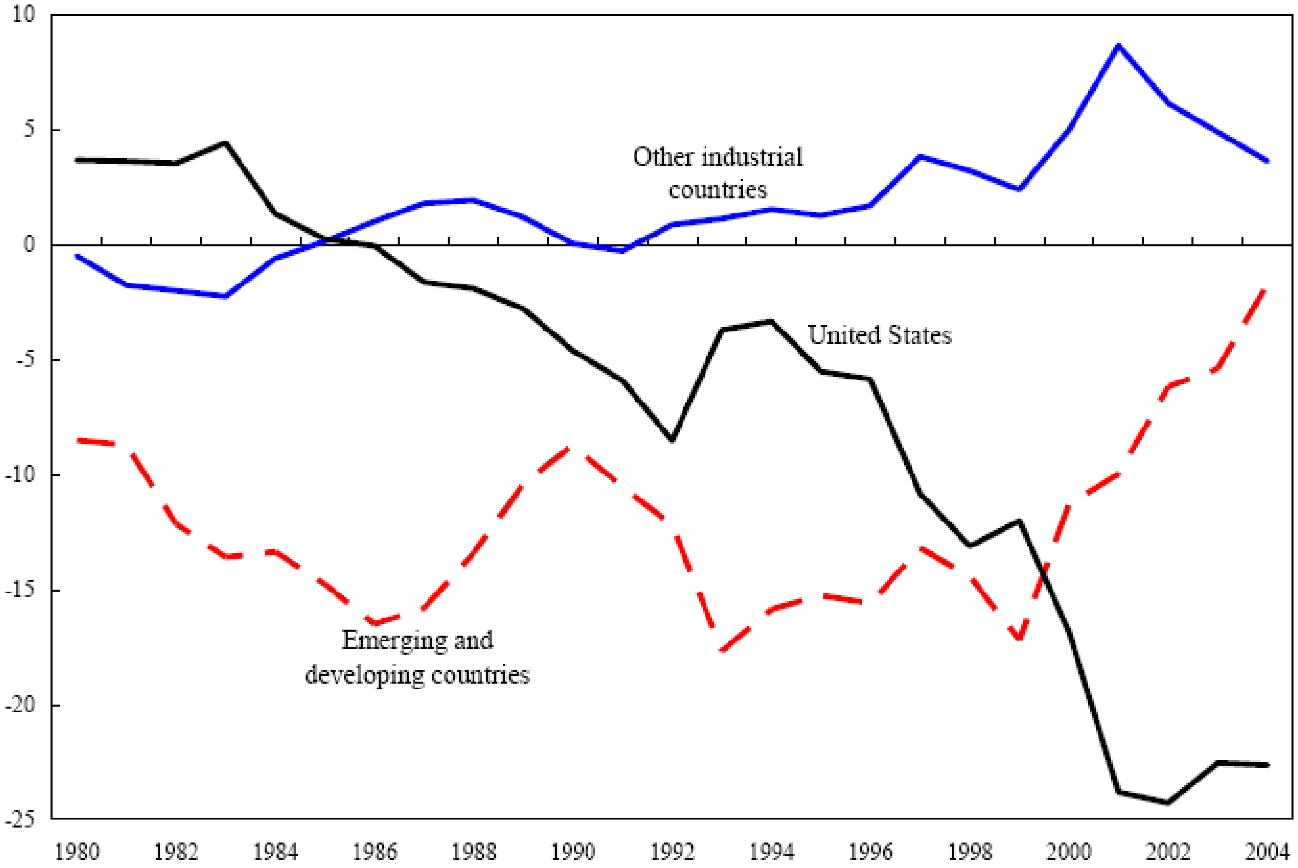
Note: Ratio of sum of foreign portfolio equity and FDI assets and liabilities to GDP.

Figure 7. External Debt and Official Reserves, Emerging Markets and Developing Group, 1970-2004



Note: The series "gross external debt" is the sum of external debt liabilities for the entire emerging market and developing country group as percent of the sum of total exports of goods and services. The series "official reserves" is the sum of official reserves for all countries of the group as a percent of the sum of total exports of goods and services.

Figure 9. Net Foreign Assets by Country Group (percent of Group GDP), 1980-2004



Note: The chart plots aggregate net foreign assets for the two country groups and the United States, divided by each group/country's GDP. The group "other industrial countries" includes all industrial countries except the United States.

Model setup

This lecture develops an example of the first models. Two country, H and F, populated by a continuum of agents normalized to 1, so that per capita and aggregate variable coincide. Starred variables are decision made by Foreign agents. Starred prices are expressed in Foreign currency.

In either country, firms use labor to produce a firm-specific intermediate good. The production functions of a Home and Foreign firm-specific good h and f are

$$\begin{aligned} Y_t(h) &= A_t \ell_t^{\theta} \\ Y_t^*(f) &= A_t^* (\ell_t^*)^{\theta} \end{aligned}$$

where A_t, A_t^* are exogenous technology shock specific each country — the only random variables in the exercise.

Intermediate goods combined into final goods

The number of goods (=# of firms) produced at time t is denoted by n_t and n_t^*. The n_t (n_t^*) intermediate goods produced at Home (Foreign) are aggregated by competitive firms into a composite final Home (Final) good, which can be used for consumption, or investment, according to the following CES functions

$$Y_{H,t} = \left(\int_0^{n_t} y(h)^{1-\frac{1}{\sigma}} \, dh \right)^{\frac{\sigma}{\sigma-1}}; \qquad Y_{F,t}^* = \left(\int_0^{n_t^*} y^*(f)^{1-\frac{1}{\sigma}} \, df \right)^{\frac{\sigma}{\sigma-1}}$$

Demand for individual goods is therefore given by

$$D_t(h) = \left(\frac{p_t(h)}{P_{H,t}} \right)^{-\sigma} D_{H,t}; \qquad D_t(f) = \left(\frac{p_t(f)}{P_{F,t}} \right)^{-\sigma} D_{F,t}$$

where demands D and price indexes are to be defined below.

Investment as creation of new firms and goods (extensive margins)

Setting up a firm to produce goods at time $t+1$ requires investing K_t units of a combination of domestic and foreign final output, as follows,

$$K_t = K_{H,t}^{\delta} K_{F,t}^{1-\delta}$$

traded at the price

$$P_{K,t} = \frac{P_H^{\delta} P_F^{1-\delta}}{\delta^{\delta} (1-\delta)^{1-\delta}}$$

K_H and K_F are the demand for the Home and Foreign final goods by the Home corporate sector. These demands are

$$K_{H,t} = \delta \frac{P_{K,t} K_t}{P_{H,t}}; \qquad K_{F,t} = (1-\delta) \frac{P_{K,t} K_t}{P_{F,t}}.$$

Note that if $\delta > 1/2$, there is home bias in investment.

Foreign firms and goods and depreciation

The corresponding expressions for the Foreign firm

$$K_t^* = \left(K_{H,t}^*\right)^{1-\delta}\left(K_{F,t}^*\right)^{\delta}$$

$$P_{K,t}^* = \frac{\left(P_H^*\right)^{1-\delta}\left(P_F^*\right)^{\delta}}{\delta^{\delta}\left(1-\delta\right)^{1-\delta}}$$

$$K_{H,t}^* = \left(1-\delta\right)\frac{P_{K,t}^* K_t^*}{P_{H,t}^*}; \qquad K_{F,t} = \delta\frac{P_{K,t}^* K_t^*}{P_{F,t}^*}$$

Note the symmetry in Home bias in investment demand.

In either country, firm capital depreciates completely in one period.

Preferences

In the Home country, utility in period t is

$$E_t \sum_{t=0}^{\infty} \beta^t U = E_t \sum_{t=0}^{\infty} \beta^t \left[\ln C_t + \kappa \left(1 - \ell_t\right)\right]$$

where C is consumption and ℓ is labor supply. The above specification (log utility and separability between consumption and leisure) is important for the results below. Consumption falls on both domestically produced and imported goods, with $\gamma > 1/2$ implying home bias:

$$C_t = C_{H,t}^{\gamma} C_{F,t}^{1-\gamma} \quad \text{with price index} \quad P_t = \frac{P_H^{\gamma} P_F^{1-\gamma}}{\gamma^{\gamma} (1-\gamma)^{1-\gamma}}$$

$$C_H = \left[\int_0^{n_t} C_t(h)^{1-\frac{1}{\sigma}} \, dh\right]^{\frac{\sigma}{\sigma-1}}; \qquad C_F^* = \left[\int_0^{n_t^*} C_t(f)^{1-\frac{1}{\sigma}} \, df\right]^{\frac{\sigma}{\sigma-1}}$$

Budget constraint: assets

Agents can trade in

- one-period discount bonds B_{t+1} at the price $E_t Q_{t,t+1}$;
- shares $s_{t+1}(h)$ of the mutual funds including n_{t+1} domestic firms h, and shares $s_{t+1}(f)$ of the n^*_{t+1} foreign firm f, at the price $q_t(h)$ and $\mathcal{E}_t q^*_t(f)$, respectively, where $\mathcal{E}_t$ is the nominal exchange rate. Recall that all firms last only one period;

- two one-period forward contracts with cash flow Θ_t in domestic currency, and Θ_t^* in foreign currency, to be defined below. Home households demand $x(h)$ and $x(f)$ units of these contracts $(= -x^*(h)$ and $-x^*(f)$, since these contracts are in zero net supply). It is convenient to define prices for these contracts $q_{x,t}$ and $q_{x,t}^*$: these prices are zero in equilibrium.

Budget identity

Households consume domestic and foreign final goods (combining n_t varieties of domestic goods, and n_t^* varieties of foreign goods). They receive labour income and dividends and coupons from their portfolio holdings of shares of domestic and foreign firms and bonds, and the cash flow from forward contracts; they pay taxes:

$$
\begin{aligned}
B_{t+1}E_tQ_{t,t+1} &+ \int_0^{n_{t+1}} s_{t+1}(h)\, q_t(h)\, dh + \mathcal{E}_t \int_0^{n_{t+1}^*} s_{t+1}(f)\, q_t^*(f)\, df \\
&+ \int_0^{n_t} C_t(h)\, p_t(h)\, dh + \mathcal{E}_t \int_0^{n_t^*} C_t(f)\, p_t^*(f)\, df \\
&= W_t\ell_t - T_t + B_t + \int_0^{n_t} s_t(h)\, \pi_t(h)\, dh + \mathcal{E}_t \int_0^{n_t^*} s_t(f)\, \pi_t^*(f)\, df \\
&\quad - \left(x_{t+1}(h)q_{x,t} + x_{t+1}(f)\mathcal{E}_t q_{x,t}^*\right) + x_t(h)\Theta_t + x_t(f)\mathcal{E}_t\Theta_t^*
\end{aligned}
$$

Household problem: first order conditions

Assume a symmetric equilibrium and the law of one price from the start:

$$
\begin{aligned}
C_H &= \gamma \frac{PC}{P_H} \qquad C_F = (1-\gamma)\frac{PC}{P_F} \qquad W = \kappa PC \\
E_t Q_{t,t+1} &= \beta E_t \frac{P_t C_t}{P_{t+1} C_{t+1}}; \\
q_{x,t} &= 0 = E_t Q_{t,t+1} \Theta_{t+1} = E_t Q_{t,t+1} \mathcal{E}_{t+1} \Theta^*_{t+1} \\
q_{H,t} &= P_{K,t} K_{H,t} = E_t \left(Q_{t,t+1} \pi_{t+1}(h) \right) \\
\mathcal{E}_t q^*_{F,t} &= \mathcal{E}_t P^*_{K,t} K^*_{F,t} = E_t \left(Q_{t,t+1} \mathcal{E}_{t+1} \pi^*_{t+1}(f) \right)
\end{aligned}
$$

The last two equations state that households finance new firms up to equalize the cost of setting up a firm with the present discounted value of profits generated by the firm. This condition presupposes *free entry of new firms*.

Firms problem: cost minimization and marginal costs

$$Min\ W_t \ell_t\left(h\right) \text{ s.t. } y_t\left(h\right) = A_{H,t}\ell_t^{\theta}(h)$$

Form the Lagrangian $W_t \ell_t\left(h\right) + \phi_t\left(y_t\left(h\right) - A_{H,t}\ell_t^{\theta}(h)\right)$:

$$W_t - \phi_t \theta A_{H,t}\ell_t^{\theta-1}(h) = 0$$

from which

$$\phi_t = \text{marginal cost} = \frac{W}{\theta A_{H,t}}\ell_t^{\theta-1}(h)$$

Firms problem: optimal pricing (flexible prices)

The demand for good h $D(h)$ falls on consumption and investment goods

$$\begin{aligned} D(h) &= C(h) + n_{t+1}K(h) + \\ & \quad C^*(h) + n^*_{t+1}K^*(h) \end{aligned}$$

Let $D_H = \int_0^{n_t} D(h)dh = n_t D(h)$ denote the corresponding demand at aggregate level. The firm problem is

$$Max \;\; p(h)D(h) - W\left(\frac{D(h)}{A_H}\right)^{\frac{1}{\theta}}$$

the first order condition is

$$0 = \left(\frac{p(h)}{P_H}\right)^{-\sigma} D_H - \sigma p(h)\frac{p(h)^{-\sigma-1}}{P_H^{-\sigma}}D_H - W_t \frac{1}{\theta A_H^{\frac{1}{\theta}}} D(h)^{\frac{1}{\theta}-1}(-\sigma)\frac{p(h)^{-\sigma-1}}{P_H^{-\sigma}}D_H$$

Optimal prices

Simplifying:

$$
\begin{aligned}
p(h) &= \frac{\sigma}{\sigma-1}\frac{W_t}{\theta A_H^{\frac{1}{\theta}}} D(h)^{\frac{1}{\theta}-1} = \frac{\sigma}{\sigma-1}\frac{W_t}{\theta A_H^{\frac{1}{\theta}}}\left(A_H \ell^{\theta}(h)\right)^{\frac{1}{\theta}-1} = \\
&= \frac{\sigma}{\sigma-1} \cdot \frac{W_t}{\theta A_H} \ell(h)^{1-\theta} = \\
&= mkp \cdot \text{marginal cost}
\end{aligned}
$$

Profits and labour income

$$\begin{aligned}
\pi_t(h) &= p(h)\, y(h) - W\ell(h) = \\
&= p(h)\, y(h) - W\left(\frac{y(h)}{A_H}\right)^{\frac{1}{\theta}} = p(h)\, y(h) - \overbrace{\frac{W}{A_H}\ell^{1-\theta}}^{\theta \text{ marginal costs}} y(h) = \\
&= p(h)\, y(h) - p(h)\, \theta \frac{\sigma - 1}{\sigma} y(h) = \\
&= p(h)\, y(h) \left[1 - \theta \frac{\sigma - 1}{\sigma}\right] \\
W_t \ell_t &= p(h)\, y(h) - profits = p(h)\, y(h) \left[\theta \frac{\sigma - 1}{\sigma}\right]
\end{aligned}$$

Hence the two are perfectly correlated.

Market clearing and numeraire

$$
\begin{aligned}
D(h) &= Y(h); \qquad D(f) = Y^*(f) \\
n_t \ell_t(h) &= \int \ell_t(j)\, dj; \qquad n_t^* \ell_t^*(f) = \int \ell_t^*(j)\, dj^* \\
B_t &= -B_t^*; \qquad x_t(h) = -x_t^*(h) \qquad x_t(f) = -x_t^*(f \\
\int_0^1 s_t(h,j)dj + \int_0^1 s_t^*(h,j^*)dj^* &= 1; \qquad \int_0^1 s_t(f,j)dj + \int_0^1 s_t^*(f,j^*)dj^* = 1
\end{aligned}
$$

Without loss of generality, set

$$W = 1; \qquad W_t^* = 1$$

All Home variable are expressed in Home wage units. All foreign variables are expressed in Foreign wage units.

RER TOT and GDP: definitions

Home terms of trade and real exchange rate are

$$TOT = \frac{p\,(h)}{\mathcal{E}_t p^*(f)}; \qquad RER = \frac{\mathcal{E}_t P_t^*}{P_t}$$

GDP is $Y_t = \int_0^{n_t} y(h)dh$ and $p(h)$ is the GDP deflator.

Complete vs. Incomplete markets (1)

If risk sharing is not perfect, in general domestic and foreign households will not have the same valuation of the cash flows of dividends. If this is the case, what discount factor should firms use to form their investment plans? To proceed in the analysis, as in Heatcote and Perri, one can arbitrarily posit that firms use some weighted average of the stochastic discount rates of Home and Foreign households. In equilibrium with perfect risk sharing, Home and Foreign discount rates will coincide.

Equilibrium

An equilibrium is a set of prices $p^*(f); p(h); RER_t; Q_{t,t+1}; Q^*_{t,t+1}; q_t; q^*_t; q_{x,t} = q^*_{x,t} = 0$;for all s^t and all $t > 0$ such that when all households and firms solve their problems taking these prices as given all markets clear.

Educated guess: there is an equilibrium with full risk sharing

$$P_tC_t = \mathcal{E}_tP_t^*C_t^*$$

and constant portfolio allocation

$$s_t = s = \frac{\delta - (2\delta - 1)\left[1 - \theta^{\frac{\sigma-1}{\sigma}}\right]}{1 - (2\delta - 1)\left[1 - \theta^{\frac{\sigma-1}{\sigma}}\right]} \quad for\ all\ t$$

where no bond and no forward contract is traded.

Verifying this conjecture takes three steps.

First, using market clearing, law of one price and demand functions, show that under the conjecture:

$$P_{H,t}Y_t - \mathcal{E}_t P^*_{F,t} Y^*_t = (2\delta - 1) \cdot \left(n_{t+1} P_{k,t} K_t - n^*_{t+1} \mathcal{E}_t P^*_{K,t} K^*_t \right) \qquad (1)$$

To wit: from market clearing

$$p(h)y(h) = p(h)C(h) + \mathcal{E} p^*(h) C^* + n_{t+1} p(h) K(h) + n^*_{t+1} \mathcal{E} p^*(h) K^*_t$$

integrate over all h, and use demand functions, to get:

$$\begin{aligned} P_H Y_H &= P_H C_H + \mathcal{E}_t P^*_H C^*_H + n_{t+1} P_H K_H + n^*_{t+1} \mathcal{E}_t P^*_H K_H \\ &= \gamma P C + (1-\gamma)\, \mathcal{E}_t P^* C^* \\ &\quad + n_{t+1} P_H \frac{\delta P_K K}{P_H} + n^*_{t+1} \mathcal{E}_t P^*_H \frac{(1-\delta)\, P^*_k K^*}{P^*_H} \end{aligned}$$

Subtract from the previous expression the analogous expression for the foreign country:

$$\begin{aligned}\mathcal{E}_t P_F Y_F &= (1-\gamma) PC + \gamma \mathcal{E}_t P^* C^* + \\ &\quad + n_{t+1} P_H \frac{(1-\delta) P_K K}{P_H} + n^*_{t+1} \mathcal{E}_t P^*_H \frac{\delta P^*_k K^*}{P^*_H}\end{aligned}$$

Recall that in our conjecture, the term in consumption differential disappears by virtue of perfect risk sharing .

Second, using the budget constraints and optimal profits and labor shares, show that

$$P_{H,t} Y_t - \mathcal{E}_t P^*_{F,t} Y^*_t = \frac{(2s-1) \cdot \left(n_{t+1} P_{k,t} K_t - n^*_{t+1} \mathcal{E}_t P^*_{K,t} K^*_t \right)}{\theta^{\frac{\sigma-1}{\sigma}} - (1-2s) \cdot \left[1 - \theta^{\frac{\sigma-1}{\sigma}} \right]} \qquad (2)$$

To wit, rewrite the aggregate budget constraint of the two countries, imposing time-invariant and symmetric portfolio shares, i.e. $s(h) = s^*(f) = s$, and substituting for profits, labor income and government budget

$$
\begin{aligned}
&P_t C_t = W_t \ell_t - T_t + s(h) n_t \pi_t(h) - s(h) n_{t+1} q_t(h) + \\
&\quad + \mathcal{E}_t (1 - s(h)) n_t^* \pi_t^*(f) - \mathcal{E}_t (1 - s(h)) n_{t+1}^* q_t^*(f) \\
&+ B_t - B_{t+1} E_t Q_{t,t+1} - \left(x_{t+1}(h) q_{x,t} + x_{t+1}(f) \mathcal{E}_t q_{x,t}^* \right) + x_t(h) \Theta_t + x_t(f) \mathcal{E}_t \Theta_t^* = \\
&= \left[\theta \frac{\sigma - 1}{\sigma} \right] P_H Y_H + s \cdot \left[1 - \theta \frac{\sigma - 1}{\sigma} \right] P_H Y_H + \\
&\mathcal{E}_t (1 - s) \left[1 - \theta \frac{\sigma - 1}{\sigma} \right] P_F^* Y_F^* - s \cdot n_{t+1} p_K K - \mathcal{E}_t (1 - s) n_{t+1}^* p_{K,t}^* K_t^* \\
&+ B_t - B_{t+1} E_t Q_{t,t+1} - \left(x_{t+1}(h) q_{x,t} + x_{t+1}(f) \mathcal{E}_t q_{x,t}^* \right) + x_t(h) \Theta_t + x_t(f) \mathcal{E}_t \Theta_t^*
\end{aligned}
$$

In the foreign country

$$\mathcal{E}_t P_t^* C_t^* = \mathcal{E}_t W_t^* \ell_t^* - \mathcal{E}_t T_t^* + (1 - s(h)) n_t \pi_t(h) - (1 - s(h)) n_{t+1} q_t(h) +$$
$$+ \mathcal{E}_t s(h) n_t^* \pi_t^*(f) - \mathcal{E}_t s(h) n_{t+1}^* q_t^*(f)$$
$$+ B_t^* - B_{t+1}^* \mathcal{E}_t E_t Q_{t,t+1}^* + x_{t+1}^*(h) q_{x,t} + x_{t+1}^*(f) \mathcal{E}_t q_{x,t}^* - x_t^*(h) \Theta_t - x_t^*(f) \mathcal{E}_t \Theta_t^* =$$
$$= \left[\theta \frac{\sigma - 1}{\sigma}\right] \mathcal{E}_t P_F^* Y_F^* + (1 - s) \left[1 - \theta \frac{\sigma - 1}{\sigma}\right] P_H Y_H$$
$$+ \mathcal{E}_t s \cdot \left[1 - \theta \frac{\sigma - 1}{\sigma}\right] P_F^* Y_F^* - (1 - s) n_{t+1} p_K K - \mathcal{E}_t s \cdot n_{t+1}^* p_{K,t}^* K_t^*$$
$$+ B_t^* - B_{t+1}^* \mathcal{E}_t E_t Q_{t,t+1}^* + x_{t+1}^*(h) q_{x,t} + x_{t+1}^*(f) \mathcal{E}_t q_{x,t}^* - x_t^*(h) \Theta_t - x_t^*(f) \mathcal{E}_t \Theta_t^*$$

Subtracting the second from the first and using market clearing in the asset market

$$
\begin{aligned}
0 \;=\; & \left(P_{H,t}Y_t - \mathcal{E}_t P^*_{F,t} Y^*_t\right)\left\{\theta\frac{\sigma-1}{\sigma} - (1-2s)\cdot\left[1-\theta\frac{\sigma-1}{\sigma}\right]\right\} \\
& -\left(n_{t+1}P_{k,t}K_t - n^*_{t+1}\mathcal{E}_t P^*_{K,t}K^*_t\right)(2s-1) \\
& +2\left(B_t - B_{t+1}E_t Q_{t,t+1}\right) \\
& -2\left(x_{t+1}(h)q_{x,t} + x_{t+1}(f)\mathcal{E}_t q^*_{x,t}\right) + 2\left(x_t(h)\Theta_t + x_t(f)\mathcal{E}_t\Theta^*_t\right)
\end{aligned}
$$

Third, show that the two expressions above (1) (2) are reconciled when the share of foreign equity in Home agents' portfolio is:

$$
\begin{aligned}
(2s-1) &= (2\delta-1)\left[\theta\frac{\sigma-1}{\sigma} - (1-2s)\cdot\left[1-\theta\frac{\sigma-1}{\sigma}\right]\right] \\
(2s-1) &= \frac{(2\delta-1)\,\theta\frac{\sigma-1}{\sigma}}{1+(2\delta-1)\left[1-\theta\frac{\sigma-1}{\sigma}\right]} \\
1-s &= \frac{1-\delta}{1-(2\delta-1)\left[1-\theta\frac{\sigma-1}{\sigma}\right]}
\end{aligned}
$$

Back-of-the-envelop calculations: set $\theta = .66$, $\sigma = .8, \delta = .8$, the share of foreign equities in portfolio is about 30 percent. Note that this expression is independent of consumption home bias γ.

A look at the literature using the model: Lucas 82 ...

Lucas (1982) assumes an endowment two-country world economy in which agents preferences are identical everywhere. He shows that perfect risk sharing can be achieved with 'perfect pooling', i.e. the national representative agent owns $1/2$ of domestic equity and $1/2$ of foreign equity. Lucas stresses that, given the structure of preferences (identical across countries), an equilibrium with efficient risk sharing can be achieved 'economizing on the number of assets', i.e. with equity shares only, instead using the full set of Arrow-Debreu securities. In the setup above, an example of parameters' specification somewhat related to Lucas' is the case $\delta = 1/2$. In this case (whatever θ, σ):

$$s = 1/2$$

...(Cole and Obstfeld and) Corsetti and Pesenti 01...

Consider a fixed number of firms and goods varieties (hence no investment). Then $K = K^* = 0$. Then perfect risk sharing requires

$$P_t C_t - \mathcal{E}_t P_t^* C_t^* = P_{H,t} Y_t - \mathcal{E}_t P_{F,t}^* Y_t^* = 0$$

It follows that terms of trade move inversely with output

$$TOT = \frac{Y}{Y^*}$$

exactly as in Cole and Obstfeld (1991), ensuring perfect risk sharing without any need for portfolio diversification. This generalizes C&O(1991) to a production economy, as in Corsetti and Pesenti (2001).

...Corsetti and Pesenti 01

In the above model, the Cobb-Douglas aggregator is symmetric but not necessarily identical across countries (there is Home bias). With Home bias in consumption, an equilibrium exists with perfect risk sharing despite no trade in international assets thanks to the specification of preferences (logC). The result does not generalize to more general specification of preferences (power utility).

In C&P (and C&O), instead, spending is symmetrical, and $RER = 1$. The solution exists also for power utility.

Heatcote and Perri 04 revisited

Look at the equilibrium expression for the share of foreign equities:

$$1 - s = \frac{1-\delta}{1-(2\delta-1)\left[1-\theta\frac{\sigma-1}{\sigma}\right]}$$

This expression shows that s and δ are positively related: the higher the home bias in investment, the higher the home bias in equity portfolio. Observe $\delta = 1$ $=>$ $s = 1$ (no international diversification). The degree of consumption home bias γ is irrelevant.

Moreover, home bias is higher in economies where the share of labor in output is lower (θ is low). Why?

Financial transmission

Suppose that expectations of future Home productivity raise investment in the Home country relative to Foreign. In general (for $\theta \in (0,1)$), a change in investment has a direct, and an indirect effect on relative consumption:

$$P_t C_t - \mathcal{E}_t P_t^* C_t^* = \left[\overset{\text{direct}}{-(2s-1)} + \overset{\text{indirect}}{(2\delta - 1)\left\{\theta \frac{\sigma - 1}{\sigma} - (1-2s) \cdot \left[1 - \theta \frac{\sigma - 1}{\sigma}\right]\right\}} \right] \left(n_{t+1} P_{k,t} K_t - n_{t+1}^* \mathcal{E}_t P_{K,t}^* K_t^* \right)$$

- the direct effect depends on the share of investment financed by each country $(2s-1)$; a high home bias in portfolio means that most of the investment is financed by Home households, at the cost of their consumption relative to Foreign consumption.

- the indirect works through relative prices and output.
  - With home bias, an increase in investment raises demand for Home output, causing appreciation. This per se raises Home consumption.
  - The Home appreciation is however contained by (a) a positive response of output and (b) the fact that part of the firms profits are paid to Foreign shareholders, whose spending is biased towards their own goods.

To maintain $\Delta\mathbb{C} = 0$, the direct and indirect effects must exactly compensated each other. Other things equal, a higher labor share θ implies that, in response to shocks, less resources are transferred abroad in the form of dividends. Pressures towards appreciation rise, so that $\Delta\mathbb{C}$ would naturally tend to be positive. The larger θ must then be matched by a higher s, increasing domestic ownership of Home equities.

Baxter and Jerman vs. *Bottazzi, Pesenti and Van Wincoop*

A traditional explanation for home bias in portfolio is explored by BP&VW, who emphasize that, for many countries, labor and capital income are negatively correlated (profits are high when wage incomes are low). If this is the case, owning a large share of domestic equity provides insurance against (undiversifiable) labor income risk: equity returns tend to be high when labor income is low.

If labor and profit income are positively correlated (as strongly argued by BJ) people can and should diversify their labor income risk by acquiring foreign assets.

In the model above, labor and capital income are perfectly correlated (see slide on distribution). Yet, the incentive to diversify one's financial portfolio abroad falls with a larger share of labor income in total income. The reason why θ matters is exclusively because of its implied effect on terms of trade movements.

Introducing friction: nominal rigidities

It is easy to show that nominal rigidities affect the correlation of profit and labor income over the business cycle: with preset prices, a positive productivity shock raises ex post markups, and lower the share of output accruing to labor.

Unless monetary authorities can and do pursue price stability, in equilibrium two assets will no longer be sufficient to support efficient risk sharing. At least two additional assets are necessary.

For simplicity, $\theta = 1$ in what follows.

A model with one period preset prices

Managers set up a firm and post a nominal price for their product, at which they stand ready to meet demand in the following period. To maximize the value of the firm, the price should be such that it delivers the highest presented discounted value of profits

$$
\begin{aligned}
& Max E_{t-1} \frac{\beta}{P_t C_t} \left[p_t(h) D_t(h) - W \left(\frac{D_t(h)}{A_{H,t}} \right) \right] \\
= \;& E_{t-1} \frac{\beta}{P_t C_t} \left[\left(\frac{p(h)}{P_H} \right)^{-\sigma} P_H D_{H,t} - W \left(\frac{p(h)}{P_H} \right)^{-\sigma} \left(\frac{D_{H,t}}{A_{H,t}} \right) \right]
\end{aligned}
$$

which yields.

$$
p(h) = \frac{\sigma}{\sigma - 1} \cdot E_{t-1} \left(\frac{W}{A_H} \right)
$$

Distribution

The shares of firms revenue $p(h)D\left(h\right)$ that goes to wages and profits are are

$$W\ell \;=\; p(h)D\left(h\right)\left[\frac{W}{A_H \cdot p(h)}\right]$$

$$\Pi\left(h\right) \;=\; p(h)D\left(h\right)\left[1-\frac{W}{A_H \cdot p(h)}\right]$$

Given prices and wages, a temporary rise in productivity lowers labor income, and raises profits.

Note however that the two are perfectly correlated if wages raise with productivity $W = A$, so that marginal costs remain constant.

For future reference, define PC$=\mu$, so that $W = \kappa PC = \kappa\mu$.

Building an equilibrium with perfect risk sharing

Following the same step as before, we conjecture an equilibrium characterized by efficient risk sharing with time invariant share of equities, and no trade in bonds. Different from before, the forward contract is essential to support this equilibrium. The educated guess for the equity share is

$$S_H = \frac{(1-\delta) + (2\delta - 1)\frac{E\beta\frac{\mu_{t-1}}{\mu_t}\left[P_H Y_H \frac{\mu_t}{A_H P_H}\right]}{E\beta\frac{\mu_{t-1}}{\mu_t} P_H Y_H}}{2(1-\delta) + (2\delta - 1)\frac{E\beta\frac{\mu_{t-1}}{\mu_t}\left[P_H Y_H \frac{\mu_t}{A_H P_H}\right]}{E\beta\frac{\mu_{t-1}}{\mu_t} P_H Y_H}}$$

which crucially depends on the ratio of labor income over total output (both in presented discounted value) $\frac{E\beta\frac{\mu_{t-1}}{\mu_t}\left[P_H Y_H \frac{\mu_t}{A_H P_H}\right]}{E\beta\frac{\mu_{t-1}}{\mu_t} P_H Y_H}$.

One way to rewrite the above is

$$S_H = \frac{(1-\delta) + (2\delta - 1)\frac{E\frac{1}{\mu_t}E\mu_t\ell + Cov\left(\frac{1}{\mu_t},\mu_t\ell\right)}{E\frac{1}{\mu_t}Y_H}}{2(1-\delta) + (2\delta - 1)\frac{E\frac{1}{\mu_t}E\mu_t\ell + Cov\left(\frac{1}{\mu_t},\mu_t\ell\right)}{E\frac{1}{\mu_t}Y_H}}$$

Other things equal, with home bias in investment $\delta > 1/2$ the share S_H is increasing in the covariance between the labor income and the inverse of nominal wages μ.

If nominal wages and thus μ are constant,

$$S_H = \frac{(1-\delta) + (2\delta - 1)\left(\frac{1}{EA_H} - \frac{Cov(A_H,\ell)}{EY_H EA_H}\right)}{2\,(1-\delta) + (2\delta - 1)\left(\frac{1}{EA_H} - \frac{Cov(A_H,\ell)}{EY_H EA_H}\right)}$$

With a constant μ the covariance between productivity shocks and employment is negative. Hence, provided $\delta > 1/2$, the share of domestic portfolio invested in domestic equities is increasing in the absolute value of this covariance.

This is essentially the point stressed by BP&VW.

If $\mu = A_H$ instead

$$S_H = \frac{(1-\delta) + (2\delta - 1)\,\frac{E\beta\frac{\mu_{t-1}}{\mu_t}[Y_H]}{E\beta\frac{\mu_{t-1}}{\mu_t}P_H Y_H}}{2\,(1-\delta) + (2\delta - 1)\,\frac{E\beta\frac{\mu_{t-1}}{\mu_t}[Y_H]}{E\beta\frac{\mu_{t-1}}{\mu_t}P_H Y_H}}$$

which yields exactly the solution with flexible prices.

Efficient risk sharing however requires the forward contract to deliver

$$\Theta_t = \left[\frac{(2S_H - 1)}{2}\left(\frac{1}{(2\delta - 1)} - 1\right) P_H Y_H - (1 - S_H)\, W\ell\right]$$

With nominal rigidities, unless monetary policy stabilizes the economy at the flex-price equilibrium, a positive productivity shocks raises markups above the flex-price level and lower labor income. The last term of the cash flow above implies that, by entering a forward, foreign investors 'pay back' the excess profits in proportion of the share they own of the domestic capital (domestic investors receive it). In fact, labor income share goes down, hence the cash flow is high.