THE INTERNATIONAL TRANSMISSION OF PRODUCTIVITY SHOCKS: CORE ANALYTICAL ISSUES

EUI 2006

Giancarlo Corsetti

Introduction (1)

How do macroeconomic shocks in one country transmit abroad?

- With *complete markets*, domestic households can insure their consumption against idiosyncratic supply shocks. In the case of a negative output shock, domestic consumers benefit from income transfers by foreign residents.

- With *trade limited to some assets*, it is possible for domestic households to borrow and lend as to smooth consumption through intertemporal trade (exchange of goods at different dates), improving their welfare.

Introduction (2)

- *In general,* positive country-specific supply shocks may cause the international price of the domestic good to fall. In this case, also foreign consumers benefit from the shock, via favorable import price movements. Even if there is no trade in financial assets, movements in relative prices may contain the consumption risk of output fluctuations, by reducing their impact on relative (domestic vs. foreign) wealth.

In this lecture, we will build a stylized two country model to study analytically the international transmission mechanism. Reference: Corsetti Dedola and Leduc (2004- in the new revised version 2006).

Outline

We will focus on productivity (endowment) shocks, and analyze their international macroeconomic transmission under

- different asset markets structure (complete markets, financial autarky, trade in bonds);

- different assumptions about trade elasticities (elasticity of substitution between Home and Foreign tradable goods);

- different degree of shock persistence.

1 A stylized model

Two-country, two-good *endowment* economy. 'Home' and 'Foreign' countries are denoted H and F, respectively.

- Let Y_{H} and Y_{F} denote Home and Foreign (tradable) output. Supply $\{Y_{\mathsf{H}}, Y_{\mathsf{F}}\}$ varies randomly.

- Households derive utility from consuming both goods.

Consumption (1)

Consumption of Home representative consumer is given by CES aggregator

$$C = C_{\mathsf{T}} = \left[a_{\mathsf{H}}^{1-\rho} C_{\mathsf{H}}^{\rho} + a_{\mathsf{F}}^{1-\rho} C_{\mathsf{F}}^{\rho} \right]^{\frac{1}{\rho}}, \qquad \rho < 1,$$

where $C_{\mathsf{H},t}$ ($C_{\mathsf{F},t}$) is the domestic consumption of Home (Foreign) produced good, a_{H} is the share of the domestically produced good in the Home consumption expenditure, a_{F} is the corresponding share of imported goods, with $a_{\mathsf{F}} = 1 - a_{\mathsf{H}}$.
$a_{\mathsf{H}} > 1/2$ indexes '**Home bias**' in consumption (preferences for local goods).

- Let ω denote the elasticity of substitution between Home and Foreign goods:

  $$\omega = (1 - \rho)^{-1}$$

  A higher ω makes goods more homogeneous.

Consumption (2)

Analogously, in the foreign country

$$C^* = C^*_{\mathsf{T}} = \left[(a^*_{\mathsf{H}})^{1-\rho} (C^*_{\mathsf{H}})^{\rho} + (a^*_{\mathsf{F}})^{1-\rho} (C^*_{\mathsf{F}})^{\rho} \right]^{\frac{1}{\rho}}, \qquad (1)$$

where $C^*_{\mathsf{H},t}$ ($C^*_{\mathsf{F},t}$) is the consumption of Home (Foreign) produced good by Foreign households. Home bias in consumption is now $a^*_{\mathsf{F}} = 1 - a^*_{\mathsf{H}} > 1/2$.

Prices

- Let $P_{\mathsf{H},t}$ ($P_{\mathsf{F},t}$) denote the price of the Home (Foreign) goods in domestic money ('domestic currency'). When starred, these prices are expressed in Foreign currency.

Nominal vs real exchange rates, and the terms of trade

- Let $\mathcal{E}$ denote the nominal exchange rate, defined as the price of one currency in terms of another.

- Let P and P^* denote the consumer price level (index).

- The **real exchange rate (RER)** is the relative price of consumption, the **terms of trade (TOT)** are the relative price of Foreign goods in terms of Home goods

$$RER = \frac{\mathcal{E}P^*}{P} \qquad\qquad TOT = \frac{P_{\mathsf{F}}}{\mathcal{E}P^*_{\mathsf{H}}}$$

Price indexes (1)

The welfare-based price index P is defined as the minimum expenditure needed to buy one unit of consumption good $C = 1$, given market prices. By setting up the minimization problem (e.g. OR 4.4.1.1 pp. 226-228), you can derive:

$$C_{\mathsf{H}} = \alpha_{\mathsf{H}} \left(\frac{P_{\mathsf{H}}}{P}\right)^{-\omega} C \qquad\qquad C_{\mathsf{F}} = \alpha_{\mathsf{F}} \left(\frac{P_{\mathsf{F}}}{P}\right)^{-\omega} C$$

where

$$P = P_{\mathsf{T}} = \left[a_{\mathsf{H}} P_{\mathsf{H}}^{\frac{\rho}{\rho-1}} + (1 - a_{\mathsf{H}})\, P_{\mathsf{F}}^{\frac{\rho}{\rho-1}}\right]^{\frac{\rho-1}{\rho}}. \qquad (2)$$

Price indexes (2)

The foreign price level P^* in foreign currency is analogously defined:

$$P^* = P^*_{\mathsf{T}} = 1\left[(1 - a^*_{\mathsf{F}})\left(P^*_{\mathsf{H}}\right)^{\frac{\rho}{\rho-1}} + a^*_{\mathsf{F}}\left(P^*_{\mathsf{F}}\right)^{\frac{\rho}{\rho-1}}\right]^{\frac{\rho-1}{\rho}}.$$

Note that (a) if the law of one price holds: $P_{\mathsf{H}} = \mathcal{E}P^*_{\mathsf{H}}$, $P_{\mathsf{F}} = \mathcal{E}P^*_{\mathsf{F}}$, and (b) if the consumption baskets are identical, namely, there is no Home Bias in the world economy: $a_H = 1 - \alpha^*_F$, then the price of consumption is equalized across countries. In this case, *purchasing power parity* (PPP) holds: $P = \mathcal{E}P^*$.

Note that $a_H = 1 - \alpha^*_F$ can be $>$1/2 ($<$1/2) if both countries like a higher (lower) share of good H than good F in their consumption baskets.

Assumptions and normalizations

In this lecture:

- we will assume that the law of one price (LOOP) holds, i.e.

$$P_{\mathsf{H}} = \mathcal{E} P^*_{\mathsf{H}}; \qquad P_{\mathsf{F}} = \mathcal{E} P^*_{\mathsf{F}}$$

- also, as we abstract from nominal issues, without loss of generality we set

$$\mathcal{E} = 1.$$

- We will assume that economies ex-ante symmetrical.

RER and TOT (1)

Write RER under law of one price

$$RER^{\frac{\rho}{\rho-1}} = \frac{\left(1-a_F^*\right) P_H^{*\frac{\rho}{\rho-1}} + a_F^* P_F^{\frac{\rho}{\rho-1}}}{a_H P_H^{*\frac{\rho}{\rho-1}} + (1-a_H) P_F^{\frac{\rho}{\rho-1}}} = \frac{\left(1-a_F^*\right) + a_F^* TOT^{\frac{\rho}{\rho-1}}}{a_H + (1-a_H) TOT^{\frac{\rho}{\rho-1}}}$$

Take a log-linear approximation around a symmetric equilibrium with $RER = TOT = 1$:

$$\widehat{RER} = (a_F^* + a_H - 1)\,\widehat{TOT}$$

where "$\widehat{\ }$" represents a variable's percentage deviation from the equilibrium.

Note that the correlation between RER and TOT depends on the degree of Home Bias in the world economy: positive with Home Bias, zero with PPP, negative without Home bias (an unrealistic situation).

Preferences and resource constraint

$$EU(C) = \sum_k \pi_s \left(\frac{C(s)}{1-\sigma} \right)^{1-\sigma}$$

where s is a state of nature and $\pi(s)$ its probability. Notation: $EX = \sum_s \pi(s)X(s)$.

Power utility function: $1/\sigma$ is the the intertemporal elasticity of substitution.

Resource constraint is:

$$Y_H = C_H + C_H^* \qquad\qquad Y_F^* = C_F + C_F^*$$

2 Complete markets (CM)

In our first world economy, we consider the case in which Households trade a complete set of state-contingent Arrow-Debreu (A-D) securities. For simplicity, consider a two-period economy.

Denote by $B(s')$ an Arrow-Debreu security (or contract) paying 1 unit of domestic currency in state s' in the second period; let $\pi(s'|s)$ be the conditional probability of state s' in the next period, given that the state s occurred in the current period. Let $Q(s'|s)$ denote the current price of $B(s')$.

CM: Consumer problem

The problem of the Home representative household is to maximize expected utility:

$$Max\ E_t \sum_{t=1}^{\infty} U(C_t) = U\left(C(s)\right) + \sum_{s'|s} \pi(s'|s) U(C\left(s'|s\right))$$

$$-\lambda(s) \left(\sum_{s'|s} Q(s'|s) B(s') - P_{\mathsf{H}}(s) Y_{\mathsf{H}}(s) + P(s) C(s) \right) +$$

$$-\sum_{s'|s} \lambda\left(s'|s\right) \left(P_{\mathsf{H}}(s'|s) Y_{\mathsf{H}}(s'|s) + B(s') - P(s'|s) C(s'|s)\right)$$

Write its Foreign analog.

CM: first order conditions

The optimal choice of consumption satisfies

$$\begin{aligned} U_c(s) &= \lambda(s)P(s) \\ U_c(s'|s) &= \lambda(s'|s)P(s'|s) \end{aligned}$$

where U_c denotes marginal utility. Note that the marginal utility of consumption in state s must be equal to the price of consumption in that state, translated into units of marginal utility $\lambda(s)$ (the multiplier). The F.O.Cs with respect to bonds $B(s')$ (one for each state) are

$$\lambda(s)Q(s\prime|s) = \pi(s'|s)\lambda\left(s'|s\right)$$

CM:fundamental pricing equation

Combining the conditions above

$$\underbrace{U_C(s)}\quad \overset{\text{units of current consumption}}{\frac{Q(s\prime|s)}{P(s)}} = \pi(s'|s)U_C(s'|s)\quad \overset{\text{units of future consumption}}{\frac{1}{P(s'|s)}}$$

In utility terms, the marginal cost of buying an extra unit of $B(s')$ consists of giving up Q/P units of consumption, evaluated at current marginal utility of consumption U_c. The marginal benefit is the expected utility from 1 unit of currency received in state s'. This consists of the expected marginal utility $(\pi(s'|s)U_C(s'|s))$ from the extra units of consumptions that one unit of currency can buy in state s' $(1/P(s'|s))$.

Fundamental risk-sharing condition (1)

Analogously, in the foreign economy

$$U_c^*(s)\frac{Q(s\prime|s)}{P^*(s)} = \pi(s'|s)U_c^*(s'|s)\frac{1}{P^*(s'|s)}$$

Combine this with the Home equation in the previous slide, and recalling that $\mathcal{E}$=1:

$$\frac{U_c(s)}{U_c^*(s)}\frac{P^*(s)}{P(s)} = \frac{U_c(s'|s)}{U_c^*(s'|s)}\frac{P^*(s'|s)}{P(s'|s)}$$

This is a crucial condition of efficient risk sharing in the international economy.

Fundamental risk-sharing condition (2)

Assume that initially the two economies are perfectly symmetric: in state s they have the same prices, same marginal utility. Then $\frac{U_C(s)}{U_C^*(s)}\frac{P^*(s)}{P(s)} = 1$. Under this condition, and using the definition of real exchange rate, we can write

$$\frac{U_C^*(s'|s)}{U_C(s'|s)} = RER\left(s'|s\right)$$

This is a defining conditions of efficient risk sharing in the international economy. The ratio between the marginal utilities of Home and Foreign consumption is proportional to the relative price of consumption (RER) across all states of nature s'.

Fundamental risk-sharing condition (3)

Rewrite the above

$$U_c^*(s'|s)\frac{1}{P^*(s'|s)} = U_c(s'|s)\frac{1}{P(s'|s)}$$

with complete markets, the marginal utility of a unit of domestic currency is equalized across countries. This is equal to $\frac{1}{P^*(s'|s)}$ unit of consumption, evaluated at current marginal utility.

Note that if PPP holds (consumption price is identical in all countries, $RER = 1$), and preferences are the same, consumption is also equalized across countries.

CM: risk sharing with power utility

With symmetry and power utility we have

$$RER\left(s'|s\right) = \frac{U_c^*(s'|s)}{U_c(s'|s)} = \left(\frac{C(s'|s)}{C^*(s'|s)}\right)^{\sigma}$$

With efficient risk sharing, domestic consumption should be higher in those states of the world in which the price of domestic consumption is relatively cheaper. In states of the world in which the price of domestic consumption is low relative to foreign consumption, domestic residents should receive contingent income transfers to 'take advantage' of the favorable goods prices.

Note that if $\sigma = 1$, complete markets imply equalization of consumption expenditure across countries in all states of nature:

$$PC = \mathcal{E}P^*C^*$$

International transmission with complete markets

Take a log-linear approximation of $RER = (C/C^*)^\sigma$ in the neighborhood of a symmetric equilibrium with $a_F^* = a_H$:

$$\sigma \left(\widehat{C} - \widehat{C^*} \right) = \widehat{RER} = (2a_H - 1)\, \widehat{TOT}$$

- Real exchange rate and relative consumption will be perfectly correlated. Correlation of relative consumption with terms of trade depends on degree of Home Bias (likely to be positive).

- Without Home Bias ($\alpha_H = a_F^* = 1/2$, PPP holds), there is no RER fluctuation no matter what happens to TOT. The rate of growth of consumption is equalized.

TOT and relative output supply (1)

Combine resource constraint and risk sharing condition

$$\frac{Y_\mathsf{H}}{Y_\mathsf{F}} = \frac{a_\mathsf{H}\left(\frac{P_\mathsf{H}}{P}\right)^{-\omega} C + (1-a_\mathsf{H})\left(\frac{P_\mathsf{H}}{P^*}\right)^{-\omega} C\left(\frac{P^*}{P}\right)^{-\sigma^{-1}}}{(1-a_\mathsf{H})\left(\frac{P_\mathsf{F}}{P}\right)^{-\omega} C + a_\mathsf{H}\left(\frac{P_\mathsf{F}}{P^*}\right)^{-\omega} C\left(\frac{P^*}{P}\right)^{-\sigma^{-1}}} = \frac{TOT^{\omega}\left[a_\mathsf{H} + (1-a_\mathsf{H})\left(\frac{P^*}{P}\right)^{\omega-\sigma^{-1}}\right]}{(1-a_\mathsf{H}) + a_\mathsf{H}\left(\frac{P^*}{P}\right)^{\omega-\sigma^{-1}}}.$$

Loglinearizing we get:

$$\widehat{Y_\mathsf{H}} - \widehat{Y_\mathsf{F}} = \omega\widehat{TOT} + \left(\frac{1}{\sigma} - \omega\right)(2a_\mathsf{H} - 1)\,\widehat{RER}$$

TOT and relative output (2)

$$\widehat{TOT} = \frac{\sigma}{\left[1 - (2a_{\mathsf{H}} - 1)^2\right] \omega\sigma + (2a_{\mathsf{H}} - 1)^2} \left(\widehat{Y_{\mathsf{H}}} - \widehat{Y_{\mathsf{F}}}\right).$$

Since $0 \leq a_{\mathsf{H}} \leq 1$, the coefficient is always positive: **a positive supply shock at home unambiguously worsens Home terms of trade.**

- Claim: the sign of the response of the terms of trade is a crucial indicator of the degree of risk sharing (see below).

Consumption and relative output

We also obtain

$$\left(\widehat{C}-\widehat{C^*}\right)^{\mathsf{CM}}=\frac{2a_{\mathsf{H}}-1}{\left[1-(2a_{\mathsf{H}}-1)^2\right]\omega\sigma+(2a_{\mathsf{H}}-1)^2}\left(\widehat{Y_{\mathsf{H}}}-\widehat{Y_{\mathsf{F}}}\right)$$

With Home Bias, the coefficient above is always positive. In response to a Home supply shock, consumption grows more at Home than abroad. Even if the Home terms of trade fall, it will never be the case that their adverse movements cause 'immiserizing growth.' In response to a positive supply shocks, domestic consumption will never fall either in absolute level, or relative to Foreign Consumption.

However, the consumption growth difference tends to fall with the elasticity of substitution among goods (ω).

3 Incomplete markets: financial autarky (FA)

If markets are incomplete, supply shocks drive a wedge between domestic and foreign wealth, leading to a much richer array of results relative to the case of efficient risk insurance. In what follows we will place (general-equilibrium) wealth effects, and their implications for relative domestic demand, at the center of our analysis.

We will first consider the case of financial autarky.

Consumption demand in financial autarky

In financial autarky, trade must be balanced $TOT \cdot C_{\mathsf{F}} = C^*_{\mathsf{H}}$ and consumption expenditure has to equal current income, i.e., $PC = Y_{\mathsf{H}} P_{\mathsf{H}}$. Using the latter, domestic demand for Home goods can be written:

$$C_{\mathsf{H}} = a_{\mathsf{H}} \left(\frac{P_{\mathsf{H}}}{P} \right)^{-\omega} C = a_{\mathsf{H}} \left(\frac{P_{\mathsf{H}}}{P} \right)^{-\omega} \frac{P_{\mathsf{H}}}{P} Y_{\mathsf{H}} = \frac{a_{\mathsf{H}}}{a_{\mathsf{H}} + (1 - a_{\mathsf{H}})\, TOT^{1-\omega}} Y_{\mathsf{H}}$$

where the elasticity of substitution across the two goods ω is the demand's price elasticity. Analogous expressions can be derived for the Foreign country.

Keep in mind that equilibrium change in $\frac{P_{\mathsf{H}}}{P}$ modifies the relative price faced by Home households as consumers, but also the value of Home output relative to Foreign output: for this reason, the income effects analyzed below are different from the effects relevant to partial equilibrium analysis.

Substitution and income effects of from international price movements (1)

$$C_{\mathsf{H}} = a_{\mathsf{H}} \left(\frac{P_{\mathsf{H}}}{P}\right)^{-\omega} C = a_{\mathsf{H}} \left(\frac{P_{\mathsf{H}}}{P}\right)^{-\omega} \frac{P_{\mathsf{H}}}{P} Y_{\mathsf{H}} = \frac{a_{\mathsf{H}}}{a_{\mathsf{H}} + (1 - a_{\mathsf{H}})\, TOT^{1-\omega}} Y_{\mathsf{H}}$$

- A fall in the relative price of the domestic tradable P_H/P (a deterioration of TOT) raises domestic demand by ω:
  - substitution effect (SE) is positive: $SE > 0$.
- But for an unchanged Y_H, consumption C falls by 1 (recall $C = Y_H P_H / P$).
  - income effect (IE) is negative $IE < 0$.

Substitution and income effects ...(2)

To see these two effects:

$$\frac{\partial C_{\mathsf{H}}}{\partial TOT} = \overbrace{\left[\omega \cdot \left(\frac{a_{\mathsf{H}}\left(1-a_{\mathsf{H}}\right)TOT^{-\omega}}{\left[a_{\mathsf{H}}+\left(1-a_{\mathsf{H}}\right)TOT^{1-\omega}\right]^{2}}Y_{\mathsf{H}}\right)\right]}^{\text{substitution effect}}$$

$$-\overbrace{\left[1 \cdot \left(\frac{a_{\mathsf{H}}\left(1-a_{\mathsf{H}}\right)TOT^{-\omega}}{\left[a_{\mathsf{H}}+\left(1-a_{\mathsf{H}}\right)TOT^{1-\omega}\right]^{2}}Y_{\mathsf{H}}\right)\right]}^{\text{income effect}} =$$

Clearly

$$\frac{\partial C_{\mathsf{H}}}{\partial TOT} > 0 \iff \omega > 1$$

Substitution and income effects ...(3)

- When $\omega > 1$, the deterioration of TOT will raise domestic demand for the Home good. SE stronger than IE in absolute value.

- when $\omega < 1$ the negative IE will more than offset SE. Thus, terms-of-trade depreciation will reduce the domestic demand for the Home tradable.

Substitution and income effects...(4)

Abroad, foreign demand for Home tradables C_H^*, instead, will always be increasing in TOT. Independently of ω, the substitution and income effects in this case are both positive. $SE^* > 0$, $IE^* > 0$:

$$C_H^* = (1 - a_H) \left(\frac{P_H^*}{P^*} \right)^{-\omega} C^* = (1 - a_H) \left(\frac{P_H^*}{P^*} \right)^{-\omega} \frac{P_F^*}{P^*} Y_F^* =$$

$$= \frac{(1 - a_H)\, TOT^{\omega}}{a_H + (1 - a_H)\, TOT^{\omega - 1}} Y_F^*$$

Substitution and income effects...(5)

Formally:

$$\frac{\partial C_{\mathsf{H}}^{*}}{\partial TOT} = \omega \left(1 - a_{\mathsf{H}}^{*}\right) TOT^{1-\omega} \frac{a_{\mathsf{H}}^{*}}{\left[\left(1 - a_{\mathsf{H}}^{*}\right) TOT^{1-\omega} + a_{\mathsf{H}}^{*}\right]^{2}} Y_{\mathsf{F}}^{*}$$

$$+ a_{\mathsf{H}}^{*} \frac{a_{\mathsf{H}}^{*}}{\left[\left(1 - a_{\mathsf{H}}^{*}\right) TOT^{1-\omega} + a_{\mathsf{H}}^{*}\right]^{2}} Y_{\mathsf{F}}^{*} =$$

$$=$$

$$SE^{*} + IE^{*} > 0$$

The shape of the world demand for Home goods $C_H + C_H^*$

Combining the above results:

- As long as the negative Income Effect in the Home country is not too strong, the world demand for Home goods $C_H + C_H^*$ will be decreasing in their relative price, i.e. increasing in TOT.

- When ω is sufficiently below 1 and the Home bias in consumption is sufficiently high (i.e., a_H is large relative to a_H^*), instead, the opposite occurs. World demand is falling in TOT.

From world demand to general equilibrium effects of endowment shocks.

Let's now trace the general equilibrium implications of our characterization of the world demand for Home tradable goods. Since $TOT = C_H^*/C_F$, taking a log-linear approximation around a symmetric equilibrium we obtain:

$$\widehat{TOT} = \frac{1}{1 - 2a_H\,(1-\omega)}\left(\widehat{Y_H} - \widehat{Y_F^*}\right)$$

$$\widehat{RER} = \frac{2a_H - 1}{1 - 2a_H(1-\omega)}\left(\widehat{Y_H} - \widehat{Y_F^*}\right)$$

Focus on the sign of the coefficient on the r.h.s. of the above expressions.

International transmission and price volatility (1)

A positive innovation to $\widehat{Y_H}$ worsens the terms of trade and depreciates the real exchange rate as long as

$$\omega > 1 - \frac{1}{2a_H}$$

In this region:

- the international transmission is positive, in the sense that a positive Home output shock benefits foreign consumers via better import prices;.

- RER and TOT volatility decreases when you raise ω.

International transmission and price volatility (2)

With Home bias, however, a positive innovation to Y_H can actually *appreciate* TOT and RER when

$$1 - 2a_H(1 - \omega) < 0 \implies 0 < \omega < 1 - \frac{1}{2a_H}$$

which is less than $1/2$.

- *This is a case of negative transmission under incomplete markets*: a positive output innovation actually hurts foreign consumers, as the price of Home goods rises.

- In this region volatility decreases when you lower ω.

Why negative spillovers from positive output shocks?

With $\omega < 1 - \frac{1}{2a_H}$ (i.e. less than 1/2 with positive Home Bias), the world demand for Home goods is upward sloping. For a positive supply shock to Y_H to be matched by an increase in world demand, the Home terms of trade needs to *appreciate*.

Strong wealth effects in incomplete markets make it possible to have *negative* terms of trade transmission.

- Note: in this simple setting, strong income effects (and an upward sloping demand) raise the possibility of multiple steady states (e.g., see the discussion in Corsetti and Dedola [JIE 2005]).

Price volatility and elasticities

We have seen above that the volatility of RER and TOT is non monotonic in ω: it is increasing in ω for $\omega < 1 - \frac{1}{2a_H}$. Then volatility becomes decreasing in ω.

An important implication is that there will be two values of ω (below and above $1 - \frac{1}{2a_H}$) that yield the same volatility of the terms of trade and real exchange rate: one associated with positive, the other associated with negative international transmission.

Moreover, the response of international relative prices to output shocks tend to become stronger as ω approaches $1 - \frac{1}{2a_H}$ from either side.

RER and relative consumption (1)

Use the balanced-trade condition to derive an expression for relative consumption as a function of the terms of trade:

$$TOT \cdot C_F = C_H^* \Longleftrightarrow \frac{C}{C^*} = RER^{\omega} TOT^{\omega-1}$$

Then, using the relation between TOT and RER), derive the log-linearized relationships between relative consumption and the real exchange rate:

$$\widehat{RER} = \frac{2a_H - 1}{2a_H\omega - 1}\left(\widehat{C} - \widehat{C^*}\right).$$

The relation between real exchange rates and relative consumption can have either sign, depending on the values of a_H and ω. Specifically, with Home Bias in consumption, the correlation C/C^* and RER will be negative for $\omega < \dfrac{1}{2a_H} < 1$.

Transmission and risk sharing (1)

Three cases:

1. For $\omega > \dfrac{1}{2a_H}$, a positive Home output shock depreciates the Home terms of trade, to the benefit of Foreign consumers (positive transmission). Relative consumption and the real exchange rate are positively correlated. Both this correlation, and international price movements, have the same sign as under complete markets. This is the conventional view of transmission.

Transmission and risk sharing (2)

2. For $\omega < 1 - \dfrac{1}{2a_H}$, the international transmission is negative. A positive Home output shocks appreciates the Home terms of trade, and the real exchange rate. Home consumption rises relative to Foreign consumption, which falls. Price movements and relative consumption - RER correlation have the opposite sign with respect to the case of complete markets.

3. The case of $1 - \dfrac{1}{2a_H} < \omega < \dfrac{1}{2a_H}$ is quite special. Transmission is positive: actually, the fall in the Home terms of trade is so large, that Foreign consumers benefit from a Home supply shocks more than domestic consumers. Foreign consumption rises relative to domestic consumption, as the Home real exchange rate depreciates.

To sum up: in response to domestic supply shocks (1)

1. if $\omega > \dfrac{1}{2a_H}$: the correlation between $\{C/C^*,\ RER\}$ is positive; Home terms of trade TOT depreciate. Positive transmission reduces consumption risk of from output shocks.

2. if $\omega < 1 - \dfrac{1}{2a_{\mathsf{H}}}$: the correlation between $\{C/C^*,\ RER\}$ is negative; TOT appreciates. Negative transmission raises consumption risk.

3. if $\dfrac{1}{2a_{\mathsf{H}}} > \omega > 1 - \dfrac{1}{2a_{\mathsf{H}}}$ the correlation between $\{C/C^*,\ RER\}$ is negative. TOT still depreciates. 'Excessively' positive transmission raises consumption risk.

Contrast complete markets and financial autarky (1)

$$\left(\widehat{C} - \widehat{C^*}\right)^{\text{FA}} = \frac{2a_H\omega - 1}{1 - 2a_H(1-\omega)}\left(\widehat{Y_H} - \widehat{Y_F^*}\right),$$

$$\left(\widehat{C} - \widehat{C^*}\right)^{\text{CM}} = \frac{2a_H - 1}{\left[1 - (2a_H - 1)^2\right]\omega\sigma + (2a_H - 1)^2}\left(\widehat{Y_H} - \widehat{Y_F^*}\right)$$

Allocations coincide when $a_H = 1/2$ (no Home bias) and $\omega = 1$ (Cobb-Douglas consumption aggregator). This is an important result derived by Cole H. and M. Obstfeld (JME 1991), “Commodity Trade and International Risk Sharing: How Much Do Financial Markets Matter?”

Contrast complete markets and financial autarky (2)

$$\frac{2a_H - 1}{2a_H\omega - 1}\left(\widehat{C} - \widehat{C*}\right)^{\text{FA}} = \left(\widehat{RER}\right)^{\text{FA}}$$

$$\sigma\left(\widehat{C} - \widehat{C*}\right)^{\text{CM}} = \left(\widehat{RER}\right)^{\text{CM}}$$

In the Cole and Obstfeld economy ($a_H = 1/2$ (no Home bias) and $\omega = 1$) the two conditions coincide.

Note however that the two conditions also coincides when parameters values happen to satisfy:

$$\frac{1}{\sigma}(2a_H - 1) = 2a_H\omega - 1.$$

Homework: is consumption risk efficiently shared in this case?

Cole and Obstfeld: terms of trade movements make trade in assets redundant

In the C&O economy, terms of trade movements automatically pool endowment (output) risk.

- When the endowment of a country rises, its price fall proportionally, leaving relative wealth unaffected.
- Domestic consumers are richer because of the higher endowment. Foreign consumers are richer because of the lower price of imports.

Implications and questions (1)

- An explanation of low international diversification of Households' and firms' portfolio (i.e. Home bias in portfolio composition)?
  - if terms of trade already provide automatic risk insurance, gains from trade in assets may be small. Small transaction costs may discourage portfolio diversification.

- Terms of trade and (tradable) output should have the same volatility. The former are more volatile than output though.

Implications and questions (2)

- In the model, spillovers of domestic productivity shocks positive. Even if the Cole-Obstfeld result is extremely fragile, it helps identifying a specific role of terms of trade in providing insurance against output risk.
  - But do productivity gains in one country benefit other countries via terms of trade movements? Do terms of trade movements reduce or magnify consumption risk of productivity shocks?

4 Incomplete markets: trade in noncontingent bonds

Now we allow agents to trade in one noncontingent bond, i.e. to borrow and lend across borders.

Wealth effects will be a function of (a) persistence of shock (b) elasticity of substitution and (c) home bias.

- First, we generalize results obtained under financial autarky, about negative transmission with a low trade elasticity for permanent shocks;

- Second, we derive a new result, specific to bond economies.

TOT and output shocks: short vs. long run effects

As in Corsetti Dedola and Leduc (2006), one can derive tractable analytical expressions by positing log utility ($\sigma = 1$) and setting the discount rate equal to 0 (the discount factor equal to 1). Under these assumptions, one can derive:

$$\widehat{TOT}_t = \frac{\overbrace{\left(\widehat{Y_{\mathsf{h},t}} - \widehat{Y_{\mathsf{h}}^{LR}}\right) - \left(\widehat{Y_{\mathsf{f},t}} - \widehat{Y_{\mathsf{f}}^{LR}}\right)}^{\text{short run}}}{1 - 4a_{\mathsf{H}}\left(1 - \alpha_{\mathsf{h}}\right)\left(1 - \omega\right)} + \frac{\overbrace{\widehat{Y_{\mathsf{h}}^{LR}} - \widehat{Y_{\mathsf{f}}^{LR}}}^{\text{long run}}}{1 - 2\alpha_{\mathsf{h}}\left(1 - \omega\right)}$$

where $\widehat{Y_{\mathsf{h}}^{LR}}$ denotes the percentage deviation of Home output from the initial steady state equilibrium in the long-run.

Permanent shocks with no dynamics

Consider the case of permanent shocks with no dynamics $\widehat{Y_{\mathsf{h},t}} = \widehat{Y_{\mathsf{h}}^{LR}}$. Agents have no incentive to smooth consumption. Clearly, the response of TOT is the same as in financial autarky:

$$\widehat{TOT} = \frac{\widehat{Y_{\mathsf{h}}^{LR}} - \widehat{Y_{\mathsf{f}}^{LR}}}{1 - 2\alpha_{\mathsf{h}}\left(1 - \omega\right)}$$

This result confirms out analysis in the previous section.

A new result: negative transmission with high elasticity and persistent shocks

However, there is now a new configuration of parameters' value which give rise to negative transmission in the short run.

Consider the case $0 < \widehat{Y_{\mathsf{h},t}} < \widehat{Y_{\mathsf{h}}^{LR}}$. With $\omega > 1$, the short-run component of the terms of trade response is unambiguously negative; the long-run component unambiguously positive. When the trade elasticity is high enough, and the difference between long-run and short-run output is large, the former component dominates.

Supply shocks with the above features induce a dynamic response of the terms of trade: appreciation in the short run, depreciation in the long run.

Why a high elasticity?

Intuitively, with $\omega > 1$ the depreciation of the terms of trade in the long run is less than proportional to the change in endowment. Hence in the long run the value of the Home output rises relative to world output. This translates into a positive wealth effect generating a short-run domestic consumption boom. Because of home bias, domestic consumption falls disproportionately on domestic goods, whose supply rises less in the short run than in the long run. Unless the short-run gains in output are already large, the domestic consumption boom creates excess demand for the Home goods, triggering an impact appreciation of the terms of trade. Over time, as the dynamic of Home output endowment fills the gap with demand, the terms of trade appreciation switches to a depreciation (also relative to the initial equilibrium).

Relative consumption and the real exchange rate

Under the simplifying assumptions stated above, we can rewrite the relation between relative consumption and the real exchange rate distinguishing a short-run from a long-run component:

$$\left(\widehat{C}_t - \widehat{C_t^*}\right) = \widehat{RER}_t + \frac{2\alpha_{\mathsf{H}}(\omega - 1)}{2\alpha_{\mathsf{H}} - 1}\widehat{RER}^{LR}.$$

Endowment shocks that generate a domestic consumption boom and appreciate the Home terms of trade will induce a negative correlation between relative consumption and the real exchange rate in the short run (recall that the real exchange rate and terms of trade move in the same direction). However, this correlation will not be perfect. A trade elasticity larger than one implies that the second term in the above expression is positive, as the real exchange rate has to depreciate in the long run.

A generalization

Write the Euler equation for international bonds in general, equating the expected rate of real depreciation to the expected growth rate in marginal utilities of consumption:

$$E_t\left(\widehat{RER}_{t+1} - \widehat{RER}_t\right) \approx E_t\left[\left(\widehat{U}^*_{c,t+1} - \widehat{U}^*_{c,t}\right) - \left(\widehat{U}_{c,t+1} - \widehat{U}_{c,t}\right)\right].$$

whereas for simplicity we abstract from the discount factor. By trading bonds, agents ensure that, in expectations, real depreciation is associated with higher consumption growth in the domestic economy relative to consumption growth abroad (precisely, lower relative growth in the marginal utility of consumption). Now, to the extent that the tight link between consumption growth rates is inherited by consumption levels, the above expression suggests that international borrowing and lending implies that the correlation between relative consumption and the real exchange rate is positive over time.

The impact response of consumption

But in a stochastic environment, the international bond is traded only after the resolution of uncertainty, and does not provide households with ex-ante insurance against country-specific income shocks — it only makes it possible to reallocate wealth and smooth consumption over time. If unexpected shocks have large wealth effects, relative consumption and the real exchange rate can move in the opposite direction on impact.

With strong wealth effects, the real exchange rate and relative consumption comove negatively on impact, but positively in the aftermath of the shock, when their joint dynamics is dictated by the equation above.

A second generalization: *From endowment economies to production economies*

A important lesson from the bond-economy is that, with incomplete markets, 'interesting dynamics' of the terms of trade, real exchange rate and relative consumption obtain when output increases gradually over time, and the elasticity of substitution is relatively high. Anticipating output and income gains in the future, consumption-smoothing agents raise current demand above output, causing a temporary appreciation of the terms of trade and the real exchange rate (and a current account deficit).

In an endowment economy, *autoregressive endowment shocks* cannot generate the above output pattern. In a production economy, however, *autoregressive productivity shocks* can, provided that they are persistent. Why? Driven by sustained productivity growth, investment increases in the Home country. As capital stock rises over time, equilibrium output correspondingly increases. See CoDeLe 2006.

5 Homework: An economy with nontradables under financial autarky

Let N denote nontradables. Introduce now consumption aggregator:

$$C = \left[a_{\mathsf{T}}^{1-\xi} C_{\mathsf{T}}^{\xi} + (1 - a_{\mathsf{T}})^{1-\xi} \, C_{\mathsf{N}}^{\xi}\right]^{\frac{1}{\xi}}$$

where T is in turn the aggregator of Tradable. The relative demand for tradables and nontradables is:

$$\frac{C_{\mathsf{T}}}{C_{\mathsf{N}}} = \frac{a_{\mathsf{T}}}{1 - a_{\mathsf{T}}} \left(\frac{P_{\mathsf{T}}}{P_{\mathsf{N}}}\right)^{-\frac{1}{1-\xi}}, \quad \frac{C_{\mathsf{T}}^{*}}{C_{\mathsf{N}}^{*}} = \frac{a_{\mathsf{T}}}{1 - a_{\mathsf{T}}} \left(\frac{P_{\mathsf{T}}^{*}}{P_{\mathsf{N}}^{*}}\right)^{-\frac{1}{1-\xi}};$$

You can derive the following price aggregator:

$$P = \left[a_{\mathsf{T}} P_{\mathsf{T}}^{\frac{\xi}{\xi-1}} + (1 - a_{\mathsf{T}}) P_{\mathsf{N}}^{\frac{\xi}{\xi-1}} \right]^{\frac{\xi-1}{\xi}};$$

Under financial autarky, the market clearing conditions are:

$$\frac{C_{\mathsf{T}}}{C_{\mathsf{N}}} = \frac{Y_{\mathsf{H}}}{Y_{\mathsf{N}}} \frac{P_{\mathsf{H}}}{P_{\mathsf{T}}}, \qquad \frac{C_{\mathsf{T}}^*}{C_{\mathsf{N}}^*} = \frac{Y_{\mathsf{F}}}{Y_{\mathsf{N}}^*} \frac{P_{\mathsf{F}}^*}{P_{\mathsf{T}}^*}.$$

The real exchange rate is:

$$\log RER = \log \frac{P_{\mathsf{T}}^*}{P_{\mathsf{T}}} + \frac{\xi - 1}{\xi} \log \frac{a_{\mathsf{T}} + (1 - a_{\mathsf{T}}) \left(\frac{P_{\mathsf{N}}^*}{P_{\mathsf{T}}^*} \right)^{\frac{\xi}{\xi-1}}}{a_{\mathsf{T}} + (1 - a_{\mathsf{T}}) \left(\frac{P_{\mathsf{N}}}{P_{\mathsf{T}}} \right)^{\frac{\xi}{\xi-1}}}.$$

Loglinearizing around a symmetric steady state yields:

$$\begin{aligned}
\widehat{\frac{P_{\mathsf{N}}}{P_{\mathsf{T}}}} &= (1-\xi)\left[\left(\widehat{Y_{\mathsf{H}}}-\widehat{Y_{\mathsf{N}}}\right)-(1-a_{\mathsf{H}})\,\widehat{TOT}\right], \\
\widehat{\frac{P_{\mathsf{N}}^{*}}{P_{\mathsf{T}}^{*}}} &= (1-\xi)\left[\left(\widehat{Y_{\mathsf{F}}}-\widehat{Y_{\mathsf{N}}^{*}}\right)+(1-a_{\mathsf{H}})\,\widehat{TOT}\right], \\
\widehat{\frac{P_{\mathsf{T}}^{*}}{P_{\mathsf{T}}}} &= (2a_{\mathsf{H}}-1)\,\widehat{TOT}, \\
\widehat{RER} &= \left[(2a_{\mathsf{H}}-1)+2\,(1-a_{\mathsf{H}})\,(1-a_{\mathsf{T}})\,(1-\xi)\right]\widehat{TOT}- \\
&\quad (1-a_{\mathsf{T}})\,(1-\xi)\left(\widehat{Y_{\mathsf{H}}}-\widehat{Y_{\mathsf{F}}}\right)+(1-a_{\mathsf{T}})\,(1-\xi)\left(\widehat{Y_{\mathsf{N}}}-\widehat{Y_{\mathsf{N}}^{*}}\right)
\end{aligned}$$

$$\widehat{TOT} = \frac{\left(\widehat{Y_{\mathsf{H}}} - \widehat{Y_{\mathsf{F}}}\right)}{1 - 2a_{\mathsf{H}}\left(1 - \omega\right)}$$

$$\begin{aligned} \widehat{RER} \;&=\; \left[\left(2a_{\mathsf{H}} - 1\right) + 2\left(1 - a_{\mathsf{H}}\right)\left(1 - a_{\mathsf{T}}\right)\left(1 - \xi\right)\right]\widehat{TOT} \\ &\quad - \left(1 - a_{\mathsf{T}}\right)\left(1 - \xi\right)\left(\widehat{Y_{\mathsf{H}}} - \widehat{Y_{\mathsf{F}}}\right) + \left(1 - a_{\mathsf{T}}\right)\left(1 - \xi\right)\left(\widehat{Y_{\mathsf{N}}} - \widehat{Y_{\mathsf{N}}^{*}}\right), \end{aligned}$$

$$\begin{aligned} \widehat{C} - \widehat{C^{*}} \;&=\; -\frac{a_{\mathsf{T}}^{1-\xi}}{a_{\mathsf{T}}^{1-\xi} + \left(1 - a_{\mathsf{T}}\right)^{1-\xi}} 2\left(1 - a_{\mathsf{H}}\right)\widehat{TOT} + \\ &\quad \frac{a_{\mathsf{T}}^{1-\xi}}{a_{\mathsf{T}}^{1-\xi} + \left(1 - a_{\mathsf{T}}\right)^{1-\xi}}\left(\widehat{Y_{\mathsf{H}}} - \widehat{Y_{\mathsf{F}}}\right) + \frac{\left(1 - a_{\mathsf{T}}\right)^{1-\xi}}{a_{\mathsf{T}}^{1-\xi} + \left(1 - a_{\mathsf{T}}\right)^{1-\xi}}\left(\widehat{Y_{\mathsf{N}}} - \widehat{Y_{\mathsf{N}}^{*}}\right). \end{aligned}$$

- If you want, derive the above conditions. The derivation is however not required to answer the following questions.

- What is the response of TOT, the relative price of nontradables and RER to a shock to the endowment of tradables?

- What is the response of the relative price of Nontradable and the real exchange rate to a positive shock to the endowment of nontradables?

  – Do you agree with the following claim: the Harrod Balassa Samuelson Hypothesis states that a positive shock to productivity in the tradable sector appreciates the real exchange rate?

- Can shocks to tradables generate a negative correlation between relative consumption and the real exchange rate?

- How about shocks to nontradables? Can they generate a negative correlation?