International Macroeconomic Fluctuations

- International business cycles

“The Facts”

Aspects of international business cycles

1. Business cycles are “similar” across rich countries

- Backus, Kehoe and Kydland, Quarterly Review, Fed of Minneapolis, 1995

- OECD economies, quarterly data, HP-filtered, 1970-1995

- Large amount of commonalities

Table 1

Business Cycles in 10 Developed Countries*

1970–mid-1990

	Volatility							Persistence:	Comovement: Correlation With Output					
	Standard Deviation		Ratio of Standard Deviation to That of Output					Auto-correlation of Output						
Country	Output	Net Exports	Consumption	Investment	Government Purchases	Employment	Productivity Shock		Consumption	Investment	Government Purchases	Net Exports	Employment	Productivity Shock
Australia	1.45%	1.23%	.66	2.78	1.28	.34	1.00	.60	.46	.68	.15	–.01	.12	.98
Austria	1.28	1.15	1.14	2.92	.36	1.23	.84	.57	.65	.75	–.24	–.46	.58	.65
Canada	1.50	.78	.85	2.80	.77	.86	.74	.79	.83	.52	–.23	–.26	.69	.84
France	.90	.82	.99	2.96	.71	.55	.76	.78	.61	.79	.25	–.30	.77	.96
Germany	1.51	.79	.90	2.93	.81	.61	.83	.65	.66	.84	.26	–.11	.59	.93
Italy	1.69	1.33	.78	1.95	.42	.44	.92	.85	.82	.86	.01	–.68	.42	.96
Japan	1.35	.93	1.09	2.41	.79	.36	.88	.80	.80	.90	–.02	–.22	.60	.98
Switzerland	1.92	1.32	.74	2.30	.53	.71	.67	.90	.81	.82	.27	–.68	.84	.93
United Kingdom	1.61	1.19	1.15	2.29	.69	.68	.88	.63	.74	.59	.05	–.19	.47	.90
United States	1.92	.52	.75	3.27	.75	.61	.68	.86	.82	.94	.12	–.37	.88	.96
Europe	1.01%	.50%	.83	2.09	.47	.85	.98	.75	.81	.89	.10	–.25	.32	.85

*All data are quarterly and have been detrended with the Hodrick-Prescott filter. All but net exports have also been logged. The specific variables included are real output, real consumption; real fixed investment; real government purchases; the ratio of net exports to output, both measured in current prices; civilian employment; and a productivity shock, which is the Solow residual, as defined in equations (1) and (2).

Figure 1:

- exports and imports as volatile as output; slight tendency for higher volatility of imports than of exports

- exports and imports both procyclical

- but imports uniformly more procyclical than exports and this explains countercyclical net-exports

TABLE 2. Moments of open economy variables

Country	s(ex)/s(y)	s(im)/s(y)	s(nx/y)	cor(ex,y)	cor(im,y)	cor(nx/y,y)
France	2.77	4.16	0.80	0.62	0.78	−0.42
	(0.19)	(0.26)	(0.06)	(0.07)	(0.04)	(0.10)
Australia	3.16	5.93	1.56	0.17	0.28	−0.22
	(0.34)	(0.91)	(0.15)	(0.12)	(0.09)	(0.09)
Canada	2.66	3.20	0.95	0.67	0.82	−0.31
	(0.22)	(0.16)	(0.07)	(0.05)	(0.05)	(0.10)
Italy	2.31	2.77	0.91	0.25	0.74	−0.54
	(0.28)	(0.22)	(0.07)	(0.09)	(0.06)	(0.08)
Japan	3.48	4.82	0.88	−0.04	0.54	−0.49
	(0.44)	(0.55)	(0.07)	(0.10)	(0.07)	(0.08)
Switzerland	1.65	2.46	1.13	0.66	0.80	−0.50
	(0.16)	(0.24)	(0.08)	(0.06)	(0.03)	(0.07)
UK	1.82	2.50	1.05	0.47	0.62	−0.31
	(0.22)	(0.22)	(0.08)	(0.08)	(0.06)	(0.09)
US	2.58	3.10	0.56	0.34	0.78	−0.40
	(0.28)	(0.22)	(0.03)	(0.08)	(0.04)	(0.08)
Germany	2.37	1.85	0.99	0.44	0.66	−0.88
	(0.25)	(0.14)	(0.07)	(0.07)	(0.06)	(0.11)
Sweden	3.02	3.42	1.59	0.33	0.26	0.03
	(0.30)	(0.37)	(0.16)	(0.10)	(0.10)	(0.10)

All data Hodrick–Prescott filtered. Standard errors computed by GMM. 's' denotes standard deviation. Numbers in parentheses are standard errors; 'cor' denotes correlation; ex: exports; im: imports; nx/y: net-exports to output.

International Comovements

Backus, Kehoe and Kydland, JPE, 1992:

- examine contemporaneous cross-country comovements amongst a sample of OECD economies

- focus on bilateral cross-correlations of output and consumption with the United States

- HP-filtered quarterly data, 1960's-1990

Country	With Same U.S. Variable		S I
	Output	Consumption	
Australia	.25	.13	
Austria	.31	.07	
Canada	.77	.65	
Finland	.02	−.01	
France	.22	−.18	
Germany	.42	.39	
Italy	.39	.25	
Japan	.39	.30	
South Africa	−.15	−.23	
Switzerland	.27	.25	
United Kingdom	.48	.43	
United States	1.00	1.00	
Europe	.70	.46	

Figure 3:

- positive cross-country comovements in bulk of cases

Ravn, JIMF, 1997

- correlations across country-pairs rather than vis-a-vis US
- 10 OECD economies, 1970-1992:4

TABLE 1. Cross country correlations of output components

Variable	Mean	Range	Positive	Negative
Output[a]	0.420	[−0.123;0.754]	44(38)	1(0)
Consumption[a]	0.283	[−0.215;0.709]	38(29)	7(2)
Government Spending[a]	0.042	[−0.285;0.435]	28(6)	17(5)
Total investments[a]	0.342	[−0.265;0.698]	41(36)	4(1)
Exports[a]	0.250	[−0.169;0.607]	36(26)	9(0)
Imports[a]	0.340	[−0.191;0.722]	40(34)	5(1)
Employment[b]	0.342	[−0.200;0.715]	35(28)	1(1)
Productivity[b]	0.376	[0.169;0.725]	36(31)	0(0)

Figure 4:

Summary of evidence on comovements:

1. Positive comovements of output components (government spending apart) and of employment within the OECD

2. Ranking of cross-country comovements: BKK - $cor\left(y, y^*\right) > cor\left(z, z^*\right)$, $cor\left(c, c^*\right)$

Single Good Competitive Real Economy with Complete Markets

(or, the simplest model works much better than you think!)

- model of intertemporal trade only: since only a single good - trade simply to smooth in response to idiosyncratic risk

- complete markets: can perfectly insure against idiosyncratic risk

- no money, competitive firms, competitive input markets

- basic model:

1. N countries, here modelled symmetrically with a representative infinitely-lived agent in each economy

2. Each country produces output using CRTS technology subject to technology shocks

3. Government in each country that finances government spending through lump-sum taxation

4. Capital internationally mobile, labor not

5. Complete set of asset markets: Costlessly verifiable, fully enforceable contracts can be written contingent on any state of nature

$$
\begin{aligned}
\max V_{i0} &= E_0 \sum_{t=0}^{\infty} \beta^t u\left(d_{it}^c c_{it}, l_{it}\right) \\
y_{it} &= \exp\left(A_{it}\right) k_{it}^{1-\alpha} n_{it}^{\alpha} \\
\sum_{i=1}^{N} \left(c_{it} + x_{it}\right) &\leq \sum_{i=1}^{N} \left(y_{it} - g_{it}\right) \\
k_{it+1} &= \left(1-\delta\right) k_{it} + v_{it} \\
x_{it} &= v_{it} + \frac{\varphi}{2}\left(\frac{v_{it} - \delta k_{it}}{k^{ss}}\right)^2 \\
l_{it} + n_{it} &= T \\
A_t &= \left[A_{1t}, A_{2t}, .., A_{Nt}\right]' \\
A_{t+1} &= RA_t + \varepsilon_{t+1} \\
\ln g_{it+1} &= \left(1-\rho_g\right) \ln g + \rho_g \ln g_{it} + u_{it} \\
\ln d_{it+1}^c &= \rho_d^c \ln d_{it}^c + h_{it}^c,
\end{aligned}
$$

Computation of Competitive Equilibrium

- exploit welfare theorems and compute Pareto optimal allocation which can be decentralized

- this is a multi-agent economy: use Negishi-Mantel algorithm - planner maximizes weighted sum of agents' utility

- constant welfare weights: complete markets implies no reallocation of wealth along equilibrium path

- welfare weights: Reflect initial wealth distribution

- normalize $\sum_i \Omega_i = 1$

- the planner's problem:

$$
\begin{aligned}
& \max \sum_{i} \Omega_i V_{i0} \\
& \text{subject to} \\
\sum_{i=1}^{N} (c_{it} + x_{it}) &\leq \sum_{i=1}^{N} (y_{it} - g_{it}) \\
y_{it} &= \exp(A_{it})\, k_{it}^{1-\alpha} n_{it}^{\alpha} \\
k_{it+1} &= (1-\delta)\, k_{it} + v_{it} \\
x_{it} &= v_{it} + \frac{\varphi}{2}\left(\frac{v_{it} - \delta k_{it}}{k^{ss}}\right)^2 \\
l_{it} + n_{it} &= T \\
& k_{i0} \text{ given}
\end{aligned}
$$

- first-order conditions:

$$
\begin{aligned}
c_{it} &: \quad \Omega_i d_{it}^c u_c\left(d_{it}^c c_{it}, l_{it}\right) = \lambda_t \\
n_{it} &: \quad \Omega_i u_l\left(d_{it}^c c_{it}, l_{it}\right) = \lambda_t \alpha \exp\left(A_{it}\right) k_{it}^{1-\alpha} n_{it}^{\alpha-1} \\
k_{it+1} &: \quad \lambda_t \left(1 + \frac{\varphi}{k^{ss}}\left(\frac{k_{it+1} - k_{it}}{k^{ss}}\right)\right) = E_t \beta \lambda_{t+1} [(1-\delta) \\
& \quad + (1-\alpha) \exp\left(A_{it+1}\right) k_{it+1}^{-\alpha} n_{it+1}^{\alpha} + \frac{\varphi}{k^{ss}}\left(\frac{k_{it+2} - k_{it+1}}{k^{ss}}\right)] \\
RC1 &: \quad \sum_{i=1}^{N}\left(c_{it} + k_{it+1} - (1-\delta) k_{it} + \frac{\varphi}{2}\left(\frac{k_{it+1} - k_{it}}{k^{ss}}\right)^2\right) \\
&= \quad \sum_{i=1}^{N}\left(\exp\left(A_{it}\right) k_{it}^{1-\alpha} n_{it}^{\alpha} - g_{it}\right) \\
RC2 &: \quad l_{it} + n_{it} = T
\end{aligned}
$$

Interpretations:

1. c_{it} condition: Marginal utility of consumption equalized across countries apart from time-invariant constant that reflect initial wealth. This is the **risk sharing** condition: Due to complete markets, the optimal allocation is the one in which marginal utility of consumption is equalized across locations since shadow price of c_{it} is equal to shadow price of c_{jt} for all i, j.

$$\underbrace{\Omega_i d_{it}^c u_c \left(d_{it}^c c_{it}, l_{it}\right)}_{\substack{\text{constant times} \\ \text{domestic marginal utility}}} = \underbrace{\Omega_j d_{jt}^c u_c \left(d_{jt}^c c_{jt}, l_{jt}\right)}_{\substack{\text{constant times} \\ \text{foreign marginal utility}}}$$

- the condition holds **state-by-state** - complete risk sharing.

- Implies complete insurance against all **idiosyncratic risk** - idiosyncratic risk does not affect λ_t

- Of course, one cannot fully insure against aggregate risk - affects the shadow price! However, even in face of aggregate risk, marginal utilities of consumption equalized across countries.

- In general, this does **NOT** imply consumption equalized across countries state-by-state:

(a) Non-separability between consumption and leisure in utility function: although marginal utility equalized state-by-state, consumption is not.

(b) Taste shocks: Affect marginal utility of consumption - and therefore the allocation of consumption across countries.

2. n_{it} condition:

- MRS between consumption and leisure equals MRT:

$$\frac{u_l\left(d_{it}^c c_{it}, l_{it}\right)}{d_{it}^c u_c\left(d_{it}^c c_{it}, l_{it}\right)} = \alpha \exp\left(A_{it}\right) k_{it}^{1-\alpha} n_{it}^{\alpha-1}$$

- Across countries the ratio of marginal utilities of leisure equals ratio of marginal productivity.

$$\frac{\Omega_i}{\Omega_j} \frac{u_l\left(d_{it}^c c_{it}, l_{it}\right)}{u_l\left(d_{jt}^c c_{jt}, l_{jt}\right)} = \frac{\alpha \exp\left(A_{it}\right) k_{it}^{1-\alpha} n_{it}^{\alpha-1}}{\alpha \exp\left(A_{jt}\right) k_{jt}^{1-\alpha} n_{jt}^{\alpha-1}}$$

- holds state-by-state due to complete markets. It is a risk sharing condition: country with higher marginal productivity are assigned higher marginal utility of leisure (ie must work more).

3. k_{it+1} condition:

- When combined with c_{it} condition

$$\begin{aligned} & d_{it}^{c} u_c \left(d_{it}^{c} c_{it}, l_{it}\right) \left(1 + \frac{\varphi}{k^{ss}} \left(\frac{k_{it+1} - k_{it}}{k^{ss}}\right)\right) \\ = \; & E_t \beta d_{it+1}^{c} u_c \left(d_{it+1}^{c} c_{it+1}, l_{it+1}\right) [(1-\delta) \\ & + (1-\alpha) \exp\left(A_{it+1}\right) k_{it+1}^{-\alpha} n_{it+1}^{\alpha} + \frac{\varphi}{k^{ss}} \left(\frac{k_{it+2} - k_{it+1}}{k^{ss}}\right)] \end{aligned}$$

- marginal utility of consumption times relative price of extra unit of capital equals discounted expected future marginal utility times the gross marginal productivity of capital

- When combined across countries:

$$\frac{\left(1+\frac{\varphi}{k^{ss}}\left(\frac{k_{it+1}-k_{it}}{k^{ss}}\right)\right)}{\left(1+\frac{\varphi}{k^{ss}}\left(\frac{k_{jt+1}-k_{jt}}{k^{ss}}\right)\right)}=E_t\frac{\lambda_{t+1}}{\lambda_{t+1}}$$

$$\frac{\left[(1-\delta)+(1-\alpha)\exp\left(A_{it+1}\right)k_{it+1}^{-\alpha}n_{it+1}^{\alpha}+\frac{\varphi}{k^{ss}}\left(\frac{k_{it+2}-k_{it+1}}{k^{ss}}\right)\right]}{\left[(1-\delta)+(1-\alpha)\exp\left(A_{jt+1}\right)k_{jt+1}^{-\alpha}n_{jt+1}^{\alpha}+\frac{\varphi}{k^{ss}}\left(\frac{k_{jt+2}-k_{jt+1}}{k^{ss}}\right)\right]}$$

- Apart from adjustment costs, expected marginal product of capital equalized across countries

- Adjustment costs means this equalization happens gradually over time

Implications

Case 1: Only technology shocks, logarithmic utility, no adjustment costs

$$u(c_{it}, l_{it}) = \ln c_{it} + B \ln (l_{it})$$

- **Consumption risk sharing now implies that**:

$$\begin{aligned} \Omega_i \frac{1}{c_{it}} &= \Omega_j \frac{1}{c_{jt}} \Rightarrow \\ c_{it} &= \frac{\Omega_i}{\Omega_j} c_{jt} \end{aligned}$$

- ie. consumption perfectly correlated across countries;

- **Intratemporal consumption-leisure condition**:

$$\frac{Bc_{it}}{1-n_{it}} = \alpha \exp\left(A_{it}\right) k_{it}^{1-\alpha} n_{it}^{\alpha-1}$$

- Rise in productivity implies that consumption leisure ratio must rise

- **Cross-country version:**

$$\frac{\Omega_i}{\Omega_j}\frac{1-n_{jt}}{1-n_{it}} = \frac{\exp\left(A_{it}\right) k_{it}^{1-\alpha} n_{it}^{\alpha-1}}{\exp\left(A_{jt}\right) k_{jt}^{1-\alpha} n_{jt}^{\alpha-1}}$$

- (recall that k_t is predetermined): Cross-country allocation of leisure/work determined by relative productivity only. An increase in A_{it}/A_{jt} means an increase in n_{it}/n_{jt}.

- **Investment condition**:

$$\frac{1}{c_{it}} = E_t \beta \frac{1}{c_{it+1}} \left[(1-\delta) + (1-\alpha) \exp\left(A_{it+1}\right) k_{it+1}^{-\alpha} n_{it+1}^{\alpha} \right]$$

- Increase in expected future marginal productivity of capital associated with expected consumption growth.

- **cross-country version:**

$$\begin{aligned} & E_t \beta \lambda_{t+1} \left[(1-\delta) + (1-\alpha) \exp\left(A_{it+1}\right) k_{it+1}^{-\alpha} n_{it+1}^{\alpha} \right] \\ = \; & E_t \beta \lambda_{t+1} \left[(1-\delta) + (1-\alpha) \exp\left(A_{jt+1}\right) k_{jt+1}^{-\alpha} n_{jt+1}^{\alpha} \right] \Rightarrow \end{aligned}$$

$$E_t \beta \lambda_{t+1} \left[\exp\left(A_{it+1}\right) k_{it+1}^{-\alpha} n_{it+1}^{\alpha} \right] = E_t \beta \lambda_{t+1} \left[\exp\left(A_{jt+1}\right) k_{jt+1}^{-\alpha} n_{jt+1}^{\alpha} \right]$$

- ie. cross-country division of capital determined by relative expected marginal utility

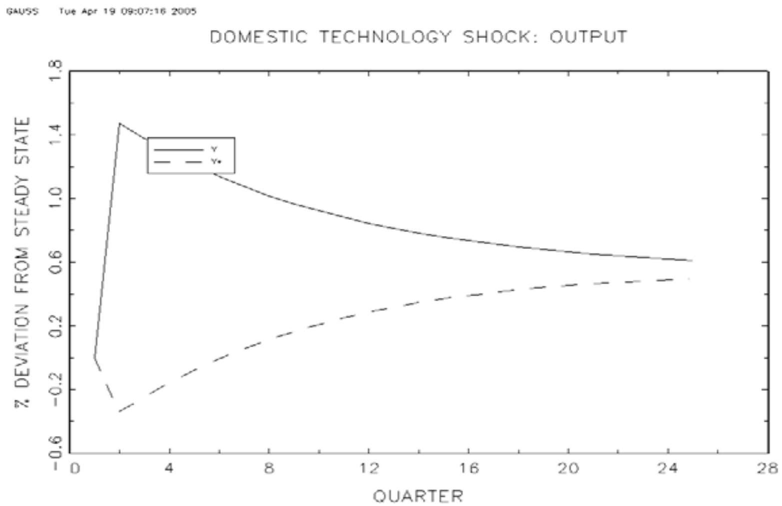
Altogether:

- Perfect cross-country consumption correlation regardless of whether idiosyncratic or aggregate risk

- non-perfectly correlated productivity shocks have a tendency for imply negative cross-country correlation of hours worked: increase in A_{it}/A_{jt} means an increase in n_{it}/n_{jt}—a strong tendency for negative cross-country correlation of output

- Strong tendency for negative cross-country comovements of investment

- More or less opposite of what is observed in the data!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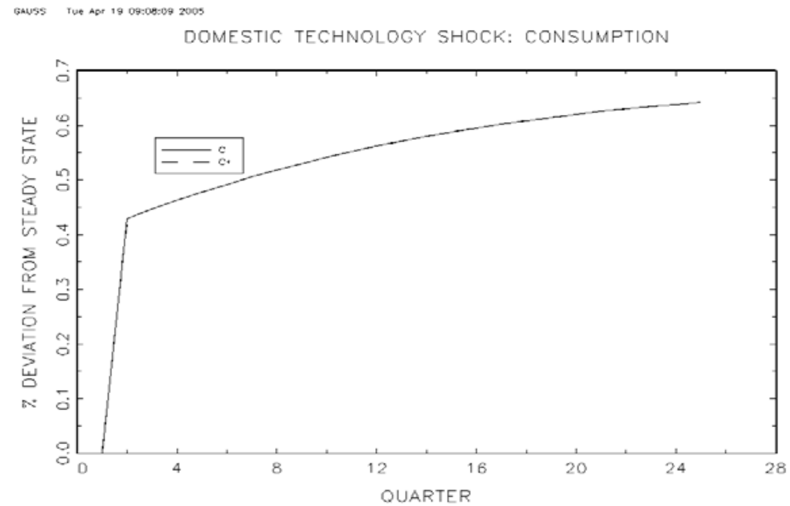
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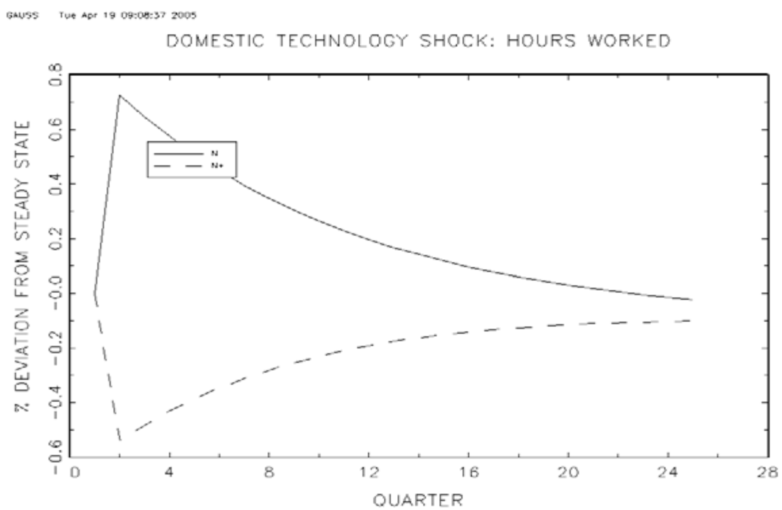
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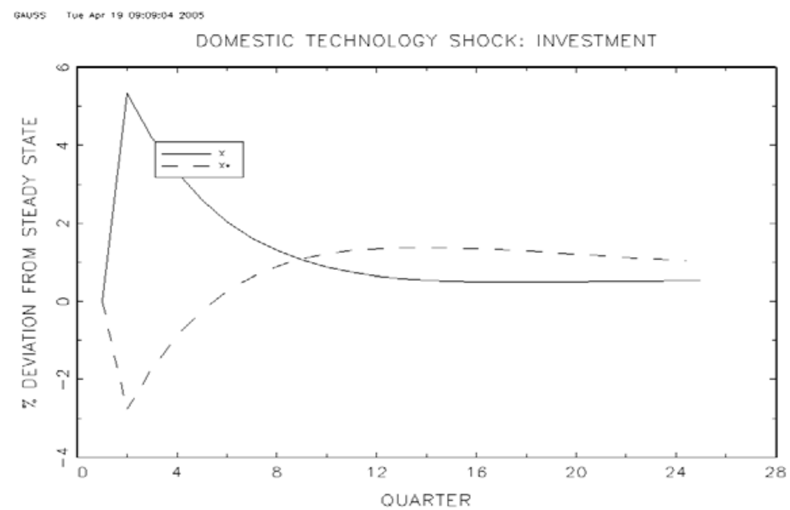
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QUARTER
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X*

Case 2: Only C-taste shocks, logarithmic utility, no adjustment costs

$$u\left(c_{it}, l_{it}\right) = d_{it}^{c} \ln c_{it} + B \ln\left(l_{it}\right)$$

- Consumption risk sharing here implies that:

$$c_{it} = \frac{\Omega_i}{\Omega_j}\frac{d_{it}^{c}}{d_{jt}^{c}} c_{jt}$$

- Relative consumption allocation determined by relative taste shock - No longer perfect consumption correlation: If d_{jt}^{c} is given, rise in d_{it}^{c} accommodated by increasing c_{it} and lowering c_{jt} (thus c_{it} rises less that d_{it}^{c})

- **Intratemporal consumption-leisure condition**:

$$\frac{Bc_{it}}{1-n_{it}} = d_{it}^{c}\alpha \,\mathsf{exp}\left(A\right) k_{it}^{1-\alpha} n_{it}^{\alpha-1}$$

- positive domestic taste shock increase ratio of domestic consumption to leisure - work harder since marginal utility of consumption is high

- **Cross-country version:**

$$\frac{\Omega_i}{\Omega_j}\frac{1-n_{jt}}{1-n_{it}} = \frac{\mathsf{exp}\left(A\right) k_{it}^{1-\alpha} n_{it}^{\alpha-1}}{\mathsf{exp}\left(A\right) k_{jt}^{1-\alpha} n_{jt}^{\alpha-1}}$$

- not affected by C-taste shock: If domestic hours worked change, so do foreign hours worked - risk sharing implication

- **Investment condition**:

$$\frac{d_{it}^{c}}{c_{it}} = E_t \beta \frac{d_{it+1}^{c}}{c_{it+1}} \left[(1-\delta) + (1-\alpha) \exp(A)\, k_{it+1}^{-\alpha} n_{it+1}^{\alpha} \right]$$

- due to increase in d_{it}^{c}/c_{it} and in hours worked, investment declines domestically

- **cross-country version:**

$$E_t \beta \lambda_{t+1} \left[\exp(A)\, k_{it+1}^{-\alpha} n_{it+1}^{\alpha} \right] = E_t \beta \lambda_{t+1} \left[\exp(A)\, k_{jt+1}^{-\alpha} n_{jt+1}^{\alpha} \right]$$

- again: this does not depend on C-taste shock - hence investment dynamics identical across countries

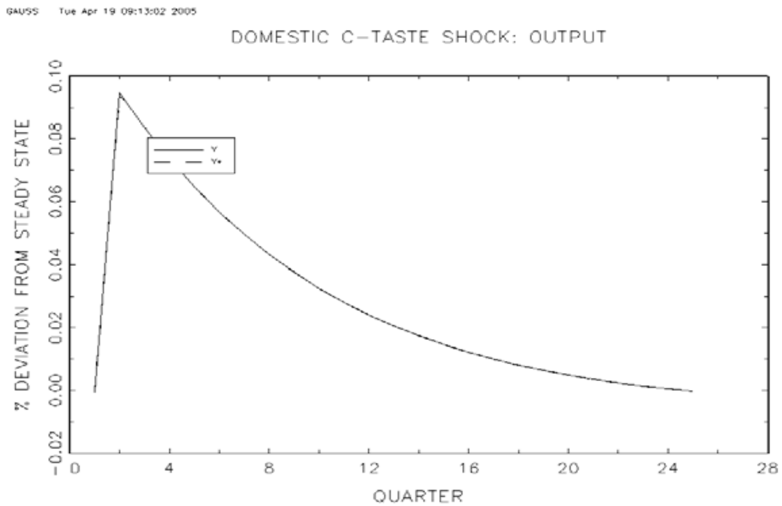
Altogether: C-Taste shocks

- imperfect, even negative consumption correlation

- positive correlation of output, hours worked, and investment (here perfect correlation)

- this brings about countercyclical net-exports and drop in both savings and investment domestically

- taste shocks, thus, may in fact be important! Seem better fit to explain pattern of cross-country correlations than technology shocks!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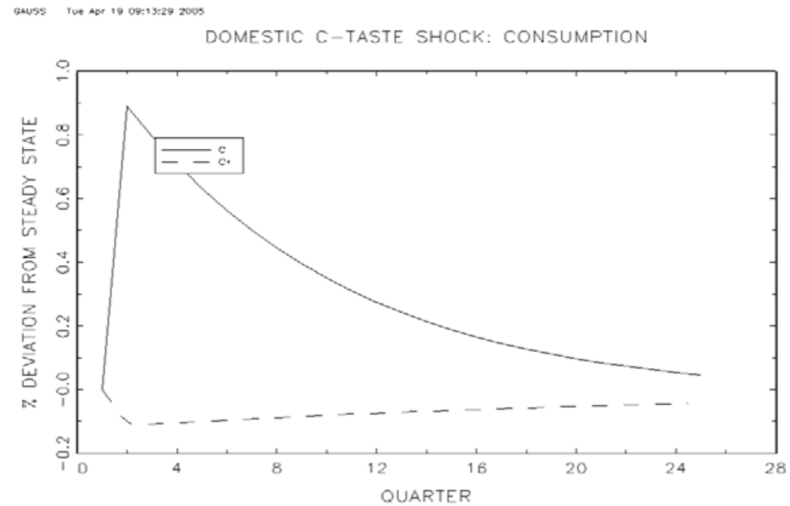
QUARTER
Y
Y*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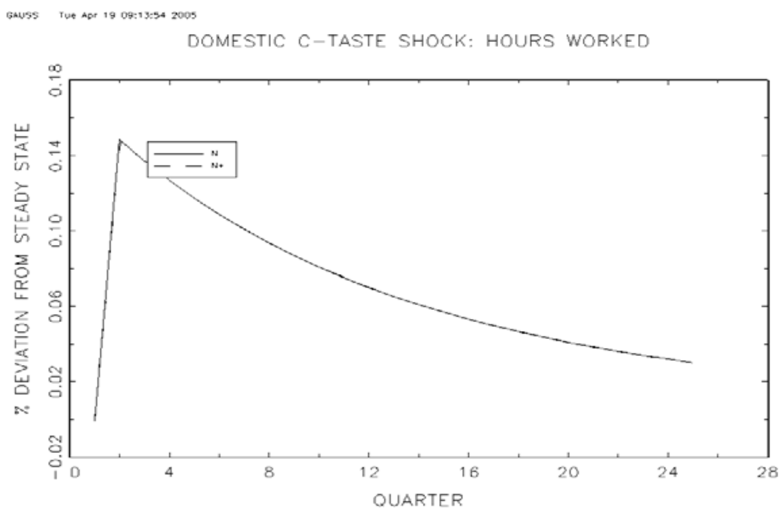
QUARTER
C
C*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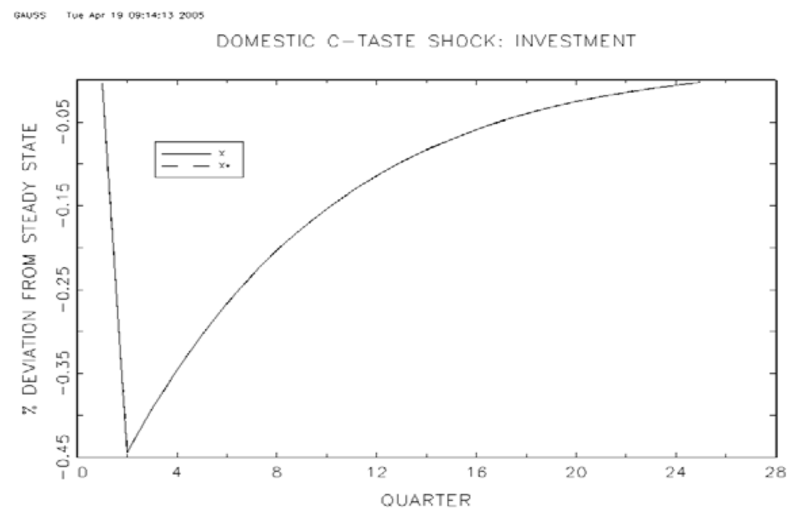
QUARTER
N
N*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DOMESTIC C−TASTE SHOCK: INVESTMENT
% DEVIATION FROM STEADY STATE
−0.45 −0.35 −0.25 −0.15 −0.05
0 4 8 12 16 20 24 28
QUARTER
X
X*

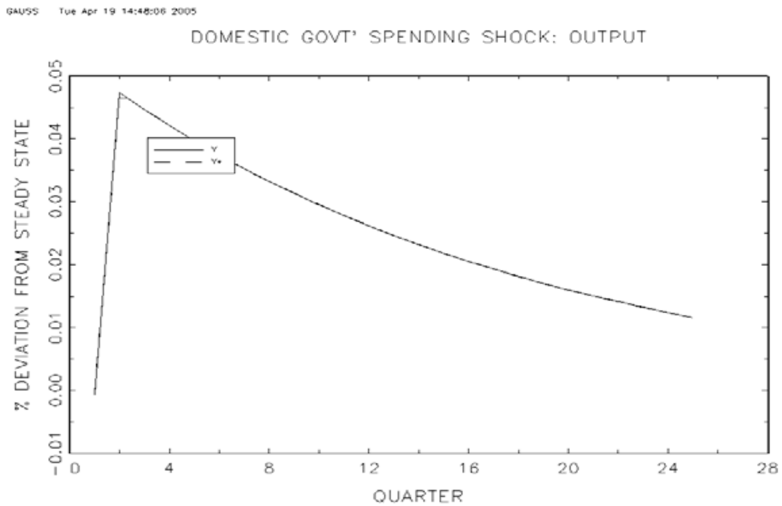
Case 3: Only government spending shocks, logarithmic utility, no adjustment costs

First-order conditions now imply that:

$$
\begin{aligned}
c_{it} &: \quad \Omega_i \frac{1}{c_{it}} = \lambda_t \\
n_{it} &: \quad \Omega_i \frac{B}{1-n_{it}} = \lambda_t \alpha \exp(A)\, k_{it}^{1-\alpha} n_{it}^{\alpha-1} \\
k_{it+1} &: \quad \lambda_t = E_t \beta \lambda_{t+1} \\
&\quad \left[(1-\delta) + (1-\alpha)\exp(A)\, k_{it+1}^{-\alpha} n_{it+1}^{\alpha}\right]
\end{aligned}
$$

- Government spending does not affect the trade off between domestic and foreign consumption, between consumption and leisure, between domestic and foreign hours worked, foreign and domestic investment

- wealth shock that affects λ_t and affect the two economies identically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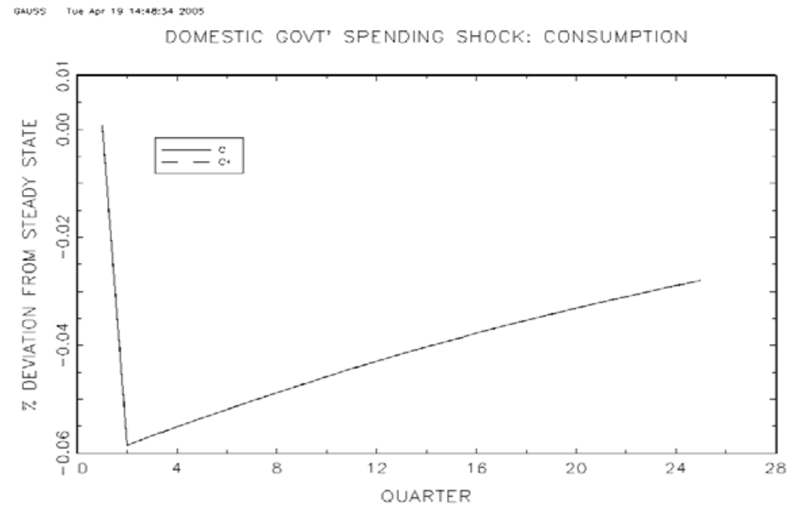
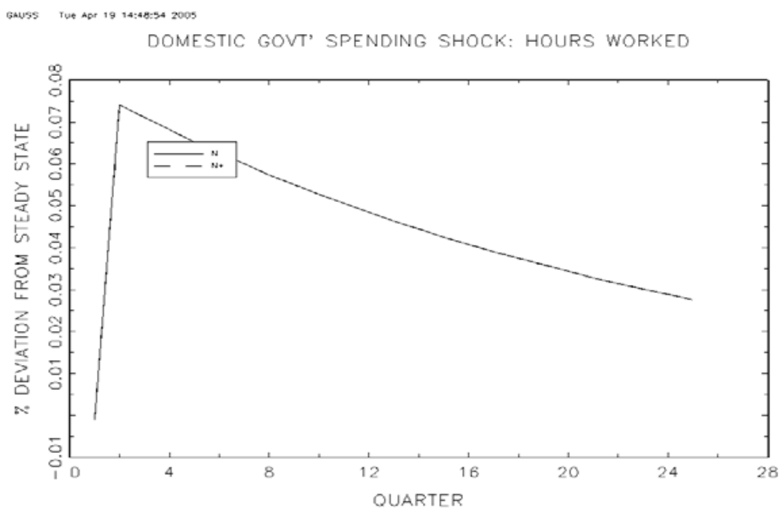
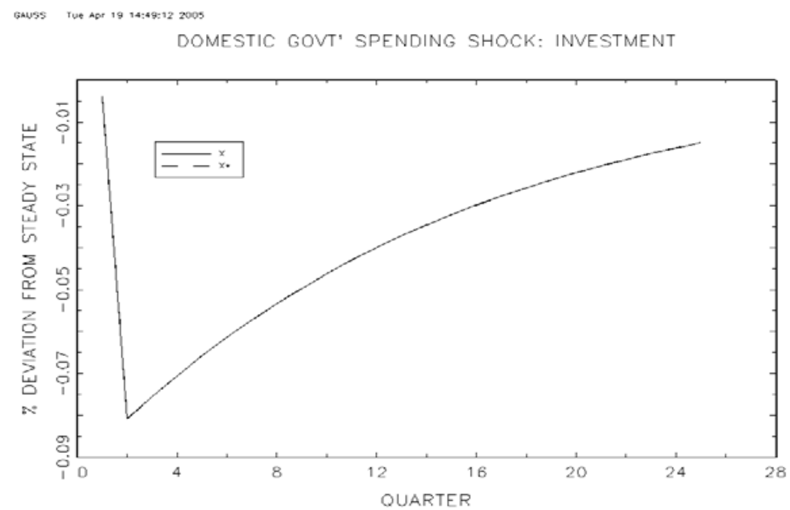
DOMESTIC GOVT' SPENDING SHOCK: OUTPUT
% DEVIATION FROM STEADY STATE
QUARTER
Y
Y*


GAUSS Tue Apr 19 14:48:34 2005
DOMESTIC GOVT' SPENDING SHOCK: CONSUMPTION
% DEVIATION FROM STEADY STATE
QUARTER
C
C*


GAUSS Tue Apr 19 14:48:54 2005
DOMESTIC GOVT' SPENDING SHOCK: HOURS WORKED
% DEVIATION FROM STEADY STATE
QUARTER
N
N*


GAUSS Tue Apr 19 14:49:12 2005
DOMESTIC GOVT' SPENDING SHOCK: INVESTMENT
% DEVIATION FROM STEADY STATE
QUARTER
X
X*

Case 4: Non-separable utility

$$u\left(d_{it}^{c}c_{it}, l_{it}\right) = \frac{1}{1-\sigma}\left[\left(d_{it}^{c}c_{it}\right)^{\theta}\left(l_{it}\right)^{1-\theta}\right]^{1-\sigma} - \frac{1}{1-\sigma}$$

First-order conditions now imply that:

$$\begin{aligned}
c_{it} &: \quad \Omega_i \theta d_{it}^{c}\left(d_{it}^{c}c_{it}\right)^{\theta-1}\left(l_{it}\right)^{1-\theta} = \lambda_t \\
n_{it} &: \quad \Omega_i \left(1-\theta\right) d_{it}^{l}\left(d_{it}^{c}c_{it}\right)^{\theta}\left(l_{it}\right)^{-\theta} = \lambda_t \alpha \exp\left(A_{it}\right) k_{it}^{1-\alpha} n_{it}^{\alpha-1} \\
k_{it+1} &: \quad \lambda_t = E_t \beta \lambda_{t+1} \\
& \quad \left[\left(1-\delta\right) + \left(1-\alpha\right)\exp\left(A_{it+1}\right) k_{it+1}^{-\alpha} n_{it+1}^{\alpha}\right]
\end{aligned}$$

Main differences:

- risk sharing condition no longer implies perfect consumption correlation in absence of C-taste shocks:

$$c_{it} = \left[\frac{\Omega_i}{\Omega_j} \left(\frac{d_{it}^c}{d_{jt}^c} \right)^{\theta} \left(\frac{l_{it}}{l_{jt}} \right)^{\theta - 1} \right]^{1/(1-\theta)} c_{jt}$$

- movements in leisure will affect cross-country consumption allocation of consumption - the country that works harder is rewarded with higher consumption

- other implications basically identical to above

Summary

	response to country specific shock to			
	A_{it}	d_{it}^{c}	g_{it}	comments
$cor\left(y,y^{*}\right)$	<0	≈ 1	≈ 1	
$cor\left(c,c^{*}\right)$	≈ 1	<0	≈ 1	
$cor\left(i,i^{*}\right)$	<0	≈ 1	≈ 1	sensitive to adjustment costs
$cor\left(n,n^{*}\right)$	<0	≈ 1	≈ 1	
$cor\left(y,nx/y\right)$	<0	<0	<0	sensitive to adjustment costs
$cor\left(s,i\right)$	>0	>0	<0	

Backus, Kehoe and Kydland, Journal of Political Economy, 1992

Main differences to above:

- time-non-separable preferences (past leisure choices affect current marginal utility)
- inventory stock in production function
- time-to-build instead of quadratic adjustment costs
- no government spending

- only productivity shocks

- two countries

Calibration: (to US vs. Europe)

- $c/y = 0.75,\ \beta = 0.99,\ \delta = 0.025,\ \alpha = 0.64,$ 4 quarters to build, $N/T = 30\%,\ \sigma = 2$

$$R = \begin{bmatrix} 0.906 & 0.088 \\ 0.088 & 0.906 \end{bmatrix}, cor\left(e_1, e_2\right) = 0.258$$

Model can account for:

- high volatility of net-exports (actually, too high)
- countercyclical net-exports
- standard business cycle features (apart from far too high investment volatility)

BUT: <u>The Comovement Puzzle:</u>

- data: $cor\,(y, y^*) > cor\,(c, c^*)$
- model: $1 \approx cor\,(c, c^*) > cor\,(y, y^*) < 0$: completely counterfactual

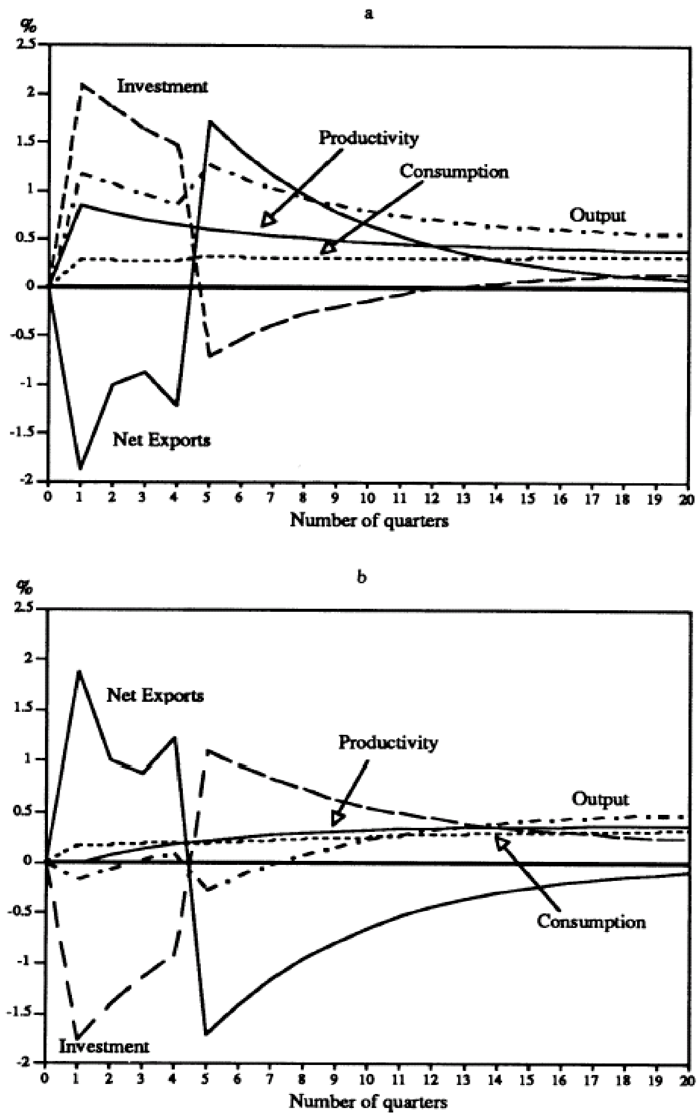


FIG. 2.—Dynamic responses to a one-standard-deviation innovation in the home ountry's technology shock in the benchmark (free-trade) economy. Productivity is ıeasured as a percentage of its steady-state value. All other variables are measured as percentage of steady-state output. *a*, Home country. *b*, Foreign country.

TABLE 4

Properties of the Benchmark Economy

A. Summary Statistics

Variable	Standard Deviation: Percentage	Standard Deviation: Relative to Output	Cross Correlation with Output at Lag k, Where k = −5	−4	−3	−2	−1	0	1	2	3	4	5
Output	1.55	1.00	−.01	.21	.25	.39	.64	1.00	.63	.39	.25	.20	−.01
	(.26)	(.00)	(.12)	(.12)	(.14)	(.14)	(.10)	(.00)	(.10)	(.14)	(.14)	(.12)	(.12)
Consumption	.62	.40	−.05	.08	.16	.30	.50	.79	.55	.36	.21	.13	.00
	(.10)	(.06)	(.16)	(.17)	(.18)	(.17)	(.14)	(.10)	(.16)	(.19)	(.20)	(.19)	(.16)
Fixed investment	16.91	10.94	.24	.40	.33	.29	.28	.27	.09	−.08	−.28	−.50	−.35
	(2.88)	(1.86)	(.12)	(.12)	(.13)	(.12)	(.09)	(.09)	(.10)	(.12)	(.11)	(.09)	(.11)
Hours	.76	.49	.01	.24	.26	.37	.59	.94	.58	.35	.24	.22	−.01
	(.12)	(.07)	(.12)	(.11)	(.14)	(.15)	(.11)	(.02)	(.11)	(.13)	(.12)	(.11)	(.12)

Figure 6:

Capital stock	1.16	.74	−.23	−.16	−.05	.11	.35	.31	.34	.47	.69	.38	.17
	(.16)	(.10)	(.11)	(.10)	(.10)	(.11)	(.12)	(.15)	(.16)	(.13)	(.08)	(.11)	(.12)
Inventory stock	3.70	2.39	.17	.33	.36	.33	.20	.42	.34	.08	−.35	−.27	−.22
	(.48)	(.31)	(.10)	(.12)	(.14)	(.13)	(.13)	(.07)	(.10)	(.10)	(.11)	(.10)	(.11)
Net exports/output	2.90	...	−.25	−.35	−.23	−.19	−.15	−.02	.14	.21	.33	.58	.36
	(.41)		(.12)	(.11)	(.12)	(.10)	(.09)	(.11)	(.11)	(.13)	(.13)	(.08)	(.11)

B. International Cross Correlations

Variable	Cross Correlation of First Variable with Second Variable at Lag k, Where k =										
	−5	−4	−3	−2	−1	0	1	2	3	4	5
Foreign and domestic output	.00	−.13	−.07	−.05	−.10	−.18	−.08	−.05	−.09	−.15	−.03
	(.18)	(.18)	(.20)	(.21)	(.20)	(.19)	(.20)	(.18)	(.17)	(.18)	(.18)
Foreign and domestic consumption	−.01	.08	.22	.40	.62	.88	.62	.40	.21	.07	−.03
	(.16)	(.17)	(.18)	(.15)	(.11)	(.04)	(.10)	(.14)	(.15)	(.15)	(.14)
Saving and investment rates	−.40	−.58	−.33	−.11	.08	.28	.30	−.31	−.36	−.45	−.27
	(.10)	(.08)	(.10)	(.09)	(.08)	(.07)	(.09)	(.12)	(.13)	(.11)	(.11)

Note.—Statistics are based on Hodrick-Prescott (1980) filtered data. Entries are averages over 50 simulations of 100 periods each; numbers in parentheses are standard deviations. Except for net exports/output, all the series are in logarithms. Here, as in Kydland and Prescott (1988), the inventory stock includes one-half of capital goods in process.

Figure 7:

Incomplete Markets

Above: Complete markets: no redistribution of wealth along equilibrium path

- incomplete markets: wealth is redistributed - hence, if wealth effects are important, complete markets may be important

How to specify incomplete markets?

- Here: Trade only in non-contingent debt contracts

- each country faces the following flow budget constraint:

$$P_t^B b_{it+1} + c_{it} + g_{it} + x_{it} = y_{it} + b_{it}$$

- buy bonds today at price P_t^B, tomorrow they pay out 1 unit of the good irrespective of the realized state of nature - here written as a zero coupon bond

- all countries can buy and sell bonds at the same price and can borrow at interest rate that is unrelated to amount of assets: integrated financial markets (UIP holds)

- The competitive equilibrium conditions with trade in bonds only:

$$
\begin{aligned}
\max V_{i0} &= E_0 \sum_{t=0}^{\infty} \beta^t u\left(d_{it}^c c_{it}, l_{it}\right) \\
y_{it} + b_{it} &= P_t^B b_{it+1} + c_{it} + g_{it} + x_{it} \\
y_{it} &= \exp\left(A_{it}\right) k_{it}^{1-\alpha} n_{it}^{\alpha} \\
k_{it+1} &= \left(1-\delta\right) k_{it} + v_{it} \\
x_{it} &= v_{it} + \frac{\varphi}{2}\left(\frac{v_{it} - \delta k_{it}}{k^{ss}}\right)^2 \\
l_{it} + n_{it} &= T
\end{aligned}
$$

- plus world bond market clearing condition:

$$
\sum_{i=1}^{N} b_{it} = 0
$$

- First-order conditions:

$$\begin{aligned}
c_{it} &: \quad d^c_{it} u_c \left(d^c_{it} c_{it}, l_{it}\right) = \lambda_{it} \\
n_{it} &: \quad u_l \left(d^c_{it} c_{it}, l_{it}\right) = \lambda_{it} \alpha \exp\left(A_{it}\right) k_{it}^{1-\alpha} n_{it}^{\alpha-1} \\
k_{it+1} &: \quad \lambda_{it} \left(1 + \frac{\varphi}{k^{ss}} \left(\frac{k_{it+1} - k_{it}}{k^{ss}}\right)\right) = E_t \beta \lambda_{it+1} [(1-\delta) \\
& \qquad + (1-\alpha) \exp\left(A_{it+1}\right) k_{it+1}^{-\alpha} n_{it+1}^{\alpha} + \frac{\varphi}{k^{ss}} \left(\frac{k_{it+2} - k_{it+1}}{k^{ss}}\right)] \\
b_{it+1} &: \quad P_t^b \lambda_{it} = E_t \beta \lambda_{it+1}
\end{aligned}$$

- plus transversality conditions:

$$\begin{aligned}
\lim_{t \to \infty} \beta^t \lambda_{it} b_{it+1} &= 0 \\
\lim_{t \to \infty} \beta^t \lambda_{it} k_{it+1} &= 0
\end{aligned}$$

Complete vs. Incomplete Markets:

- With incomplete markets, countries no longer able to enter into risk sharing agreements - allocations now subject to redistribution of wealth and subject to idiosyncratic risk

- The latter is reflected in the appearance of λ_{it} rather than λ_t in the first-order conditions - an increase in y_{it} together with a decrease in y_{jt} now affects the allocation since λ_{it} and λ_{it} are affected while this is not the case under complete markets

- Thus the wealth effect of idiosyncratic shocks is the key difference

Implications:

1. Consumption allocation

- Combining c_{it} and b_{it+1} conditions gives us:

$$E_t \beta \frac{d^c_{it+1} u_c \left(d^c_{it+1} c_{it+1}, l_{it+1} \right)}{d^c_{it} u_c \left(d^c_{it} c_{it}, l_{it} \right)} = P^b_t$$

- Expected MRS between current and future consumption equals relative price

- Due to bond market integration this then implies that:

$$E_t \beta \frac{d^c_{it+1} u_c \left(d^c_{it+1} c_{it+1}, l_{it+1} \right)}{d^c_{it} u_c \left(d^c_{it} c_{it}, l_{it} \right)} = E_t \beta \frac{d^c_{jt+1} u_c \left(d^c_{jt+1} c_{jt+1}, l_{jt+1} \right)}{d^c_{jt} u_c \left(d^c_{jt} c_{jt}, l_{jt} \right)}$$

- Thus: marginal rates of substitution are equalized across countries

- How does this differ from complete markets?

- Smoothing vs. Risk sharing: Incomplete markets implies consumption smoothing through borrowing and lending, Complete markets implies risk sharing through transfers of goods and leisure

- Consider the example:

$$u\left(d_{it+1}^{c} c_{it+1}^{IM}, l_{it+1}^{IM}\right) = \ln c_{it} + \theta \ln\left(l_{it}\right)$$

- and assume equal welfare weights so we get that:

$$\begin{aligned} CM &: \quad c_{it}^{CM} = c_{jt}^{CM} \\ IM &: \quad E_t \beta \frac{c_{it}^{IM}}{c_{it+1}^{IM}} = E_t \beta \frac{c_{jt}^{IM}}{c_{jt+1}^{IM}} \end{aligned}$$

- while consumption levels are equalized under CM, expected consumption growth rates are equalized under IM

- but IM condition does **NOT** tie down level of consumption - this is a reflection of the wealth effect

- consider a log-linearization:

$$
\begin{aligned}
CM &: \widehat{c}_{it}^{CM} = \widehat{c}_{jt}^{CM} \Rightarrow \\
CM &: \widehat{c}_{it+1}^{CM} - \widehat{c}_{it}^{CM} = \widehat{c}_{jt+1}^{CM} - \widehat{c}_{jt}^{CM} \\
IM &: E_t \widehat{c}_{it+1}^{IM} - \widehat{c}_{it}^{IM} = E_t \widehat{c}_{jt+1}^{IM} - \widehat{c}_{jt}^{IM}
\end{aligned}
$$

- CM: the **realized** marginal rate of substitution is equalized across countries. IM: the **expected** marginal rate of substitution is equalized under IM

- equalization of realized MRS implies that expected MRSs are also equalized, BUT the opposite is not true

- The last condition can also be written as:

$$\widehat{c}_{it+1}^{IM} - \widehat{c}_{it}^{IM} + \xi_{it+1} = \widehat{c}_{jt+1}^{IM} - \widehat{c}_{jt}^{IM} + \xi_{jt+1}$$

- where CM implies that $\xi_{it+1} = \xi_{jt+1} = 0$, IM only implies that $E_t\xi_{it+1} = E_t\xi_{jt+1} = 0$

- This means that levels of consumption can drift apart under IM but not under CM

- This happens through “initial” impact under IM

Non-stationarity

- Model implies "permanent effects" of temporary shocks: Random walk of bond holdings

- And initial conditions matter

- Note that CM model doesn't pin down steady state wealth either ....

- Where does the problem come from? The domestic and foreign Euler equations imply that in steady-state $P^B = \beta$ - nothing ties down the steady-state level of consumption

This can be "solved" various ways:

- endogenous discount factor:

$$
\begin{aligned}
\max V_{i0} &= E_0 \sum_{t=0}^{\infty} \beta^t \theta_t u\left(c_{it}, l_{it}\right) \\
\theta_0 &= 1 \\
\theta_{t+1} &= \beta\left(c_{it}, l_{it}\right) \\
\beta_c, \beta_l &< 0
\end{aligned}
$$

- portfolio rebalancing costs

$$
P_t^B b_{it+1} + c_{it} + g_{it} + x_{it} = y_{it} + b_{it} - \frac{\psi}{2}\left(b_{it} - \overline{b}\right)^2
$$

- debt elastic interest rates

$$
\begin{aligned}
P_{it}^{B} b_{it+1} + c_{it} + g_{it} + x_{it} &= y_{it} + b_{it} \\
P_{it}^{B} &= P_{t}^{B} - q\left(b_{it}\right) \\
q' &> 0
\end{aligned}
$$

The properties of model with and without these features are very similar

- Data: Canada, Model 1a: endogenous discount factor, Model 1b: endogenous discount factor (external model), Model 2: debt elastic interest rate, Model 3: portfolio rebalancing costs, model 3: complete markets, model 4: standard incomplete markets model

- Models are calibrated at the annual frequency

Observed and implied second moments						
	Data	Model 1	Model 1a	Model 2	Model 3	Model 4
Volatilities:						
std(y_t)	2.8	3.1	3.1	3.1	3.1	3.1
std(c_t)	2.5	2.3	2.3	2.7	2.7	1.9
std(i_t)	9.8	9.1	9.1	9	9	9.1
std(h_t)	2	2.1	2.1	2.1	2.1	2.1
std$\left(\frac{tb_t}{y_t}\right)$	1.9	1.5	1.5	1.8	1.8	1.6
std$\left(\frac{ca_t}{y_t}\right)$		1.5	1.5	1.5	1.5	
Serial correlations:						
corr(y_t, y_{t-1})	0.61	0.61	0.61	0.62	0.62	0.61
corr(c_t, c_{t-1})	0.7	0.7	0.7	0.78	0.78	0.61
corr(i_t, i_{t-1})	0.31	0.07	0.07	0.069	0.069	0.07
corr(h_t, h_{t-1})	0.54	0.61	0.61	0.62	0.62	0.61
corr$\left(\frac{tb_t}{y_t}, \frac{tb_{t-1}}{y_{t-1}}\right)$	0.66	0.33	0.32	0.51	0.5	0.39
corr$\left(\frac{ca_t}{y_t}, \frac{ca_{t-1}}{y_{t-1}}\right)$		0.3	0.3	0.32	0.32	
Correlations with output:						
corr(c_t, y_t)	0.59	0.94	0.94	0.84	0.85	1
corr(i_t, y_t)	0.64	0.66	0.66	0.67	0.67	0.66
corr(h_t, y_t)	0.8	1	1	1	1	1
corr$\left(\frac{tb_t}{y_t}, y_t\right)$	−0.13	−0.012	−0.013	−0.044	−0.043	0.13
corr$\left(\frac{ca_t}{y_t}, y_t\right)$		0.026	0.025	0.05	0.051	

Figure 8:

Baxter and Crucini, IER, 1995, and Baxter, Handbook of Int'l Economics, 1995:

Examine implications for cross-country comovements in incomplete markets model

- no taste shocks
- Cobb-Douglas preferences
- Two specifications of productivity shock processes: (1) As BKK 1992, (2) Permanent shocks with no spill-overs

TABLE 3

TREND STATIONARY SHOCKS

(1) Results for complete markets economy
(2) Results for economy trading noncontingent bonds and goods only

	Standard deviation		Relative standard deviation		Persistence		Correlation w/y, lag 0			Other correlations	
	(1)	(2)	(1)	(2)	(1)	(2)	(1)	(2)		(1)	(2)
Output	2.01	1.99	1.00	1.00	0.75	0.76	1.00	1.00	y, y^*	-0.04	0.06
Consumption	0.97	0.98	0.48	0.49	0.81	0.81	0.82	0.84	c, c^*	0.95	0.92
Investment	3.72	3.55	1.85	1.79	0.73	0.74	0.98	0.97	i, i^*	0.02	0.12
Labor	1.07	1.02	0.53	0.51	0.73	0.72	0.91	0.91	N, N^*	−0.70	−0.67
Wage	1.13	1.14	0.56	0.57	0.80	0.80	0.92	0.93	w, w^*	0.75	0.72
Net exports	0.57	0.59	0.29	0.30	0.80	0.80	0.65	0.65	s, i	0.95	0.94
Bonds	0.00	3.22	0.00	1.62	0.00	0.98	0.00	0.23	w, N	0.66	0.69

Figure 9:

- Incomplete and complete markets allocations very similar

- Basically, the induced wealth effects are "too small" to make much difference

Figure 7: Trend stationary shocks with spillovers

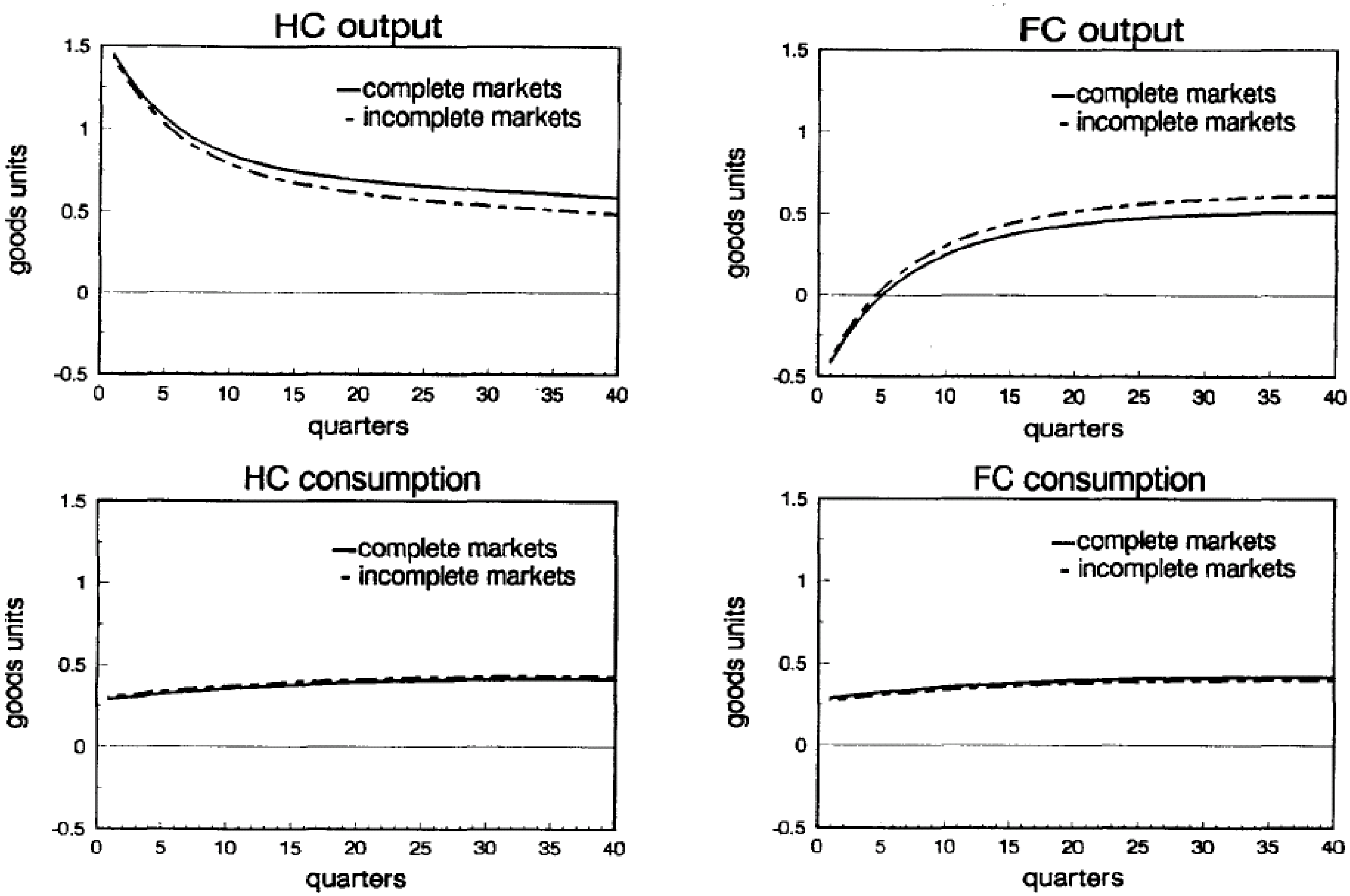


Figure 10:

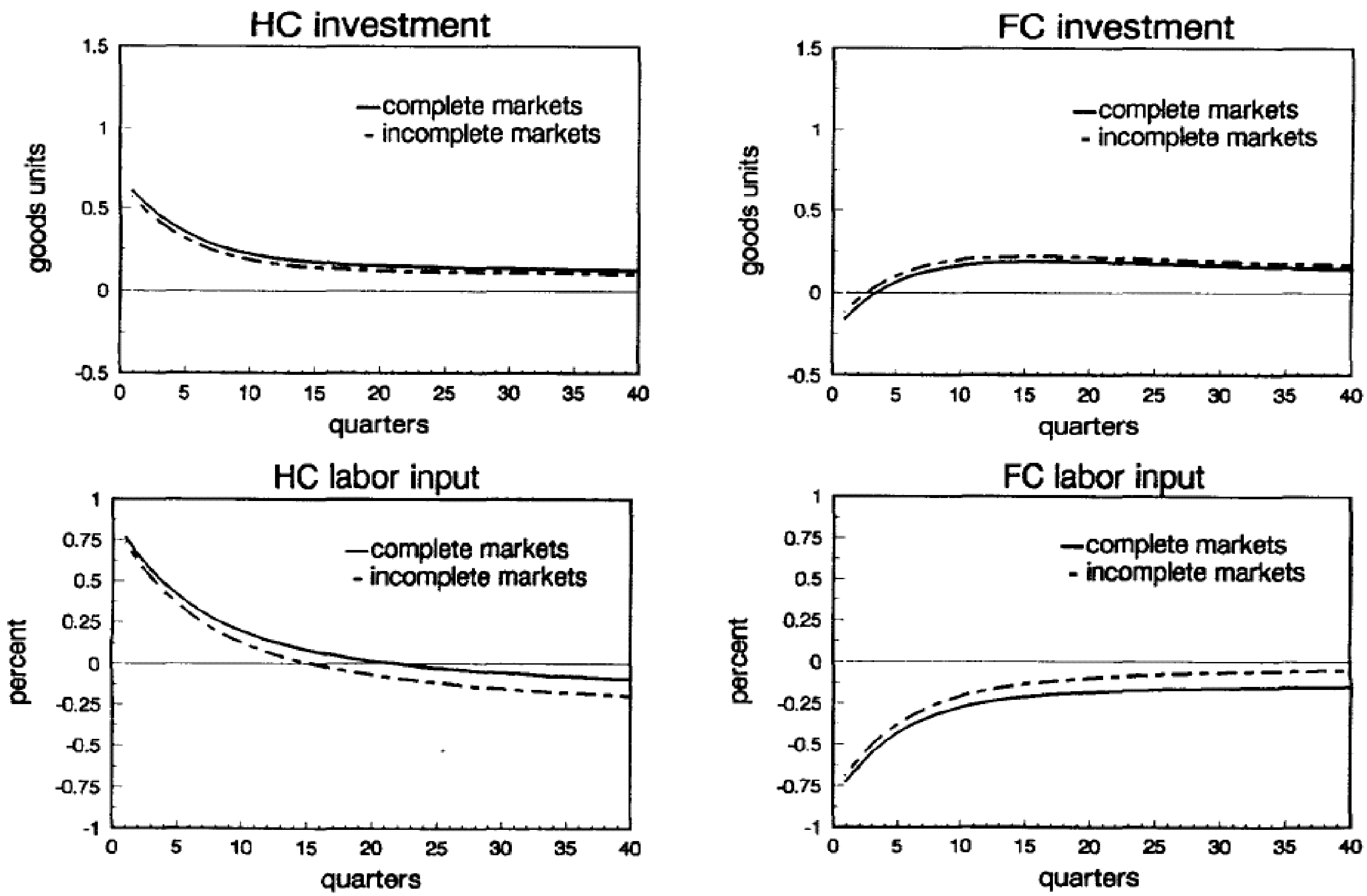

HC investment
1.5
1
0.5
0
-0.5
goods units
—complete markets
- incomplete markets
0
5
10
15
20
25
30
35
40
quarters
FC investment
1.5
1
0.5
0
-0.5
goods units
—complete markets
- incomplete markets
0
5
10
15
20
25
30
35
40
quarters
HC labor input
1
0.75
0.5
0.25
0
-0.25
-0.5
-0.75
-1
percent
—complete markets
- incomplete markets
0
5
10
15
20
25
30
35
40
quarters
FC labor input
1
0.75
0.5
0.25
0
-0.25
-0.5
-0.75
-1
percent
—complete markets
- incomplete markets
0
5
10
15
20
25
30
35
40
quarters

Figure 11:

- When the shocks are more persistent, IM vs. CM matters more

- Hence: It's not market completeness as such that matters, but the interaction between market completeness and the persistence of shocks