

INTRODUCTION TO INTERNATIONAL FINANCE

ABC of asset pricing, gains from trade in risky assets, comparative advantages and pattern of trade across periods and states of nature, complete vs. incomplete markets

Questions:

- Is trading in international financial markets beneficial?
- What are the determinants of international portfolio composition?

This introduction will provide a first pass at these questions based on Lars Svensson, “Trade in Risky Assets”, AER 1988, and OR 1.1 (1.2 and 1.3.4.), 5.1, 5.2.3.

A simple setup

- Two countries: Home and Foreign (foreign country is denoted with an asterisk). Each is inhabited by a national representative agent.
- Two periods: t and $t + 1$.
- One good, endowment economy. Output in period t is known: Y_t and Y_t^* . Output in period $t + 1$ is a random variable. Let the state of the economy be $s = Y_{t+1}, Y_{t+1}^*, s = 1, 2, \dots, S$. Probability distributions over the state of the economy is common knowledge.

Asset Markets

There are N assets, with $n = 1, 2, \dots, N$. Assume that each asset is freely traded at time t at the equilibrium price q_n , and gives the owner a claim to the cashflow $\mathcal{R}_n(s)$.

Examples of cash flows:

- – $\mathcal{R}_n(s) = 1$ for all s (sure bonds);
 - $\mathcal{R}_n(s') = 1$ if $s = s'$, 0 otherwise (Arrow Debreu securities);
 - $\mathcal{R}_n(s) = Y_{t+1}$ is the Home country' mutual fund.

Preferences and budget constraint

Assuming that net asset positions are initially zero, let B_n denote the *net* position in (=purchase of) asset B_n taken in period t by the Home country representative agent, which can be positive (the agent buys — or import — the asset) or negative (the agent sells — or exports — the asset).

The problem of the Home representative household is

$$\begin{aligned} \text{Max} \quad & U(C_t) + \beta E_t U(C_{t+1}) \\ \text{s.t.} \quad & C_t = Y_t - \sum_n q_n B_n \\ & C_{t+1}(s) = Y_{t+1}(s) + \sum_n \mathcal{R}_n(s) B_n \end{aligned}$$

where E_t is the expectations operator conditional on information at time t .

ABC of asset pricing

Let us write the problem of the Home representative agent as

$$\text{Max}_{\mathbf{B}} U(Y_t - \sum_n q_n B_n) + \beta E_t U(Y_{t+1} + \sum_n \mathcal{R}_n B_n)$$

The first order condition with respect to the n -th asset yields the euler equation

$$q_n U'(C_t) = \beta E_t [U'(C_{t+1}) \mathcal{R}_n]$$

Interpreting: at an optimum, the cost of purchasing one unit of security n evaluated in terms of current marginal utility, must equate the expected future benefits of its payoff, also evaluated in terms of future marginal utility (including time preferences).

Rearranging the euler equation, we can express the price of an asset as the present discounted value of its payoff. The discount factor is stochastic

$$q_n = E_t \left[\overbrace{\frac{\beta U'(C_{t+1})}{U'(C_t)}}^{\text{stochastic discount factor}} \underbrace{\mathcal{R}_n}_{\text{asset payoff}} \right]$$

The above must hold in equilibrium.

In the case of a **sure bond**, the asset payoff is identical in all states of nature. Hence the price of a bond is

$$q_{bond} = \frac{E_t [\beta U'(C_{t+1})]}{U'(C_t)}$$

The real return on the bond is instead defined as

$$q_{bond} \equiv \frac{1}{1+r} \implies r = \frac{1}{q_{bond}} - 1.$$

which is generally referred to as the real interest rate (the price of certain future consumption in terms of present consumption).

In general, we can decompose the **price** of an asset n as the present discounted value of expected payoffs, and a risk adjustment. Since:

$$E(XY) = EX \cdot EY + Cov(X, Y)$$

we can write:

$$\begin{aligned} q_n &= \frac{E_t [\beta U'(C_{t+1})] E_t [\mathcal{R}_n]}{U'(C_t)} + \frac{\beta Cov_t [U'(C_{t+1}), \mathcal{R}_n]}{U'(C_t)} \\ &= \frac{E_t [\beta U'(C_{t+1})]}{U'(C_t)} \left\{ E_t [\mathcal{R}_n] + \frac{\beta Cov_t [U'(C_{t+1}), \mathcal{R}_n]}{E_t [\beta U'(C_{t+1})]} \right\} \end{aligned}$$

Using the definition of real interest rate, and defining Π_n as a ‘measure of risk,’ proportional to the **covariance** between marginal utility and the asset’s payoff, we can finally write

$$q_n = \frac{1}{1+r} \left\{ \overbrace{E_t [\mathcal{R}_n]}^{\text{expected payoff}} - \overbrace{\Pi_n}^{\text{risk adjustment}} \right\}.$$

The price of a risky asset is *decreasing in ‘risk.’*

The above pricing formula can be rearranged in terms of **expected return**. Namely, dividing by q :

$$\begin{aligned}
 1 &= \frac{1}{1+r} \left\{ \left(\frac{E_t[\mathcal{R}_n]}{q_n} \right) - \frac{\Pi_n}{q_n} \right\} \\
 &= \frac{1}{1+r} \left\{ \underbrace{\text{expected return}}_{(1+E_t r_n)} - \overbrace{\frac{\Pi_n}{q_n}}^{\text{risk}} \right\}
 \end{aligned}$$

which yields

$$E_t r_n = r + \frac{\Pi_n}{q_n}$$

The expected return of risky asset relative to the return on a non-contingent bond (the 'excess return') is *increasing in risk*.

A closer look at Π_n :

$$\Pi_n = -\frac{\beta \text{Cov}_t [U'(C_{t+1}), \mathcal{R}_n]}{E_t [\beta U'(C_{t+1})]}$$

- If the payoff \mathcal{R}_n tends to be high in states of nature where marginal utility of consumption $U'(C_{t+1})$ is high (C_{t+1} is low), the asset tends to be valuable and low-risk.
- Conversely, if the payoff tends to be low when the $U'(C_{t+1})$ is high, the asset is risky and less valuable.

Note that a bond with a non-contingent payoff has $\Pi_{bond} = 0$). However, such bond is not necessarily less risky than other assets. There could be assets with a negative Π_n traded at a lower price than a bond.

The Svensson model

We now consider two important results (gains from trade and comparative advantages) using the Svensson's framework.

To characterize the two-country equilibrium: the foreign representative household solves a similar problem as the above. Let m denote the external deficit in the goods market (net import) of the Home country in period t . Note that $m_t = -CA_t$, and the fundamental BOP equation implies

$$m_t = - \sum_n q_n B_n = -\mathbf{q}\mathbf{B}.$$

where \mathbf{q} and \mathbf{B} are vectors. Market clearing for each asset and the goods market implies:

$$\begin{aligned}\mathbf{B} + \mathbf{B}^* &= \mathbf{0} \\ m + m^* &= 0.\end{aligned}$$

Note the (Home) representative agent problem can be written in a slightly different way, stressing utility from trade. Namely, writing indirect utility as a function of m and the vector \mathbf{B} :

$$\mathcal{W}(m, \mathbf{B}) \equiv$$

$$\max_{m, \mathbf{B}} \left[U(Y_t + m) + \beta E_t U(Y_{t+1} + \sum_n \mathcal{R}_n B_n) \right]$$

Result 1: Gains from trade

- Expected utility in a trade equilibrium is at least as large as expected utility in an autarky equilibrium.

In other word, we want to show that

$$u^T \geq u^A$$

where superscripts T and A denote the trade and the autarky equilibrium, respectively.

To prove the above proposition, it is useful to define the *balance of payment function* $\mathcal{B}[p, q, u]$ as the following expenditure function

$$\mathcal{B}[p, \mathbf{q}, u] = \min [pm + \mathbf{q}\mathbf{B} \mid \mathcal{U}(m, \mathbf{B}) \geq u]$$

(this defines m and \mathbf{B} minimizing the expenditure needed to deliver a given utility level u at the good price p and asset prices \mathbf{q}).

Taking C_t to be the numeraire, so that $p = 1$, a **trade equilibrium** is

$$\begin{aligned}\mathcal{B} [1, \mathbf{q}^T, u^T] &= 0 \\ m^T &= \frac{\partial \mathcal{B} [1, \mathbf{q}^T, u^T]}{\partial p} \\ \mathbf{B}^T &= \frac{\partial \mathcal{B} [1, \mathbf{q}^T, u^T]}{\partial \mathbf{q}}\end{aligned}$$

The first equation is just the BOP. The other two equations state that the derivative of the expenditure function with respect to the price of a good or an asset (\mathcal{B}_p or \mathcal{B}_q) is the demand for that good or asset.

An **autarky equilibrium** instead is

$$\begin{aligned}\mathcal{B} [1, \mathbf{q}^A, u^A] &= 0 \\ m^A &= 0 \\ \mathbf{B}^A &= 0\end{aligned}$$

- Proof of 'gains from trade'

$$\begin{aligned}\mathcal{B} [1, \mathbf{q}^T, u^T] &= 0 \\ &= m^A + \mathbf{q}^T \mathbf{B}^A \geq \mathcal{B} [1, \mathbf{q}^T, u^A]\end{aligned}$$

- First equality follows from the definition of BOP.
- Second equality follows from $m^A = 0$ and $\mathbf{B}^A = \mathbf{0}$.
- Without international trade, the expected utility level is u^A . But at the trade equilibrium price \mathbf{q}^T , the minimum expenditure required to reach u^A cannot be higher than 0 (and will be smaller if \mathbf{q}^T differ from \mathbf{q}^A). Since the expenditure function is increasing in utility, it must be that $u^T \geq u^A$. This completes the proof.

To sum up:

- The above proposition states that a trade equilibrium is at least as good as an autarky equilibrium.
- It does **not** say that trade in 'more asset' is weakly better than trade in 'less assets.' This is because the introduction of a new asset may change the price vector q . If after the introduction of the new asset previously imported assets become more expensive, the negative terms of trade effect (which makes an agent 'poorer') may outweigh gains from trade.

See also OR 5.1.7.

Result 2: **The law of comparative advantage**

Which asset will the Home country import/export?

The law of comparative advantage states:

- On average, the Home country will import assets whose autarky prices are higher in the Home country than in the Foreign country.

$$\sum_n (q_n^A - q_n^{*A}) B_n^T \geq 0$$

Note that this law may not hold for an individual asset — it only holds on average.

To prove the above proposition, note that for the Home country:

$$m^T + \mathbf{q}^A \mathbf{B}^T \geq B(\mathbf{1}, \mathbf{q}^A, u^T) \geq B(\mathbf{1}, \mathbf{q}^A, u^A) = 0$$

First inequality: $\{m^T, \mathbf{B}^T\}$ yield u^T , but do not necessarily minimize expenditure at autarky prices \mathbf{q}^A .

The second inequality follows from gains from trade, i.e., $u^T \geq u^A$.

Above we have shown that for the Home country

$$m^T + \mathbf{q}^A \mathbf{B}^T \geq 0$$

By the same token, for the Foreign country

$$m^{*T} + \mathbf{q}^{*A} \mathbf{B}^{*T} \geq 0$$

which, using market equilibrium, becomes

$$-m^T - \mathbf{q}^{*A} \mathbf{B}^T \geq 0$$

Adding through

$$\begin{aligned} (m^T + \mathbf{q}^A \mathbf{B}^T) + (-m^T - \mathbf{q}^{*A} \mathbf{B}^T) &= \\ (\mathbf{q}^A - \mathbf{q}^{*A}) \mathbf{B}^T &\geq 0 \end{aligned}$$

QED.

On the determinants of comparative advantages and patterns of trade

Importing (=buying) an asset means buying future (possibly contingent) consumption in exchange for present consumption. Now, the law of comparative advantages tells us that a country will tend to import assets whose autarky prices are relative expensive. This suggests that, by looking at the determinants of assets' autarky prices q^A , we can say something about the direction of trade across time and states of nature.

To derive q^A , we evaluate asset prices at the autarky allocation, that is, $m^A = 0$ and $B^A = 0$, or $C_t = Y_t$ and $C_{t+1} = Y_{t+1}$. Hence

$$q_n^A = \frac{E_t [\beta U'(Y_{t+1}) \mathcal{R}_n]}{U'(Y_t)}$$

- If q_n^A and q_n^{*A} are identical for all n , there is no basis for trade across time and states of nature (although in more general models with heterogeneous goods there could be trade in different goods).
- But differences in q^A, q^{*A} can (and will) arise for a variety of reasons, including differences in asset payoffs, time preferences, risk aversion, endowment, subjective beliefs etc.

Examples: **The real interest rate and intertemporal trade**

Suppose that international trade is limited to a non-contingent bond (paying $\mathcal{R}_n(s) = 1$ in all states of nature s).

The law of comparative advantage tells us that the Home country will import the bond when

$$q_{bond}^A > q_{bond}^{*A} \iff r^A < r^{*A}.$$

This means that the country will export period 1 consumption in exchange for period 2 consumption: it has a comparative advantage in producing current consumption.

Using the autarky equilibrium conditions

$$q_{bond}^A - q_{bond}^{*A} = \frac{E_t [\beta U'(Y_{t+1})]}{U'(Y_t)} - \frac{E_t [\beta^* U^{*'}(Y_{t+1}^*)]}{U^{*'}(Y_t^*)}$$

we can derive examples whereas, everything else equal, $q_{bond}^A \geq q_{bond}^{*A}$:

- $\beta > \beta^*$ (Home country is more patient than Foreign)
- $Y_t > Y_t^*$ (Home country output is currently higher than Foreign)
- Y_{t+1} is stochastically dominated by Y_{t+1}^* to the first order (see definition), which implies $E_t Y_{t+1} > E_t Y_{t+1}^*$.
- and

Pattern of trade in risky assets

Assume that parameters are such that $r^A = r^{*A}$. Assume that only asset n is traded internationally. Then the Home country will import this asset if

$$\begin{aligned}\Pi_n &< \Pi_n^* \\ &\Leftrightarrow \\ \text{Cov}_t [U'(Y_{t+1}), \mathcal{R}_n] &> \text{Cov}_t [U'(Y_{t+1}^*), \mathcal{R}_n]\end{aligned}$$

The asset payoff is more positively correlated with the marginal utility of consumption at Home than abroad.

Note that, in general the above is NOT implied by a higher correlation between asset payoffs and output

$$\text{Cov}_t [Y_{t+1}, \mathcal{R}_n] > \text{Cov}_t [Y_{t+1}^*, \mathcal{R}_n]$$

This is only true in special cases. For instance, let $U = -e^{-\gamma C}$ and output and cash-flows be jointly normally distributed. Then

$$\Pi_n = \gamma \text{Cov}_t [Y_{t+1}, R_j]$$

Clearly, the world equilibrium allocation will depend on the structure of the asset markets.

In general, it is very difficult to derive such allocation — involving the joint determination of optimal consumption and portfolio diversification. Even in the simple case above, an analytical solution exists exclusively for particular specifications of the asset markets.

In what follows, we will focus on the case in which markets are complete, i.e. in which agents can freely trade an Arrow Debreu Security corresponding to every future state of nature.

Note that the fact that AD securities as such are not quoted in financial markets per se does not imply that markets are incomplete. This is because it may be possible to construct AD securities by combining existing securities in specific portfolios, each with a payoff contingent on one state of nature only.

COMPLETE MARKETS

Following OR chapter 5 page 269-285

- Assume S states of nature $s = 1, 2, \dots, S$, occurring with probability $\pi(s)$, with $\sum_S \pi(s) = 1$.
- Assume a complete set of Arrow-Debreu (AD) securities.
- Let $B(s)$ the net stock of the 'state s Arrow-Debreu security' owned by the Home representative individual at the end of period t . At time $t + 1$, the payoff of this security is 1 in state s , 0 otherwise.
- Let $p(s)/(1 + r)$ denote the price of the 'state s Arrow-Debreu security', quoted in terms of period t consumption. Hence $p(s)$ is the $t + 1$ price of period $t + 1$ consumption in state s .

We can write the Home representative agent problem as

$$\begin{aligned}
 & \text{Max} && U(C_t) + \beta \sum_S \pi(s) U(C_{t+1}(s)) \\
 & \text{s.t.} && C_t = Y_t - \sum_S \frac{p(s)}{1+r} B(s) \\
 & && C_{t+1}(s) = Y_{t+1}(s) + B(s), \quad s = 1, 2, \dots, S
 \end{aligned}$$

The FOC condition w. r. to the AD security is

$$\frac{p(s)}{1+r} U'(C_t) = \pi(s) \beta U'(C_{t+1}(s))$$

to be interpreted as a euler equation.

Note that trading in a complete set of AD securities ensures that the stochastic discount rate is identical for all individuals since

$$\frac{p(s)}{1+r} = \frac{\pi(s) \beta U'(C_{t+1}(s))}{U'(C_t)}$$

The price of an asset can be written as

$$q_n = \sum_S \frac{p(s)}{1+r} \mathcal{R}_n(s) = \sum_S \frac{\pi(s) \beta U'(C_{t+1}(s))}{U'(C_t)} \mathcal{R}_n(s)$$

Note that a non contingent bond can be built by forming a portfolio with one unit of each AD security. The price of this portfolio will be

$$\sum_S \frac{p(s)}{1+r} = q_{bond} = \frac{1}{1+r}$$

implying

$$\sum_S p(s) = 1$$

Combining the FOC conditions above for s and s'

$$\frac{\pi(s)\beta U'(C_{t+1}(s))}{\pi(s')\beta U'(C_{t+1}(s'))} = \frac{p(s)}{p(s')}$$

the marginal rate of substitution between consumption in the two states must be equal to the relative price of consumption in the two states.

With non-state contingent preferences, consumption is identical across states of nature if

$$\frac{\pi(s)}{\pi(s')} = \frac{p(s)}{p(s')}$$

When this holds, AD security prices are 'actuarially fair.' Agents can fully insure their consumption.

Taking log of the above, and differentiating

$$d \log \left(\frac{p(s)}{p(s')} \right) = \rho(C(s)) \cdot d \log C(s) + \rho(C(s')) \cdot d \log C(s')$$

where ρ is the Arrow-Pratt coefficient of relative risk aversion (a measure of the degree of concavity of the utility function)

$$\rho(C) = -\frac{CU''(C)}{U'(C)}$$

Risk aversion

Risk aversion follows from strict concavity of the utility function: the degree of concavity of the UF measures the extent of risk aversion.

Assume that ρ is constant for all levels of C . Then with complete markets we can write

$$d \log \left(\frac{C(s')}{C(s)} \right) = \frac{1}{\rho} d \log \left(\frac{p(s)}{p(s')} \right)$$

The inverse of the coefficient of relative risk aversion is the elasticity of substitution between state-contingent consumption with respect to their state prices. The higher ρ , the lower the consumption response to a change in the relative price of consumption across states.

The role of the current account with complete and incomplete markets: a useful simple example (see OR 5.2.3)

Assume two endowment economies (countries) each with a national representative agent. These agents are identical in everything but endowment at time t . In other words, preferences are identical; output at time $t + 1$ is constant at some exogenously given level

$$Y_{t+1} = Y_{t+1}^* = \Omega$$

The only difference is that in period t there are two (symmetric) states of the world:

$$\begin{aligned} s &= \{Y_t = \Omega + \varepsilon, Y_t^* = \Omega - \varepsilon\} \\ s' &= \{Y_t = \Omega - \varepsilon, Y_t^* = \Omega + \varepsilon\} \end{aligned}$$

Case of incomplete markets

Suppose that asset markets open at t , **after** the shock is realized. Trade in securities cannot provide any consumption-insurance: the only shock in the economy is already realized and future output is deterministic.

Yet we have seen above that, everything else equal, a positive realization of the output shock in the Home country will imply $r^A < r^{*A}$: the Home country will tend to import (buy) assets at t , running a CA surplus and accumulating foreign assets.

$$CA_t = \Omega + \varepsilon - C_t > 0$$

corresponding to

$$CA^* = \Omega - \varepsilon - C_t^* < 0$$

In other words, the Home country will export period t consumption (of which it is relatively better endowed), against a promise to receive period $t + 1$ consumption.

Although asset trade does not allow agents to insure consumption, it does allow them to smooth consumption via intertemporal trade.

- At time t consumption C_t will increase above C_t^* , but this by less than in the absence of trade.
- At time $t + 1$, $C_{t+1} > Y_{t+1}$, while $C_{t+1}^* < Y_{t+1}^*$, as Foreign residents will have to make asset income payments to Home residents. In the absence of trade, the two agents will just consume their identical endowments.

Case of complete markets

Suppose now that assets markets open before the shock is realized and markets are complete.

With identical preferences and symmetric shocks, the AD securities prices will be actuarially fair. National representative agents will find it optimal to insure their consumption stream completely. In particular, they will perfectly pool output risk.

After trading in the assets markets, when the shock is realised at time t , perfect pooling implies

$$C_t = C_t^* = \Omega$$

while $C_{t+1} = C_{t+1}^*$ will be unaffected.

We note that the Home (and therefore Foreign) Current Account is identically equal to zero. In case of a positive shock to Home output:

$$CA_t = \underbrace{TB_t}_{\varepsilon} + NFI_t = 0 = \underbrace{NFI_t}_{-\varepsilon}$$

net exports of goods NFI payments from Home to Foreign

Incomplete markets

Intertemporal trade improves allocation over autarky. Current account deficits/surpluses (international borrowing and lending) allow national resident to smooth consumption.

Complete markets

Current account deficits and surpluses are irrelevant for consumption smoothing.

Note: this does not imply that CA is zero. The current account still record changes in foreign ownership of new capital stock. Moreover, with complete markets, there are subtle accounting issues.