MACROECONOMIC INTERDEPENDENCE AND THE TRANSMISSION MECHANISM

A CLOSE-UP ANALYSIS

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Introduction

In this lecture, we will study the transmission mechanism of aggregate shocks using a simple general equilibrium model. Our main goal is to understand the interaction of relative price and financial channels of transmission, mostly focusing on endowment shocks.

Role for heterogeneity: stylized two-country (or two-region) two-goods endowment economy (see section 2 of Corsetti Dedola and Leduc 2008 and Viani 2010), under different asset markets structures:

- complete markets
- incomplete markets: (a) financial autarky and (b) trade in bonds.

1 A stylized model

Two-country, two-good *endowment* economies, ‘Home’ and ‘Foreign’, denoted H and F, respectively.

- Posit perfect specialization. Let Y_{H} and Y_{F} denote Home and Foreign (tradable) output. Supply $\{Y_{\mathsf{H}}, Y_{\mathsf{F}}\}$ varies randomly.

- In each country, continuum of households derive utility from consuming both goods. Preferences are also subject to shocks.

- All prices are flexible.

Consumption (1)

Consumption of Home representative consumer is given by CES aggregator

$$C = C_{\mathsf{T}} = \left[a_{\mathsf{H}}^{1/\phi} C_{\mathsf{H}}^{\frac{\phi-1}{\phi}} + a_{\mathsf{F}}^{1/\phi} C_{\mathsf{F}}^{\frac{\phi-1}{\phi}} \right]^{\frac{\phi}{\phi-1}}, \qquad \phi > 0,$$

where $C_{\mathsf{H},t}$ ($C_{\mathsf{F},t}$) is the domestic consumption of Home (Foreign) produced good, a_{H} is the share of the domestically produced good in the Home consumption expenditure, a_{F} is the corresponding share of imported goods, with $a_{\mathsf{F}} = 1 - a_{\mathsf{H}}$.
$a_{\mathsf{H}} > 1/2$ indexes '**Home bias**' in consumption (preferences for local goods).

- ϕ denotes the elasticity of substitution between Home and Foreign goods: higher ϕ makes goods more homogeneous.

Consumption (2)

Analogously, in the foreign country

$$C^* = C^*_{\mathsf{T}} = \left[(a^*_{\mathsf{H}})^{1/\phi} (C^*_{\mathsf{H}})^{\frac{\phi-1}{\phi}} + (a^*_{\mathsf{F}})^{1/\phi} (C^*_{\mathsf{F}})^{\frac{\phi-1}{\phi}} \right]^{\frac{\phi}{\phi-1}}, \tag{1}$$

where $C^*_{\mathsf{H},t}$ ($C^*_{\mathsf{F},t}$) is the consumption of Home (Foreign) produced good by Foreign households. Home bias in consumption is now $a^*_{\mathsf{F}} = 1 - a^*_{\mathsf{H}} > 1/2$.

Prices

- Let $P_{\mathsf{H},t}$ ($P_{\mathsf{F},t}$) denote the price of the Home (Foreign) goods in 'domestic currency'. When starred, these prices are expressed in F- currency.

Exchange rates, and relative prices

- Let $\mathcal{E}$ denote the 'nominal exchange rate,' defined as the price of the H currency in terms of the F currency.

- The **terms of trade (TOT)** is the relative price of Foreign goods imported by the H-economy in terms of Home goods exported to the F-economy:

$$TOT = \frac{P_{\mathsf{F}}}{\mathcal{E} P^*_{\mathsf{H}}}$$

  In our economy the TOT coincides with the relative price of tradables (need not to in general, since some of the tradable goods produced by an economy may not be exported).

Goods demand and the price of consumption

Let P and P^* denote the price of domestic consumption in each economy. The welfare-based price index P is defined as the minimum expenditure needed to buy one unit of consumption good $C = 1$, given market prices. By setting up the minimization problem (e.g. OR 4.4.1.1 pp. 226-228), you can derive the demand by H households for each good and the consumption price P

$$C_{\mathsf{H}} = a_{\mathsf{H}} \left(\frac{P_{\mathsf{H}}}{P} \right)^{-\phi} C \qquad\qquad C_{\mathsf{F}} = a_{\mathsf{F}} \left(\frac{P_{\mathsf{F}}}{P} \right)^{-\phi} C$$

$$P = P_{\mathsf{T}} = \left[a_{\mathsf{H}} P_{\mathsf{H}}^{1-\phi} + (1 - a_{\mathsf{H}}) P_{\mathsf{F}}^{1-\phi} \right]^{\frac{1}{1-\phi}}$$

The foreign analog of P is :

$$P^* = P_{\mathsf{T}}^* = \left[(1 - a_{\mathsf{F}}^*) (P_{\mathsf{H}}^*)^{1-\phi} + a_{\mathsf{F}}^* (P_{\mathsf{F}}^*)^{1-\phi} \right]^{\frac{1}{1-\phi}} .$$

Real exchange rate

The relative price of consumption is the **real exchange rate (RER)**, $RER = \frac{\mathcal{E}P^*}{P}$.

Note that (a) if the law of one price holds: $P_{\mathsf{H}} = \mathcal{E}P^*_{\mathsf{H}}$, $P_{\mathsf{F}} = \mathcal{E}P^*_{\mathsf{F}}$, and (b) if the consumption baskets are identical, namely, there is no Home Bias in the world economy: $a_H = 1 - a^*_F$, then the price of consumption is equalized across countries. In this case, **purchasing power parity** (PPP) holds: $P = \mathcal{E}P^*$.

Note also that $a_H = 1 - a^*_F$ can be $>$1/2 ($<$1/2) if both countries like a higher (lower) share of good H than good F in their consumption baskets.

Preferences and budget constraint

Preferences are identical in both economies. In H these are

$$E_t \sum_{s=0}^{\infty} \beta^s U\left(C_{t+s}; \zeta_{C,t+s}\right) = E_0 \sum_{s=0}^{\infty} \beta^s \zeta_{C,t+s} \frac{C_{t+s}^{1-\sigma} - 1}{1-\sigma}$$

where $1/\sigma$ is the the intertemporal elasticity of substitution and $\zeta_{C,t}$ is a taste shock. The individual flow budget constraint is:

$$B_{\mathsf{H},t+1} + \int q_{\mathsf{H},t}\left(s_{t+1}\right) \mathcal{B}_{\mathsf{H},t+1}\left(s_{t+1}\right) ds_{t+1} \leq (1+i_t) B_{\mathsf{H},t} + \mathcal{B}_{\mathsf{H},t} + P_{\mathsf{H},t} Y_{H,t} - P_{\mathsf{H},t} C_{\mathsf{H},t} - P_{\mathsf{F},t} C_{\mathsf{F},t},$$

where $\mathcal{B}_{\mathsf{H},t}$ is the holdings of state-contingent claims, priced at $q_{\mathsf{H},t}$, paying off one unit of domestic currency in the realized state of the world as of t, s_t, and i_t is the yield on a domestic nominal bond $B_{\mathsf{H},t}$, paid at the beginning of period t in domestic currency but known at time $t-1$.

Resource constraint and total demand for H(F) goods

Resource constraint is:

$$Y_H = C_H + C_H^* \qquad\qquad Y_F^* = C_F + C_F^*$$

Holding the law of one price, total demand for good H and G can be written as:

$$Y_H = \left(\frac{P_{\mathsf{H},t}}{P_t}\right)^{-\phi}\left(a_{\mathsf{H}}C_t + a_{\mathsf{H}}^* RER_t^{\phi} C_t^*\right)$$

$$Y_F^* = \left(\frac{P_{\mathsf{F},t}}{P_t}\right)^{-\phi}\left(a_{\mathsf{F}}C_t + RER_t^{\phi} a_{\mathsf{F}}^* C_t^*\right).$$

Normalizations and log-linearization

In this lecture: we will assume that the law of one price (LOOP) holds, i.e.

$$P_{\mathsf{H}} = \mathcal{E}P_{\mathsf{H}}^{*}; \qquad P_{\mathsf{F}} = \mathcal{E}P_{\mathsf{F}}^{*}$$

Recall that with home bias there are deviations from PPP. Also, as we abstract from nominal issues, without loss of generality we set $\mathcal{E} = 1$.

Assuming that economies ex-ante symmetrical, we will proceed by taking log-linear approximations around a symmetric equilibrium (so that $RER = TOT = 1$). Notation: "$\widehat{\ }$" will represent a variable's percentage deviation from the equilibrium. For endogenous variables, the ˆ becomes a ˜ in a flexible price allocation; accompanied by 'fb' if the allocation is first best.

RER and TOT

So, relate RER to TOT under the law of one price

$$RER^{1-\phi} = \frac{\left(1 - a_F^*\right) {P_H^*}^{1-\phi} + a_F^* P_F^{1-\phi}}{a_H {P_H^*}^{1-\phi} + (1 - a_H) P_F^{1-\phi}} = \frac{\left(1 - a_F^*\right) + a_F^* TOT^{1-\phi}}{a_H + (1 - a_H) TOT^{1-\phi}}$$

Taking a log-linear approximation around a symmetric equilibrium

$$\begin{aligned} \widehat{RER} &= (a_F^* + a_H - 1) \widehat{TOT} \\ & \quad \text{under our assumption} \\ \widetilde{RER} &= (2a_H - 1) \widetilde{TOT} \end{aligned}$$

Note that the correlation between RER and TOT depends on the degree of Home Bias in the world economy: positive with Home Bias, zero with PPP, negative without a bias for foreign goods.

2 Complete markets (CM)

Focus first on a market allocation in which Households trade a complete set of state-contingent Arrow-Debreu (A-D) securities. Given the problem at hands, by the first welfare theorem we know that the market allocation will be first best — first best variable will be denoted with both ˜ and fb.

For the next few slides, it is convenient to rewrite the household problem in the following recursive formulation. Let by $B(s') = \mathcal{B}_{\mathsf{H,t+1}}(s_{t+1})$ be an Arrow-Debreu security (or contract) paying 1 unit of domestic currency in state s' in the next period; let $\Pr(s'|s)$ be the conditional probability of state s' in the next period, given that the state s occurred in the current period. Let $q(s'|s) = q_{\mathsf{H},t}(s_{t+1})$ denote the current price of $B(s')$.

CM: Consumer problem

The Home representative household maximizes:

$$
\begin{aligned}
&Max\ U\left(C(s), \zeta_C(s)\right) + \sum_{s'|s} \mathsf{Pr}(s'|s) U(C\left(s'|s\right), \zeta_C\left(s'|s\right)) \\
&-\lambda(s)\left(\sum_{s'|s} q(s'|s)B(s') - P_{\mathsf{H}}(s)Y_{\mathsf{H}}(s) + P(s)C(s)\right) + \\
&-\sum_{s'|s} \lambda\left(s'|s\right)\left(P_{\mathsf{H}}(s'|s)Y_{\mathsf{H}}(s'|s) + B(s') - P(s'|s)C(s'|s)\right).
\end{aligned}
$$

The optimal choice of consumption satisfies

$$\begin{aligned} U_C(s) &= \lambda(s)P(s) \\ U_C(s'|s) &= \lambda(s'|s)P(s'|s) \end{aligned}$$

where U_c denotes marginal utility. Note that the marginal utility of consumption in state s must be equal to the price of consumption in that state, translated into units of marginal utility $\lambda(s)$ (the multiplier). The F.O.Cs with respect to bonds $B(s')$ (one for each state) are

$$\lambda(s)q(s\prime|s) = \beta \,\mathsf{Pr}(s'|s)\lambda\left(s'|s\right)$$

Combining the conditions above,

$$U_C(s) \underbrace{\frac{q(s\prime|s)}{P(s)}}_{\text{units of current consumption}} = \beta \,\mathsf{Pr}(s'|s)U_C(s'|s) \underbrace{\frac{1}{P(s'|s)}}_{\text{units of future consumption}}$$

In utility terms, the marginal cost of buying an extra unit of $B(s')$ consists of giving up q/P units of consumption, evaluated at current marginal utility of consumption U_C. The marginal benefit is the expected utility from 1 unit of currency received in state s'. This consists of the expected marginal utility $(\text{Pr}(s'|s)U_C(s'|s))$ from the extra units of consumptions that one unit of currency can buy in state s' $(1/P(s'|s))$.

Analogously, in the foreign economy

$$U_C^*(s)\frac{Q(s\prime|s)}{P^*(s)} = \beta \, \text{Pr}(s'|s)U_C^*(s'|s)\frac{1}{P^*(s'|s)}$$

Combine this with the Home equation in the previous slide. By the law of one price for bonds (and recalling that $\mathcal{E}$=1):

$$\frac{U_C(s)}{U_C^*(s)}\frac{P^*(s)}{P(s)} = \frac{U_C(s'|s)}{U_C^*(s'|s)}\frac{P^*(s'|s)}{P(s'|s)}$$

This is the key condition of efficient risk sharing.

The ratio between the marginal utilities consumption across households is proportional to the relative price of consumption (RER) across all states of nature s':

$$\frac{U_C^*(s'|s)}{U_C(s'|s)} \text{ proportional to } RER\left(s'|s\right)$$

The constant of proportionality depends on the initial conditions. Assuming that the two economies are initially perfectly symmetric: $\frac{U_C^*(s)}{U_C(s)} = \frac{P^*(s)}{P(s)}$. Relative to the initial condition, efficient risk sharing implies no reallocation of wealth along the equilibrium path: $\lambda = \lambda^*$, for all s'.

Rewrite the above (for symmetric initial conditions)

$$U_C^*(s'|s)\frac{1}{P^*(s'|s)} = U_C(s'|s)\frac{1}{P(s'|s)}.$$

With complete markets, the marginal utility of a unit of domestic currency is equalized across countries. This is equal to $\frac{1}{P^*(s'|s)}$ unit of consumption, evaluated at current marginal utility.

- This must be true independently of transaction costs, deviations from the law of one price, nominal rigidities and other distortions in the good markets (of course with distortions CM does not ensure that the allocation is first best).

- In the special case of PPP (consumption price is identical across all consumers and $RER = 1$), if preferences are the same, consumption is also equalized across countries.

For given preferences, when risk sharing is complete, domestic consumption should be higher in those states of the world in which the price of domestic consumption is relatively cheaper. In states of the world in which the price of domestic consumption is low relative to foreign consumption, domestic residents should receive contingent income transfers to 'take advantage' of the favorable goods prices. *With power utility*

$$RER\left(s'|s\right) = \frac{U_C^*(s'|s)}{U_C(s'|s)} = \left(\frac{C(s'|s)}{C^*(s'|s)}\right)^{\sigma} \frac{\zeta_C^*\left(s'|s\right)}{\zeta_C\left(s'|s\right)}$$

Transmission with complete markets

Take a log-linear approximation of $(C/C^*)^\sigma \left(\zeta_C^*/\zeta_C\right) = RER$ in the neighborhood of a symmetric equilibrium with $a_F^* = a_H$:

$$\sigma\left(\widetilde{C}^{fb} - \widetilde{C^*}^{fb}\right) + (\widehat{\zeta_C^*} - \widehat{\zeta_C}) = \widetilde{RER}^{fb} = (2a_H - 1)\,\widetilde{TOT}^{fb}$$

- If no preference shocks, real exchange rate and relative consumption will be perfectly correlated. Correlation of relative consumption with terms of trade depends on degree of Home Bias (likely to be positive).

- Without Home Bias ($a_H = a_F^* = 1/2$, PPP holds), there is no RER fluctuation no matter what happens to TOT. The rate of growth of consumption is only driven by taste shocks.

CM: *TOT and relative output supply (1)*

Combine resource constraint (recall that $C^*_H = \left(\frac{P_{\mathsf{H},t}}{P^*_t}\right)^{-\phi} a^*_{\mathsf{H}} \mathcal{E}^{\phi} C^*_t$) and the risk sharing condition:

$$\frac{Y_{\mathsf{H}}}{Y_{\mathsf{F}}} = \frac{a_{\mathsf{H}}\left(\frac{P_{\mathsf{H}}}{P}\right)^{-\phi} C + (1-a_{\mathsf{H}})\left(\frac{P_{\mathsf{H}}}{P^*}\right)^{-\phi} C\left(\frac{P^*}{P}\frac{\zeta_C}{\zeta^*_C}\right)^{-\sigma^{-1}}}{(1-a_{\mathsf{H}})\left(\frac{P_{\mathsf{F}}}{P}\right)^{-\phi} C + a_{\mathsf{H}}\left(\frac{P_{\mathsf{F}}}{P^*}\right)^{-\phi} C\left(\frac{P^*}{P}\frac{\zeta_C}{\zeta^*_C}\right)^{-\sigma^{-1}}} = \frac{TOT^{\phi}\left[a_{\mathsf{H}} + (1-a_{\mathsf{H}})\left(\frac{P^*}{P}\right)^{\phi-\sigma^{-1}}\left(\frac{\zeta_C}{\zeta^*_C}\right)^{-\sigma^{-1}}\right.}{(1-a_{\mathsf{H}}) + a_{\mathsf{H}}\left(\frac{P^*}{P}\right)^{\phi-\sigma^{-1}}\left(\frac{\zeta_C}{\zeta^*_C}\right)^{-\sigma^{-1}}}$$

Log-linearizing we get:

$$\widetilde{Y_{\mathsf{H}}}^{fb} - \widetilde{Y_{\mathsf{F}}}^{fb} = \phi \widetilde{TOT}^{fb} + \left(\frac{1}{\sigma} - \phi\right)(2a_{\mathsf{H}} - 1)\, \widetilde{RER}^{fb} + \frac{1}{\sigma}(2a_{\mathsf{H}} - 1)\left(\widehat{\zeta_C} - \widehat{\zeta^*_C}\right)$$

CM: *TOT and relative output (2)*

$$\left\{\left[1-(2a_{\mathsf{H}}-1)^2\right]\phi\sigma+(2a_{\mathsf{H}}-1)^2\right\}\widetilde{TOT}^{fb}=\sigma\left(\widetilde{Y_{\mathsf{H}}}^{fb}-\widetilde{Y_{\mathsf{F}}}^{fb}\right)-(2a_{\mathsf{H}}-1)\left(\widehat{\zeta_C}-\widehat{\zeta_C^*}\right)$$

Since $0 \leq a_{\mathsf{H}} \leq 1$, the first coefficient is always positive: with constant preferences, **a positive shock to Y_{H} unambiguously worsens the Home terms of trade**, to the benefits of foreign consumers.

In equilibrium output risk is thus shared not only through the 'financial channel', but also through the 'relative price' channel. The two are however strictly interconnected. As shown below, relative prices are an endogenous function of the structure of financial markets.

CM: *Consumption and relative output*

For given preferences $\left(\widehat{\zeta_C} = \widehat{\zeta_C^*} = 0\right)$

$$\left(\widetilde{C}^{fb} - \widetilde{C^*}^{fb}\right) = \frac{2a_{\mathsf{H}} - 1}{\left[1 - (2a_{\mathsf{H}} - 1)^2\right]\phi\sigma + (2a_{\mathsf{H}} - 1)^2}\left(\widetilde{Y_{\mathsf{H}}}^{fb} - \widetilde{Y_{\mathsf{F}}}^{fb}\right)$$

With Home Bias, the coefficient above is always positive. In response to a Home supply shock, consumption grows more at Home than abroad. Even if the Home terms of trade fall, it will never be the case that their adverse movements cause 'immiserizing growth.' In response to a positive supply shocks, domestic consumption will never fall either in absolute level, or relative to Foreign Consumption.

However, the consumption growth difference tends to fall with the elasticity of substitution among goods (ϕ). With $\phi \to \infty$, $\widetilde{C}^{fb} \to \widetilde{C^*}^{fb}$.

3 Incomplete markets: financial autarky (FA)

If markets are incomplete, shocks drive a wedge between domestic and foreign wealth, leading to a much richer array of results relative to the case of efficient risk insurance. In what follows we will place (general-equilibrium) wealth effects, and their implications for relative domestic demand, at the center of our analysis.

We first shut down financial flows assuming financial autarky. This allows us to derive a close-up analysis of the role of relative price adjustment in shock transmission. We impose that external trade must be balanced each period,

$$TOT \cdot C_{\mathsf{F}} - C^*_{\mathsf{H}} = 0$$

implying that consumption expenditure has to equal current income, i.e., $P_t C_t - P_{H,t} Y_{H,t} = 0$.

Relative wealth and the relative demand gap or imbalance (1)

As resources cannot be transferred over time, set up the following Lagrangian:

$$Max_{\{c,\chi\}}\zeta_{C,t}\frac{C_t^{1-\sigma}}{1-\sigma}+\chi_t\left(P_{H,t}Y_t-P_tC_t\right)$$

where χ is the multiplier attached to the FA budget constraint. The F.O.C. $\zeta_{C,t}C^{-\sigma}=\chi_tP_t$ — for the foreign country $\zeta^*_{C,t}\left(C^*_t\right)^{-\sigma}=\chi^*_tP^*_t$. Combining

$$\frac{\zeta_{C,t}}{\zeta^*_{C,t}}\left(\frac{C}{C^*_t}\right)^{-\sigma}\frac{P^*_t}{P_t}=\frac{\zeta_{C,t}}{\zeta^*_{C,t}}\left(\frac{C}{C^*_t}\right)^{-\sigma}RER_t=\frac{\chi_t}{\chi^*_t}$$

The marginal utility of consumption cannot be expected to be equalized across states of nature. There is a gap relative to the perfect risk sharing condition $\frac{\zeta_{C,t}}{\zeta^*_{C,t}}\left(\frac{C}{C^*_t}\right)^{-\sigma}RER_t=0$.

Relative wealth and the relative demand gap or imbalance (2)

Log-linearising around a symmetric steady state (in which relative wealth is constant), define the DGAP as in Viani 2010

$$DGAP = \left(\widetilde{RER}\right) + (\widehat{\zeta_C^*} - \widehat{\zeta_C}) - \sigma\left(\widehat{C} - \widehat{C^*}\right)$$

Country-specific shocks open a gap proportional to the shadow value of the current income across the two countries. This is measured in terms of PPP- and preference-adjusted differential between national consumption.

- A positive gap means that the shadow value of Home income is higher, hence Home consumers are relatively poorer. Relative to efficiency (complete market) the cross-country allocation of consumption is imbalanced: too much consumption in the Foreign country.

Consumption demand in financial autarky

To understand wealth effect, using $PC = Y_{\mathsf{H}}P_{\mathsf{H}}$, write the domestic demand for Home goods as:

$$C_{\mathsf{H}} = a_{\mathsf{H}}\left(\frac{P_{\mathsf{H}}}{P}\right)^{-\phi}C = a_{\mathsf{H}}\left(\frac{P_{\mathsf{H}}}{P}\right)^{-\phi}\frac{P_{\mathsf{H}}}{P}Y_{\mathsf{H}} = \frac{a_{\mathsf{H}}}{a_{\mathsf{H}} + (1-a_{\mathsf{H}})\,TOT^{1-\phi}}Y_{\mathsf{H}}$$

where the elasticity of substitution across the two goods ϕ is the demand's price elasticity. Analogous expressions can be derived for the Foreign country.

Key: in equilibrium changes in $\frac{P_{\mathsf{H}}}{P}$ modify not only the relative price faced by Home households as consumers, but also the value of Home output relative to Foreign output: for this reason, the income effects analyzed below are different from the effects relevant to partial equilibrium analysis.

Substitution and income effects from international price movements (1)

$$C_{\mathsf{H}} = a_{\mathsf{H}} \left(\frac{P_{\mathsf{H}}}{P} \right)^{-\phi} C = a_{\mathsf{H}} \left(\frac{P_{\mathsf{H}}}{P} \right)^{-\phi} \frac{P_{\mathsf{H}}}{P} Y_{\mathsf{H}} = \frac{a_{\mathsf{H}}}{a_{\mathsf{H}} + (1 - a_{\mathsf{H}})\, TOT^{1-\phi}} Y_{\mathsf{H}}$$

- A fall in the relative price of the domestic tradable P_H/P (a deterioration of TOT) raises domestic demand by ϕ:
  - substitution effect (SE) is positive: $SE > 0$.
- But for an unchanged Y_H, consumption C falls by 1 (recall $C = Y_H P_H / P$).
  - income effect (IE) is negative $IE < 0$.

Substitution and income effects ...(2)

To see these two effects:

$$\frac{\partial C_{\mathsf{H}}}{\partial TOT} = \overbrace{\left[\phi \cdot \left(\frac{a_{\mathsf{H}} \left(1 - a_{\mathsf{H}}\right) TOT^{-\phi}}{\left[a_{\mathsf{H}} + \left(1 - a_{\mathsf{H}}\right) TOT^{1-\phi}\right]^{2}} Y_{\mathsf{H}} \right)\right]}^{\text{substitution effect}}$$

$$- \overbrace{\left[1 \cdot \left(\frac{a_{\mathsf{H}} \left(1 - a_{\mathsf{H}}\right) TOT^{-\phi}}{\left[a_{\mathsf{H}} + \left(1 - a_{\mathsf{H}}\right) TOT^{1-\phi}\right]^{2}} Y_{\mathsf{H}} \right)\right]}^{\text{income effect}} =$$

Clearly

$$\frac{\partial C_{\mathsf{H}}}{\partial TOT} > \quad 0 \iff \phi > 1$$

- When $\phi > 1$, the deterioration of TOT will raise domestic demand for the Home good. SE stronger than IE in absolute value.

- when $\phi < 1$ the negative IE will more than offset SE. Thus, terms-of-trade depreciation will reduce the domestic demand for the Home tradable.

Substitution and income effects...(3)

Abroad, foreign demand for Home tradables C_H^*, instead, will always be increasing in TOT. Independently of ϕ, the substitution and income effects in this case are both positive. $SE^* > 0$, $IE^* > 0$:

$$C_H^* = (1 - a_H)\left(\frac{P_H^*}{P^*}\right)^{-\phi} C^* = (1 - a_H)\left(\frac{P_H^*}{P^*}\right)^{-\phi} \frac{P_F^*}{P^*} Y_F^* =$$

$$=$$

$$\frac{(1 - a_H)\,TOT^{\phi}}{a_H + (1 - a_H)\,TOT^{\phi-1}} Y_F^*$$

The world demand for Home goods $C_H + C_H^*$

Combining the above results:

- As long as the negative Income Effect in the Home country is not too strong, the world demand for Home goods $C_H + C_H^*$ will be decreasing in their relative price, i.e. increasing in TOT.

- When ϕ is sufficiently below 1 and the Home bias in consumption is sufficiently high (i.e., a_H is large relative to a_H^*), instead, the opposite occurs. World demand is falling in TOT.

From world demand to general equilibrium effects of endowment shocks.

Let's now trace the general equilibrium implications of our characterization of the world demand for Home tradable goods. Since $TOT = C^*_H/C_F$, taking a log-linear approximation around a symmetric equilibrium we obtain:

$$\widetilde{TOT} = \frac{1}{1 - 2a_H (1 - \phi)} \left(\widetilde{Y_H} - \widetilde{Y^*_F}\right)$$

$$\widetilde{RER} = \frac{2a_H - 1}{1 - 2a_H(1 - \phi)} \left(\widetilde{Y_H} - \widetilde{Y^*_F}\right)$$

Focus on the sign of the coefficient on the r.h.s. of the above expressions. To study the behavior of the economy, we derive three important thresholds for ϕ: $\phi(TOT)$ and $\phi(CORR)$ and $\phi(GAP)$.

International transmission and price volatility: deriving the $\phi(TOT)$ threshold

A positive innovation to $\widehat{Y_H}$ worsens the terms of trade and depreciates the real exchange rate as long as

$$\phi > \phi(TOT) = 1 - \frac{1}{2a_H}$$

$\phi(TOT)$ denotes the elasticity threshold at which the response of TOT switches sign. When $\phi > \phi(TOT)$.

- a positive Home output shock benefits foreign consumers via better import prices;.
- RER and TOT volatility is *decreasing* in ϕ.

International transmission and price volatility

With Home bias, however, a positive innovation to Y_H can actually *appreciate* TOT and RER when

$$0 < \phi < \phi(TOT)$$

Note that $\phi(TOT)$ is less than 1/2.

- a positive output innovation actually hurts foreign consumers, as the price of Home goods rises.

- In this region volatility is *increasing* in ϕ.

A 'perverse' terms of trade transmission of positive output shocks?

With $\phi < \phi(TOT)$ (i.e. less than 1/2 with positive Home Bias), for a positive supply shock to Y_H to be matched by an increase in world demand, the Home terms of trade needs to *appreciate*. Because of Home bias, the income of the H economy needs to be boosted by an increase in their supply price for absorbing the higher endowment of good H.

Strong wealth effects in incomplete markets make it possible for the terms of trade movements to hurt foreign consumers in response to positive output gains.

- Important note: strong income effects raise the possibility of multiple steady states (e.g., see the discussion in Corsetti and Dedola [JIE 2005]).

Price volatility and elasticities

We have seen above that the volatility of RER and TOT is non-monotonic in ϕ: it is increasing in ϕ for $\phi < \phi(TOT)$, decreasing otherwise.

=> An important implication is that there will be two values of ϕ (below and above $1 - \frac{1}{2a_H}$) that yield the same volatility of the terms of trade and real exchange rate: one associated with positive, the other associated with negative international transmission.

Moreover, the response of international relative prices to output shocks tend to become stronger as ϕ approaches $\phi(TOT)$ from either side.

RER and relative consumption

Use the balanced-trade condition to express relative consumption as a function of the terms of trade:

$$TOT \cdot C_F = C_H^* \Longleftrightarrow \frac{C}{C^*} = RER^{\phi} TOT^{\phi-1}$$

Then, using the relation between TOT and RER, derive the log-linearized relationships between relative consumption and the real exchange rate:

$$\widetilde{RER} = \frac{2a_H - 1}{2a_H\phi - 1}\left(\tilde{C} - \widetilde{C^*}\right).$$

The relation between real exchange rates and relative consumption can have either sign, depending on the values of a_H and ϕ. Specifically, with Home Bias in consumption, the correlation C/C^* and RER will be negative for ϕ below the threshold $\frac{1}{2a_H} = \phi(CORR) < 1$. Note $\phi(CORR) > \phi(TOT)$.

1. For $\phi > \phi(CORR)$, a positive Home output shock depreciates the Home terms of trade, to the benefit of Foreign consumers (positive transmission). Relative consumption and the real exchange rate are positively correlated. Both this correlation, and international price movements, have the same sign as under complete markets. This is the conventional view of transmission.

2. For $\phi < \phi(TOT)$, a positive Home output shocks appreciates the Home terms of trade, and the real exchange rate. Home consumption rises relative to Foreign consumption, which falls. Price movements and relative consumption — RER correlation have the opposite sign with respect to the case of complete markets.

3. The case of $\phi(TOT) < \phi < \phi(CORR)$: the fall in the Home terms of trade is so large, that Foreign consumers benefit from a Home supply shocks more

than domestic consumers. Foreign consumption rises relative to domestic consumption, as the Home real exchange rate depreciates.

To sum up: in response to positive domestic supply shocks

1. if $\phi > \phi(CORR)$: the correlation between $\{C/C^*, RER\}$ is positive; Home terms of trade TOT depreciate.

2. if $\phi < \phi(TOT)$: the correlation between $\{C/C^*,\ RER\}$ is negative; TOT appreciates.

3. if $\phi(TOT) > \phi > \phi(CORR)$ the correlation between $\{C/C^*,\ RER\}$ is negative. TOT still depreciates.

Complete markets versus financial autarky

The FA allocation clearly differs from CM. Compare the above with:

$$\begin{aligned}
(2a_{\mathsf{H}}\phi - 1)\,\widetilde{RER} &= (2a_{\mathsf{H}} - 1)\left(\widetilde{C} - \widetilde{C}^{*}\right). \\
(\eta + \sigma)\,\widetilde{Y}_{H} &= (\sigma - 1)\,(1 - a_{\mathsf{H}})\,\widetilde{TOT} + \widehat{\zeta}_{C} \\
(\eta + \sigma)\,\widetilde{Y}_{F} &= (\sigma - 1)\,(1 - a_{\mathsf{H}})\left(-\widetilde{TOT}\right) + \widehat{\zeta}_{C}^{*} \\
(1 - 2a_{\mathsf{H}}\,(1 - \phi))\,\widetilde{TOT} &= \widetilde{Y}_{H} - \widetilde{Y}_{F}
\end{aligned}$$

The two allocation may happen to coincide under special configurations of parameters/shocks. Specifically, for the FA allocation to coincide with the CM one it must be the case that DGAP is identically equal to zero:

$$\sigma\left(\widetilde{C} - \widetilde{C^{*}}\right) - \left(\widetilde{RER}\right) - \left(\widehat{\zeta_{C}} - \widehat{\zeta_{C}^{*}}\right) = 0$$

Using the following relations under FA

$$\widetilde{RER} = (2a_{\mathsf{H}} - 1)\,\widetilde{TOT} = \frac{2a_{\mathsf{H}} - 1}{1 - 2a_{\mathsf{H}}(1-\phi)}\left(\widetilde{Y}_H - \widetilde{Y}_F\right),$$

$$\left(\widetilde{C}_t - \widetilde{C}_t^*\right) = (2a_{\mathsf{H}}\phi - 1)\,\widetilde{TOT} = \frac{2a_{\mathsf{H}}\phi - 1}{1 - 2a_{\mathsf{H}}(1-\phi)}\left(\widetilde{Y}_H - \widetilde{Y}_F\right),$$

the DGAP under FA can be written as

$$\frac{\sigma(2a_{\mathsf{H}}\phi - 1) - (2a_{\mathsf{H}} - 1)}{1 - 2a_{\mathsf{H}}(1-\phi)}\left(\widetilde{Y}_H - \widetilde{Y}_F\right) - \left(\widehat{\zeta}_C - \widehat{\zeta}_C^*\right) = 0.$$

- In general, there is NO combination of parameters for which the DGAP is zero in FA, for **both** endowment and taste shocks. There are however different combinations of parameters such that the FA allocation coincides with the CM one conditional on each shock in isolation.

- One particular case is discussed by Cole and Obstfeld (JME 91) and Helpman and Razin 87: shocks are only to endowment (($\widehat{\zeta_C^*} = \widehat{\zeta_C} = 0$), consumption baskets are identical (in our case: $a_H = 1/2$ no Home bias), and $\phi = 1$ (Cobb-Douglas consumption aggregator). In the *Cole and Obstfeld case, if supply shocks only, terms of trade movements make trade in assets redundant* (when the endowment of a country rises, its price falls proportionally, leaving relative wealth unaffected). In this case the DGAP ($DGAP = \widetilde{RER} - \sigma\left(\widehat{C} - \widehat{C^*}\right)$) is zero by virtue of terms of trade adjustment.

A generalization of Cole and Obstfeld: the threshold $\phi(DGAP)$

According to the linearized expressions above the DGAP is zero in a FA allocations with endowment shocks only if

$$\phi\sigma = \frac{1}{2a_H}(2a_H + \sigma - 1)$$

Viani 2010: The D-GAP is positive, zero or negative depending on whether the trade elasticity happens to be above, equal to, or below the following threshold:

$$\phi(DGAP) = \frac{2a_H - 1 + \sigma}{2a_H\sigma} > \phi(CORR)$$

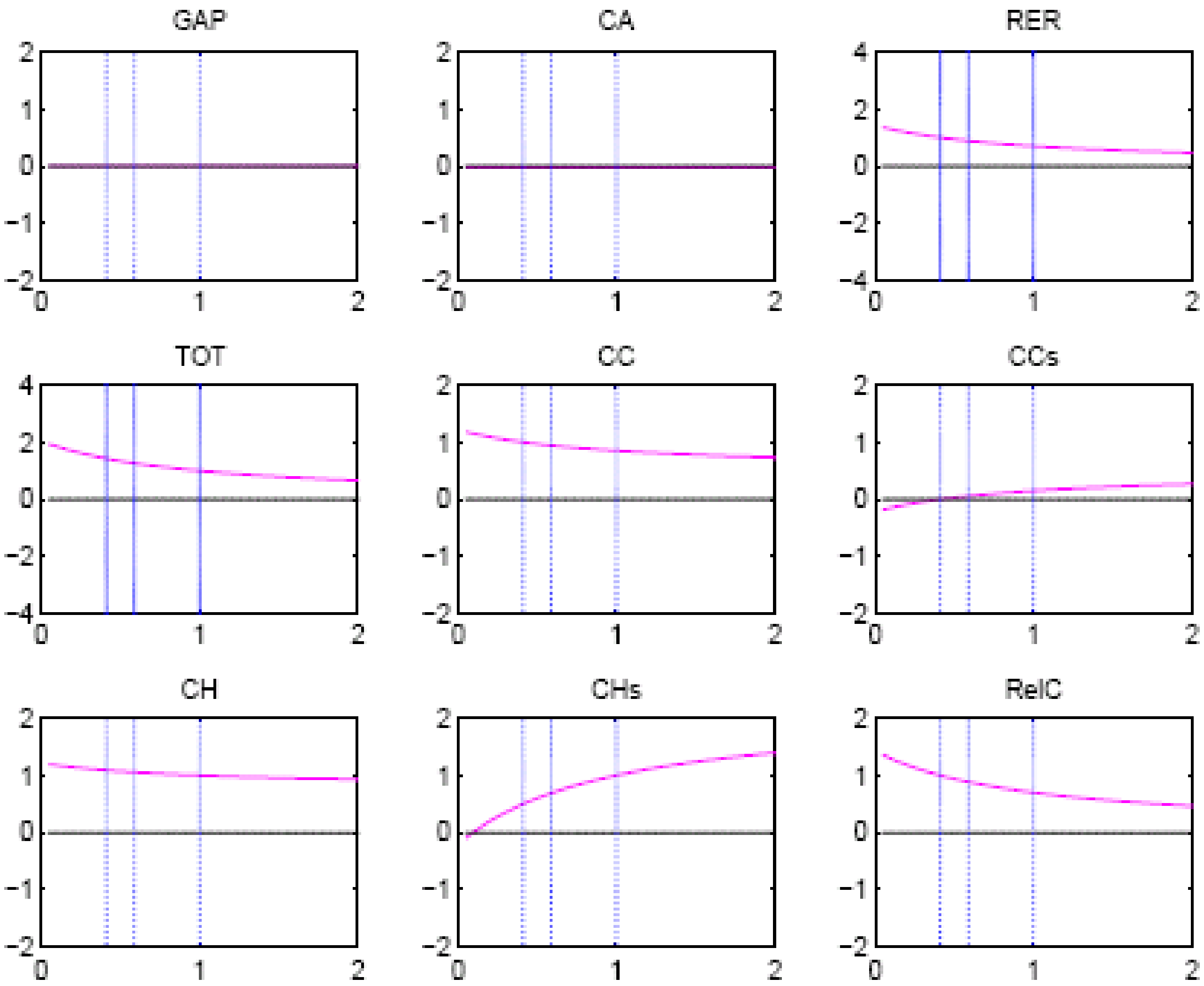
- for $\phi < \phi(DGAP)$, movements in price are in excess of what is needed to insure consumption risk. for $\phi > \phi(DGAP)$ they are too small.

Contrast complete markets and financial autarky

The following figures show contemporaneous response of CA, RER, TOT, C, C*, C_H, C_H^*, C/C^* to endowment shocks, against various values of the elasticity ϕ.

- In CM (first figure): TOT (RER) always depreciate, C/C^* always increases. Volatility is monotonic in the elasticity (high for a low ϕ). Note that for this reason C^* may actually fall, yet consistent with risk sharing.

- In FA (second figure): Positive and negative transmission. DGAP can be negative or positive. Correlation of relative consumption and the real exchange rate switches sign.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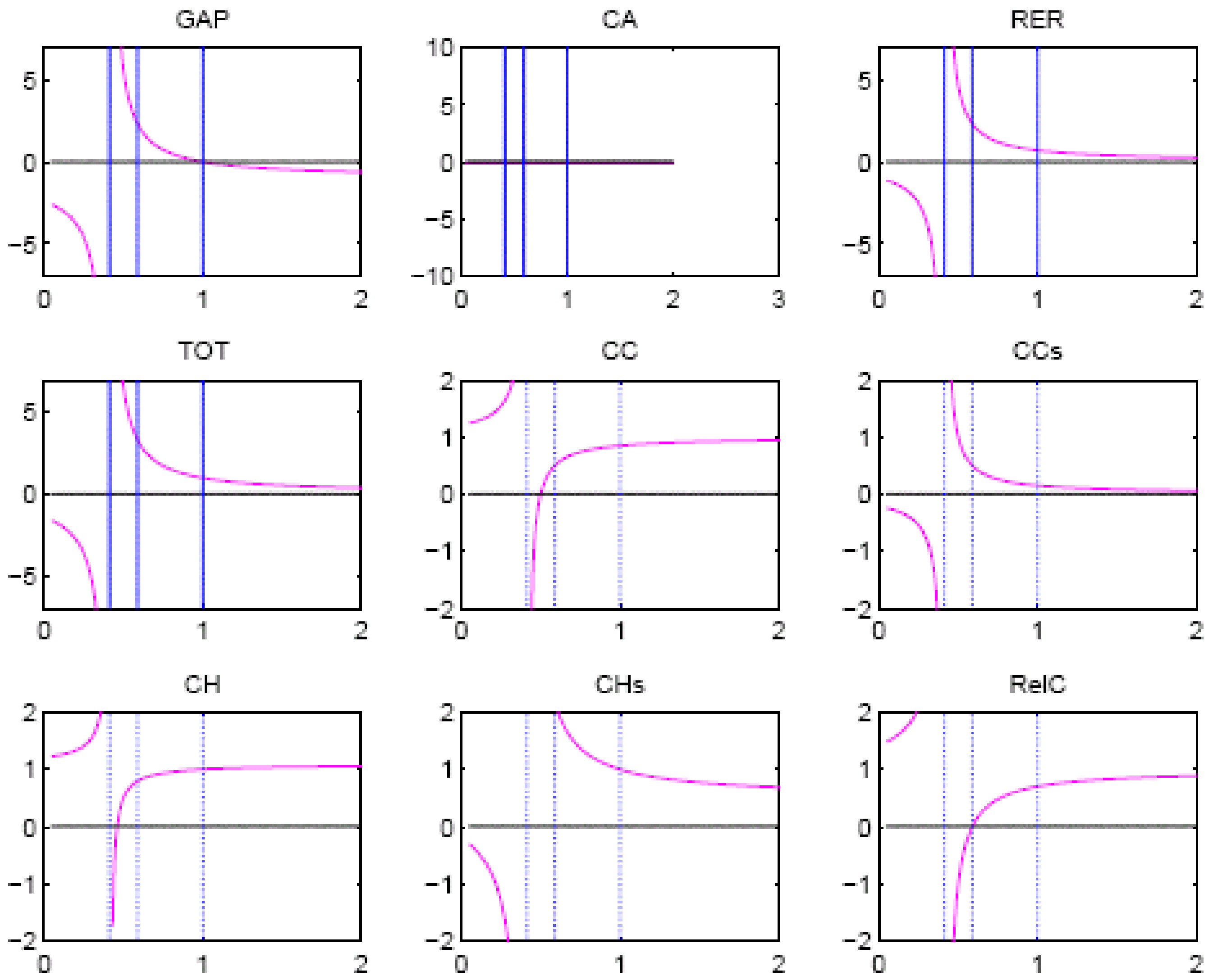
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On the economics of relative-price transmission under FA

- $\phi > \phi(GAP)$, $D - GAP < 0$ (the Home economy is relatively rich ex post in response to a positive supply shock as $\chi_t < \chi_t^*$), the Home terms of trade depreciates.
  - With a high degree of substitutability between H and F goods, only a small depreciation is needed to clear the good markets after the rise in the supply of the H good. But the fall in the price of the H variety is inefficiently small: it does not transfer enough purchasing power to Foreign consumers to generate complete production risk sharing. The terms of trade volatility is also too small.

- $\phi(TOT) < \phi < \phi(GAP)$, $D - GAP > 0$ (H is relative poor ex post as $\chi_t > \chi_t^*$). The Home terms of trade depreciate.
  - With a low degree of substitutability, a strong depreciation of the terms of trade is required for the global good market to absorb the increased supply of the H good. The price of the H variety falls so much in equilibrium that, relative to efficiency, too much purchasing power is transferred to F agents. Excessive volatility of the terms of trade makes F agents relatively richer.

- $\phi < \phi(TOT) < \phi(GAP)$, $D - GAP < 0$ (H is relative rich ex post as $\chi_t < \chi_t^*$). The Home terms of trade appreciate.
  - Price movements magnify the wedge between the relative value of endowments on top and beyond changes in relative quantities.

4 A bridge to the analysis of intertemporal trade

The Transfer problem

Historically, the problem arises with the Ohlin vs. Keynes debate, on the consequences of imposing war reparations on Germany. The issue is whether generating surpluses to pay for war reparations would increase their effective economic costs because of adverse effects on the terms of trade.

Let TR denote transfers from F to Home and write

$$\begin{aligned} P_t C_t &= P_{H,t} Y_{H,t} + P_t TR_t \qquad P_t^* C_t^* = P_{F,t} Y_{F,t} - P_t TR_t \\ &=> \widetilde{TOT} = \frac{2a_H}{(a_H - 1)(2a_H(\phi - 1) + 1)} \widehat{TR} \end{aligned}$$

where we have log-linearized around an equilibrium with TR=0. Note:

$$if\ \phi < \phi(TOT): \quad \partial TOT/\partial TR > 0$$

the country which transfers resources to the other is made relatively richer by the improvement of its terms of trade. Otherwise, it is made relatively poor.

An introduction to consumption smoothing via intertemporal trade

Starting in financial autarky, let's calculate *the shadow intertemporal price of consumption under no trade in asset.* This would be the price at t for a one unit of consumption at $t+1$, at which consumers would find it optimal to consume the autarky allocation:

$$q_t^{FA} = E_t\left[\beta \frac{U_{c,t+1}^{FA}}{U_{c,t}^{FA}} \frac{P_t^{FA}}{P_{t+1}^{FA}}\right] = E_t\left[\beta \frac{U_c\left(\frac{P_{H,t+1}Y_{H,t+1}}{P_{t+1}}\right)}{U_c\left(\frac{P_{H,t}Y_{H,t}}{P_t}\right)} \frac{P_t^{FA}}{P_{t+1}^{FA}}\right]$$

q^{FA} is the price of the bond at which agents find it optimal not to trade bonds across borders. Now, perform the following *experiment*: holding all prices (P and q) fixed, suppose that there is a temporary increase in Home endowment at t (holding $t+1$ endowment fixed). This clearly lowers current marginal utility of consumption relative to the expected future one:

$$U_c\left(\frac{P_{H,t}}{P_t}\left(Y_{H,t}+\Delta Y_{H,t}\right)\frac{1}{P_t^{FA}}\right) \text{ falls relative to } E_t\left[\beta U_c\left(\frac{P_{H,t+1}}{P_{t+1}}Y_{H,t+1}\right)\frac{1}{P_{t+1}^{FA}}\right]$$

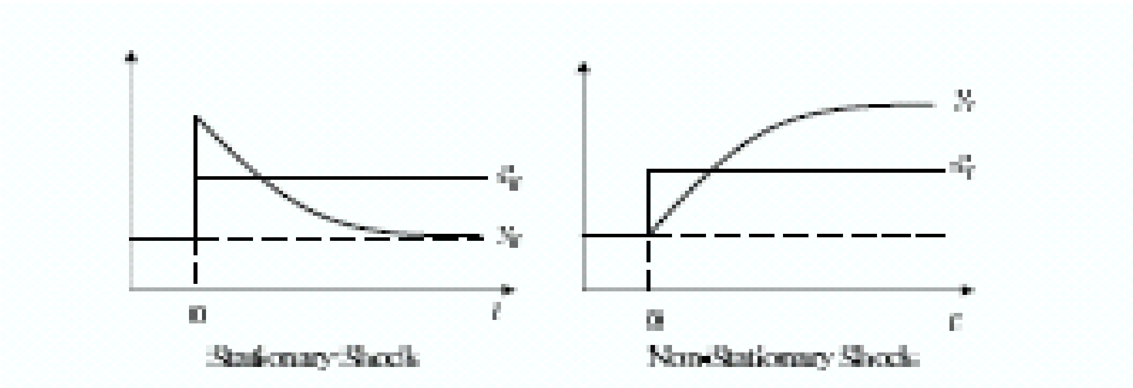
The H consumer could raise its felicity by transferring resources intertemporally at the at the price q, as to increase future consumption relative to today's.

This experiment underlies a *popular view of intertemporal trade:*

- the positive endowment shock makes the home country relatively richer ex post;

- resources flow from the ex-post richer to the ex-post poorer country.

Graphs like the following (from Uribe) are often used as a synthesis of current account theory — based on models with infinite elasticity of substitution between Home and Foreign goods and a given intertemporal price of consumption:



But a positive shock to endowment also changes the equilibrium relative prices. We can envision two issues

- First, we have seen cases under financial autarky, in which the country experiencing a positive supply shock may end up relatively poorer ex-post. Countries hit by positive output shocks may end up borrowing from abroad.

- Second, in light of the transfer problem, it is far from obvious that resources will necessarily flow from ex-post rich to ex-post poor countries. The problem is that, with a low elasticity, when a country which is rich because of more output and stronger terms of trade try to lend abroad, the terms of trade may appreciate further!

5 Incomplete markets: trade in noncontingent bonds

We now allow agents to trade in one noncontingent bond, i.e. to borrow and lend across borders. Assume a one period, discount bond denominated in domestic currency, traded at the price $q = \frac{1}{1+i}$ (in terms of H-consumption):

$$q_{\mathsf{H},t} B_{\mathsf{H},t+1} \leq B_{\mathsf{H},t} + P_{\mathsf{H},t} Y_{H,t} - P_t C_t$$

In a world equilibrium, market clearing requires $B_{\mathsf{H},t} + B^{*}_{\mathsf{H},t} = 0$. Note: the change in net foreign wealth is the current account: $q_{\mathsf{H},t}\left(B_{\mathsf{H},t+1} - B_{\mathsf{H},t}\right) = P_{\mathsf{H},t} Y_{H,t} - P_t C_t + (1 - q_{\mathsf{H},t}) B_{\mathsf{H},t}$. The right hand side is the sum of trade balance (output minus demand), and the net factor income from abroad.

The Home household problem

$$Max\ E_t \sum_{s=0}^{\infty} \left[\begin{array}{c} \beta^s \zeta_{C,t+s} \frac{C_t^{1-\sigma}-1}{1-\sigma} + \\ \upsilon_{t+s} \left[-q_{\mathsf{H},t+s} B_{\mathsf{H},t+s+1} + B_{\mathsf{H},t+s} + P_{\mathsf{H},t+s} Y_{H,t+s} - P_{t+s} C_{t+s} \right] \end{array} \right]$$

is solved by

$$\begin{aligned} \zeta_{C,t} C_t^{-\sigma} &= \upsilon_t P_t \\ q_{\mathsf{H},t} &= \beta E_t \left[\frac{\upsilon_{t+1}}{\upsilon_t} \right] \end{aligned}$$

For simplicity, abstract from taste shocks, and combine the above with the foreign analog:

$$E_t \left[\frac{\zeta_{C,t+1} C_{t+1}^{-\sigma}}{\zeta_{C,t} C_t^{-\sigma}} \frac{P_t}{P_{t+1}} \right] = E_t \left[\frac{\zeta^*_{C,t+1} \left(C^*_{t+1} \right)^{-\sigma}}{\zeta^*_{C,t} \left(C^*_t \right)^{-\sigma}} \frac{P^*_t}{P^*_{t+1}} \right]$$

Assuming that the economy is initially at a steady state with zero bond holding:

$$-\sigma E_t\left(\widetilde{C}_{t+1}\right)+\sigma E_t\left(\widetilde{C^*}_{t+1}\right)+E_t\widetilde{RER}_{t+1}+\overbrace{\sigma\widetilde{C}_t-\sigma\widetilde{C}^*_t-\widetilde{RER}_t}^{=0\text{ by initial conditions}}=0$$

suggests that in this economy optimally choose to 'close the Dgap in expectations', as they cannot do this state by state.

$$E_t\left[Dgap_{t+1}\right]=E_t\widetilde{RER}_{t+1}-\sigma E_t\left(\widetilde{C}_{t+1}-\widetilde{C^*}_{t+1}\right)=0$$

Nontraded risk and wealth wedges (1)

By the law of one price in the market for nominal risk-free bond (i.e. with perfect capital market integration) intertemporal trade equalizes the marginal cost in utility terms of forgoing $q_{\mathsf{H},t}$ units of currency of country i today to the marginal benefit of enjoying one unit tomorrow.

$$\begin{aligned} q_{\mathsf{H},t} &= E_t\left[\beta \frac{U_{c,t+1}}{U_{c,t}} \frac{P_t}{P_{t+1}}\right] = E_t\left[\beta^* \frac{U^*_{c,t+1}}{U^*_{c,t}} \frac{P^*_t \mathcal{E}_t}{P^*_{t+1} \mathcal{E}_{t+1}}\right] \\ &=> E_t\left(\widetilde{RER}_{t+1} - \widetilde{RER}_t\right) \approx E_t\left[\left(\widetilde{U}^*_{c,t+1} - \widetilde{U}^*_{c,t}\right) - \left(\widetilde{U}_{c,t+1} - \widetilde{U}_{c,t}\right)\right]. \end{aligned}$$

By trading bonds, agents ensure that, in expectations, real depreciation is associated with higher consumption growth in the domestic economy relative to consumption growth abroad (precisely, lower relative growth in the marginal utility of consumption) — a version of the DGAP in growth rate.

Nontraded risk and wealth wedges (2)

But in a stochastic environment, the international bond is traded only after the resolution of uncertainty, and does not provide households with ex-ante insurance against country-specific income shocks — it only makes it possible to reallocate wealth and smooth consumption over time. Namely, ex post:

$$\frac{U_{c,t+1}}{U_{c,t}}\frac{P_t}{P_{t+1}} - \frac{U^*_{c,t+1}}{U^*_{c,t}}\frac{P^*_t\mathcal{E}_t}{P^*_{t+1}\mathcal{E}_{t+1}} = \varepsilon_{t+1}$$

where ε should be uncorrelated with any random variable on which the payoff of existing (traded) assets can be conditioned. Unexpected shocks thus have effects on relative wealth. These can be so strong that relative consumption and the real exchange rate move in the opposite direction on impact — with trade in bond, they will co-move positively in the aftermath of the shock, when their joint dynamics is dictated by the equation in the previous slide.

Plan of the rest of the slides

Relative to the previous analysis, we now have two possible ways to comprehend strong relative wealth effects from idiosyncratic shocks.

1. Strong income effects from low intratemporal elasticity (complementarity)

2. Anticipation of future income gains: persistent shocks (diffusion, news), with high intratemporal elasticity

We consider these possibilities in turn

5.1 Income effects from low intratemporal elasticity

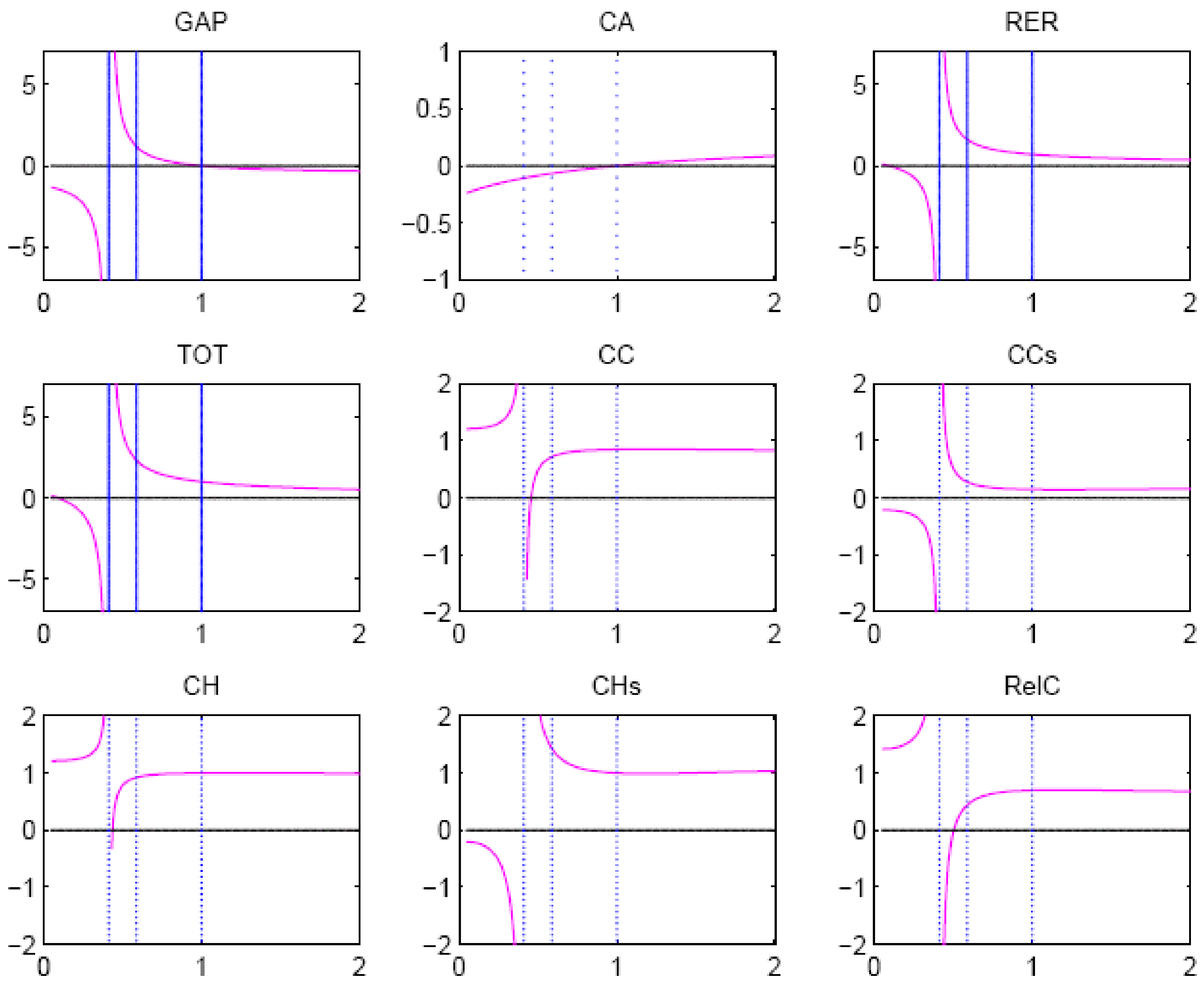
Consider a temporary positive shock to Home endowment. How does consumption smoothing by Home households affect the current account? To answer this question, what matters is the relative wealth effect of the shock, depending on the equilibrium response of the terms of trade.

- If the TOT worsen moderately (high elasticity), home households are relative richer and thus increase their consumption today, but not as much as output. The country runs a current account surplus: a positive net foreign asset position allows residents to raise optimally future consumption.

- However, if the terms of trade worsen a lot (relatively low elasticity), residents are *relatively worse off*: they will need to borrow against future income to smooth consumption.

- And yet, for a very low elasticity, the TOT may strengthen: relative wealth move by more than relative output. While being relatively richer, Home residents borrow to raise current consumption! Why? If they try to lend, the financial flow would actually strengthen the terms of trade even more, amplifying the relative wealth effect (see the transfer problem above).

Remarkably, we learn the sign of the current account looking at the threshold $\phi(GAP)$! The graph to follow is a powerful synthesis of current account theory for the case of temporary shock to output (quantity).


GAP
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RelC

The current account

Recall the graphs from Uribe for the high elasticity case. With a high elasticity, consumption is less volatile than output in response to temporary output disturbances (implying that the trade balance is pro-cyclical), more volatile than output in response to non-stationary shocks (implying that the TB is counter-cyclical).

It should be clear by now that consumption and current account dynamics may be quite different once wealth effects from price movements are modelled.

We have already discuss the case of temporary shocks. We now turn to non-stationary shocks.

5.2 Anticipation of future income gains

Under financial autarky, we have discussed how strong wealth effects could be generated by the model if the elasticity of substitution is low enough. However, there is also a second, most intuitive way, to comprehend strong wealth effects: they can be generated by persistent shocks translating into an increase in the present discount value of own output.

To treat this point analytically, it is convenient to resort to a tractable specification of the model, obtained by positing log utility ($\sigma = 1$) and $\beta = 1$. Also, we only consider two periods, dubbed the short run (today) and the long run (the future).

TOT and output shocks: short vs. long run effects

Let $\widehat{Y_{\mathsf{h}}^{LR}}$ denotes the percentage deviation of Home output from the initial steady state equilibrium in the long-run. Under the above assumptions, one can derive:

$$\widetilde{TOT} = \frac{\overbrace{\left(\widetilde{Y_{\mathsf{H}}} - \widetilde{Y_{\mathsf{H}}^{LR}}\right) - \left(\widetilde{Y_{\mathsf{F}}} - \widetilde{Y_{\mathsf{F}}^{LR}}\right)}^{\text{short run}}}{1 - 4a_{\mathsf{H}}\left(1 - a_{\mathsf{H}}\right)\left(1 - \phi\right)} + \frac{\overbrace{\widetilde{Y_{\mathsf{H}}^{LR}} - \widetilde{Y_{\mathsf{F}}^{LR}}}^{\text{long run}}}{1 - 2a_{\mathsf{H}}\left(1 - \phi\right)}$$

Consider the case of permanent shocks with no dynamics $\widetilde{Y_{\mathsf{H},t}} = \widetilde{Y_{\mathsf{H}}^{LR}}$. Agents have no incentive to smooth consumption. Clearly, the response of TOT is the same as in financial autarky, confirming our analysis under FA:

$$\widetilde{TOT} = \frac{\widetilde{Y_{\mathsf{H}}^{LR}} - \widetilde{Y_{\mathsf{F}}^{LR}}}{1 - 2a_{\mathsf{H}}\left(1 - \phi\right)}$$

A new result: negative transmission with high elasticity and persistent shocks

However, there is now a new configuration of parameters' value which give rise to short-run appreciation. Consider the case $0 < \widetilde{Y_{\mathsf{H},t}} < \widetilde{Y_{\mathsf{H}}^{LR}}$. With $\phi > 1$, the short-run component of the terms of trade response is unambiguously negative; the long-run component unambiguously positive. When the trade elasticity is high enough, and the difference between long-run and short-run output is large, the former component dominates.

Supply shocks with the above features induce a dynamic response of the terms of trade: appreciation in the short run, depreciation in the long run.

Why a high elasticity?

Intuitively, with $\phi > 1$ the depreciation of the terms of trade in the long run is less than proportional to the change in endowment. Hence in the long run the value of the Home output rises relative to world output. This translates into a positive wealth effect generating a short-run domestic consumption boom.

Because of home bias, domestic consumption falls disproportionately on domestic goods, whose supply rises less in the short run than in the long run. Unless the short-run gains in output are already large, the domestic consumption boom creates excess demand for the Home goods, triggering an impact appreciation of the terms of trade. Over time, as the dynamic of Home output endowment fills the gap with demand, the terms of trade appreciation switches to a depreciation (also relative to the initial equilibrium).

Relative consumption and the real exchange rate

Under the simplifying assumptions stated above, we can rewrite the relation between relative consumption and the real exchange rate distinguishing a short-run from a long-run component:

$$\left(\tilde{C} - \widetilde{C^*}\right) = \widetilde{RER}^{SR} + \frac{2a_{\mathsf{H}}(\phi - 1)}{2a_{\mathsf{H}} - 1}\widetilde{RER}^{LR}.$$

Endowment shocks that generate a domestic consumption boom and appreciate the Home terms of trade will induce a negative correlation between relative consumption and the real exchange rate in the short run (recall that the real exchange rate and terms of trade move in the same direction). However, this correlation will not be perfect. A trade elasticity larger than one implies that the second term in the above expression is positive, as the real exchange rate has to depreciate in the long run. From the above, it is straightforward to derive:

$$\left(\widetilde{C} - \widetilde{C^*}\right) = (2a_H - 1)\cdot$$
$$\left[\frac{\left(\widetilde{Y_{\mathsf{H}}} - \widetilde{Y_{\mathsf{H}}^{LR}}\right) - \left(\widetilde{Y_{\mathsf{F}}} - \widetilde{Y_{\mathsf{F}}^{LR}}\right)}{1 - 4a_{\mathsf{H}}\left(1 - a_{\mathsf{H}}\right)\left(1 - \phi\right)} + \left(1 + \frac{2a_{\mathsf{H}}\left(\phi - 1\right)}{2a_{\mathsf{H}} - 1}\right)\frac{\widetilde{Y_{\mathsf{H}}^{LR}} - \widetilde{Y_{\mathsf{F}}^{LR}}}{1 - 2a_{\mathsf{H}}\left(1 - \phi\right)}\right]$$

A note on ‘news shock’: *comparison CM and bond economy (with production) when shocks to H productivity is diffusion*

The following figure shows a impulse responses to a productivity shocks in a production version of our economy (labor is the only factor of production) for the low elasticity case (goods complementarity).

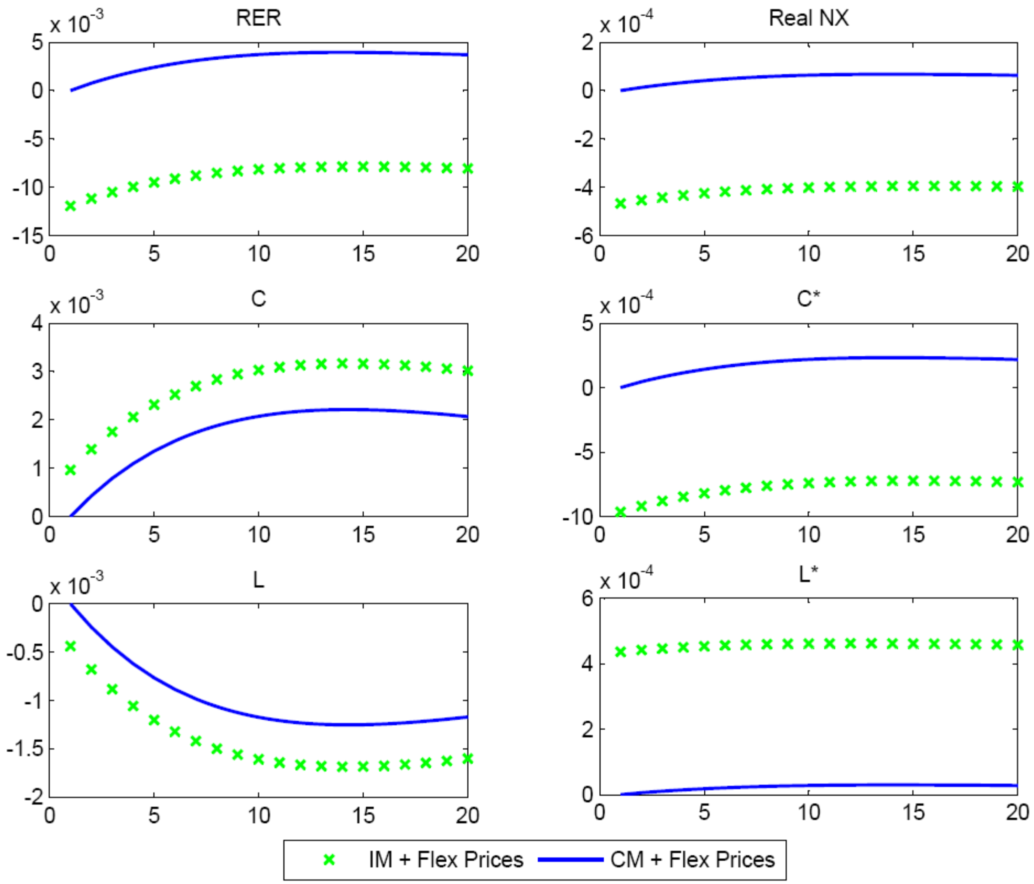
The shock is a diffusion: ‘news shock’ about future productivity.

Relative to first best with CM, the bond economy is characterized by overborrowing, persistent misalignment and imbalances in demand.

Lessons for production economies

A important lesson from the bond-economy is that, with incomplete markets, 'interesting dynamics' of the terms of trade, real exchange rate and relative consumption obtain when output increases gradually over time, and the elasticity of substitution is relatively high. Anticipating output and income gains in the future, consumption-smoothing agents raise current demand above output, causing a temporary appreciation of the terms of trade and the real exchange rate (and a current account deficit).

Note: in an endowment economy, *autoregressive endowment shocks* cannot generate the above output pattern. In a production economy, however, *autoregressive productivity shocks* can, provided that they are persistent. Why? Driven by sustained productivity growth, investment increases in the Home country. As capital stock rises over time, equilibrium output correspondingly increases.



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Real NX
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C
x 10^-3
C*
x 10^-4
L
x 10^-3
L*
x 10^-4
IM + Flex Prices
CM + Flex Prices