# Macro3 EUI 2009: Recap of lecture 1;2-6

- Models of money and their properties

- Empirics

- New Keynesian Models

## Models of Money

- We looked at the following three models:

1. Cash-in-advance: Money are used because a subset of the goods cannot be purchased on credit - financial market friction

2. Money-in-the-utility-function: Money are valued directly because the yield utility - utility based

3. Transactions technology: Money allows for “cheaper” consumption - saves on transactions costs - technology based

## Basic set-up

- Assume: Endowment economy with fixed endowment $y$ every period

- We look at competitive economy: Private sector agents take all prices and all aggregate variables for given

- Private sector budget constraint:

$$c_t + \frac{b_{t+1}}{R_t} + \frac{m_{t+1}}{p_t} = y - \tau_t + b_t + \frac{m_t}{p_t}$$

- Need to assume positive nominal interest rates in order to have bounded budget set

## 1. Cash-In-Advance

- Consumption is a cash good - cannot be purchased on credit

- Agents first visit consumption goods market and after that the asset market

- CIA constraint:

$$p_t c_t \leq m_t$$

- asset market constraint:

$$c_t + \frac{b_{t+1}}{R_t} + \frac{m_{t+1}}{p_t} = y - \tau_t + b_t + \frac{m_t}{p_t}$$

- From the first-order conditions

$$
\begin{aligned}
R_t &= \frac{u_c(c_t) - \lambda_{mt}}{\beta\left(u_c(c_{t+1}) - \lambda_{mt+1}\right)} \\
\frac{m_t}{p_t} &= c_t
\end{aligned}
$$

- Nominal interest rate acts as a tax on consumption

- "Consumption velocity" is constant - quantity theory

## 2. MIU Models

- Money yields utility directly:

$$\max \sum_{t=0}^{\infty} \beta^t u\left(c_t, \frac{m_{t+1}}{p_t}\right)$$

- From the first-order conditions:

$$\begin{aligned} R_t &= \frac{u_c\left(c_t, \frac{m_{t+1}}{p_t}\right)}{\beta u_c\left(c_{t+1}, \frac{m_{t+2}}{p_{t+1}}\right)} \\ \frac{u_{m/p}\left(c_t, \frac{m_{t+1}}{p_t}\right)}{u_c\left(c_t, \frac{m_{t+1}}{p_t}\right)} &= \frac{i_t}{1+i_t} \Rightarrow \\ \frac{m_{t+1}}{p_t} &= F^{MIU}\left(c_t, i_t\right) \end{aligned}$$

- Real interest rate equals IMRS - but: IMRS depends on real money balances

- Money demand depends positively on consumption and negatively on the nominal interest rate

## 3. Transactions Costs

- Cash balances make purchases of goods less costly - fewer trips to the bank

$$\max \sum_{t=0}^{\infty} \beta^t u\left(c_t, 1 - s_t\right) \tag{1}$$

$$s_t = H\left(c_t, \frac{m_{t+1}}{p_t}\right) \tag{2}$$

- First-order conditions imply that:

$$R_t = \frac{1}{\beta} \frac{u_c\left(c_t, l_t\right) - H_c\left(c_t, \frac{m_{t+1}}{p_t}\right) u_l\left(c_t, l_t\right)}{u_c\left(c_{t+1}, l_{t+1}\right) - H_c\left(c_{t+1}, \frac{m_{t+2}}{p_{t+1}}\right) u_l\left(c_{t+1}, l_{t+1}\right)}$$

$$
\begin{aligned}
\frac{H_{m/p}\left(c_t, \frac{m_{t+1}}{p_t}\right) u_l\left(c_t, l_t\right)}{\left(u_c\left(c_t, l_t\right) - H_c\left(c_t, \frac{m_{t+1}}{p_t}\right) u_l\left(c_t, l_t\right)\right)} &= \frac{i_t}{1 + i_t} \\
\frac{m_{t+1}}{p_t} &= F\left(c_t, i_t\right)
\end{aligned}
$$

- The real interest rate equals the IMRS corrected for marginal disutility of shopping.

**Did we leave out other ways?**

- Yes: A few other alternatives. From macro perspective, the one important one that we left out was "Limited Participation" - similar to CIA but assumes that firms must pay salaries before receiving revenues from production

- Such models can generate the "liquidity effect"

- Suppose that firms need to borrow the labor cost from banks at the beginning of the period. At the end of the period, they then repay the bank including the interest

- The loans market equilibrium condition is then:

$$p_t w_t n_t = m_t + \tau_t$$

- For a given wage bill, $p_t w_t n_t$, a monetary injection ($\tau_t > 0$) then drives down the nominal interest rate

- This makes production cheaper and stimulates labor demand which leads to short-run non-neutrality

- This model also implies super non-neutrality

## The Optimum Quantity of Money

- In the shopping time model, the socially optimal quantity of money is the one that minimizes the shopping time

- This requires that $R_m$ needs to be set equal to $R$ : the optimum requires deflating the economy at the rate of the real interest rate - the Friedman rule

$$i = \frac{R}{R_m} - 1 = 0$$

- In the shopping time economy without a satiation point, this can only be attained “approximately”

- Same in CIA model, for the CIA constraint to be binding with equality

**Neutrality, Super Neutrality, Super super ...**

- What happens after:

1. Once and for all change in the money supply? If no real variables affected - neutrality

2. Permanent change in money growth rate? If no real variables (apart from real cash balances) affect - super-neutrality

- Consider competitive MIU model with production:

$$\begin{aligned} V &= \sum_{t=0}^{\infty} \beta^t u\left(c_t, m_{t+1}/p_t, l_t\right) \\ y_t &= f\left(k_t, n_t\right) \\ l_t + n_t &= 1 \end{aligned}$$

- The steady-state must satisfy the conditions:

$$
\begin{aligned}
k &: \quad f_k\left(k^{ss}, 1-l^{ss}\right) = 1/\beta - (1-\delta) \\
m &: \quad \frac{u_{m'/p}\left(c^{ss}, g_m\left(m/p\right)^{ss}, l^{ss}\right)}{u_c\left(c^{ss}, g_m\left(m/p\right)^{ss}, l^{ss}\right)} = \frac{g_m/\beta - 1}{g_m/\beta} \\
n &: \quad \frac{u_l\left(c^{ss}, g_m\left(m/p\right)^{ss}, l^{ss}\right)}{u_c\left(c^{ss}, g_m\left(m/p\right)^{ss}, l^{ss}\right)} = f_n\left(k^{ss}, 1-l^{ss}\right) \\
c &: \quad c^{ss} = f\left(k^{ss}, 1-l^{ss}\right) - \delta k^{ss} \\
\tau &: \quad \tau^{ss} = \left(g_m - 1\right)\left(m/p\right)^{ss}
\end{aligned}
$$

- Neutrality: No real effects of a once and for all change in the money stock - price level affects only prices

- Superneutrality: In general - superneutrality does not hold: $g_m$ affects steady state labor supply, and therefore output, the capital stock and consumption

- Superneutrality requires the marginal rate of substitution between leisure and consumption be independent of real money balances:

$$u\left(c, m'/p, l\right) = u^{1}(c, l) + u^{2}\left(m'/p\right)$$

or

$$u\left(c, m'/p, l\right) = u^{1}(c, l)\, u^{2}\left(m'/p\right)$$

## Dynamics: The Effects of Stochastic Money Growth Rate Shocks

- Model:

$$\begin{aligned} V_0 &= E_0 \sum_{t=0}^{\infty} \beta^t u\left(c_t, m_{t+1}/p_t, l_t\right) \\ l_t &= 1 - n_t \\ u_t &= \frac{\left[a c_t^{1-b} + (1-a)\left(m_{t+1}/p_t\right)^{1-b}\right]^{(1-\theta)/(1-b)}}{1-\theta} + \phi \frac{l_t^{1-d}}{1-d} \end{aligned}$$

- Money growth rate process:

$$g_{mt} = (1-\gamma)\, g_m^{ss} + \gamma g_{mt-1} + \varepsilon_{mt}$$

- The equilibrium must then fulfill the following conditions:

$$
\begin{aligned}
&: \quad k_t^{\alpha} n_t^{1-\alpha} = c_t + k_t - (1-\delta)\, k_t \\
&: \quad R_t = [\alpha y_{t+1}/k_{t+1} + (1-\delta)] \\
&: \quad a c_t^{-b} \left[ a c_t^{1-b} + (1-a)\,(m_{t+1}/p_t)^{1-b} \right]^{(1-\theta)/(1-b)-1} \\
= &\quad \beta a E_t c_{t+1}^{-b} \left[ a c_{t+1}^{1-b} + (1-a)\,(m_{t+2}/p_{t+1})^{1-b} \right]^{(1-\theta)/(1-b)-1} R_t \\
&: \quad \frac{\phi\,(1-n_t)^{-d}}{a c_t^{-b} \left[ a c_t^{1-b} + (1-a)\,(m_{t+1}/p_t)^{1-b} \right]^{(1-\theta)/(1-b)-1}} = (1-\alpha)\, y_t/n_t
\end{aligned}
$$

$$
: \quad \frac{1-a}{a} \left( \frac{m_{t+1}/p_t}{c_t} \right)^{-b} = \frac{i_t}{1+i_t}
$$

$$
: \quad m_{t+1}/p_t = \frac{g_{mt}}{p_t/p_{t-1}} m_t/p_{t-1}
$$

## Implications

**1. Dichotomy: When** $\theta = b$, the real allocation is independent of monetary shocks — labor supply unaffected by real cash balances

**2. Expected Inflation**: When $\theta \neq b$, **superneutrality does not hold** and money growth rate shocks affect the economy through **expected inflation**

- when $\gamma = 0$: shocks do not alter expected inflation and nominal interest rates (neutrality and superneutrality)
- But when $\gamma > 0$ money shocks matter through its' effect on expected inflation: $E_t \frac{p_{t+1}}{p_t}$ will be affected and this changes the nominal interest rate which in turn affects real cash balances and consumption etc..

**3. The lack of a liquidity effect**:

- An autocorrelated positive money growth rate shock means that the current shock increases expected future inflation - therefore the nominal interest rate increases! In other words, there is no "liquidity effect" (negative response of the nominal interest rate to the money supply)

**4. Output, Money and Labor Supply**

- Recall the "labor supply" equation:

$$\frac{\phi\left(1-n_t\right)^{-d} n_t}{a c_t^{-b}\left[a c_t^{1-b}+\left(1-a\right)\left(m_{t+1}/p_t\right)^{1-b}\right]^{(1-\theta)/(1-b)-1}}=\left(1-\alpha\right) y_t$$

- Then positive money growth rate shock decreases $\left(m_{t+1}/p_t\right)$

- Sign of response of labor supply depends on preferences (complementarity of real balances and consumption in a Pareto-Edgeworth sense):

$$\begin{aligned} n \text{ decreases if } b &< \theta \\ n \text{ increases if } b &> \theta \end{aligned}$$

- Generality? Compare with CIA model with the same structure.

- Here we get that::

$$
\begin{aligned}
c &: \frac{1}{c_t} = \lambda_t + \mu_t \\
n &: A(1-n_t)^{-d} = \lambda_t (1-\alpha) y_t / n_t \\
k' &: \lambda_t = \beta E_t \lambda_{t+1} [\alpha y_{t+1}/k_{t+1} + 1 - \delta] \\
b' &: Q_t \lambda_t = \beta E \lambda_{t+1} \\
m' &: \lambda_t = \beta E_t \frac{p_t}{p_{t+1}} \left[\lambda_{t+1} + \mu_{t+1}\right] \\
CIA &: c_t = m_{t+1}/p_t \\
BC &: y_t = c_t + k_{t+1} - (1-\delta) k_t
\end{aligned}
$$

- $g_{mt}$ is serially uncorrelated: $\varepsilon_t^m$ have no real effects: the price level changes (as MIU). Again it's expected inflation that matters.

- When $g_{mt}$ is positively serially correlated: Expected future inflation rises in response to positive money growth rate shock

- This increases $\mu$ (multiplier on CIA) which lowers consumption (see f.o.c. for $c$), which lowers $n$ (see f.o.c. for $n$), which lowers output.

- The reason is that inflation acts as a tax on consumption - higher expected inflation leads to a substitution from consumption (the cash good) to leisure (the 'credit' good).

- In sum: flexible price, competitive models may give rise to real effects of monetary shocks - but they work through expected inflation. Under plausible circumstances, positive money growth rate shocks lead to lower consumption, more leisure, and lower output.

- And in CIA and MIU models: No liquidity effect. This can be generated by limited participation, however.

## Empirics

**Questions:**

(a) Long run relationships between money, inflation, and output

- one-to-one relationship between money growth and inflation in the long-run. Not true in the short-run unless high-inflationary environment
- across all countries, in the long-run money growth seems not to affect output growth but appears no to be the case for OECD
- across all countries, in the long-run, inflation appears not to affect output growth

(b) The business cycle effects of monetary policy **shocks**

- money and real output positively correlated at the business cycle frequencies

- but: such a correlation might be due to reverse causality so we cannot conclude that this means that an expansionary monetary policy shock leads to an increase in output

- Thus, we need to identify *monetary policy shocks*

- In a regression framework this means that we need exclusion restrictions

- Christiano, Eichenbaum and Evans, Bernanke and Gertler (and many others) assume a monetary policy reaction function:

$$S_t = f\left(z_{it}\right) + e_t$$

- $S_t$ :The monetary policy indicator (often specified as the relevant interest rate - Federal Funds rate for the US, or as an indicator of open market operations)

- $e_t$ : the monetary policy shock (the part that is not due to endogenous response to $z_{it}$)

- $f(z_{it})$ a linear function of a vector of variables $z_{it}$

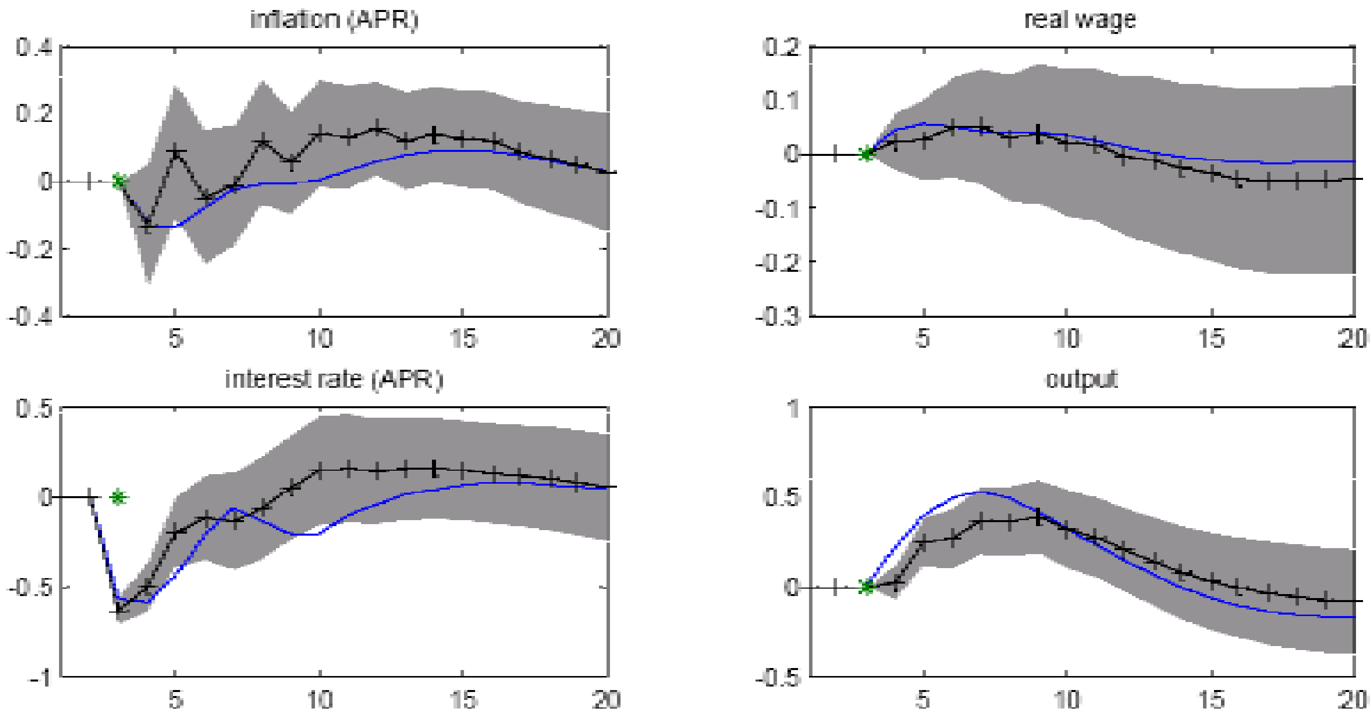
- $z_{it}$ can then be interpreted as the Central Bank's information set: It contains those variables that the CB observes today and reacts to when setting the monetary policy instrument

- $z_{it}$ should include lagged values of relevant variables and it might include contemporaneous values of some variables but not of others

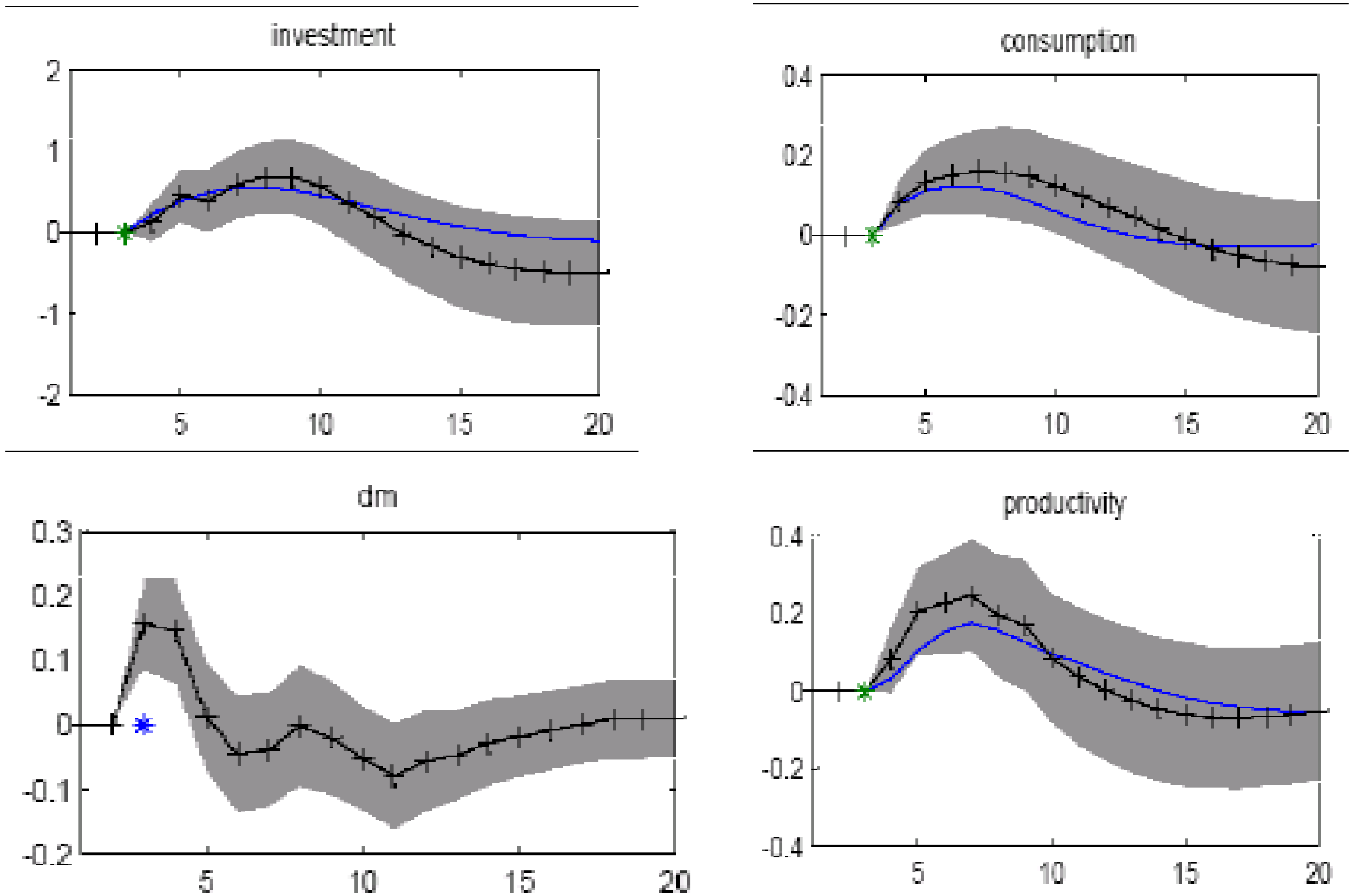
- Christiano, Eichenbaum and Evans assume, for example, that the CB can observe **current output and inflation** - they are in the information set

- This therefore implies that **current** output and inflation, for example, **do not react to the current value of** $S_t$ - but future values of output and inflation, for example, might be affected

- given such assumptions, a VAR technique can be used to estimate the dynamic effects of monetary policy shocks



inflation (APR)
0.4
0.2
0
-0.2
-0.4
5
10
15
20
real wage
0.2
0.1
0
-0.1
-0.2
-0.3
5
10
15
20
interest rate (APR)
0.5
0
-0.5
-1
5
10
15
20
output
1
0.5
0
-0.5
5
10
15
20



investment
consumption
dm
productivity

- large and persistent positive effects on output and its components

- inflation increases persistently but with a significant delay

- Seems incompatible with the type of models we have looked at earlier (but perhaps not with the limited participation model)

## Nominal Rigidities

- We constructed a simple model with staggered contracts. To do this we introduced monopolistic competition

- Calvo: Individual price setters only some times get the opportunity to reset prices

- Were prices flexible, firms would set prices as:

$$p_{it} = \mu p_t mc_t$$

- $\mu$ is the mark-up which is constant in the flexible price equilibrium because of constant price elasticity demand

- $mc_t$ are real marginal costs which are independent of scale due to constant returns to scale

- $p_t mc_t$ are thus nominal marginal costs

- But firms can only sometimes reset their prices

- with probability $\gamma$ they do not get the chance to re-optimize

- The optimal reset price is:

$$p_{it}^* = \frac{1/\rho}{1/\rho - 1} \frac{E_t \sum_{\tau=t}^{\infty} \gamma^{\tau-t} R_{t,\tau} mc_\tau p_\tau^{1/\rho} y_\tau}{E_t \sum_{\tau=t}^{\infty} \gamma^{\tau-t} R_{t,\tau} p_\tau^{1/\rho-1} y_\tau}$$

- When prices are not fully flexible firms need to be forward looking and prices depend on a weighted average of future expected marginal costs

- Since some prices cannot be adjusted, there is short-run non-neutrality

- We can express the NK model as:

$$
\begin{aligned}
\widehat{y}_t &= -\frac{1}{\sigma}\left(\widehat{i}_t - E_t\widehat{\pi}_{t+1}\right) + E_t\widehat{y}_{t+1} \\
\widehat{mc}_t &= \left(\sigma + d\frac{n^{ss}}{1-n^{ss}}\right)\widehat{y}_t - \left(1 + \phi\frac{n^{ss}}{1-n^{ss}}\right) z_t \\
\widehat{m}_{t+1} - \widehat{p}_t &= \frac{\sigma}{b}\widehat{y}_t - \frac{1}{b}\widehat{i}_t \\
\widehat{\pi}_t &= \frac{(1-\gamma\beta)(1-\gamma)}{\gamma}\widehat{mc}_t + \beta E_t\widehat{\pi}_{t+1}
\end{aligned}
$$

- The first of these equations is referred to as the forward looking IS curve in the NK literature, the second equation determines marginal costs, the third equation is the money demand equation (which then combined with a money supply equation is sometimes referred to as an LM curve), the fourth equation is the forward looking Phillips curve

- Two instructive ways of re-formulating the Phillips curve:

$$\begin{aligned}
\pi_t &= \frac{(1-\gamma\beta)(1-\gamma)}{\gamma} E_t \sum_{\tau=t}^{\infty} \beta^{\tau-t} \widehat{mc}_\tau \\
\widehat{\pi}_t &= \frac{(1-\gamma\beta)(1-\gamma)}{\gamma} \left(\sigma + d\frac{n^{ss}}{1-n^{ss}}\right) y_t^{gap} + \beta E_t \widehat{\pi}_{t+1}
\end{aligned}$$

- the first of these tells us that inflation is forward looking - while prices are sticky, inflation is not. Inflation is determined by future expected real marginal costs.

- the second says that we can express the Phillips curve in terms of the output gap - output is above (below) its flexible price level is inflationary (deflationary). also reveals the difference vis-à-vis old style Phillips curve - its' future inflation that matters, and the right output measure is the deviation from flexible price output

- The monetary transmission mechanism: works through the real interest rate - solving the "IS" curve forwards:

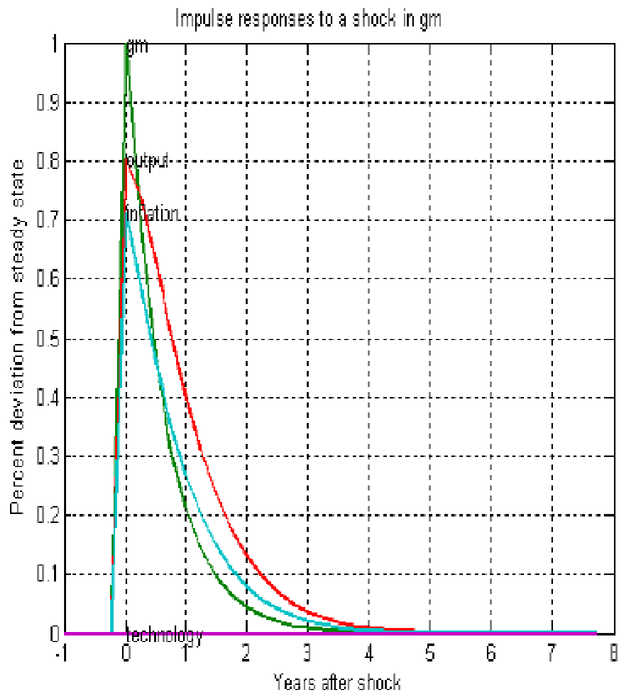
$$\widehat{y}_t = -\frac{1}{\sigma} \sum_{s=0}^{\infty} E_t \left( \widehat{i}_{t+s} - E_t \widehat{\pi}_{t+1+s} \right)$$

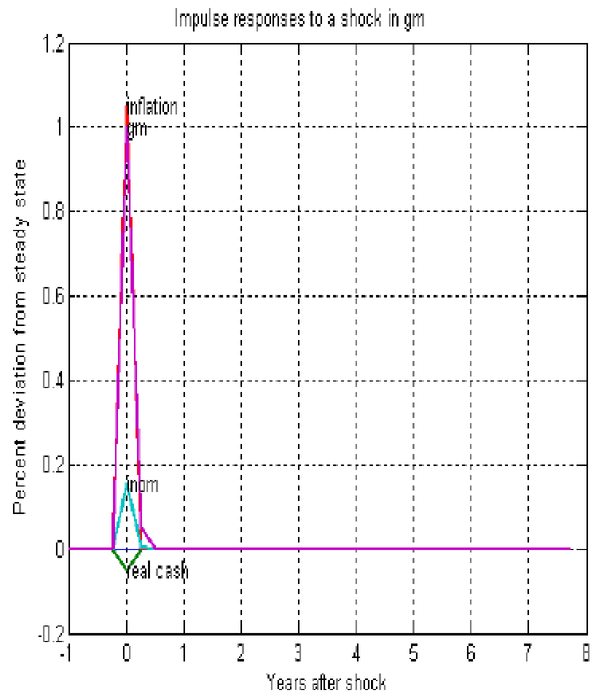
- The path of output is determined by the real interest rate - a lower real interest rate increases current output relative to future expected output because it makes current consumption cheap relative to future consumption

- the effects of monetary policy shocks: Assume that:

$$\widehat{g}_{mt} = \rho_m \widehat{g}_{mt-1} + e_t^m$$

- Results when $\gamma = 3/4$ so that prices stay fixed for one year on average



- We get large and persistent output effects - but (i) no inflation persistence, and (ii) effects hinge on large extent of price stickiness

- Monetary policy rules: We saw problems in local indeterminacy and self-fulfilling inflation expectations

- We have seen that this is the case if the interest rate is set as follows:

$$\begin{aligned} \widehat{i}_t &= \rho_i \widehat{i}_{t-1} + e_t^i \\ \rho_i &< 1 \end{aligned}$$

- To address this: Real interest rate must decline in response to (expected) inflation — the Taylor principle

- Is the NK Phillips curve “consistent” with the data?

- Gali and Gertler generalize the Phillips curve slightly by allowing for rule-of-thumb producers:

$$\begin{aligned}
\widehat{\pi}_t &= (1-\omega)\frac{(1-\gamma\beta)(1-\gamma)}{\gamma}\widehat{mc}_t + \varsigma_f E_t \widehat{\pi}_{t+1} + +\varsigma_b \widehat{\pi}_{t-1} \\
\varsigma_f &= \frac{\beta\gamma}{\gamma+\omega(1-\gamma(1-\beta))} \\
\varsigma_b &= \frac{\omega}{\gamma+\omega(1-\gamma(1-\beta))}
\end{aligned}$$

- Notice that we get the standard NK Phillips curve when $\omega = 0$

- There is a significant share of producers $(\omega)$ that appear to use rule-of-thumb pricing - this is concerning. But the model forecasts well.

- **Indexation:** above - firms need to keep constant prices if not given the chance to re-optimize

Table 2
Estimates of the new hybrid Phillips curve

| | $\omega$ | $\theta$ | $\beta$ | $\gamma_b$ | $\gamma_f$ | $\lambda$ |
|---|---|---|---|---|---|---|
| GDP deflator | | | | | | |
| (1) | 0.265 | 0.808 | 0.885 | 0.252 | 0.682 | 0.037 |
| | (0.031) | (0.015) | (0.030) | (0.023) | (0.020) | (0.007) |
| (2) | 0.486 | 0.834 | 0.909 | 0.378 | 0.591 | 0.015 |
| | (0.040) | (0.020) | (0.031) | (0.020) | (0.016) | (0.004) |
| Restricted $\beta$ | | | | | | |
| (1) | 0.244 | 0.803 | 1.000 | 0.233 | 0.766 | 0.027 |
| | (0.030) | (0.017) | | (0.023) | (0.015) | (0.005) |
| (2) | 0.522 | 0.838 | 1.000 | 0.383 | 0.616 | 0.009 |
| | (0.043) | (0.027) | | (0.020) | (0.016) | (0.003) |
| NFB deflator | | | | | | |
| (1) | 0.077 | 0.830 | 0.949 | 0.085 | 0.871 | 0.036 |
| | (0.030) | (0.016) | (0.019) | (0.031) | (0.018) | (0.008) |
| (2) | 0.239 | 0.866 | 0.957 | 0.218 | 0.755 | 0.015 |
| | (0.043) | (0.025) | (0.021) | (0.031) | (0.016) | (0.006) |

- But - might be easy just to index - update preset price with lagged inflation if not given the chance to re-optimize

- We get that:

$$\widehat{\pi}_t - \widehat{\pi}_{t-1} = \beta \left( E_t \widehat{\pi}_{t+1} - \widehat{\pi}_t \right) + \frac{(1 - \beta\gamma)(1 - \gamma)}{\gamma} \widehat{mc}_t$$

- we get inflation persistence since:

$$\widehat{\pi}_t = \frac{\beta}{1 + \beta} E_t \widehat{\pi}_{t+1} + \frac{1}{1 + \beta} \widehat{\pi}_{t-1} + \frac{(1 - \beta\gamma)(1 - \gamma)}{\gamma (1 + \beta)} \widehat{mc}_t$$

- i.e. the Phillips curve relates current inflation to past and future inflation

- Did we leave out other important stuff?

1. Other models of sticky prices:

- Taylor staggered contracts - fixed contract length and staggering of firms - but hard to handle

- Other possibility is that it is costly to change prices. Large literature on "menu costs" - a small fixed cost of changing prices.

- An easier way is Rotemberg (1982) - assumes that it is costly to change prices. A producer sets the price to minimize:

$$E_t \sum_{j=0}^{\infty} \beta^j R_{t,t+j} \left[ \left( p_{t+j} - p_{t+j}^{opt} \right)^2 + \chi \left( p_{t+j} - p_{t+j-1} \right)^2 \right]$$

- $p_{t+j}^{opt}$ is the optimal (flexible price) price, $\chi$ is a measure of price rigidity - when $\chi = 0$ firms will set $p_t = p_t^{opt}$, the larger is $\chi$ the more sticky are prices

- One can show that this gives rise to a Phillips curve similar to the one derived earlier:
$$\widehat{\pi}_t = \beta E_t \widehat{\pi}_{t+1} + \frac{1/\rho}{\chi} \widehat{mc}_t$$
recall $1/\rho$ is elasticity of substitution (markup is $1/(1-\rho)$).

- Hence: the business cycle properties are exactly like the Calvo model that we looked at!

- You can 'translate' price stickiness in one model into the other comparing $\frac{1/\rho}{\chi}$ and $\frac{(1-\gamma\beta)(1-\gamma)}{\gamma}$.

- However, there is an important difference. In Rotemberg, prices and production is symmetric among firms. In Calvo they are not. This is relevant for assessing the welfare implication of shocks.

- Also left out information based theories

2. Other sources of rigidities:

- We assumed that prices are rigid - perhaps wages are more rigid than prices?

- Perhaps “information” is sticky rather than prices themselves?

3. Real rigidities:

- We looked at very simplified model - no capital, no other frictions

- These rigidities might be important too.

- Specifically: interaction between nominal and real rigidities may affect price and macro dynamics for any given $\gamma$