

Openness and the case for flexible exchange rates*

Giancarlo Corsetti

European University Institute, University of Rome III and CEPR

First draft: November 2005

This revision February 2006

Abstract

Models of stabilization in open economy traditionally emphasize the role of exchange rates as a substitute for nominal price flexibility in fostering relative price adjustment. This view has been recently criticized on the ground that, to the extent that prices are sticky in local currency, the exchange rate does not play the stabilizing role envisioned by the received wisdom. An important question is whether, for this very reason, stabilization policies should limit exchange rate movements, or even eliminate them altogether. In this paper, I re-assess this issue by extending the Corsetti and Pesenti (2001) model to allow for home bias in consumption — so that I can exploit the advantages of closed-form solutions. While this extension leaves most properties of the model unaffected, home bias implies that the real exchange rate in an efficient equilibrium is not constant, but fluctuates with the terms of trade. The weight that monetary authorities optimally place on stabilizing domestic marginal costs is increasing in Home bias: with asymmetric shocks, fixed exchange rates are incompatible with efficient monetary rules. Yet, the adverse welfare consequences of exchange rate movements constrain the optimal intensity of monetary responses to domestic shocks. Openness matters: in our specification each country produces an equal share of the world value added; the lower the import content of consumption, the higher the exchange rate volatility implied by optimal stabilization rules. In relatively closed economy, optimal monetary rules tend to converge, regardless of the nature of nominal rigidities in the exports market.

JEL classification: E31, E52, F42

Keywords: optimal monetary policy, nominal rigidities, exchange rate pass-through, exchange rate regimes, international policy cooperation.

*Preliminary version. I thank Maurice Obstfeld for comments. Corsetti's work on this paper is part of the Pierre Werner Chair Programme on Monetary Union, at the Robert Schuman Center of the European University Institute. Financial support from the programme is gratefully acknowledged.

1. Introduction

According to the received wisdom about optimal stabilization policy in the presence of country-specific shocks, exchange rate movements are an essential mechanism to regulate international relative prices adjustment in the presence of nominal rigidities. Thus optimal monetary rules should not oppose, but favor exchange rate movements as a way to overcome inefficiency due to price stickiness (e.g. Friedman 1953, Mundell 1963). Recent literature, however, has questioned the received wisdom, by stressing evidence on the stability of import price in local currency. Indeed, to the extent that import prices are sticky in the importer currency because of nominal frictions,¹ exchange rate movements do not perform the stabilizing role envisioned by the traditional model of international transmission. Namely, with enough nominal frictions in local currency, nominal depreciation improves (rather than worsening) a country's terms of trade (by raising revenues in domestic currency from sales abroad). There are no expenditure switching effects from exchange rate movements: consumer prices are essentially unresponsive to the exchange rate. When domestic monetary authorities try to stabilize the output gap, exchange rate changes tend to move the economy away from the efficient allocation. For this reason, it is now well understood that, with nominal frictions in local currency, domestic and foreign monetary authorities should optimally stabilize some weighted average of domestic and foreign producers' marginal costs. Such policy tends to reduce exchange rate volatility relative to the case of inward-looking stabilization of domestic output gaps (see Corsetti and Pesenti (2005b) for a stylized analytical and graphical discussion of the literature on this issue).

These results are typically derived in the framework of models which can be solved in closed form after Corsetti and Pesenti (2001) and Obstfeld and Rogoff (2002), in turn indebted to Cole and Obstfeld (1991). In these models, the assumption of identical preferences of consumers in different countries implies that, when export prices are set in local currency, the optimal monetary stance is perfectly symmetric at Home and abroad. In other words, when monetary policy is optimally conducted, the exchange rate does not fluctuate at all in response to contingent shocks. Devereux and Engel (2003) — henceforth DE — suggests a strong interpretation of this result, as a new and distinct argument in favor of fixed exchange rates.² Loosely speaking, since with nominal rigidities in local currency exchange rate movements do not perform any role as automatic stabilizers of relative prices (as emphasized by the received wisdom), it is efficient to eliminate their fluctuations altogether.³

In this paper I will reconsider the case for exchange rate flexibility extending previous work with Paolo Pesenti model to economies that differ in their degree of openness. The new version of the Corsetti-Pesenti model incorporates home bias in consumption preferences, yet can still be solved in closed form. The crucial assumption is that consumption preferences in the Home and Foreign country are Cobb-Douglas with symmetric weights on goods produced domestically and abroad (i.e. the degree of home bias is identical). As in the original version of the model,

¹There is a considerable debate regarding the cause of local-currency price stability of imports. Burstein, Neves, and Rebelo (2003) and Corsetti and Dedola (2005) stress that an important contributing factor is the presence of distribution services intensive in local inputs. Corsetti, Dedola and Leduc (2005a) show that allowing for distribution improves the performance of models with nominal rigidities.

²In the economy examined by DE, however, it may still be optimal to let the exchange rate depreciate along a trend, if countries have different preferences over inflation rates in the long run.

³Note that this argument is independent of whether such regime is effective in limiting discretion by central banks, so to enhance credibility of their anti-inflationary stance.

terms of trade movements minimize consumption risk of productivity shocks even if there is no international market for assets. Different from the original Corsetti-Pesenti formulation, with flexible prices the real exchange rate is no longer constant, but moves in proportion to the terms of trade. The stronger the home bias (i.e. the closer the economy), the stronger the correlation between real exchange rate and the terms of trade.

Because of home bias, efficient stabilization policy cannot be generally implemented if the exchange rate is fixed. We have mentioned above that, with prices sticky in local currency, efficient monetary rules will stabilize a weighted average of domestic and foreign marginal costs (output gaps), with the same weights of domestic and imported goods as in the consumption price index. When there is home bias, domestic monetary authorities will place more weight on their national output gap and marginal costs. Hence, unless shocks are always symmetric, the welfare-maximizing monetary stance will be different across countries, implying some degree of exchange rate volatility. Even if with local currency prices exchange rate movements do not generate any efficient adjustment in relative prices, a low weight of imports in consumption tends to raise the importance of domestic policy trade-offs in optimal policy design. Obviously, the adverse welfare consequences of exchange rate movements will constrain the intensity of the optimal policy response to domestic shocks.

Openness matters: as in our specification each country produces 1/2 of the world value added, optimal monetary rules turn off exchange rate variability in the limiting case of no home bias in domestic expenditure — so that each national household spends 1/2 of its income on national and foreign goods. This cross-border symmetry in consumption and production structures is essential to the DE result: moving away from this limiting case in any direction will restore the desirability of flexible exchange rates. In particular, raising the degree of home bias will lead monetary authorities to place more weight on domestic marginal costs stabilization. As the monetary stance in each country becomes more responsive to domestic productivity shocks, the volatility of the exchange rate correspondingly rises. Overall, the relation between openness and exchange rate volatility is non linear in the home bias. A strong bias for Foreign goods would also correspond to high exchange rate volatility, although optimal policies would be mainly concerned with marginal costs of Foreign firms.

This paper complements the analysis by Duarte and Obstfeld (2004) — henceforth DO — who emphasize that in the DE economy fixed exchange rates are efficient because the real exchange rate would be constant with flexible prices. DO consider a version of the same model augmented with a nontraded good sector in each country (as in Obstfeld and Rogoff 2002), allowing for economy-wide shocks hitting both sectors symmetrically. Since shocks to nontraded good sectors are not perfectly correlated across countries, optimal monetary rules will not imply the same monetary stance at Home and abroad. In general, policymakers face a trade-off between stabilizing the relative price of nontraded goods and traded goods, and the relative prices of domestic and foreign tradables (i.e., stabilizing marginal costs of firms operating in different sectors). Adopting a fixed exchange rate regime would impose an excessively strict constraint on policy making. In this respect, while the argument in this paper does not rely on sectoral differences, the essence of the mechanism is very similar.

This paper is organized as follows. Section 2 revisits the analysis in Corsetti-Pesenti introducing home bias in preferences. Section 3 characterizes the efficient allocation, while Sections 4 and 5 analyze optimal stabilization rules. Section 5 concludes. A full and detailed derivation of the model is presented in the Appendix.

2. The Corsetti-Pesenti model with home bias

The model consists of two countries, each specialized in the production of one good. Technology is linear in labor only, with random productivity. There is monopolistic competition in production, and firms are subject to nominal rigidities: they preset the price of their product for one period. Preferences are additive and separable: utility from the consumption basket is in logarithmic form, disutility from labor is linear. The elasticity of substitution between domestic and foreign goods is equal to one, i.e. the consumption aggregator is Cobb-Douglas. As in Cole and Obstfeld (1991), this implies that, in a flex price equilibrium, consumption risk is efficiently shared via terms of trade movements, independently of the presence of assets markets. Corsetti and Pesenti (2001) shows that the same argument goes through (and the model can be solved in closed form) also in production economies, and in the presence of nominal rigidities.

Since the specification of this model is well-known, in the text I will only discuss the novel feature I introduce in the specification — leaving analytical details to Corsetti and Pesenti (2005b) and the appendix. Namely, let C_t and C_t^* denote consumption at Home and in the Foreign country. Let C_H and C_F denote consumption of Home and Foreign goods by domestic households — C_H^* and C_F^* are similarly defined. Consider the following consumption aggregator:

$$C = C_H^\gamma C_F^{1-\gamma} \quad C^* = (C_H^*)^{1-\gamma} (C_F^*)^\gamma \quad (2.1)$$

In each country, a fraction γ of consumption expenditure falls on domestically produced goods. As long as $\gamma > 1/2$, there will be home bias in consumption. Namely, let P denote the price level in the Home country, while P_H is the price of the Home produced consumption good. Then, the optimal consumption plan by the Home representative household implies:

$$P_H C_H = \gamma P C \quad (2.2)$$

By the same token, the optimal consumption expenditure on domestic goods by the foreign representative household will be

$$P_F C_F = \gamma P^* C^* \quad (2.3)$$

where prices in foreign currency and foreign variables are denoted with an asterisk. Note that the degree of home bias is symmetric across border.

The above is an important improvement over the original version of my model with Paolo Pesenti. In its original version, the specification has identical consumption aggregators across countries. In other words, domestic and foreign households have identical preferences over the consumption of Home (Foreign) goods: hence the share of private expenditure falling on the Home goods will be identical in either country. In this paper, I show that this assumption is not necessary to derive a closed form solution. This extension of the Corsetti-Pesenti framework removes an important constraint limiting the scope of the model, as regards its ability to address issues related to openness.

Home bias is the only new feature that I introduce in the model specification. For this reason, the solution of the new model is mostly unaffected, except for the behavior of the real exchange rate. To see this, the domestic welfare-based CPI in the Home country is:

$$P_t = \frac{1}{\gamma^\gamma (1-\gamma)^{1-\gamma}} P_{H,t}^\gamma P_{F,t}^{1-\gamma} \quad (2.4)$$

whereas $P_{H,t}$ and $P_{F,t}$ are the usual CES price indexes aggregating prices of different varieties (brands) of Home and Foreign goods. The welfare-based CPI in the Foreign country is instead

$$P_t^* = \frac{1}{\gamma^\gamma (1-\gamma)^{1-\gamma}} (P_{H,t}^*)^\gamma (P_{F,t}^*)^{1-\gamma} \quad (2.5)$$

Let \mathcal{E} denote the nominal exchange rate, and TOT the Home terms of trade, i.e. the price of imports by the Home country in terms of the price of its exports (TOT* are similarly defined). With home bias, the real exchange rate can be written as:

$$RER \equiv \frac{\mathcal{E}P^*}{P} = \left(\frac{1}{TOT} \right)^{1-\gamma} (TOT^*)^\gamma \quad (2.6)$$

In general, the real exchange rate will not be constant, but will fluctuate with the terms of trade. Consumption is equalized across countries in PPP terms, not in quantities.

It is instructive to look at possible equilibria differing in the pricing assumptions. With flexible prices, for instance, the Home terms of trade are the reciprocal of the Foreign terms of trade ($\mathcal{E}P_F^*/P_H = [(P_H/\mathcal{E})/P_F^*]^{-1}$). Hence:

$$RER = (TOT)^{2\gamma-1} = \left(\frac{P_F}{P_H} \right)^{2\gamma-1} = \left(\frac{\mathcal{E}P_F^*}{P_H} \right)^{2\gamma-1} \quad (2.7)$$

Observe that, without home bias ($\gamma = 1/2$), terms of trade movements do not impinge on the real exchange rate, which is identically equal to one. As the degree of home bias increases, the comovement between the real exchange rate and the terms of trade becomes stricter. Thus, given the stochastic process driving productivity shocks and therefore the equilibrium terms of trade, the volatility of the real exchange rate will be increasing with the degree of home bias: the closer the economy, the higher the volatility of RER. This relation appears to be consistent with empirical evidence, e.g. Hau (2002).

The above formula also applies to an equilibrium with nominal rigidities, whereas exports prices are preset in the producers' currency — the case of 'Producer Currency Pricing' or 'PCP.' The terms of trade will move one to one with the nominal exchange rate. The real exchange rate, instead, will move by a fraction of the terms of trade:

$$\widehat{RER} = (2\gamma - 1) \widehat{TOT} = (2\gamma - 1) \widehat{\mathcal{E}} \quad (2.8)$$

where a $\widehat{}$ denotes the percent change of a variable. Once again: other things equal, the closer a country, the stronger the correlation between RER and TOT; without home bias, the real exchange rate would be constant.

When export prices are preset in the currency of the market of destination — this is the Local Currency Pricing, or LCP case — the Home and Foreign terms of trade are not the reciprocal of each other. The real exchange rate is:

$$RER = \left(\frac{\mathcal{E}P_H^*}{P_F} \right)^{1-\gamma} \left(\frac{P_F^*}{P_H/\mathcal{E}} \right)^\gamma = \mathcal{E} \left(\frac{P_H^*}{P_F} \right)^{1-\gamma} \left(\frac{P_F^*}{P_H} \right)^\gamma \quad (2.9)$$

Note that, with sticky prices (PCP or LCP), the real exchange rate moves together with the nominal one. Thus, the volatility of the real exchange rate depends on the characteristics of optimal monetary policy in the two countries.

The Corsetti-Pesenti model in its original formulation can be derived as a special case of the above setting $\gamma = 1/2$. The appendix shows that, after allowing for home bias, most properties of the model remain largely unaffected.

3. Efficient allocation with positive terms of trade spillovers

This section analyzes the efficient allocation in the model with home bias in preferences, and relates it to the flex-price equilibrium allocation. We start by describing technology, resource constraints and preferences. Let ℓ denote labor and Z labor productivity in the Home economy. With linear production function, the resource constraint of the economy is

$$\begin{aligned} Z\ell &= C_H + C_H^* \\ Z^*\ell^* &= C_F + C_F^* \end{aligned}$$

Abstracting from utility from money holding (see appendix), Households' preferences are

$$\begin{aligned} \sum_{s=t}^{\infty} E_{t-1} U(\cdot) &= \sum_{s=1}^{\infty} E_{t-1} [\ln C_s - \kappa \ell_s] \\ \sum_{s=t}^{\infty} E_{t-1} U(C^*, \ell^*) &= \sum_{s=1}^{\infty} E_{t-1} [\ln C_s^* - \kappa \ell_s^*] \end{aligned} \quad (3.1)$$

As in Cole and Obstfeld (1991), the planner problem consists of maximizing some weighted average of the two. By taking equal weights, the solution implies

$$\begin{aligned} C_H^{P.O.} &= \gamma \frac{Z}{\kappa} & (C_H^*)^{P.O.} &= (1 - \gamma) \frac{Z}{\kappa} \\ C_F^{P.O.} &= (1 - \kappa) \frac{Z^*}{\kappa} & (C_F^*)^{P.O.} &= \gamma \frac{Z^*}{\kappa} \end{aligned} \quad (3.2)$$

where *P.O.* stands for Pareto Optimum. In this allocation consumption risk sharing is ensured by the fact that world supply of each product is consumed by households in proportion of their 'bias' towards it, i.e.

$$\begin{aligned} C^{P.O.} &= \Gamma (Z)^\gamma (Z^*)^{1-\gamma} \frac{1}{\kappa} \\ (C^*)^{P.O.} &= \Gamma (Z)^{1-\gamma} (Z^*)^\gamma \frac{1}{\kappa} \end{aligned} \quad (3.3)$$

where $\Gamma = \gamma^\gamma (1 - \gamma)^\gamma$. Clearly in general $C \neq C^*$. The Home to Foreign consumption ratio is

$$\left(\frac{C}{C^*} \right)^{P.O.} = \left(\frac{Z}{\kappa} \right)^{2\gamma-1} \left(\frac{Z^*}{\kappa} \right)^{1-2\gamma} \quad (3.4)$$

Labor is always at its efficient level $\ell^{P.O.} = (\ell^*)^{P.O.} = 1/\kappa$, which is constant.

A flexible price equilibrium supports the same allocation of consumption as above, once output and consumption are scaled down by a constant proportional to firms' markup — i.e.

consumption and employment are lower than efficient due to monopolistic distortions in production. With CES preferences over different varieties/brands of the same good, firms set prices by charging a constant markup over marginal costs. Let θ denote the elasticity of substitution among varieties of the Home (Foreign) goods, and MC denote marginal costs, consisting of unit labor costs. In a symmetric equilibrium

$$\begin{aligned} p(h) &= \mathcal{E}p^*(h) = \frac{\theta}{\theta - 1}MC = P_H \\ p^*(f) &= p(f)/\mathcal{E} = \frac{\theta}{\theta - 1}MC^* = P_F^* \end{aligned}$$

With competitive labor markets, the first order conditions of the Home representative households imply that equilibrium nominal wages are proportional to PC (P^*C^* in the foreign country). Using these results together with the resource constraint, and the fact that consumers spend a constant share of their income on each good, we obtain that equilibrium employment is constant:

$$\ell_t = \ell_t^* = \frac{\theta - 1}{\theta\kappa} \equiv \bar{\ell} < \frac{1}{\kappa} \quad (3.5)$$

Due to monopolistic distortions, employment will be a fraction $\frac{\theta - 1}{\theta}$ of its Pareto-optimal level.

Assume efficient consumption risk sharing, which in our symmetric environment means $PC = \mathcal{E}P^*C^*$. Then, using the trade balance and consumption demand functions, it is easy to show that the value of consumption is identically equal to the value of output, i.e. $PC = P_H Y_H$. It follows that the overall level of consumption is

$$\begin{aligned} C &= \Gamma(Z)^\gamma (Z^*)^{1-\gamma} \frac{1}{\kappa} \frac{\theta - 1}{\theta} = \frac{\theta}{\theta - 1} C^{P.O.} \\ C^* &= \Gamma(Z)^{1-\gamma} (Z^*)^\gamma \frac{1}{\kappa} \frac{\theta - 1}{\theta} = \frac{\theta}{\theta - 1} (C^*)^{P.O.} \end{aligned}$$

expressions implying exactly the same ratio of Home to Foreign consumption as in an efficient equilibrium. It also follows that consumption demand for each good will be the efficient share of available output. For the Home good demand, we can write:

$$PC = P_H Y_H \Rightarrow \frac{P_H C_H}{\gamma} = P_H Y_H \Rightarrow C_H = \gamma Y_H \quad (3.6)$$

Similar expressions can be derived for C_F^* , as well as for C_H^* and C_F , whereas the latter will be a fraction $(1 - \gamma)$ of the corresponding output.

I have derived the above allocation under the assumption of perfect risk sharing. Following the steps laid out by Corsetti and Pesenti (2001), as discussed in the appendix one can show that the allocation with efficient risk sharing is supported in equilibrium in a world economy with no trade in international assets (provided that there is no predetermined stock of international debt in the economy). Terms of trade movements insure that the consumption risk due to productivity shocks is efficiently shared.

4. Optimal stabilization policies and exchange rate flexibility

This and the next section characterize optimal policy stabilization rules in economies with nominal frictions. The main goal is to study the role of openness in shaping optimal monetary rules and therefore exchange rate flexibility. In particular, we will show that, with home bias ($\gamma > 1/2$), no solution to the policy makers' problem is compatible with a fixed exchange rate regime — although optimal rules may somewhat limit currency fluctuations. In doing so, we will revisit and generalize some of the results established by the literature.⁴ As in previous work, I will not specify the instruments used by the policymakers. I will maximize welfare relative to an indicator of monetary stance defined as

$$\mu = PC; \quad \mu^* = P^*C^*$$

assuming that (whatever the instrument used) monetary authorities have perfect commitment. In the text I will also abstract from the demand for money, which is however discussed in appendix. Note that, using the above indicator, perfect risk sharing implies that the nominal exchange rate is simply the ratio of Home to Foreign monetary stance,

$$\mathcal{E}_t = \frac{\mu_t}{\mu_t^*}. \quad (4.1)$$

and nominal wages are $W = \kappa\mu$ and $W^* = \kappa\mu^*$.

Define expected utility as $\mathcal{W} = E(U)$ and $\mathcal{W}^* = E(U^*)$. In the absence of international coordination, Home policymakers determine their optimal monetary stance by maximizing \mathcal{W} with respect to μ while taking μ^* as given. Foreign authorities behave in the same way. The two monetary stances define the following Nash equilibrium:

$$\mu_{Non-Coop} = \arg \max_{\mu} \mathcal{W}; \quad \mu_{Non-Coop}^* = \arg \max_{\mu^*} \mathcal{W}^* \quad (4.2)$$

To characterize cooperative policymaking, instead, we posit that policymakers jointly maximize an equally weighted average of Home and Foreign welfare:

$$\{\mu_{Coop}, \mu_{Coop}^*\} = \arg \max_{\mu, \mu^*} \left[\frac{1}{2} \mathcal{W} + \frac{1}{2} \mathcal{W}^* \right] \quad (4.3)$$

whereas the weights coincide with the size of each country.

A useful property of the Corsetti-Pesenti specification is that, independently of nominal rigidities as well as of home bias, labor is always equal to its natural rate in expectations:

$$E_{t-1}(\ell_t) = \bar{\ell}. \quad (4.4)$$

In expected terms, the economy always operates at the constant level of employment characterizing the allocation with flexible prices (although actual employment may fluctuate with shocks).

⁴An incomplete list discussing optimal policies in open economies without using models which can be solved in closed form includes Benigno and Benigno (2004), Clarida Gali and Gertler (2001), Galí and Monacelli (2005), Kollmann (2002, 2004), Monacelli (2003), Monacelli and Faia (2004), Smets and Wouters (2002), Sutherland (2005).

Thanks to this property, expected utility is not a function of monetary policy rules. Thus welfare can be analyzed by focusing on consumption only:

$$\begin{aligned}\mathcal{W} &= E_{t-1} \ln C_t + \text{constant (independent of } \mu) \\ &= E_{t-1} [\ln \mu_t - \ln P_t] + \text{constant (independent of } \mu)\end{aligned}$$

A second useful property is that optimally preset prices can be written as a markup over expected marginal costs, expressed in the appropriate currency. For the PCP economy, for instance, we have:

$$\begin{aligned}p(h) &= \mathcal{E}p^*(h) = \frac{\theta}{\theta-1} E_{t-1} \left(\frac{\kappa \mu}{Z} \right) = P_H \\ p^*(f) &= p(f)/\mathcal{E} = \frac{\theta}{\theta-1} E_{t-1} \left(\frac{\kappa \mu^*}{Z^*} \right) = P_F^*\end{aligned}\tag{4.5}$$

The law of one price holds: once goods prices are expressed in the same currency, identical goods have the same price. Moreover, the exchange rate pass-through is complete: import prices move one to one with the exchange rate.

Before examining the economy with LCP, it is helpful to analyze briefly optimal policy in model with producer currency pricing. This allows us to present a straightforward case for optimal policy rules which are completely “inward looking”, in the sense that policymakers focus exclusively on stabilizing the domestic output gap, so that, independently of home bias, exchange rate in a fully stabilized economy fluctuates with relative productivity innovations across countries.

When exports are priced in the currency of the producer, the policy problems at Home and in the Foreign country are:

$$\begin{aligned}\max_{\mu_t} E_{t-1} (\ln C_t) &= E_{t-1} [\ln \mu_t - \gamma \ln P_{H,t} - (1-\gamma) \ln \mathcal{E}_t P_{F,t}^*] + \text{constant} \\ \max_{\mu_t^*} E_{t-1} (\ln C_t^*) &= E_{t-1} [\ln \mu_t^* - (1-\gamma) \ln P_{H,t}/\mathcal{E}_t - \gamma \ln P_{F,t}^*] + \text{constant}\end{aligned}\tag{4.6}$$

Substituting out the exchange rate and abstracting from constant terms we can also write:

$$\begin{aligned}\max_{\mu_t} E_{t-1} (\ln C_t) &= E_{t-1} \left[\gamma \ln \mu_t - \gamma \ln E_{t-1} \left(\frac{\mu}{Z} \right) + (1-\gamma) \ln \mu_t^* - (1-\gamma) \ln E_{t-1} \left(\frac{\mu^*}{Z^*} \right) \right] + \dots \\ \max_{\mu_t^*} E_{t-1} (\ln C_t^*) &= E_{t-1} \left[\gamma \ln \mu_t^* - \gamma \ln E_{t-1} \left(\frac{\mu^*}{Z^*} \right) + (1-\gamma) \ln \mu_t - (1-\gamma) \ln E_{t-1} \left(\frac{\mu}{Z} \right) \right] + \dots\end{aligned}$$

It is easy to show that optimal policy rules must satisfy:

$$\frac{1}{\mu_t} - \frac{1/Z_t}{E_{t-1}(\mu_t/Z_t)} = 0; \quad \frac{1}{\mu_t^*} - \frac{1/Z_t^*}{E_{t-1}(\mu_t^*/Z_t^*)} = 0\tag{4.7}$$

Home monetary policy responds one-to-one to real shocks, stabilizing Home firms’ marginal costs. A positive innovation in Home productivity is matched by an increase in domestic demand (via an expansionary monetary stance), which also depreciates the currency. As domestic goods become cheaper in the world economy (the Home terms of trade deteriorate), Foreign demand for

Home goods rises as well. The exchange rate movement is efficient, in the sense that international relative prices move in the same direction as in the flexible price economy studied in the previous section — essentially the point stressed by Clarida Gertler and Galí (2001).

As is well known, with PCP, implementing optimal policy rules at Home and in the Foreign country supports a flex-price allocation. This can be easily verified by noting that the above first-order conditions are solved by $\mu_t = \varsigma_t Z_t$ and $\mu_t^* = \varsigma_t^* Z_t^*$, whereas ς_t and ς_t^* are some time-contingent variables, providing the scale to nominal variables in each national economy. Policy rules satisfying these conditions would completely stabilize marginal costs. Demand moves with productivity, making sure that no output/employment gap is ever opened, and making import prices move efficiently with the exchange rate. Note that policymakers optimally stabilize the GDP deflator, but not the CPI, which fluctuates to accommodate relative price movements.

With PCP, home bias is not relevant in policy design. Independently of γ , the best policy strategy consists of focusing on the domestic output gap, i.e. it is ‘inward looking.’ In a Nash equilibrium, the nominal exchange rate fluctuates with relative productivity shocks — driving movements in the terms of trade. The real exchange rate however is less volatile, depending on home bias.

An additional property of this PCP economy is that maximizing jointly the Home and Foreign expected utility would not alter the attributes of optimal policy rules relative to the Nash equilibrium. While the Home and Foreign objective functions are not identical, combining them with symmetric welfare weights and symmetric home bias parameters yields:

$$\max_{\mu, \mu^*} \left[\frac{1}{2} \mathcal{W} + \frac{1}{2} \mathcal{W}^* \right] = E_{t-1} \left[\ln \mu_t - \ln E_{t-1} \left(\frac{\mu}{Z} \right) + \ln \mu_t^* - \ln E_{t-1} \left(\frac{\mu^*}{Z^*} \right) \right] + \dots \quad (4.8)$$

It is easy to verify that the first order conditions for a cooperative equilibrium are the same as in a Nash equilibrium: there are no gains from coordination — a result similar to Obstfeld and Rogoff (2002) and Corsetti and Pesenti (2005a).

5. Openness, volatility and the international dimension of optimal monetary policy

The PCP model has been questioned by pointing to empirical evidence documenting the extent of the local currency price stability of imports — which is incompatible with the exchange rate playing a role as stabilizer of relative prices between imports and domestic goods. While there is a considerable debate on the relative importance of real and nominal factors in determining the local currency price stability of imports, I will cast my argument abstracting from real factors altogether.⁵ As discussed in Corsetti and Pesenti [2005b], nominal frictions in local currency are an argument in favor of an international dimension in the optimal design of monetary policy rules. Below, I will formally link this argument to the degree of openness in the economy.

In an economy with LCP, $p(h)$ and $p^*(f)$ will still be optimally determined charging a fixed markup over expected marginal costs in Home and Foreign currency, respectively. Export prices

⁵These include distributive trade, difference in preferences generating differences in elasticities across markets, vertical and horizontal interactions in non-competitive markets, or other factors creating scope for optimal price discrimination.

are however different:

$$\begin{aligned} p(f) &= \frac{\theta}{\theta - 1} E_{t-1} \left(\frac{\kappa \mu^* \mathcal{E}}{Z^*} \right) = \frac{\theta}{\theta - 1} E_{t-1} \left(\frac{\kappa \mu}{Z^*} \right) = P_F \\ p^*(h) &= \frac{\theta}{\theta - 1} E_{t-1} \left(\frac{\kappa \mu}{Z \mathcal{E}} \right) = \frac{\theta}{\theta - 1} E_{t-1} \left(\frac{\kappa \mu^*}{Z} \right) = P_H^* \end{aligned}$$

What matters is expected marginal costs expressed in the currency of the importing countries. Clearly, the law of one price does not necessarily hold: exchange rate fluctuations will generally drive prices at Home and abroad apart (implicitly, we are assuming that arbitrage in the goods market is not feasible). Moreover, as prices are sticky in local currency, exchange rate pass-through is zero.

In the LCP case, it is well understood that nominal depreciation in response to a positive domestic productivity shock moves relative prices in the opposite direction with respect to the flexible price equilibrium. The terms of trade appreciate, rather than worsening, so that the international transmission of monetary policy is negative. There are no efficient expenditure switching effects. As exchange rate movements are not efficient, we will see below that the best conduct for monetary policy is to stabilize a weighted average of domestic and foreign marginal costs.⁶

When export prices are preset in local currency, the policy problem can be written as

$$\begin{aligned} \max_{\mu_t} E_{t-1} (\ln C_t) &= E_{t-1} [\ln \mu_t - \gamma \ln P_{H,t} - (1 - \gamma) \ln P_{F,t}] + \text{constant} & (5.1) \\ \max_{\mu_t^*} E_{t-1} (\ln C_t^*) &= E_{t-1} [\ln \mu_t^* - (1 - \gamma) \ln P_{H,t}^* - \gamma \ln P_{F,t}^*] + \text{constant} \end{aligned}$$

that is

$$\begin{aligned} \max_{\mu_t} E_{t-1} (\ln C_t) &= E_{t-1} \left[\ln \mu_t - \gamma \ln E_{t-1} \left(\frac{\mu_t}{Z_t} \right) - (1 - \gamma) \ln E_{t-1} \left(\frac{\mu_t}{Z_t^*} \right) \right] + \dots & (5.2) \\ \max_{\mu_t^*} E_{t-1} (\ln C_t^*) &= E_{t-1} \left[\ln \mu_t^* - (1 - \gamma) \ln E_{t-1} \left(\frac{\mu_t^*}{Z_t} \right) - \gamma \ln E_{t-1} \left(\frac{\mu_t^*}{Z_t^*} \right) \right] + \text{constant} \end{aligned}$$

Observe that in the LCP case there are no monetary spillovers, i.e. the objective function of each country does not depend on Foreign monetary policy. The first order conditions of the

⁶Observe that, according to the above expressions, optimal export prices preset in local currency fall with the covariance between firms' productivity and the monetary stance of the import country. The logic is straightforward. Suppose that the importing country expands its money supply, depreciating its currency: the movement of the exchange rate hurts exporters by reducing the revenue in their own currency for each sale abroad. Unless marginal costs of these firms happen to be temporarily low (productivity is temporarily high), the monetary shock tends to reduce their profit margin of firms exporting into the country. By the same token, a monetary contraction by the importing country in periods when exporters marginal costs are low tends to raise profit margins above their optimal level. Bringing these considerations together, a positive covariance between Home monetary stance and productivity of foreign firms turns out to destabilize profits. When this covariance increases, firms optimally react by raising prices and lowering, on average, their sales abroad. The lower the covariance between μ and Z^* , the higher the preset prices of imports.

policy problem now are

$$\begin{aligned} \frac{1}{\mu_t} - \gamma \frac{1/Z_t}{E_{t-1}(\mu_t/Z_t)} - (1-\gamma) \frac{1/Z_t^*}{E_{t-1}(\mu_t/Z_t^*)} &= 0 \\ \frac{1}{\mu_t^*} - (1-\gamma) \frac{1/Z_t}{E_{t-1}(\mu_t^*/Z_t)} - \gamma \frac{1/Z_t^*}{E_{t-1}(\mu_t^*/Z_t^*)} &= 0 \end{aligned} \quad (5.3)$$

Optimal monetary stances stabilize a weighted average of domestic and foreign marginal costs — with weights given by the CPI weights of domestic and import good prices.⁷

Home bias is now relevant for policy design. With home bias, the two first order conditions above cannot be solved by $\mu = \mu^*$, unless shocks are completely symmetric. This observation establishes that in general a fixed exchange rate is not part of efficient stabilization rules.

Nonetheless, optimal stabilization rules generally imply a lower exchange rate variability relative to the PCP economy. To see this, combine the first order condition above with the expression for the exchange rate, obtaining

$$\mathcal{E}_t = \frac{\mu_t}{\mu_t^*} = \frac{Z_t^*}{Z_t} \frac{(1-\gamma) \frac{1}{E_{t-1}(\mu_t^*/Z_t)} + \gamma \frac{Z_t}{Z_t^*} \frac{1}{E_{t-1}(\mu_t^*/Z_t^*)}}{\gamma \frac{Z_t^*}{Z_t} \frac{1}{E_{t-1}(\mu_t/Z_t)} + (1-\gamma) \frac{1}{E_{t-1}(\mu_t/Z_t^*)}} \quad (5.4)$$

Taking the limiting case $\gamma \rightarrow 1$, so that each country becomes effectively closed to trade, the optimal policy rules will prescribe $\mu = \varsigma Z$, and $\mu^* = \varsigma Z^*$, and the nominal exchange rate will fluctuate proportionally to productivity shocks, as in the PCP case:

$$\lim_{\gamma \rightarrow 1} \mathcal{E}_t = \lim_{\gamma \rightarrow 1} \frac{\mu_t}{\mu_t^*} = \frac{Z_t}{Z_t^*} \frac{\frac{1}{E_{t-1}(\mu_t^*/Z_t^*)}}{\frac{1}{E_{t-1}(\mu_t/Z_t)}} = \frac{\varsigma_t Z_t}{\varsigma_t^* Z_t^*} \quad (5.5)$$

This establishes that optimal stabilization policy in an economy with a small import share in domestic demand will tend to be ‘inward-looking’, even if import prices are preset in the local currency. The argument for an international dimension in monetary policy — reflecting the need to stabilize at the margin the markups of foreign firms exporting in the domestic economy — is strong in economies without pronounced home bias in consumption. Starting from the extreme case $\gamma = 1$, raising openness tends to raise the reaction of domestic monetary policy to productivity shocks abroad, implying lower exchange rate volatility. However, for relatively closed economy, the policy prescription for economy with LCP tends to converge to the prescription for an economy with PCP.

In our symmetric world economy optimal policies with an international dimension translate into very limited exchange rate variability for values of γ around 1/2. In the limiting case $\gamma \rightarrow 1/2$ (the case of no home bias) optimal policy rules actually imply that the exchange rate is not contingent on productivity shocks at all. In this limiting case, the model essentially

⁷Rewriting the above model incorporating non-traded goods, whereas the latter goods have equal weight in the utility function, and considering only one economy-wide shock per country, as in DO, one can derive very similar first order conditions for the policy problem. The main difference is that the parameter γ would be replaced by $\gamma/2$. This establishes a nice functional equivalence between the analysis above and Duarte and Obstfeld (2004).

becomes identical to the original Corsetti-Pesenti formulation — and the case analyzed by DE. The point stressed by this literature is that, for relatively stable exchange rates to be an attribute of optimal monetary policy, there must be substantial cross-border symmetry in both consumption and production structures — i.e., the consumption weight on the goods produced in a country must by and large correspond to the share of that country in world value added. In our economy, this share is $1/2$.⁸

Nonetheless, the limiting case discussed above is obviously not an argument in favor of a fixed exchange rate regime. For instance, one could observe that, even when $\gamma \rightarrow 1/2$, countries may have different preferences over inflation. This would lead monetary authorities to let the exchange rate depreciate predictably over time, at a rate equal to the desired inflation differential (i.e. in proportion to $(\varsigma_{t+1}/\varsigma_t)/(\varsigma_{t+1}^*/\varsigma_t^*)$).

Openness *per se* is not an argument in favor of limiting exchange rate flexibility. The relation between openness and exchange rate variability (implied by optimal stabilization policy) is in fact non linear. To see this, consider the limit for $\gamma \rightarrow 0$: households strongly prefer imported goods over domestic goods. In this economy, monetary policy makers are mostly concerned with the marginal costs of foreign producers selling in the home market. The optimal monetary rules are such that $\mu = \zeta Z^*$, and $\mu^* = \zeta^* Z$. The exchange rate still varies with relative productivity shocks, as above, although with a different sign:

$$\lim_{\gamma \rightarrow 0} \mathcal{E}_t = \lim_{\gamma \rightarrow 0} \frac{\mu_t}{\mu_t^*} = \frac{Z_t^*}{Z_t} \frac{E_{t-1}(\mu_t^*/Z_t)}{1} = \frac{\varsigma_t}{\varsigma_t^*} \frac{Z_t^*}{Z_t} \frac{1}{E_{t-1}(\mu_t/Z_t^*)}$$

This example shows that in economies that are very open to external trade,⁹ a strong “international dimension” in monetary policy does not necessarily imply a very low exchange rate volatility.¹⁰

In all these cases, because of the absence of policy spillovers noted above, there are no gains from policy coordination. Regardless of γ , maximizing a weighted average of the above objective functions will yield exactly the same optimal rules as above. As discussed by Corsetti and Pesenti (2005a), nominal friction in local currency implies that monetary shocks have large ex post spillovers on employment and output, but not on consumption. The spillovers on employment are however inconsequential on the design of monetary rules under commitment, because optimal pricing by firms is such that employment will always be constant at its natural rate in expectations (a consequence of preference specification). Without spillovers of domestic monetary policy on consumption abroad, there is no ground for cooperative policy to improve

⁸It is worth emphasizing that the exchange rate would be optimally constant in a world of n symmetric countries, each specialized in the production of a single type of tradable goods, provided that the consumption basket of each individual consumer is symmetrically defined over all the national goods produced in the world.

⁹An economy with a very large import content in consumption is also analyzed by Galí and Monacelli (2003), who however only consider the case of PCP.

¹⁰It is tempting at this point to draw some conclusions about optimal currency areas. A long-lasting question in European monetary study is why European countries have traditionally shown aversion to flexible exchange rates. Based on the result above, a possible answer lies in the symmetry between production and consumption patterns in relatively open economies. However, if monetary union and further market integration raise the degree of specialization, making these economies even more open and breaking symmetry, the equilibrium exchange rate would become more variable. A common monetary policy would lose appeal over time.

welfare: optimal monetary rules are identical in a Nash and in a cooperative equilibrium.¹¹

6. Conclusion

Recent literature has amply debated whether nominal price frictions could motivate a reconsideration of the received wisdom on optimal stabilization policy in open economy. To the extent that the nominal prices of imports are sticky in local currency, there is an argument for choosing policy targets less inward-oriented, depending on the degree of openness of the economy. A large import content of domestic expenditure creates a trade-off between stabilizing the marginal costs of domestic and foreign producers. Because of this trade-off, optimal policies will generally imply a lower exchange rate volatility relative to the case of “inward-looking” policies.

In this text I provide a simple analytical treatment of this argument, extending previous work with Paolo Pesenti, so as to allow for home bias in consumption. Home bias is enough to show that fixed exchange rates would impose undue constraints to the conduct of stabilization policy. Hence, I present a case restoring the desirability of flexible exchange rates without relying on a sectoral dimension of shocks emphasized in Duarte and Obstfeld (2004), or other economic features generating domestic policy trade-offs (as emphasized for instance by Devereux and Engel 2004).

With strong home bias, optimal monetary rules tend to be inward looking, generating exchange rates that move in proportion to the fundamental shocks hitting the Home and the Foreign economy. The exchange rate volatility implied in optimal rule, however, falls as the two economies become more symmetric: welfare-maximizing monetary authorities tend to target similar averages of domestic and foreign goods. A non-contingent exchange rate may result in the limiting case in which households have identical preferences across border, and the weight of each national good is identical to the share of world value added produced by the corresponding country — but this can hardly be considered a case in favor of a fixed exchange rate regime.

A general question raised by this literature, is the extent to which the international dimension matters in the design of optimal stabilization rules. The framework presented in this paper suggests that exchange rate variability may be ‘tolerated’ in an optimized equilibrium — given the weight of domestic policy trade-offs — in economies that are relatively closed to trade. The model however only predicts a low degree of equilibrium exchange rate variability (less than proportional to productivity shocks), whereas the transmission of supply shocks is positive. In a more general model, the welfare implications of exchange rate variability (as well as its volatility)

¹¹In Canzoneri et al. (2005), asymmetric shocks to the nontraded good sector generate potentially sizeable gains from international policy coordination. This raises the question of whether there are positive gains from coordination in DO. The answer is negative. In the text, we have seen a case of no gains from cooperation in LCP economies with tradables only, with or without home bias. The extension of the same result to the DO economy with nested Cobb-Douglas consumption aggregators is straightforward. There will still be no monetary spillovers across countries: independently of sectoral shocks, cooperative rules cannot improve upon the Nash equilibrium with optimal stabilization rules. The presence of monetary spillovers explains instead the gains from cooperation in the PCP version of the same economy, studied by Canzoneri et al. (2005). In either the non-cooperative or the cooperative equilibrium, monetary authorities are inward looking, in the sense that they only stabilize some weighted average of marginal costs in the traded and the nontraded good sectors of the national economy. However, relative to the Nash solution, optimal monetary rules under cooperation place a larger weight on stabilizing the traded good sector. This is because, with producer currency pricing, the international spillovers from monetary policy are positive. In a Nash equilibrium, the Home monetary authorities fail to take into account these spillovers when solving the policy problem. A cooperative setting addresses this failure.

may be much more pronounced. Then, even when an economy is relative closed to trade, optimal monetary stabilization may implicitly curb exchange rate movements. Quantitative analyses are clearly needed to address this question.¹²

References

- [1] Benigno, Pierpaolo and Gianluca Benigno. 2004. “Designing Targeting Rules for International Policy Cooperation”, forthcoming in *Journal of Monetary Economics*.
- [2] Burstein, Ariel T., João Neves, and Sergio Rebelo. 2003. “Distribution Costs and Real Exchange Rate Dynamics During Exchange-Rate-Based Stabilizations,” *Journal of Monetary Economics*, 50, 1189-1214.
- [3] Canzoneri, Matthew B., Robert Cumby and Behzad Diba. 2005. “The Need for International Policy Coordination: What’s Old, What’s New, What’s Yet to Come?”, *Journal of International Economics*, 66, 363-384
- [4] Clarida Richard, Jordi Galí, and Mark Gertler. 2001. Optimal Monetary Policy in Open vs. Closed Economies: An Integrated Approach. *American Economic Review Papers and Proceedings* 91 (2), 248-252.
- [5] Cole, Harold L., Maurice Obstfeld, 1991. Commodity Trade and International Risk Sharing: How Much Do Financial Markets Matter?. *Journal of Monetary Economics*, 28, 3-24.
- [6] Corsetti Giancarlo, and Paolo Pesenti. 2001. Welfare and Macroeconomic Interdependence. *Quarterly Journal of Economics* 116 (2), 421-446.
- [7] Corsetti, Giancarlo, and Paolo Pesenti. 2002. Self-Validating Optimum Currency Areas. National Bureau of Economic Research Working Paper No. 8783.
- [8] Corsetti, Giancarlo, and Paolo Pesenti. 2005a. “International Dimensions of Optimal Monetary Policy.”, *Journal of Monetary Economics*, 52, 281-305.
- [9] Corsetti, Giancarlo, and Paolo Pesenti. 2005b. “The simple geometry of transmission and stabilization in closed and open economy”, National Bureau of Economic Research working paper no. 11341, and Center for Economic Policy Research discussion paper no. 5080.
- [10] Corsetti, Giancarlo and Luca Dedola. 2005. “Macroeconomics of International Price Discrimination,” *Journal of International Economics*, 67, 129-156.
- [11] Corsetti, Giancarlo, Luca Dedola, and Sylvain Leduc. 2005., “SDGE Models of High Exchange Rate Volatility and Low Pass-through,” Center for Economic Policy Research Discussion Paper No. 5377.
- [12] Devereux, Michael B., and Charles Engel. 2003. Monetary Policy in the Open Economy Revisited: Price Setting and Exchange Rate Flexibility. *Review of Economic Studies* 70, 765-783.

¹²This is the topic of ongoing work with Luca Dedola and Sylvain Leduc (see Corsetti et al. 2005b).

- [13] Devereux, Michael B., and Charles Engel. 2004. Expenditure Switching vs. Real Exchange Rate Stabilization: Competing Objectives for Exchange Rate Policy. mimeo, Department of Economics, University of British Columbia.
- [14] Duarte Margarida and Maurice Obstfeld. 2004. "Monetary Policy in the Open Economy Revisited: the case for flexible exchange rate restored", mimeo
- [15] Friedman, Milton. 1953. The Case for Flexible Exchange Rates, in: *Essays in Positive Economics* (Chicago, IL: University of Chicago Press), 157-203.
- [16] Galí, Jordi, and Tommaso Monacelli. 2005. "Monetary Policy and Exchange Rate Volatility in a Small Open Economy. *The Review of Economic Studies*, 72, 707-734.
- [17] Hau, Harald. 2002. "Exchange Rate Volatility and Economic Openness: Theory and Evidence", *Journal of Money, Credit and Banking*, 34, 611-630.
- [18] Kollmann, Robert. 2002. "Monetary Policy Rules in the Open Economy: Effects on Welfare and Business Cycle", *Journal of Monetary Economics*, 49, 989-1015.
- [19] Kollmann, Robert. 2004. "Monetary Policy Rules in an Interdependent World", Center for Economic Policy Research Discussion Paper no. 4012.
- [20] Mundell, Robert A.. 1963. Capital Mobility and Stabilization Policy under Fixed and Flexible Exchange Rates. *Canadian Journal of Economics and Political Science* 29, 475-85.
- [21] Monacelli, Tommaso and Ester Faia. 2004. "Ramsey Monetary Policy and International Relative Prices", European Central Bank w.p. 344.
- [22] Monacelli, Tommaso. 2003. "Monetary Policy in a Low Pass-Through Environment", forthcoming, *Journal of Money Credit and Banking*.
- [23] Obstfeld, Maurice, and Kenneth Rogoff. 2003. "Risk and Exchange Rates" in Elhanan Helpman and Efraim Sadka, editors, "Contemporary Economic Policy: Essays in Honor of Assaf Razin, Cambridge University Press.
- [24] Obstfeld, Maurice, and Kenneth Rogoff. 2002. "Global Implications of Self-Oriented National Monetary Rules." *Quarterly Journal of Economics* 117, 503-36.
- [25] Smets, Frank and Raf Wouters. 2002. "Openness, imperfect exchange rate pass-through and monetary policy", *Journal of Monetary Economics*, 49, 947-981.
- [26] Sutherland, Alan. 2005. "Incomplete Pass-through, and the Welfare Effects of Exchange Rate Variability". *Journal of International Economics*, 65, 375-399.

Appendix 1. Derivation and analysis of the equilibrium

This appendix derives the solution of the model presented in the text, shows that the assumption of complete markets is irrelevant as regards the equilibrium allocation, and briefly describes properties of the international transmission of shocks under alternative assumptions about nominal rigidities. For the sake of comparison, it closely follows the appendix in Corsetti and Pesenti (2005b).

There are two symmetric countries, Home and Foreign. In each country there are households, firms, and a government. Home households and firms are defined over a continuum of unit mass, with indexes $j \in [0, 1]$ and $h \in [0, 1]$. Foreign households and firms are also defined over a continuum of unit mass, with indexes $j^* \in [0, 1]$ and $f \in [0, 1]$.

Households are immobile across countries and they own national firms. Firms in each country specialize in the production of a country-specific good. Each firm produces a variety (brand) of the national good which is an imperfect substitute to all other varieties under conditions of monopolistic competition. Labor market is competitive. Markets are complete.

Households problem The one-period utility of household j is:

$$U_t(j) = \ln C_t(j) - \kappa \ell_t(j) + \chi \ln \frac{M_t(j)}{P_t} \quad (\text{A.1})$$

where $C_t(j)$ is now a Cobb-Douglas basket (that is, a CES basket with unit elasticity) of the Home and Foreign goods:

$$C_t(j) = C_{H,t}(j)^\gamma C_{F,t}(j)^{1-\gamma} \quad (\text{A.2})$$

and $C_{H,t}(j)$ and $C_{F,t}(j)$ are CES baskets of, respectively, Home and Foreign varieties (for simplicity with identical elasticity θ):

$$C_{H,t}(j) = \left(\int_0^1 C_t(h, j)^{1-\frac{1}{\theta}} dh \right)^{\frac{\theta}{\theta-1}} \quad C_{F,t}(j) = \left(\int_0^1 C_t(f, j)^{1-\frac{1}{\theta}} df \right)^{\frac{\theta}{\theta-1}} \quad (\text{A.3})$$

Note that the degree of substitution between domestic goods and imports is lower than the degree of substitution among varieties ($1 < \theta$). Since γ will be the share of consumption spending falling on domestic goods, any $\gamma > 1/2$ implies home bias in consumption.

The one-period utility flow of Foreign household j^* is:

$$U_t^*(j^*) = \ln C_t^*(j^*) - \kappa \ell_t^*(j^*) + \chi \ln \frac{M_t^*(j^*)}{P_t^*} \quad (\text{A.4})$$

$C_t^*(j^*)$ will be a Cobb-Douglas basket with symmetric home bias:

$$C_t^*(j^*) = C_{H,t}^*(j^*)^{1-\gamma} C_{F,t}^*(j^*)^\gamma \quad (\text{A.5})$$

where $C_{H,t}^*(j^*)$, $C_{F,t}^*(j^*)$ are CES baskets of, respectively, Home and Foreign varieties:

$$C_{H,t}^*(j^*) = \left(\int_0^1 C_t^*(h, j^*)^{1-\frac{1}{\theta}} dh \right)^{\frac{\theta}{\theta-1}} \quad C_{F,t}^*(j^*) = \left(\int_0^1 C_t^*(f, j^*)^{1-\frac{1}{\theta}} df \right)^{\frac{\theta}{\theta-1}} \quad (\text{A.6})$$

This implies that, for given Home-currency prices of the varieties, $p_t(h)$ and $p_t(f)$, the utility-based CPI, P_t , is defined as:

$$P_t = \frac{1}{\gamma^\gamma (1-\gamma)^{1-\gamma}} P_{H,t}^\gamma P_{F,t}^{1-\gamma} \quad (\text{A.7})$$

for the Home country, whereas:

$$P_{H,t} = \left(\int_0^1 p_t(h)^{1-\theta} dh \right)^{\frac{1}{1-\theta}} \quad P_{F,t} = \left(\int_0^1 p_t(f)^{1-\theta} df \right)^{\frac{1}{1-\theta}}. \quad (\text{A.8})$$

For the Foreign country, instead, we will have

$$P_t^* = \frac{1}{\gamma^\gamma (1-\gamma)^{1-\gamma}} (P_{H,t}^*)^\gamma (P_{F,t}^*)^{1-\gamma}$$

with

$$P_{H,t}^* = \left(\int_0^1 p_t^*(h)^{1-\theta} dh \right)^{\frac{1}{1-\theta}} \quad P_{F,t}^* = \left(\int_0^1 p_t^*(f)^{1-\theta} df \right)^{\frac{1}{1-\theta}}. \quad (\text{A.9})$$

The individual demand curves for varieties h and f are standard. For the Home country we have:

$$C_t(h, j) = \gamma \left(\frac{p_t(h)}{P_{H,t}} \right)^{-\theta} \left(\frac{P_{H,t}}{P_t} \right)^{-1} C_t(j) \quad (\text{A.10})$$

$$C_t(f, j) = (1-\gamma) \left(\frac{p_t(f)}{P_{F,t}} \right)^{-\theta} \left(\frac{P_{F,t}}{P_t} \right)^{-1} C_t(j) \quad (\text{A.11})$$

and the optimal composition of nominal spending is:

$$P_{H,t} C_{H,t}(j) = \gamma P_t C_t(j); \quad P_{F,t} C_{F,t}(j) = (1-\gamma) P_t C_t(j) \quad (\text{A.12})$$

For the Foreign country we have instead:

$$C_t(h, j^*) = (1-\gamma) \left(\frac{p_t^*(h)}{P_{H,t}^*} \right)^{-\theta} \left(\frac{P_{H,t}^*}{P_t^*} \right)^{-1} C_t^*(j^*) \quad (\text{A.13})$$

$$C_t(f, j^*) = \gamma \left(\frac{p_t^*(f)}{P_{F,t}^*} \right)^{-\theta} \left(\frac{P_{F,t}^*}{P_t^*} \right)^{-1} C_t^*(j^*) \quad (\text{A.14})$$

$$P_{H,t}^* C_{H,t}^*(j^*) = \gamma P_t^* C_t^*(j^*); \quad P_{F,t}^* C_{F,t}^*(j^*) = (1-\gamma) P_t^* C_t^*(j^*) \quad (\text{A.15})$$

The Home representative household j own the portfolio of Home firms, hold the Home currency, M , receive wages, $W_t \ell_t(j)$, and profits from the firms, $\mathcal{P}_t(j)$, and pay non-distortionary (lump-sum) net taxes $NETT$, denominated in Home currency. With complete markets, it also owns Arrow-Debreu securities.

The optimality conditions for the Home representative households with respect to $C_t(j)$, $M_t(j)$ and $\ell_t(j)$ are

$$\frac{1}{C_t(j)} - D_t(j)P_t = 0 \quad (\text{A.16})$$

$$\frac{\chi}{M_t(j)} - D_t(j) + \beta E_t D_{t+1}(j) = 0 \quad (\text{A.17})$$

$$-\kappa + W_t D_t(j) = 0 \quad (\text{A.18})$$

where $D_t(j)$ is the Lagrangian multiplier associated with the flow budget constraint at time t . From (A.16), $D_t(j)$ measures the increase in household j 's utility (shadow price) associated with one additional unit of nominal wealth. Workers equate the marginal rate of substitution between consumption and leisure, $\kappa C_t(j)$, to the real wage in consumption units, W_t/P_t . Note that, with a common CPI index, the previous expression implies equalization of consumption across agents, or:

$$C_t(j) = C_t, \quad D_t(j) = D_t, \quad Q_{t,t+\tau}(j) = Q_{t,t+\tau}. \quad (\text{A.19})$$

The problem of the foreign representative household is similarly defined. Similar conditions hold for the Foreign representative household.

Let \mathcal{E}_t denote the nominal exchange rate (defined as units of Home currency per unit of Foreign currency). With complete markets the rate of growth of marginal utility is equal to the rate of real depreciation (the rate of growth of the real exchange rate):

$$\frac{\partial U_{t+1}/\partial C_{t+1}}{\partial U_t/\partial C_t} = \frac{\partial U_{t+1}^*/\partial C_{t+1}^*}{\partial U_t^*/\partial C_t^*} \frac{\mathcal{E}_t P_t^*/P_t}{\mathcal{E}_{t+1} P_{t+1}^*/P_{t+1}} \quad (\text{A.20})$$

In our setup this condition becomes:

$$\frac{P_t C_t}{P_{t+1} C_{t+1}} = \frac{\mathcal{E}_t P_t^* C_t^*}{\mathcal{E}_{t+1} P_{t+1}^* C_{t+1}^*} \quad (\text{A.21})$$

Define

$$\mu = PC \quad \text{and} \quad \mu^* = P^* C^*$$

we can write

$$\frac{\mu_t}{\mu_{t+1}} = \frac{\mathcal{E}_t \mu_t^*}{\mathcal{E}_{t+1} \mu_{t+1}^*} \quad (\text{A.22})$$

Iterating the above expression we can rewrite the above with respect to some initial date 0:

$$\mu_t = \left(\frac{\mu_0}{\mathcal{E}_0 \mu_0^*} \right) \mathcal{E}_t \mu_t^* = \text{constant} \cdot \mathcal{E}_t \mu_t^* \quad (\text{A.23})$$

In a symmetric world, Home and Foreign consumption are ex ante identical, hence the constant in the above expression is equal to one. The equilibrium exchange rate is therefore equal to the ratio of Home to Foreign monetary stance:

$$\mathcal{E}_t = \frac{\mu_t}{\mu_t^*} \implies P_t C_t = \mathcal{E}_t P_t^* C_t^*. \quad (\text{A.24})$$

Technology and resource constraints The production functions in the two countries are linear in labor:

$$Y_t(h) = Z_t \ell_t(h) \qquad Y_t^*(f) = Z_t^* \ell_t^*(f) \qquad (\text{A.25})$$

where Z_t and Z_t^* are two country-specific productivity processes. Note that the resource constraint for Home variety h is now:

$$Y_t(h) = \int_0^1 C_t(h, j) dj + \int_0^1 C_t^*(h, j^*) dj^* \qquad (\text{A.26})$$

and similarly for Foreign variety f :

$$Y_t^*(f) = \int_0^1 C_t(f, j) dj + \int_0^1 C_t^*(f, j^*) dj^* \qquad (\text{A.27})$$

Aggregating across j -agents we obtain total Home demand for variety h :

$$\int_0^1 C_t(h, j) dj = \left(\frac{p_t(h)}{P_{H,t}} \right)^{-\theta} \int_0^1 C_{H,t}(j) dj = \left(\frac{p_t(h)}{P_{H,t}} \right)^{-\theta} C_{H,t} \qquad (\text{A.28})$$

Similarly, total Foreign demand for variety h is obtained by aggregating over j^* -agents:

$$\int_0^1 C_t^*(h, j^*) dj^* = \left(\frac{p_t^*(h)}{P_{H,t}^*} \right)^{-\theta} \int_0^1 C_{H,t}^*(j^*) dj^* = \left(\frac{p_t^*(h)}{P_{H,t}^*} \right)^{-\theta} C_{H,t}^* \qquad (\text{A.29})$$

so that Home firm h faces the following demand schedule for its product:

$$Y_t(h) = \left(\frac{p_t(h)}{P_{H,t}} \right)^{-\theta} \left(\frac{P_{H,t}}{P_t} \right)^{-1} \gamma C_t + \left(\frac{p_t^*(h)}{P_{H,t}^*} \right)^{-\theta} \left(\frac{P_{H,t}^*}{P_t^*} \right)^{-1} (1 - \gamma) C_t^* \qquad (\text{A.30})$$

Similarly we can obtain total demand for Foreign variety f .

Price setting Home firm h minimizes costs $W_t \ell_t(h)$ subject to the above technology: the Lagrangian multiplier associated with this problem is the nominal marginal cost $MC_t(h)$, equal to:

$$MC_t(h) = MC_t = \frac{W_t}{Z_t} \qquad (\text{A.31})$$

or using the FOC with respect to ℓ :

$$MC_t = \frac{\kappa \mu_t}{Z_t}$$

Firms operating under conditions of monopolistic competition take into account the downward-sloping demand for their product (A.30) and set prices to maximize their value. Firms are small, in the sense that they ignore the impact of their pricing and production decisions on aggregate variables and price indexes.

Home firm h 's nominal profits can be written as:

$$\begin{aligned}
\mathcal{P}_t(h) &= p_t(h) \int_0^1 C_t(h, j) dj + \mathcal{E}_t p_t^*(h) \int_0^1 C_t(h, j^*) dj^* - W_t \ell_t(h) \\
&= p_t(h) \int_0^1 C_t(h, j) dj + \mathcal{E}_t p_t^*(h) \int_0^1 C_t(h, j^*) dj^* \\
&\quad - \frac{W_t}{Z_t} \left(\int_0^1 C_t(h, j) dj + \int_0^1 C_t^*(h, j^*) dj^* \right) \\
&= (p_t(h) - MC_t) \int_0^1 C_t(h, j) dj + (\mathcal{E}_t p_t^*(h) - MC_t) \int_0^1 C_t(h, j^*) dj^* \\
&= (p_t(h) - MC_t) \left(\frac{p_t(h)}{P_{H,t}} \right)^{-\theta} C_{H,t} + (\mathcal{E}_t p_t^*(h) - MC_t) \left(\frac{p_t^*(h)}{P_{H,t}^*} \right)^{-\theta} C_{H,t}^*
\end{aligned} \tag{A.32}$$

Consider first the case of an economy with **flexible prices**. Home firms set prices to maximize $\mathcal{P}_t(h)$ with respect to $p_t(h)$ and $p_t^*(h)$. This implies:

$$p_t(h) = \mathcal{E}_t p_t^*(h) = \frac{\theta}{\theta - 1} MC_t \tag{A.33}$$

Both prices are equal to the marginal cost augmented by a constant markup $\theta/(\theta - 1)$. The law of one price holds, as the same good h sells at the same price in both markets when expressed in terms of the same currency.

Suppose now firms are subject to **nominal rigidities**: at time $t - 1$, firms preset the price(s) at which they sell their goods in the Home country at time t (prices are preset for one period only). They choose the preset price which maximizes the value of the firm, i.e. expected discounted profits $E_{t-1}(Q_{t-1,t} \mathcal{P}_t(h))$. The first order condition for the Home good is:

$$E_{t-1}(Q_{t-1,t} \left(\frac{p_t(h)}{P_{H,t}} \right)^{-\theta} C_{H,t}) = \frac{\theta}{p_t(h)} E_{t-1}(Q_{t-1,t} (p_t(h) - MC_t) \left(\frac{p_t(h)}{P_{H,t}} \right)^{-\theta} C_{H,t}) \tag{A.34}$$

Recalling that $Q_{t-1,t} = \beta P_{t-1} C_{t-1} / P_t C_t$, $C_{H,t} = \gamma P_t C_t / P_{H,t}$, and observing that all prices $p_t(h)$ are symmetric, thus

$$p_t(h) = P_{H,t} : p_t(h) = P_{H,t} = \frac{\theta}{\theta - 1} E_{t-1}(MC_t)$$

$p(h)$ is a markup over expected marginal costs.

What about the **Foreign-currency price** $p_t^*(h)$? Logically, this price can be set in two different ways, depending on the specific currency in which Home exports are priced.

First, we consider the case of **'producer currency pricing' (PCP)**: exports are priced and invoiced in domestic (producer's) currency. The firm h maximizes $E_{t-1}(Q_{t-1,t} \mathcal{P}_t(h))$ with

respect to $\mathcal{E}_t p_t^*(h)$, setting the price of variety h according to:

$$E_{t-1}(Q_{t-1,t} \left(\frac{\mathcal{E}_t p_t^*(h)}{\mathcal{E}_t P_{H,t}^*} \right)^{-\theta} C_{H,t}^*) = \tag{A.35}$$

$$\frac{\theta}{\mathcal{E}_t p_t^*(h)} E_{t-1}(Q_{t-1,t} (\mathcal{E}_t p_t^*(h) - MC_t) \left(\frac{\mathcal{E}_t p_t^*(h)}{\mathcal{E}_t P_{H,t}^*} \right)^{-\theta} C_{H,t}^*)$$

Rearranging:

$$\mathcal{E}_t p_t^*(h) = \frac{\theta}{\theta - 1} \frac{E_{t-1} \left(Q_{t-1,t} MC_t \left(\frac{\mathcal{E}_t p_t^*(h)}{\mathcal{E}_t P_{H,t}^*} \right)^{-\theta} C_{H,t}^* \right)}{E_{t-1} \left(Q_{t-1,t} \left(\frac{\mathcal{E}_t p_t^*(h)}{\mathcal{E}_t P_{H,t}^*} \right)^{-\theta} C_{H,t}^* \right)} \tag{A.36}$$

Recalling that $Q_{t-1,t} = \beta P_{t-1} C_{t-1} / P_t C_t$, $C_{H,t}^* = (1 - \gamma) \mathcal{E}_t P_t^* C_t^* / (\mathcal{E}_t P_{H,t}^*)$, and observing that all prices $\mathcal{E}_t p_t^*(h)$ are symmetric, thus $\mathcal{E}_t p_t^*(h) = \mathcal{E}_t P_{H,t}^*$, we obtain:

$$\mathcal{E}_t p_t^*(h) = \mathcal{E}_t P_{H,t}^* = \frac{\theta}{\theta - 1} E_{t-1} (MC_t) \tag{A.37}$$

Foreign-currency prices $P_{H,t}^*$ move one-to-one with the nominal exchange rate, leaving the export price $\mathcal{E}_t P_{H,t}^*$ unchanged when expressed in Home currency. In other words, there is full exchange rate pass-through. The law of one price holds. Domestic goods have the same price (in the same currency) everywhere. No arbitrage is possible.

Consider next a model with **'local currency pricing' (LCP)**: the export price is preset in Foreign currency. The firm h maximizes expected discounted profits $E_{t-1}(Q_{t-1,t} \mathcal{P}_t(h))$ with respect to $p_t^*(h)$. The first order condition is:

$$E_{t-1}(Q_{t-1,t} \mathcal{E}_t \left(\frac{p_t^*(h)}{P_{H,t}^*} \right)^{-\theta} C_{H,t}^*) = \tag{A.38}$$

$$= \frac{\theta}{p_t^*(h)} E_{t-1}(Q_{t-1,t} (\mathcal{E}_t p_t^*(h) - MC_t) \left(\frac{p_t^*(h)}{P_{H,t}^*} \right)^{-\theta} C_{H,t}^*)$$

which can be written as:

$$p_t^*(h) = P_{H,t}^* = \frac{\theta}{\theta - 1} E_{t-1} \left(\frac{MC_t}{\mathcal{E}_t} \right) \tag{A.39}$$

This is in general different from $\frac{P_{H,t}^*}{\mathcal{E}_t}$, i.e. exchange rate movements will induce deviations from the law of one price. Home export prices expressed in Foreign currency do not move when the exchange rate changes. Pass through is zero. Note the implicit assumption that arbitrage in the goods market is not possible.

Government and monetary policy indicator There is no public spending: the government uses seigniorage revenues and taxes to finance transfers. The public budget constraint is simply:

$$M_t - M_{t-1} + \int_0^1 NETT_t(j) dj = 0 \quad (\text{A.40})$$

and in equilibrium money supply equals demand, or $M_t = \int_0^1 M_t(j) dj$.

As in Corsetti and Pesenti (2005a,b), we take μ and μ^* as our indicator of monetary stance at Home and abroad.

A synthesis of the model The resource constraint for the Home output is:

$$\begin{aligned} Z_t \ell_t &= C_{H,t} + C_{H,t}^* = \left(\gamma \frac{P_t C_t}{P_{H,t}} + (1 - \gamma) \frac{P_t^* C_t^*}{P_{H,t}^*} \right) \\ &= \left(\gamma \frac{P_t C_t}{P_{H,t}} + (1 - \gamma) \frac{P_t C_t}{\mathcal{E}_t P_{H,t}^*} \right) = P_t \left(\frac{\gamma}{P_{H,t}} + \frac{1 - \gamma}{\mathcal{E}_t P_{H,t}^*} \right) C_t \end{aligned} \quad (\text{A.41})$$

Define the variable τ as

$$\frac{1}{\tau_t} \equiv P_t \left(\frac{\gamma}{P_{H,t}} + \frac{1 - \gamma}{\mathcal{E}_t P_{H,t}^*} \right) \quad (\text{A.42})$$

The resource constraint can then be written synthetically as:

$$C_t = Z_t \ell_t \tau_t \quad (\text{A.43})$$

The variable τ_t is an index of international spillovers, reflecting the macroeconomic impact of fluctuations of relative prices and terms of trade on the Home economy. Similarly, for the Foreign economy

$$C_t^* = Z_t^* \ell_t^* \tau_t^* \quad (\text{A.44})$$

and:

$$\frac{1}{\tau_t^*} \equiv P_t^* \left(\frac{\gamma}{P_{F,t}^*} + \frac{1 - \gamma}{P_{F,t} / \mathcal{E}_t} \right) \quad (\text{A.45})$$

Now, using the resource constraint with optimal prices, it is easy to see that, absent nominal rigidities, there is full employment in both economies regardless of the shocks:

$$\ell_t = \ell_t^* = \frac{\theta - 1}{\theta \kappa} = \bar{\ell} \quad (\text{A.46})$$

In the presence of nominal rigidities, instead, full employment holds only on average:

$$E_{t-1}(\ell_t) = E_{t-1}(\ell_t^*) = \bar{\ell} \quad (\text{A.47})$$

regardless of export pricing.

Now, given the exogenous variables $Z_t, Z_t^*, \mu_t, \mu_t^*$ and given prices $P_{H,t}, P_{F,t}, P_{H,t}^*, P_{F,t}^*$, the macroeconomics of the two-country model is described by the system of 13 equations in 13 endogenous variables $\mathcal{E}_t, P_t, P_t^*, C_t, C_t^*, \tau_t, \tau_t^*, \ell_t, \ell_t^*, C_{H,t}, C_{F,t}, C_{H,t}^*,$ and $C_{F,t}^*$, shown below:

$$\begin{aligned}
P_t &= \frac{1}{\gamma^\gamma(1-\gamma)^{1-\gamma}} P_{H,t}^\gamma P_{F,t}^{1-\gamma} & \mathcal{E}_t &= \mu_t/\mu_t^* \\
\mu_t &= P_t C_t & P_t^* &= \frac{1}{\gamma^\gamma(1-\gamma)^{1-\gamma}} (P_{H,t}^*)^{1-\gamma} (P_{F,t}^*)^\gamma \\
\frac{1}{\tau_t} &\equiv P_t \left(\frac{\gamma}{P_{H,t}} + \frac{1-\gamma}{\mathcal{E}_t P_{H,t}^*} \right) & \mu_t^* &= P_t^* C_t^* \\
C_t &= Z_t \ell_t \tau_t & \frac{1}{\tau_t^*} &\equiv P_t^* \left(\frac{\gamma}{P_{F,t}^*} + \frac{1-\gamma}{P_{F,t}^*/\mathcal{E}_t} \right) \\
P_{H,t} C_{H,t} &= \gamma P_t C_t & C_t^* &= Z_t^* \ell_t^* \tau_t^* \\
P_{H,t}^* C_{H,t}^* &= (1-\gamma) P_t^* C_t^* & P_{F,t} C_{F,t} &= (1-\gamma) P_t C_t \\
& & P_{F,t}^* C_{F,t}^* &= \gamma P_t^* C_t^*
\end{aligned}$$

To close the model, we need to specify a pricing assumption. In the case of **flexible prices** we have

$$\begin{aligned}
P_{H,t} &= \frac{\theta\kappa}{\theta-1} \frac{\mu_t}{Z_t} & P_{F,t} &= \mathcal{E}_t P_{F,t}^* = \mathcal{E}_t \frac{\theta\kappa}{\theta-1} \frac{\mu_t^*}{Z_t^*} \\
P_{H,t}^* &= \frac{P_{H,t}}{\mathcal{E}_t} = \frac{1}{\mathcal{E}_t} \frac{\theta\kappa}{\theta-1} \frac{\mu_t}{Z_t} & P_{F,t}^* &= \frac{\theta\kappa}{\theta-1} \frac{\mu_t^*}{Z_t^*}
\end{aligned} \tag{A.48}$$

With nominal rigidities and **PCP** (export prices are set in the producer's currency) we have:

$$\begin{aligned}
P_{H,t} &= \frac{\theta\kappa}{\theta-1} E_{t-1} \left(\frac{\mu_t}{Z_t} \right) & P_{F,t} &= \mathcal{E}_t P_{F,t}^* = \mathcal{E}_t \frac{\theta\kappa}{\theta-1} E_{t-1} \left(\frac{\mu_t^*}{Z_t^*} \right) \\
P_{H,t}^* &= \frac{P_{H,t}}{\mathcal{E}_t} = \frac{1}{\mathcal{E}_t} \frac{\theta\kappa}{\theta-1} E_{t-1} \left(\frac{\mu_t}{Z_t} \right) & P_{F,t}^* &= \frac{\theta\kappa}{\theta-1} E_{t-1} \left(\frac{\mu_t^*}{Z_t^*} \right)
\end{aligned} \tag{A.49}$$

Finally, with nominal rigidities and **LCP** (export prices are set in the consumer's currency), we have:

$$\begin{aligned}
P_{H,t} &= \frac{\theta\kappa}{\theta-1} E_{t-1} \left(\frac{\mu_t}{Z_t} \right) & P_{F,t} &= \frac{\theta\kappa}{\theta-1} E_{t-1} \left(\frac{\mu_t}{Z_t^*} \right) \\
P_{H,t}^* &= \frac{\theta\kappa}{\theta-1} E_{t-1} \left(\frac{\mu_t^*}{Z_t^*} \right) & P_{F,t}^* &= \frac{\theta\kappa}{\theta-1} E_{t-1} \left(\frac{\mu_t^*}{Z_t^*} \right)
\end{aligned} \tag{A.50}$$

Irrelevance of the complete market assumption Following the same logic as in Corsetti and Pesenti (2001), it can be shown that the allocation is the same if financial markets are incomplete, as long as in the economy there is no outstanding debt inherited from the past. In particular, suppose there is international trade in one bond, denominated in domestic currency. The first order conditions of the Home and Foreign agents with respect to bond holdings can be written

$$\begin{aligned}
\frac{1}{\mu_t} &= \beta(1+i_t) E_t \left(\frac{1}{\mu_{t+1}} \right) \\
\frac{1}{\mathcal{E}_t \mu_t^*} &= \beta(1+i_t) E_t \left(\frac{1}{\mathcal{E}_{t+1} \mu_{t+1}^*} \right)
\end{aligned}$$

where i_t is the nominal interest rate. Combining these two conditions yields the following

$$\frac{\mathcal{E}_t \mu_t^*}{\mu_t} = \frac{E_t \left(\frac{1}{\mu_{t+1}} \right)}{E_t \left(\frac{1}{\mathcal{E}_{t+1} \mu_{t+1}^*} \right)}$$

the uncovered interest parity conditions. On the other hand, the expressions for the current account at Home and abroad are

$$\begin{aligned} B_{t+1} &= \mu - P_H Y_H \\ -B_{t+1} &= \mathcal{E}_t \mu^* - \mathcal{E}_t P_F^* Y_F^* \end{aligned}$$

where we assume that the inherited stock of debt from the past is zero, It is easy to verify that $B_t = 0$ and $\mathcal{E}_t = \frac{\mu_t}{\mu_t^*}$ solve the above equations.

International transmission With **flexible prices** we have

$$\begin{aligned} \ell &= \bar{\ell}; \Rightarrow Y_H = Z \bar{\ell} \\ C &= Z \bar{\ell}^\tau = \frac{Z \bar{\ell}}{\gamma^\gamma (1-\gamma)^{1-\gamma}} \left(\frac{P_H}{P_F} \right)^{1-\gamma} = \frac{Z \bar{\ell}}{\gamma^\gamma (1-\gamma)^{1-\gamma}} \left(\frac{Z^*}{Z} \right)^{1-\gamma} = \frac{Z^\gamma (Z^*)^{1-\gamma}}{\gamma^\gamma (1-\gamma)^{1-\gamma}} \bar{\ell} \\ C^* &= Z^* \bar{\ell}^* \tau^* = \frac{Z^* \bar{\ell}^*}{\gamma^\gamma (1-\gamma)^{1-\gamma}} \left(\frac{P_F^*}{P_H^*} \right)^{1-\gamma} = \frac{Z^{1-\gamma} (Z^*)^\gamma}{\gamma^\gamma (1-\gamma)^{1-\gamma}} \bar{\ell}^* \end{aligned}$$

Transmission of productivity shocks is ‘positive.’ As Home country is better off because of higher productivity, Foreign also benefit via an improvements of their terms of trade. Nominal shocks are obviously neutral. Consumption is not equalized across countries. Instead:

$$\frac{C}{C^*} = \frac{\mathcal{E} P^*}{P} = \left(\frac{P_F}{P_H} \right)^{2\gamma-1}$$

With **nominal rigidities and PCP**

$$\ell_t = \frac{\mu_t/Z_t}{E_{t-1}(\mu/Z)} \bar{\ell}, \quad \ell_t^* = \frac{\mu_t^*/Z_t^*}{E_{t-1}(\mu_t^*/Z_t^*)} \bar{\ell} \quad (\text{A.51})$$

$$\tau_t = \gamma^\gamma (1-\gamma)^{1-\gamma} \left(\frac{E_{t-1}(\mu_t/Z_t)}{E_{t-1}(\mu_t^*/Z_t^*)} \frac{1}{\mathcal{E}_t} \right)^{1-\gamma}, \quad \tau_t^* = \gamma^\gamma (1-\gamma)^{1-\gamma} \left(\frac{E_{t-1}(\mu_t^*/Z_t^*)}{E_{t-1}(\mu_t/Z_t)} \mathcal{E}_t \right)^{1-\gamma} \quad (\text{A.52})$$

$$\begin{aligned} C_t &= \gamma^\gamma (1-\gamma)^{1-\gamma} \bar{\ell} \frac{\mu_t^\gamma (\mu_t^*)^{1-\gamma}}{[E_{t-1}(\mu_t/Z_t)]^\gamma [E_{t-1}(\mu_t^*/Z_t^*)]^{1-\gamma}} \\ C_t &= \gamma^\gamma (1-\gamma)^{1-\gamma} \bar{\ell}^* \frac{\mu_t^{1-\gamma} (\mu_t^*)^\gamma}{[E_{t-1}(\mu_t/Z_t)]^{1-\gamma} [E_{t-1}(\mu_t^*/Z_t^*)]^\gamma} \end{aligned} \quad (\text{A.53})$$

Home productivity shocks only affect Home employment (labor ‘gap’). Monetary policies have spillovers on consumption, but not on output abroad. A depreciation of E_t deteriorates the Home terms of trade: monetary transmission is positive. Consumption moves together but not proportionally.

Under PCP, the terms of trade $P_F/\mathcal{E}P_H^*$ are equal to $P_F^*\mathcal{E}/P_H$. Since P_H and P_F^* are preset, the Home terms of trade worsen with a nominal depreciation of the Home currency (i.e. a higher

\mathcal{E}). When the Home currency weakens, Home goods are cheaper relative to Foreign goods in both the Home and the Foreign country. As demand shifts in favor of the goods with the lowest relative price, world consumption of Home goods increases relative to consumption of Foreign goods. These are referred to as ‘*expenditure switching effects*’ of exchange rate movements.

With **LCP** instead we have

$$\ell_t = \left(\frac{\gamma \mu_t / Z_t}{E_{t-1}(\mu_t / Z_t)} + \frac{(1 - \gamma) \mu_t^* / Z_t}{E_{t-1}(\mu_t^* / Z_t)} \right) \bar{\ell} \quad (\text{A.54})$$

$$\ell_t^* = \left(\frac{\gamma \mu_t^* / Z_t^*}{E_{t-1}(\mu_t^* / Z_t^*)} + \frac{(1 - \gamma) \mu_t / Z_t^*}{E_{t-1}(\mu_t / Z_t^*)} \right) \bar{\ell} \quad (\text{A.55})$$

Productivity only affects domestic employment. Monetary policies have spillovers on output and employment overseas. Since prices are preset in local currency, a depreciation of \mathcal{E}_t improves the Home terms of trade $P_F / \mathcal{E} P_H^*$: it increases Home exporters’ sales revenue and reduces Foreign exporters’ sales revenue, without effects on consumer prices. Thus, a depreciation of \mathcal{E}_t has now a positive impact on τ_t and negative on τ_t^* – the opposite of the PCP case:

$$\tau_t = \frac{\gamma^\gamma (1 - \gamma)^{1-\gamma} \left(\frac{E_{t-1}(\mu_t / Z_t)}{E_{t-1}(\mu_t / Z_t^*)} \right)^{1-\gamma}}{\gamma + (1 - \gamma) \frac{E_{t-1}(\mu_t / Z_t)}{E_{t-1}(\mu_t^* / Z_t)} \frac{1}{\mathcal{E}_t}}$$

$$\tau_t^* = \frac{\gamma^\gamma (1 - \gamma)^{1-\gamma} \left(\frac{E_{t-1}(\mu_t^* / Z_t^*)}{E_{t-1}(\mu_t^* / Z_t)} \right)^{1-\gamma}}{\gamma + (1 - \gamma) \frac{E_{t-1}(\mu_t^* / Z_t^*)}{E_{t-1}(\mu_t / Z_t^*)} \mathcal{E}_t}$$

There are no monetary spillovers on consumption. A home monetary shock raise C at Home and ℓ^* abroad: ‘beggar-thy-neighbor’ transmission of monetary policy.

$$C_t = \gamma^\gamma (1 - \gamma)^{1-\gamma} \bar{\ell} \frac{\mu_t}{[E_{t-1}(\mu_t / Z_t)]^\gamma [E_{t-1}(\mu_t / Z_t^*)]^{1-\gamma}} \quad (\text{A.56})$$

$$C^* = \gamma^\gamma (1 - \gamma)^{1-\gamma} \bar{\ell}^* \frac{\mu_t^*}{[E_{t-1}(\mu_t^* / Z_t^*)]^\gamma [E_{t-1}(\mu_t^* / Z_t)]^{1-\gamma}} \quad (\text{A.57})$$

With prices preset in local currency, exchange rate fluctuations do not affect the relative price faced by importers and consumers. There is no ‘*expenditure switching effect*’ of exchange rate movements.